Optimal Headways and Slack Times at Multiple-Route Timed-Transfer Terminals

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ABSTRACT

Schedule synchronization may significantly reduce transfer delays at terminals where various transit routes interconnect. Since vehicle arrivals are stochastic, slack times in the vehicle schedules may be desirable to reduce the probability of missed connections, even at the cost of some additional waiting times for vehicles, drivers, and some non-transferring passengers. Total system costs are formulated to assess the effectiveness of coordinated operations under various demand and traffic conditions. Numerical integration is used to compute transfer delays due to probabilistic vehicle arrival distributions. Afterwards, a newly developed headway clustering algorithm is used to jointly optimize headways and slack times for all coordinated routes. The results show that as demand decreases, increased coordination is desirable. They also show the value of coordination with integer-ratio headways rather than a single common headway for routes with widely differing characteristics. The results from sensitivity analysis also show that coordination is not worth attempting, even in low demand and high headway cases, if arrival randomness becomes excessive.

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1. INTRODUCTION

In transit networks with multiple interconnected routes, delays may be considerably reduced if the arrivals of vehicles from different routes at transfer terminals can be synchronized, at least to some extent. Complete synchronization may be greatly hindered by the network configuration even if vehicle arrivals at transfer terminals could be deterministic. Since arrivals are actually stochastic, due to variable demand, driver and traffic characteristics, synchronization may be quite difficult to achieve. To reduce the probability of missed connections, it may be desirable to include slack times in the vehicle schedules even at the cost of some additional waiting times for vehicles, drivers, and some non-transferring passengers.

Three options are considered here for operating multiple routes through a transfer terminal. These are (1) uncoordinated operation of multiple routes whose headways are only optimized independently for each route, (2) coordinated operation of multiple routes with a common headway, and (3) coordinated operation of multiple routes with integer-ratio headways. For analyzing such transportation systems, the total system cost may be classified into two major categories : Non-transfer cost includes vehicle running cost, user waiting cost in origin terminals and user in-vehicle cost. Transfer cost is the waiting cost of vehicles and passengers in transfer terminals.

Schedule coordination implies that the headways on various routes should either be equal to a common headway "cycle" or be integer multiples of that cycle. This reduces overall costs although non-transfer costs may increase. When demand is high, such as in peak periods, service headways should be short and transfer cost are typically dominated by other costs. Therefore, uncoordinated option will be preferable. Conversely, coordinated operation will be preferred when demand is low, such as in off-peak periods. The threshold in the passenger volume between uncoordinated operation and coordinated operation will be determined by the trade-offs between the non-transfer cost and transfer cost. The main objectives of this study are to identify which option is preferable under what circumstances and to optimize headways and slack times for each route if the coordinated operation is preferred.

Some descriptive studies of timed transfer systems, without formal optimization, have been published. Vuchic (1983) provided detailed descriptions and a systematic classification of timed transfer systems, without mathematical models to optimize system characteristics. Bakker, Calkin, and Sylvester (1990) provided an interesting discussion of the concepts and considerations involved in designing a timed transfer network for the bus system in Austin, Texas. In that analysis slack times (and other important variables, such as headways) were set judgmentally rather than optimized mathematically.

When timed transfers are optimized with deterministic models, slack times to allow for uncertain vehicle arrivals cannot be endogenously optimized. Therefore, deterministic models tend to focus on schedule synchronization in a transit network. Purely deterministic models to schedule connections and minimize delays were proposed by Rapp and Gehner (1976) and by Salzborn (1980). Kwan (1988) introduced an interactive heuristic approach embedded in a computer system called HINT for coordinating joint headways. Keudel (1988) developed algorithms for computer-aided network design (DIANA) and minimization of transfer times in networks (FABIAN) to optimize the transit systems. DIANA is used to generate a network with minimum operating costs, while FABIAN is used to minimize signal delays by optimizing the signal settings within a signalized street network. Klemt and Stemme (1988) used computer aided scheduling to minimize the total transfer waiting times in a schedule synchronization problem. They presented a simple and intelligible heuristic approach to reduce the computation time significantly. Daganzo (1990) examined the implications of scheduling in various ways the inbound and outbound vehicle routes serving a transportation terminal. The results from a small example with ten inbound routes and two outbound routes showed that solutions with all the headways equal often minimize the total logistic cost.

Several probabilistic distributions have been considered for vehicle arrivals at transit stations. Turnquist (1978) suggested that the distribution of bus arrival times at a point along a route is lognormal. A limited empirical study produced results consistent with such theoretical expectations. Talley and Becker (1987) and Hall (1985) used exponential distributions to compute the probability that buses will be more than x minutes early and more than y minutes late.

Guenthner and Hamat (1990) showed that adjusted bus arrival times follow a gamma distribution, but the proposed gamma distribution does not fit at all times since buses in the morning and evening peaks tend to arrive late but the midday buses tend to arrive early.

A few researchers have developed probabilistic models that account for the stochastic nature of passenger demand and vehicle travel times, but typically at the expense of analytic features such as optimization, realistic networks or realistic objective functions. For instance, Hall (1985) developed what seems to be the best analytic optimization model found to date for timed transfer systems. Its main weaknesses seem to be the use of exponential distributions for vehicle arrivals and the neglect of costs in an objective function which simply minimized wait time for transferring passengers. Exponential arrival time distributions seem unrealistic for transit operation but their use makes Hall's model analytically tractable. Lu (1990) formulated an analytic model for two routes to optimize slack time with stochastic arrivals. The formulation did not consider the joint probability of stochastic arrivals on the two routes. Bookbinder and Desilets (1992) combined a simulation procedure with an optimization model to determine the optimal departure time of buses in a transit network with decentralized transfers. That approach may consider both deterministic and stochastic bus travel times when the scheduled headways are treated as fixed on each line. The results exhibited the problems of optimizing transfers with a deterministic bus-travel-time assumption, if travel times are in fact stochastic. Lee and Schonfeld (1991) developed a numerical model for optimizing slack times for simple systems with transfers between one bus route and one rail line which can work with any arrival distributions (including irregular empirical distributions). Some analytic results were derived for empirical discrete and Gumbel distributions of bus arrival time. The sensitivity of optimal slack times to various factors was analyzed using normally distributed arrivals. The results showed that the optimal slack times become zero when service headways are small and/or when standard deviations of arrivals exceed certain levels.

A review of the above studies indicates that deterministic models for optimizing time transfer in transit systems have received considerable attention. However, no stochastic models for time transfer optimization among multiple routes have been found. The present paper seeks to fill this void.

2. SYSTEM DEFINITION

The system modelled consists of multiple transit routes interconnected at one transfer terminal (as shown in Figure 1). Bus routes, rail transit routes and various other kinds of transit routes may be included in any combination.

For uncoordinated operation, only a separate headway is optimized independently for each route. For coordinated operation, the headways and slack times for each route are jointly optimized. Coordination requires either one common headway for all routes or integer multiples of a basic headway cycle on each route, although the optimal slack times for different routes are unrestricted.

In transfer terminals, passengers typically walk between the vehicle loading positions. The required dwell times for vehicles may depend on expected inflows and outflows, door characteristics, and the distance between the stopped vehicles, but not on pre-planned headways and slack times in the schedule. Therefore, costs for dwell time would drop out of the objective function when w and H are optimized. Lee and Schonfeld (1991) analyzed the a transfer process with a time-space diagram which justifies focusing on scheduled departure times and neglecting the minimum required dwell times in the present model.

Finally, it is assumed here that delays on any route are independent from those on the other routes (although speeds and delay distributions may still vary over time). This assumption greatly simplifies the treatment of joint probabilities and is reasonable when routes operate independently in different environments. This assumption may be relaxed if delays are correlated due to traffic conditions, demand characteristics, or control actions.

3. THE TOTAL COST FUNCTION

Based on the system definition, models for uncoordinated and coordinated operations are developed. All variables are defined in Table 1. The total system costs for each type of operation are formulated below :

3.1 Uncoordinated Operation

The total cost includes non-transfer cost and transfer cost:

$$C_U=C_N+C_F \quad (1)$$

where

C_U= total system cost of uncoordinated operation ($/min.)

C_N= non-transfer cost ($/min.)

C_F= transfer cost ($/min.)

The non-transfer cost includes vehicle running cost, user waiting cost in origin terminals, and user in-vehicle cost:

$$C_N= C_o+C_w+C_v \quad (2)$$

where

C_o= vehicle running cost ($/min.)

C_w=user waiting cost in origin terminals ($/min.)

C_v= user in-vehicle cost ($/min.)

Table 1 Variable Definitions

Symbol	Definition	Baseline Value
B_i	vehicle operating cost ($/veh.min.)	0.667
C_C	total system cost for coordinated operation ($/min.)	-
C_d	waiting cost of transfer passengers from early to late vehicles ($/min.)	-
C_F	total transfer cost ($/min.)	-
C_m	missed connection cost ($/min.)	-
C_N	total non-transfer cost ($/min.)	-
C_o	vehicle running cost ($/min.)	-
C_p	inter-cycle transfer delay cost due to unequal integer-ratio headways ($/min.)	-
C_s	slack delay cost due to slack time ($/min.)	-
C_U	total system cost of uncoordinated operation ($/min.)	-
C_v	user in vehicle cost ($/min.)	-
C_w	user waiting cost ($/min.)	-
D_i	length of route i (miles)	-
d_i	average passenger travel distance along route i (miles)	-
$E(x)$	expected value of a random variable x	-
$f(t_i)$	probability density function for the vehicle arrival time on route i	-
H_i	vehicle headway of route i (minutes/veh.)	-
h_i^*	independently optimized headway for route i (mins/veh.)	-
i	route index	-
j	index for route to which passengers transfer	-
m	positive integer multiples of a basic headway cycle	-
n	number of routes	-
N_i	fleet size for route i (vehicles)	-
Q_i	total transfer passengers of route i (pass./min.)	-
q_i	total passenger demand of route i (pass./min.)	-
Q_{ij}	transfer passenger volume from route i to route j (pass./min.)	-
S	vehicle size (seats/veh.)	-
$\bar{r}_i$	average wait time of randomly arriving passengers for transferring to route i. (minutes)	-
t_i	vehicle arrival time of route i (minutes)	-
u	time value of passenger waiting time ($/pass.min.)	0.2
$V(x)$	variance of a random variable x	-
V_i	average speed on route i (mile/min.)	-
v	value of passenger in-vehicle time ($/pass.min.)	0.1
w_i	slack time on route i (minutes)	-
y	basic headway "cycle" (minutes)	-
z_{ij}	average transfer delays from route i to route j with integer-ratio headways (minutes)	-
$*$	superscript indicating optimal value	-

The vehicle running cost of each route is the product of the required fleet size N_i and vehicle operating cost B_i. The total vehicle running cost for all routes C_o is the sum of the vehicle running costs for each route:

$$C_o=\sum_{i=1}^{n} N_i B_i=\sum_{i=1}^{n} [2D_iB_i/(V_iH_i)] \tag{3}$$

where

n =number of routes

B_i = vehicle operating cost on route i (\$/vehicle minute)

H_i = vehicle headway on route i (minute/vehicle)

N_i = fleet size for route i (vehicles)

D_i = length of route i (miles)

V_i = average speed on route i (mile/min.)

According to our system definition, the user waiting cost C_w only includes waiting time before the initial boarding. The user waiting cost in transfer terminals will be included in the passenger transfer cost. Assuming that passengers arrive at the origin terminal uniformly over time and that vehicle capacity is adequate, the average wait time for the initial boarding may be approximated as half the service headway on that route. The resulting user waiting cost in origin terminals can be formulated as

$$C_w=\sum_{i=1}^{n} q_i\, u\, (0.5H_i) \tag{4}$$

where

u = value of passenger waiting time (\$/pass.min.)

q_i = total passenger volume on route i (pass./min.)

Those assumptions may be relaxed, if necessary, to provide a more accurate estimate of the average wait time.

The user in-vehicle cost is the value of time spent by passengers in the vehicles :

$$C_v=\sum_{i=1}^{n} 2d_i q_i v/V_i \qquad (5)$$

where

v = time value of passenger in in-vehicle time (\$/pass.min.)

d_i = average travel distance of boarding passengers of route i (miles/pass.)

Therefore, the total non-transfer cost is the summation of the three cost components in Eq. 2.

$$C_N=\sum_{i=1}^{n} [2D_iB_i/(V_iH_i)+0.5q_iuH_i+2vq_id_i/V_i\,] \qquad (6)$$

In such uncoordinated operation, the passengers arrive randomly at transfer terminals with respect to the scheduled departure times of their next (pick-up) vehicles. The pick-up vehicles arrive according to a probability distribution whose mean is the scheduled arrive time and whose variance can be determined empirically. In such a situation, the average wait time for the pick-up vehicle may be estimated (Osuna and Newell, 1972) as

$$\bar{r}_i=E(r_i)=0.5E(H_i)\{1+V(H_i)/[E(H_i)]^2\} \qquad (7)$$

where

$\bar{r}_i$= average wait time for randomly arriving passengers for transferring to route i. (minutes)

$E(H_i)$=expected value of the headway on route i.

$V(H_i)$=variance of the headway on route i.

The total transfer cost in this uncoordinated system is the total cost of waiting time in the terminal for all transfer passengers. This total waiting cost C_F is the product of the average waiting cost and the total number of transfer passengers.

$$C_F = \sum_{i=1}^{n} \sum_{j=1}^{n} Q_{ij}\, u\, \bar{r}_i \qquad (8)$$

where

C_F = total transfer cost (\$/min.)

Q_{ij} = the transfer passengers from route i to route j (pass./min.)

Eqs. 6 and 8 can be substituted into Eq. 1 to obtain the total system cost for an uncoordinated operation.

3.2 Coordinated Operation

The non-transfer cost for a coordinated operation is the same as that for an uncoordinated operation (Eq.2-6). The total transfer cost function includes all costs components that are sensitive to the slack time and headway. Three such components can be identified. These are (1) the slack delay cost of vehicles and passengers (due to holding the vehicles, their drivers, and non-transferring passengers for a scheduled slack time w), (2) the missed connection cost of transfer passengers (i.e., the value of the time spent waiting for the next suitable vehicle), and (3) the waiting cost of transfer passengers from early to late vehicles (i.e. the value of time spent by passengers in waiting for transfers from early to late vehicles).

3.2.1 Coordinated Operation with One Common Headway

The total transfer cost for a coordinated operation with a common headway is formulated as follows:

$$C_F=C_s+C_m+C_d \qquad (9)$$

C_s = slack delay cost ($/min.)

C_m = missed connection cost for transfer passengers ($/min.)

C_d = connection cost of transfer passengers from early to late vehicles ($/min.)

It should be noted that according to our terminology "delay" can occur when successful connections (i.e., connections between the intended vehicles) are made. "Missed connection" costs result from "unsuccessful" connections (i.e., in connections to the next arriving vehicle on the intended route). The slack delay cost C_s includes the user time for Q_i passengers per minute and the supplier cost for $1/H_i$ vehicles per minute subjected to the added slack time w_i for each route :

$$C_s = \sum_{i=1}^{n} (Q_i u+B_i/H_i)w_i \qquad (10)$$

where

Q_i =total transfer passengers of route i (passengers/min)

Only one dispatching strategy whereby vehicles do not wait for other vehicles that arrive behind schedule is considered here for pre-planned scheduling. The missed connection cost for transfer passengers C_m is determined from the joint probability distributions for vehicle arrivals on any coordinated pair of routes. Since vehicles arrivals were assumed to vary independently from other vehicle arrivals in our system definition, the joint probabilities of arrivals may be obtained by simply multiplying the probabilities obtained separately from the two vehicle arrival distributions. The probability of missed connections must include the following two cases regarding the two vehicles involved in a connection : (1) the delivering vehicle i arrives late while the pick-up vehicle j is not late (2) both vehicles are late, but the delivering vehicle i arrives after the pick-up vehicle j leaves. The missed connection cost C_m is then :

$$C_m= \sum_{i=1}^{n} \sum_{j=1}^{n} Q_{ij}uH_j \left[\int_{w_i}^{H_i} f(t_i)\, dt_i \int_{-H_j}^{w_j} f(t_j)\, dt_j + \int_{w_i}^{H_i} f(t_i)\, dt_i \int_{w_j}^{t_i-w_i+w_j} f(t_j)\, dt_j \right] \qquad (11)$$

where

w_j = slack time of route j at the transfer terminal (minutes)

t_i = vehicle arrival time on route i (minutes)

$f(t_i)$= probability density function for the vehicle arrival time on route i

t_j = vehicle arrival time on route j (minutes)

$f(t_j)$= probability density function for the vehicle arrival time on route j

Figure 2 shows the joint probability of missed connections for transfer passengers on some pair of routes. In Figure 2, A is marked as the probability that the delivering vehicle i arrives late, B is the probability that the pick-up vehicle j is not late, C is the probability that the delivering vehicle i arrives late and after the pick-up vehicle j when j is also late, and D is the probability that the pick-up vehicle j arrives late but still earlier than the delivering vehicle i.

The connection cost C_d in Eq. 12 accounts for delays to vehicles and their passengers when connections are successfully made but the pick-up vehicle arrives behind schedule. The probability of that connection delay must include the following two cases regarding the two vehicles involved in a connection : (1) the delivering vehicle i arrives early while the pick-up vehicle j is late (2) both vehicles are late but the delivering vehicle i arrives before the pick-up vehicle j. This dispatching delay cost is expressed as follows :

$$C_d=\sum_{i=1}^{n} \sum_{j=1}^{n} Q_{ij}u \left[\int_{-H_i}^{w_i} f(t_i)\, dt_i \int_{w_j}^{H_j} (t_j-w_j)\, f(t_j)\, dt_j + \int_{w_i}^{H_i} f(t_i)\, dt_i \int_{t_i-w_i+w_j}^{H_j} (t_j-w_j+w_i-t_i)\, f(t_j) dt_j \right] \qquad (12)$$

Figure 3 shows the joint probability of the dispatching delay incurred by transfer passengers on each pair of routes. In Figure 3, E is marked as the probability that the delivering vehicle i arrives early, F is the probability that the pick-up vehicle j is late, G is the probability that the delivering vehicle i arrives late but before the pick-up vehicle j, and H is the probability that the pick-up vehicle j arrives late and after the delivering vehicle i when i is also late.

Eqs.10-12 obtained above can be substituted into Eq. 9 to determine the total transfer cost.

3.2.2 Coordinated Operation with Integer-Ratio Headways

In this case, the non-transfer cost is the same as in the uncoordinated system (Eqs.2-6). One more cost component, the inter-cycle transfer delay cost C_p, is included in the total transfer cost, as there will be some fraction of transfer passengers who must wait for one or more additional cycles because the scheduled vehicle arrivals on their delivery and pick-up routes do not coincide:

$$C_F=C_s+C_d+C_m+C_p \tag{13}$$

where

C_p= transfer cost between cycles due to unequal integer-ratio headways ($/min.)

In this model, each route maintains its optimized slack time at the same value through all cycles. In later versions that slack time may change, depending on which routes connect in a given cycle. The present model also assumes that vehicle loads on any route are Q_iH_i, i.e. the volume multiplied by the headway. This implies that passenger boardings on any one vehicle are not affected by the scheduled connections in the transfer terminal. Later model versions may use more elaborate demand models to reflect passenger preference for and knowledge of immediate connections. The missed connection cost and the dispatching delay cost for transfer passengers from early to late vehicles C_m and C_d (Eqs. 11-12) are modified as follows :

$$C_m=\sum_{i=1}^{n}\sum_{j=1}^{n} Q_{ij}\, u\, y \left[\int_{w_i}^{H_i} f(t_i)\, dt_i \int_{-H_j}^{w_j} f(t_j)\, dt_j + \int_{w_i}^{H_i} f(t_i)\, dt_i \int_{w_j}^{t_i-w_i+w_j} f(t_j)\, dt_j\right] \tag{11a}$$

$$C_d=\sum_{i=1}^{n}\sum_{j=1}^{n} Q_{ij}\, u\, y / H_j \left[\int_{-H_i}^{w_i} f(t_i)\, dt_i \int_{w_j}^{H_j} (t_j-w_j)\, f(t_j)\, dt_j + \int_{w_i}^{H_i} f(t_i)\, dt_i \int_{t_i-w_i+w_j}^{H_j} (t_j-w_j+w_i-t_i)\, f(t_j)\, dt_j\right] \tag{12a}$$

Eq. 13 is appropriate if, as expected, the probability of a vehicle arrival delay larger than the basic cycle y is small enough to be negligible. The slack delay cost C_s and the dispatching delay cost C_d are the same as in the common headway case (Eq. 9-11). The cost of transferring between cycles C_p can be formulated as

$$C_p=\sum_{i=1}^{n} \sum_{j=1}^{n} u\, Q_{ij}\, z_{ij} \qquad (14)$$

where

z_{ij}= average waiting time for passengers transfering from route i to route j at the transfer terminal, given integer-ratio headways (mins.)

For integer-ratio headways, the z_{ij} may be easily estimated by assuming that the volumes of passenger arrivals on incoming vehicles do not depend on transfer delays. Otherwise, (i.e. if passengers favor schedules with immediate connections) some more complex demand models should be applied. Figure 4 shows how z_{ij} may be determined. For example, when the integer ratio of two headways is 1 to 2 (e.g. H_i/H_j=1y/2y=1/2), z_{ji} should be zero and z_{ij} should be 0.5y (half the passengers arriving at the transfer terminal wait y minutes). If the integer ratio of two headways is 1 to 3 (e.g. H_i/H_j=1y/3y=1/3), z_{ji} should be zero and z_{ij} should be y (one-third of the transfer passengers wait 2y and another one-third wait y minutes). If the integer ratio of two headways is 2 to 3 (e.g. H_i/H_j=2y/3y=2/3), z_{ji} should be 0.5y (half the passengers arriving at the transfer terminal wait y minutes) and z_{ij} should be y (one-third of the passengers arriving at the transfer terminal wait 2y and another one-third wait y minutes).

In the general case, z_{ij} may be formulated as

$$z_{ij}= g\, y\, [(H_j/2gy)-0.5] \qquad (15)$$

where

g= the greatest common factor of H_i/y and H_j/y

y= the basic headway cycle

H_j= headway of route j

Eqs. 10,11a,12a and 14 can be substituted into Eq. 13 to obtain the total transfer cost for a coordinated operation with integer-ratio headways.

4. OPTIMAL SLACK TIME AND HEADWAY

After formulating the components of total cost C as functions of the headways H_i and slack times w_i, the optimal values of H_i and w_i can be sought numerically, since the probability distributions are too complex for analytic integration. Thus, numerical integration of interacting probabilistic vehicle arrival distributions is used to compute transfer delays. Such numerical integration is much faster and more precise than simulation. Afterwards, the following newly developed algorithm is used to jointly optimize headways and slack times for all coordinated routes. The algorithm determines whether the optimal solution is a single common headway or integer-ratio headways.

The algorithm starts by determining optimal headways independently for each route, ranking routes according to their independently optimal headways, and grouping them into initial clusters with headways "nearest" to the integer multiples of the cycle y, where y itself is an optimizable variable. "Nearest" is based on relative rather than absolute distance. Thus if y is the basic cycle and m is any positive integer, an independently optimal headway h_i^* would be initially grouped with the routes whose headways are set at my if

$$\sqrt{(m-1)m}\; y < h_i^* \leq \sqrt{m(m+1)}\; y \qquad (16)$$

The above inequality thus determines the boundaries between route clusters whose headways are integer multiples of the cycle y.

The clustering process adjusts headways, without changing the initial route rankings based on independently optimized headways. Such ranking inversions or "swaps" invariably worsen the objective function, unless the rankings are already equal (in which case swaps are meaningless) or certain route pairs exchange disproportionate fractions of passengers. The stability of rankings is a special feature of this problem which can be exploited to greatly reduce the computations in the clustering process.

The independently optimal headways h_i^* which separately minimize the total cost for each route can be obtained with the following well known "Square Root Formula" :

$$h_i^* = \sqrt{2 D_i B_i / (q_i u V_i)} \qquad (17)$$

This result can be obtained by analytically minimizing a total cost function, such as an expanded Eq. 1, with respect to each headway h_i.

For an initial "round" cycle y (e.g. 0.5, 1, 1.5, 2,..., 12, 15, 20, 30 minutes, obtained as 60 minutes are divided by an integer number of hourly departures), the proposed procedure starts with an initial solution and seeks improvements by shifting the boundaries between clusters by one route at a time until no further improvement can be obtained for that cycle. Then, the slack times are optimized for the optimal headways obtained. Finally, the process is repeated for a new round cycle y until no further improvement is obtainable by increasing or decreasing the cycle.

The iterative search procedures for the optimal cycle and slack times can be described as follows :

1. Determine the independently optimal headways for each route h_i^* with Eq. 17.

2. Rank the routes by sorting them in the order of increasing headway h_i^*.

3. Start with an initial cycle y below the smallest h_i^*.

4. Determine the boundaries between the clusters with headways my according to Eq. 16 and group the routes accordingly.

5. Assume zero slack times for each route ($w_i^*=0$) and compute the total system cost for the initial combination of headways from step 4.

6. Increase or decrease the first boundary (between route with headways 1y and 2y), while other boundaries are fixed, as long as the total system cost can be reduced.

7. Repeat step 6 one boundary at a time for every boundary as long as the total system cost can be reduced. After considerring all boundaries from 1 to (m-1), additional iterations are run as long as significant further improvements are obtained.

8. Apply a multidimensional optimization algorithm such as the IMSL routine UMIMF (1987) to determine the optimal slack times (w_i*) for each route, given the optimized integer-ratio headways from step 7.

9. Repeat steps 6-7 with w_i^* to determine the best new integer-ratio headways.

10. If the new solution (with w_i^*) is different from the old one(with w_i=0), repeat alternatively step 8 with the new headways to determine new w_i^* and then step 9 to optimize headways until no significant further improvement is obtained.

11. Repeat steps 4-10 by increasing or decreasing the "round" cycle y until no significant further improvement is obtained.

The resulting objective function, although convex on a macroscopic level, may have slight steps as routes are reassigned to different clusters. The search in step 11 must proceed far enough to overcome the effects of such minor local optima.

At the conclusion of step 11, the results include the optimal cycle, the optimal headway for each route as an integer multiple of that cycle, and the optimal slack time for each route.

5. NUMERICAL RESULTS

Numerical results were computed mainly for the purpose of investigating the sensitivity of optimal slack times and headways to various factors such as the passenger time value, passenger transfer volumes, vehicle operation cost, and standard deviations of vehicle arrival times.

When using a normal distribution of vehicle arrival times, the probability density function is

$$f(t)=[1/(2\pi)^{0.5}s]\ \exp(-0.5t^2/s^2) \tag{18}$$

in which the mean and mode are 0, and the standard deviation is s. Although a normal distribution is analytically untractable for deriving closed-form solutions, it can be used here to compute the results numerically.

A multidimensional optimization method, the IMSL routine UMINF (1987), which uses a Quasi-Newton method with a finite-difference gradient to find the minimum of a function f(X) of n variables is used here to optimize the slack times for each route.

Two numerical examples are considered. The first is a three-route operation through a transfer terminal. The purpose of this simple example is to explore the relations among variables and particular parameters through sensitivity analysis. The second example is a ten-route operation through a transfer terminal. This more complex example is used to show that many routes may be coordinated and jointly optimized.

The baseline parameter values shown in Table 1 were selected for the numerical analysis, because they seemed reasonable and typical. The total passenger volume and transfer passenger volume for each route in the three-route and ten-route examples are shown in Tables 2 and 3.

With the above baseline values, the comparison of the optimal results for each option in the three-route example is shown in Table 4. It shows that coordinated operation with a common headway is preferable in this case. The optimal slack times for the three routes are computed to be 0.032, 0.056, 0.99 minutes, respectively, and the common headway is 10.6 minutes. For practical application, the "round" headway closest the optimum is 10 minutes, for which slack times are optimized at 0.028, 0.051, and 0.96 minutes. The threshold demand between coordinated and uncoordinated operation for the three-route example is illustrated in Figure 5. In this case, coordination is preferable at demand levels up to 1.09 times the baseline demand and uncoordinated operation is preferable at higher demand levels. Figures 6 and 7 show the relations among the total cost, common headway, and optimal slack time. In Figure 6, the optimal slack times are zero when the headways are small and increase at a decreasing rate beyond certain critical headways (7.5 minutes for route 3 and 10 minutes for routes 1 and 2). It should be remembered that Figure 6 is computed for normally distributed vehicle arrivals with a baseline standard deviation for each route. Baseline values are also used for other parameters. In Figure 7, the total cost function C is U-shaped with respect to the headway and reaches a minimum when H=10.6 minutes. The total cost may be reduced if other decision variables (in this case, the slack times w_i) can be reoptimized. The relations between the slack time and the cost components of the transfer cost function are shown in Figure 8. This figure clearly shows that the optimal slack time represents a tradeoff between the missed connection cost C_m and the dispatching delay cost C_d, both of which decrease with w, and the slack delay cost C_s, which increases with w. At high values of w, C_m and C_d approach zero while C_s still increases. That limits to a finite value of the magnitude of the optimal value of w.

Figure 9a shows how the passenger time value u influences the optimal slack time w*, in this case after headways are reoptimized, as shown in Figure 9b. The optimal slack times w_i^* start increasing above zero beyond a critical value of passenger time. The w_i^* first increases as the passengers time value u increases. Then, since the higher u favors lower headways, the combined effect of smaller headway and higher passenger time value reduces slack times w_i^* until they reach zero. The optimal slack time for route 3 is significantly greater than for routes 1 and 2, since route 3 has a lower demand and higher variance of arrival times. Figure 9b shows how the optimal common headway decreases as the time value of passengers increases.

Figures 10a and 10b show the effect of transfer volumes on the optimal slack times and the optimal headways. For example, w_3* decreases as transfer volumes from route 3, Q_{3j}, increase. The w_3* curves for different value of transfer volumes to route 3, Q_{i3}, tend to converge to zero as Q_{3j} increases. That is because the tradeoffs in the transfer cost function which yield w_3* are increasingly dominated by Q_{3j} and the optimal headways become too small to justify coordination, while Q_{i3} and the vehicle operating cost B become relatively negligible. Figure 11 shows the relations between the transfer passenger volume and optimal slack time without re-optimizing the common headway, which remains 10.6 minutes. When both the passenger volumes Q_{3j} from route 3 to other routes and the volumes Q_{i3} from other routes to route 3 are very low, the optimal slack time for route 3 is zero. As Q_{3j} increases while Q_{i3} stays very small (e.g. Q_{i3}=0.2 passengers per minute), w_3^* increases at a decreasing rate since the common headway stays the same. As Q_{3j} increases while Q_{i3} is high (e.g. Q_{i3}=1.0 or 1.5 passengers per minute), w_3^* decreases at a decreasing rate. The w_3^* curves for different Q_{i3} values tend to converge to a positive value instead of zero as the Q_{j3} volumes increase due to the constant headways.

The effect of the vehicle operating cost on the optimal slack time w* is shown in Figure 12. It is reasonable that for a given passenger volume, w* should decrease at a decreasing rate as the cost of delaying vehicles and the optimal headway increases. Figures 13 shows the effects of standard deviations of arrival times on optimal slack times. The slopes of the w* curves are determined by the slopes of the normal distributions and by the re-optimization of headways as standard deviations change. Thus, in Figure 13, w* first increases as the standard deviation increases. At first, the additional uncertainty provides economic justification for a larger safety factor, i.e. slack time. However, as the standard deviation approaches a significant fraction of the headway, it becomes preferable to reduce slack time, and allow a higher probability of missed connections in the "tail" of the vehicle arrivals distribution. Beyond a certain critical standard deviation, the optimal slack time w* should be zero, implying that as vehicle arrivals become more uncertain and headway magnitudes do not produce excessive missed connection costs, it becomes uneconomical to leave any safety factors in a schedule. Conversely, slack time is most feasible and desirable when arrival uncertainties are low and headways are large. The intersection of the w_1^* and w_2^*

curves has no particular significance. It occurs due to the combined effects of the demand and the standard deviation on the optimal slack time (the demand on route 1 is higher and the standard deviation of vehicle arrivals on route 1 is also larger than on route 2).

The comparison of the optimal results with and without coordination for the ten-route example is shown in Table 5. The results show that if the headways for various routes are relatively large and similar (i.e. if the variance of headways among routes is small), a coordinated operation with a common headway is preferred. A coordinated operation with integer-ratio headways is preferable when the headways are relatively large and quite different (i.e. with a large variance of headways among routes). Coordinated operation with integer-ratio headways is preferable in the ten route example since the optimal headways for all routes are large and the variance of headways is also large.

6. CONCLUSIONS AND EXTENSIONS

Whether coordinated operation is advantageous in transit operation depends mainly on the passenger demand. If demand is high, the optimal headways are too small to make coordinated operation worthwhile. Conversely, at low demand coordinated operation is preferable, even at the expense of slack times in schedules. In a coordinated operation, a single optimal common headway H^* is a compromise among the optimal headways for each route. That compromise may be considerably alleviated, at the expense of some increased transfer delays, if integer-ratio headways are used. The optimal scheduled slack time w^* for vehicles interchanging passengers at a transfer terminal is the result of tradeoffs among several supplier cost and user cost components expressed in the total transfer cost functions (Eqs. 8, 9, 10, and 11). As w increases, the slack delay cost to vehicles and passengers increases while missed connection costs and dispatching delay costs decrease. The values of H^* and w^* for each route can be determined with the proposed algorithm.

The conclusions from the numerical results may be summarized as follows :

1. Uncoordinated operation is preferred when the demand is sufficiently high to keep the independently optimal headways h_i^* low.

2. Coordinated operation with a common headway is preferred when the headways h_i^* are relatively large and their variance is small.

3. Coordinated operation with integer-ratio headways is preferred when at least some h_i^* are relatively large and their variance is large.

The results show some interesting but reasonable ways in which the optimal slack times for vehicles vary with such variables as headways, vehicle arrival time variances, transfer volumes, passenger time values and vehicle operating costs. In particular, the results show that as vehicle arrival variances increase (Fig. 13), optimal slack times eventually drop back to zero. This implies that coordination is not worth attempting, even in low demand and high headway cases, if arrival randomness becomes excessive.

The proposed numerical approach and the iterative search algorithm can efficiently solve an apparently large combinatorial problem with many interconnecting routes by exploiting the special structure of this problem (i.e. that the headway rankings based on independent headway optimization are not significantly changed by the clustering process).

The following extensions to this work seem worth pursuing :

1. Use real data, including empirical discrete arrival distributions, in the numerical analysis.

2. Optimize a schedule that varies with demand over time, in which headways are mostly uncoordinated during higher demand periods and coordination increases as demand decreases (possibly one route at a time).

3. Adapt for other types of transportation terminals such as airline hubs and container ports.

4. Optimize real-time dispatching decisions based on predicted vehicle arrival delays.

5. Consider the effects of demand elasticity on maximum profit or welfare objectives.

6. Consider networks with multiple transfer terminals.

Table 2a Input Data for Three-Route Example

Route	Demand (pass./min.)	Distance (miles)	Speed (mph)	Standard Deviation (minutes)
1	2.0	7.5	20	2.5
2	2.5	10.0	20	3.0
3	1.5	12.5	20	3.5

Table 2b Input Data for Ten-Route Example

Route	Demand (pass./min.)	Distance (miles)	Speed (mph)	Standard Deviation (minutes)
1	2.42	10.0	20	2.0
2	1.68	10.0	20	1.5
3	1.21	10.0	20	1.5
4	0.77	10.0	20	2.0
5	0.49	10.0	20	2.5
6	0.37	10.0	20	2.5
7	0.23	10.0	20	3.0
8	0.18	10.0	20	3.5
9	0.15	10.0	20	3.0
10	0.12	10.0	20	3.0

Table 3a Transfer Passenger Volumes for Three-Route Example (passengers/minute)

From\ To	1	2	3
1	0	1.5	0.5
2	1.5	0	1.0
3	0.5	1.0	0

Table 3b Transfer Passenger Volumes for Ten-Route Example (passengers/minute)

From\To	1	2	3	4	5	6	7	8	9	10
1	0.00	0.78	0.56	0.36	0.23	0.17	0.11	0.08	0.07	0.06
2	0.68	0.00	0.34	0.22	0.14	0.10	0.07	0.05	0.04	0.03
3	0.46	0.32	0.00	0.15	0.09	0.07	0.04	0.03	0.03	0.02
4	0.27	0.19	0.14	0.00	0.06	0.04	0.03	0.02	0.02	0.01
5	0.17	0.12	0.08	0.05	0.00	0.03	0.02	0.01	0.01	0.01
6	0.12	0.09	0.06	0.04	0.03	0.00	0.01	0.01	0.01	0.01
7	0.08	0.05	0.04	0.02	0.02	0.01	0.00	0.01	0.00	0.00
8	0.06	0.04	0.03	0.02	0.01	0.01	0.01	0.00	0.00	0.00
9	0.05	0.03	0.02	0.02	0.01	0.01	0.00	0.00	0.00	0.00
10	0.04	0.03	0.02	0.01	0.01	0.01	0.00	0.00	0.00	0.00

Table 4. Comparison of Uncoordinated and Coordinated Operation for Three-Route Example

Route	Uncoordinated Operation		Coordinated Operation	
	Headway	Slack Time	Headway	Slack Time
1	8.66	-	10.56	0.032
2	8.95	-	10.56	0.056
3	12.91	-	10.56	0.991
C_o	11.812		11.326	
C_w	5.906		6.360	
C_v	35.25		35.25	
C_N	52.968		52.936	
C_s	-		0.237	
ϵ_m	-		4.021	
C_d	-		1.876	
C_F	6.444		6.134	
C	59.412		59.070	

Table 5. Comparison of Uncoordinated and Coordinated Operation for Ten-Route Example

Route	Uncoordinated Operation		Coordinated Operation	
	Headway	Slack Time	Headway	Slack Time
1	9.09	-	18.00	0.96
2	10.91	-	18.00	0.99
3	12.86	-	18.00	1.14
4	16.12	-	18.00	1.39
5	20.21	-	18.00	1.88
6	23.26	-	18.00	2.16
7	29.50	-	36.00	2.36
8	33.34	-	36.00	2.75
9	36.52	-	36.00	3.03
10	40.84	-	36.00	3.75
C_o	17.265		17.787	
C_w	13.843		14.940	
C_v	45.720		45.720	
C_N	76.828		78.447	
C_s	-		1.546	
C_m	-		6.837	
C_d	-		3.577	
C_p	-		1.431	
C_F	16.731		13.391	
C	93.559		91.837	

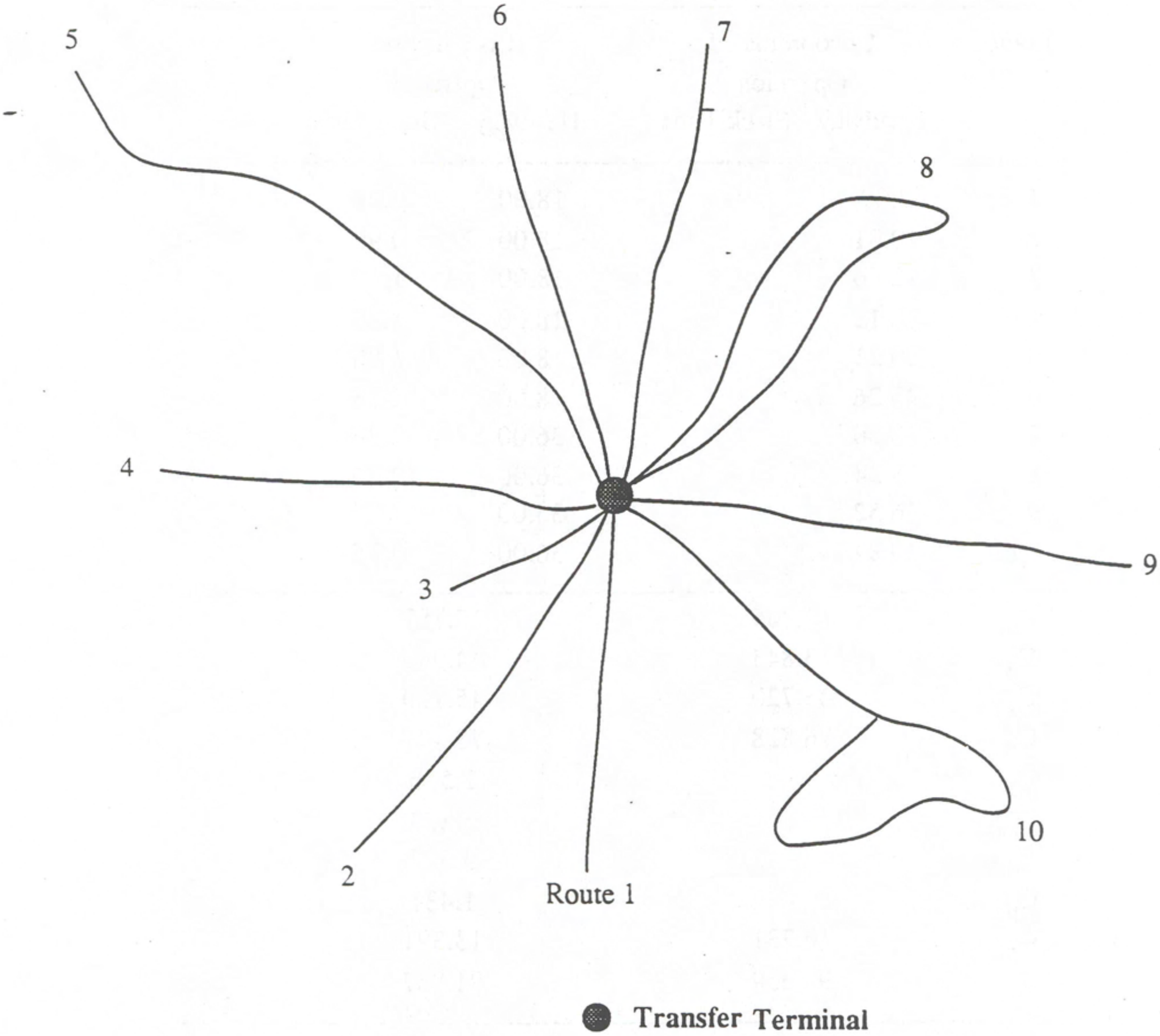

Figure 1 : Transportation Network With Multiple Routes Connecting at a Transfer Terminal

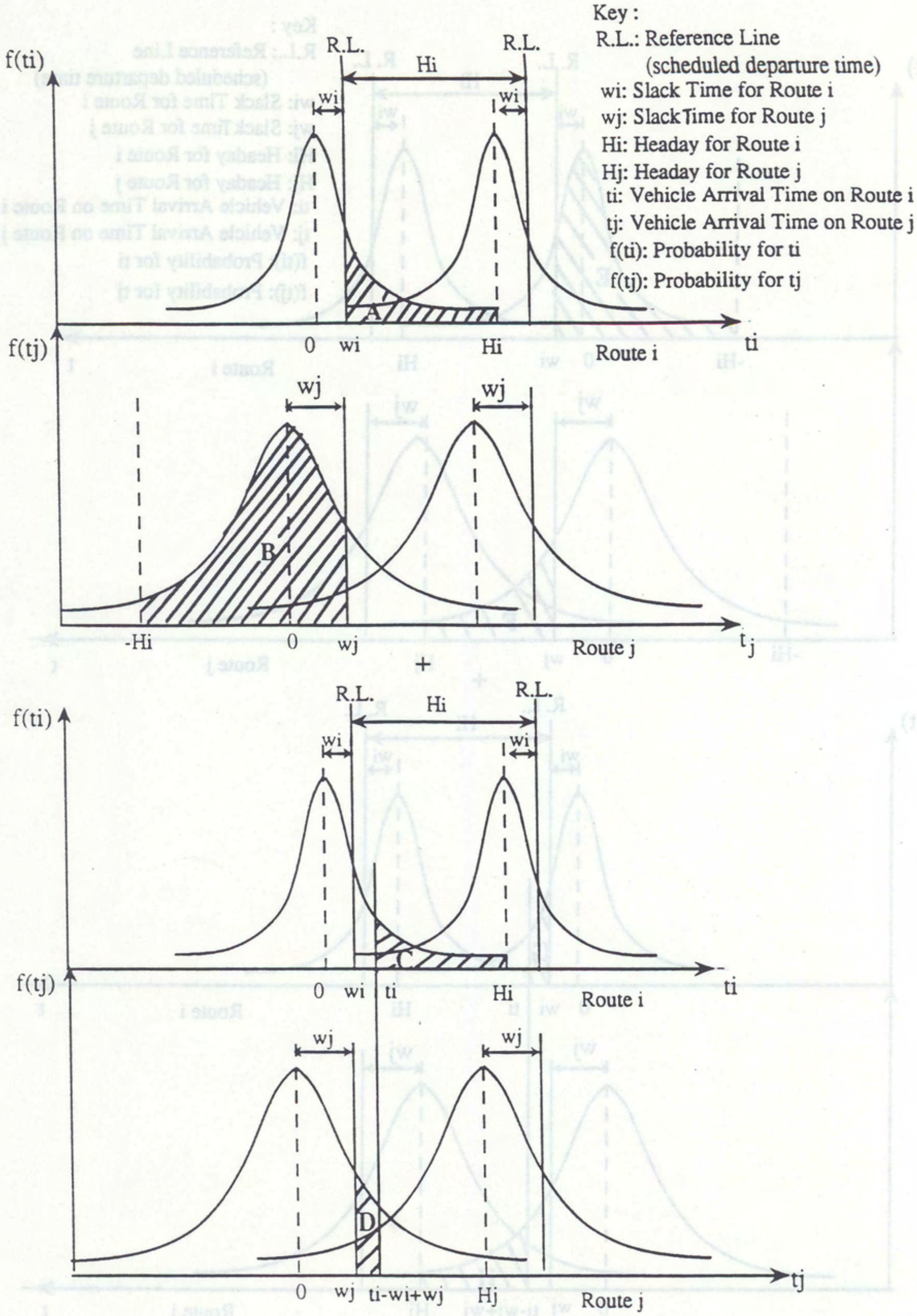

Figure 2: The Joint Probability of Missed Connections for Passengers Transferring from Route i to Route j

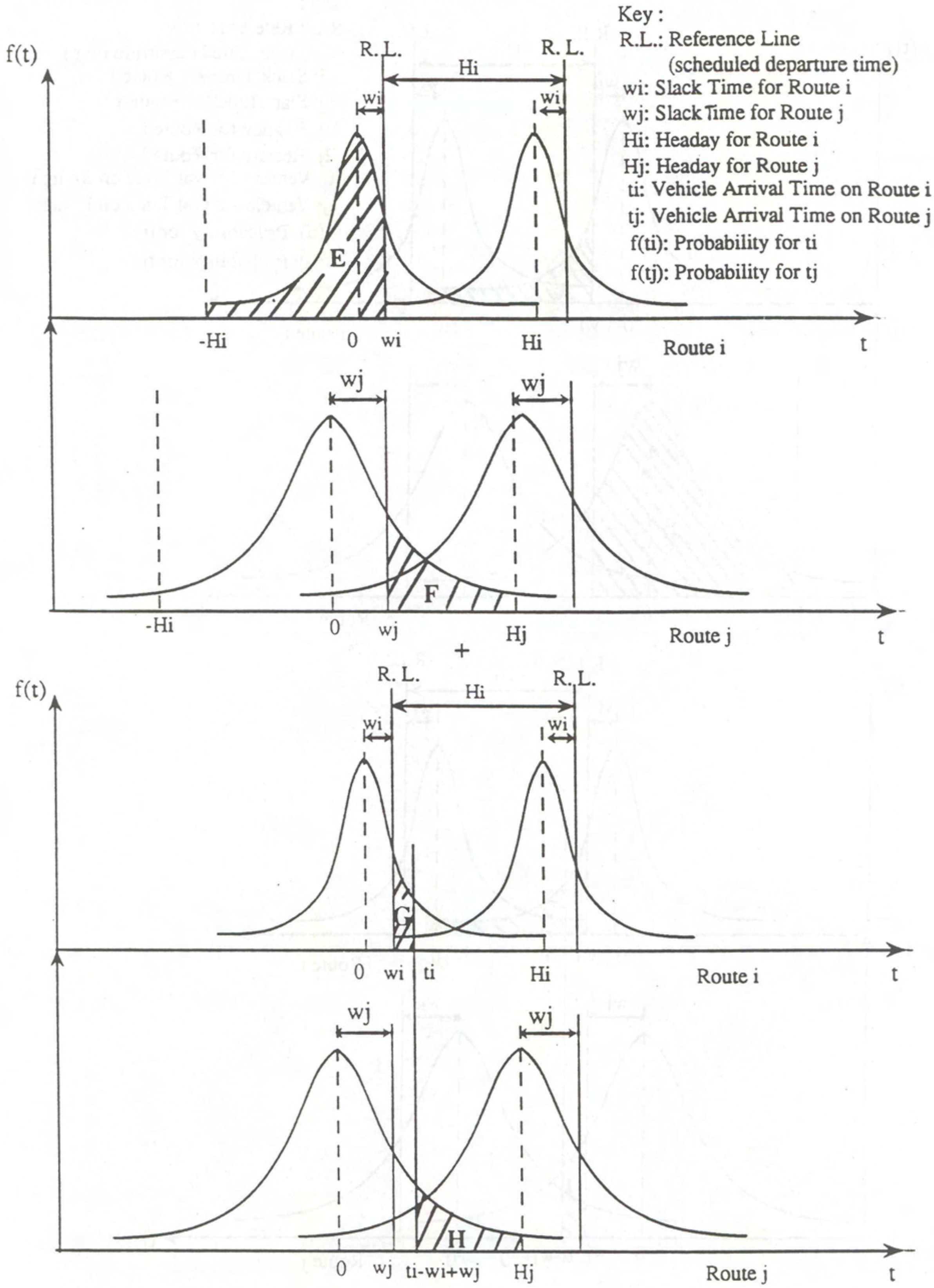

Figure 3: The Joint Probability of Dispatching Delays for Passengers Transferring from Route i to Route j

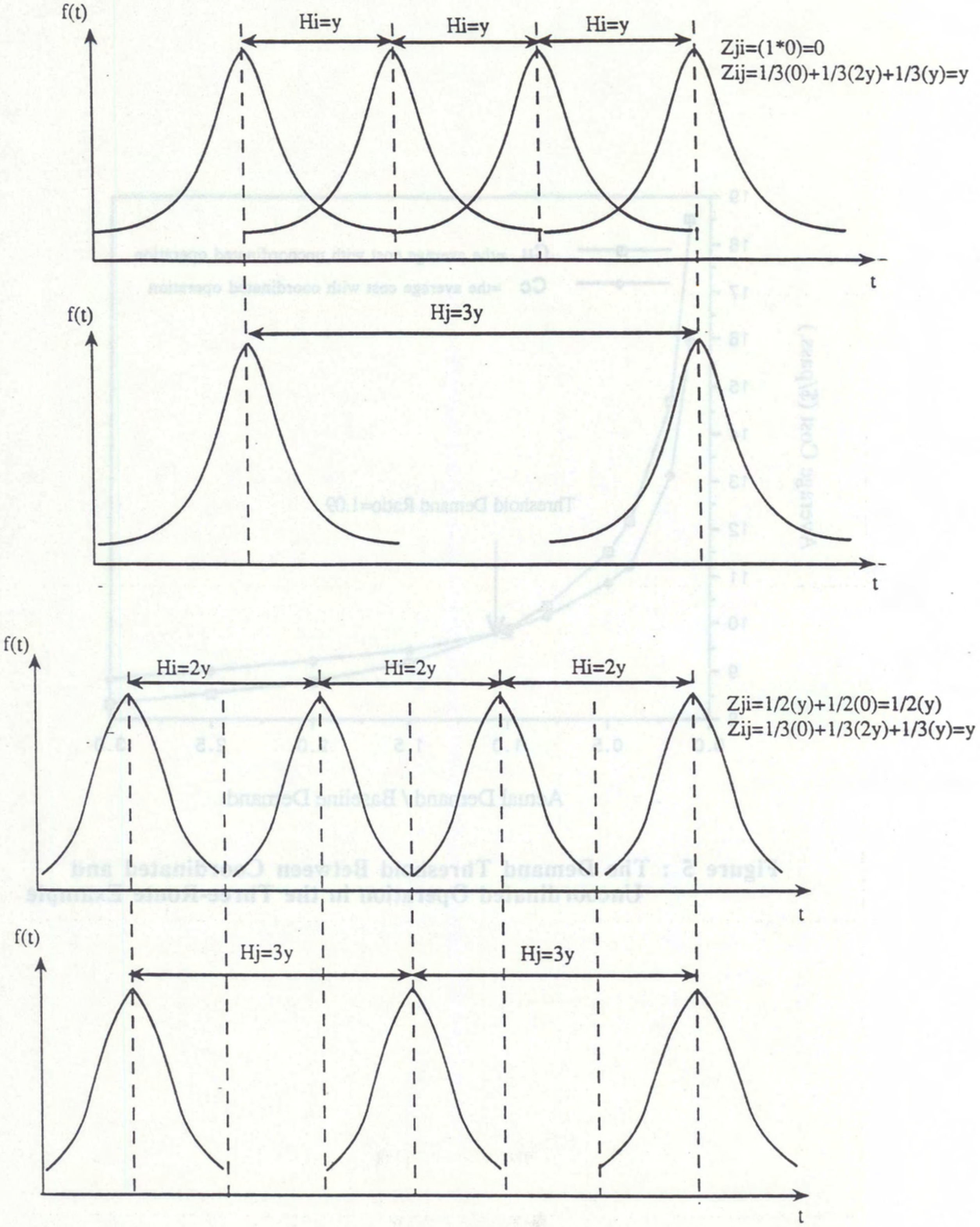

Figure 4: The Inter-Cycle Transfer Delays for Passengers Transferring Between Routes with Integer-Ratio Headways

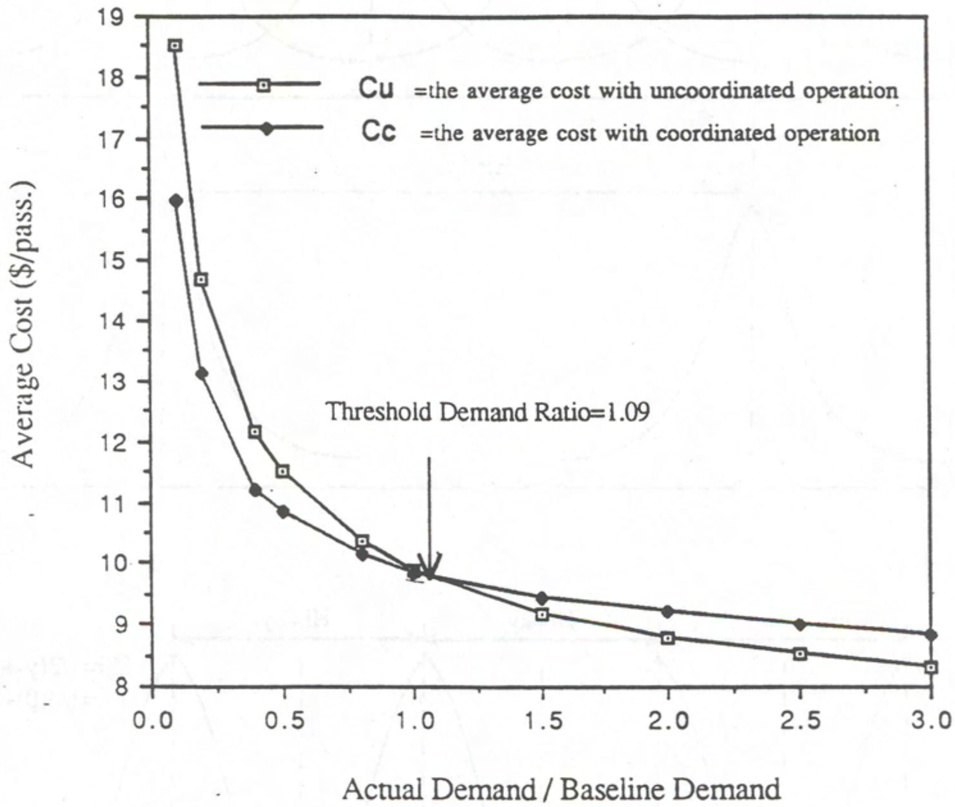

Figure 5 : The Demand Threshold Between Coordinated and Uncoordinated Operation in the Three-Route Example

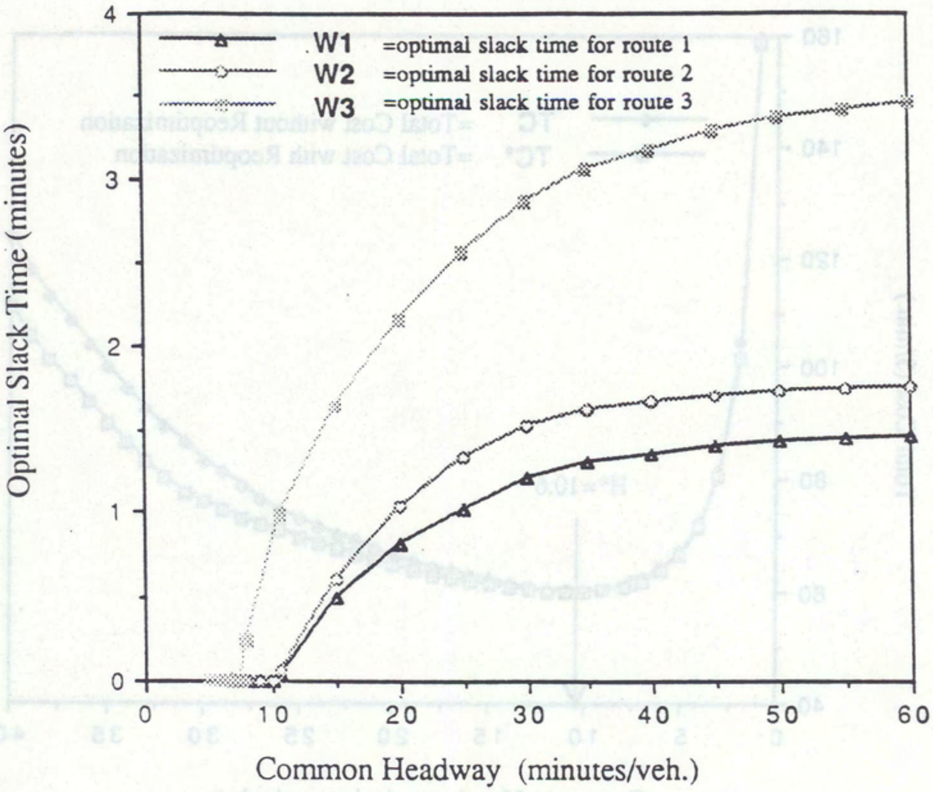

Figure 6 : Effect of the Common Headway on the Optimal Slack Time

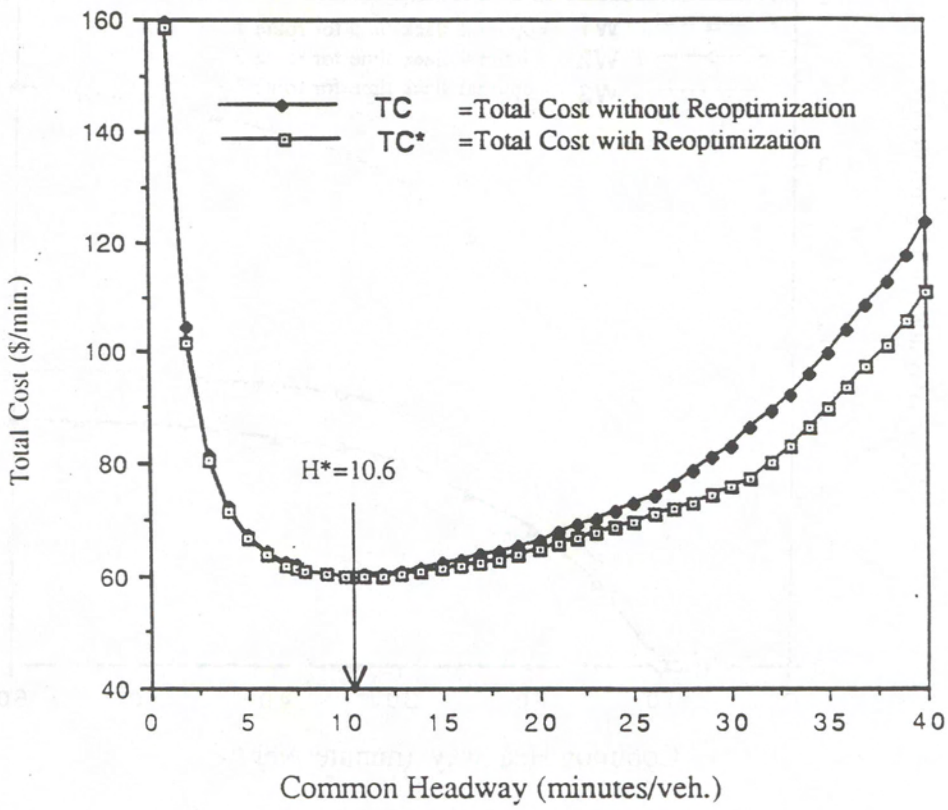

Figure 7 : Relations Between Total Cost and the Common Headway

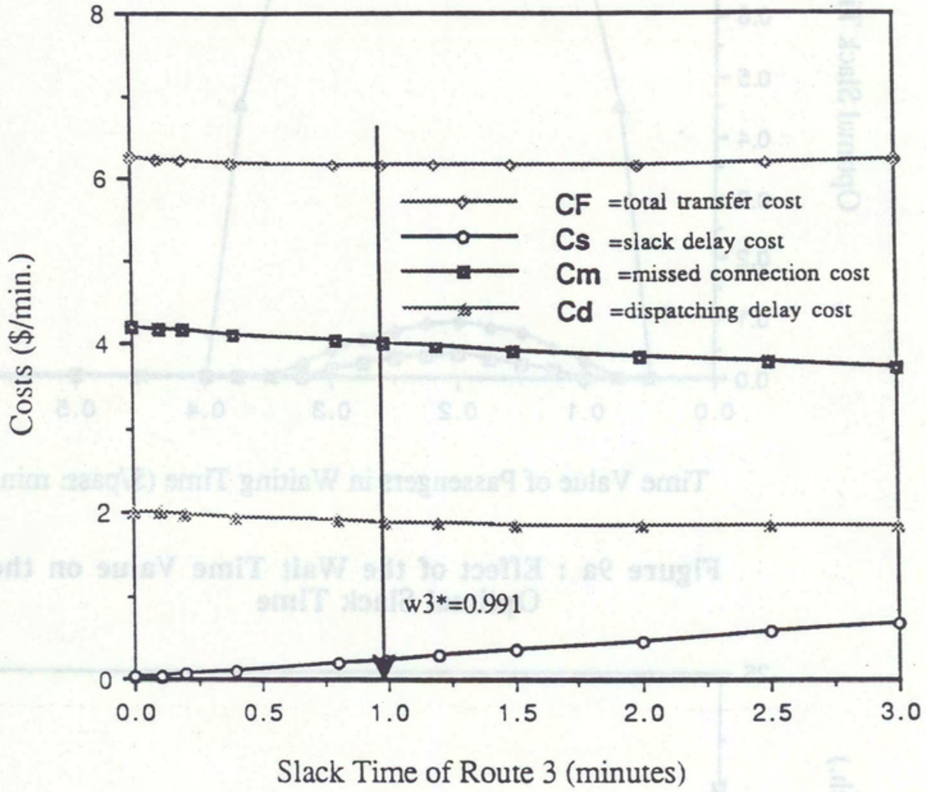

Figure 8 : The Relations Between Slack Time and the Components of Transfer Cost for the Three-Route Example

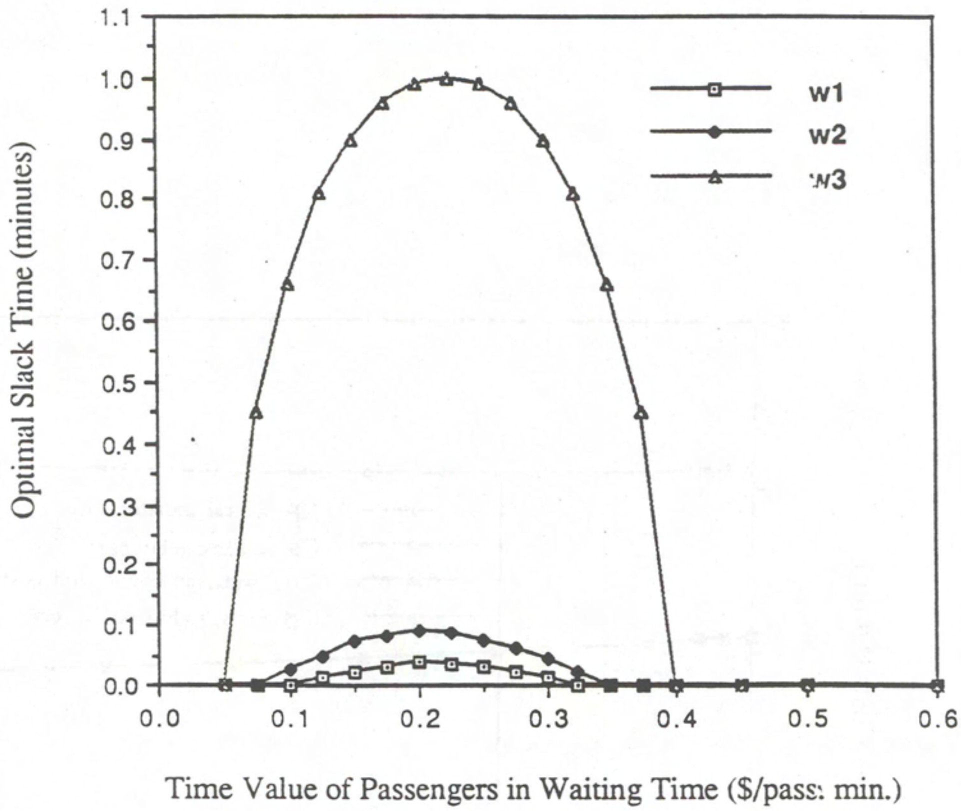

Figure 9a : Effect of the Wait Time Value on the Optimal Slack Time

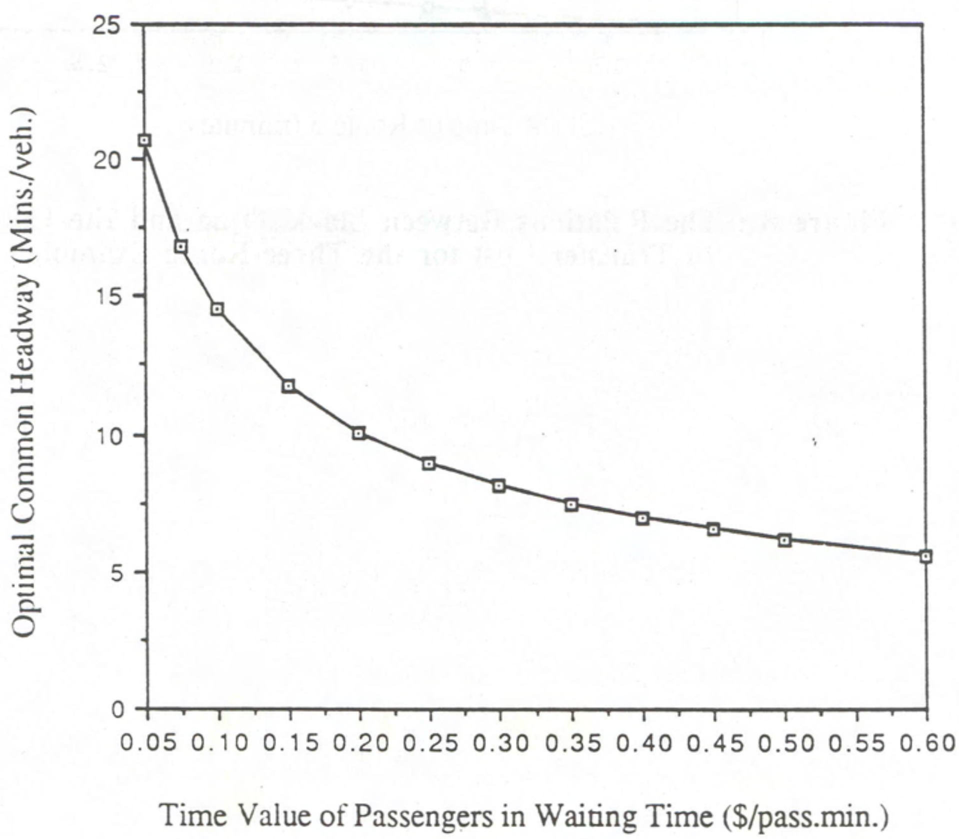

Figure 9b : Effect of the Wait Time Value on the Optimal Common Headway

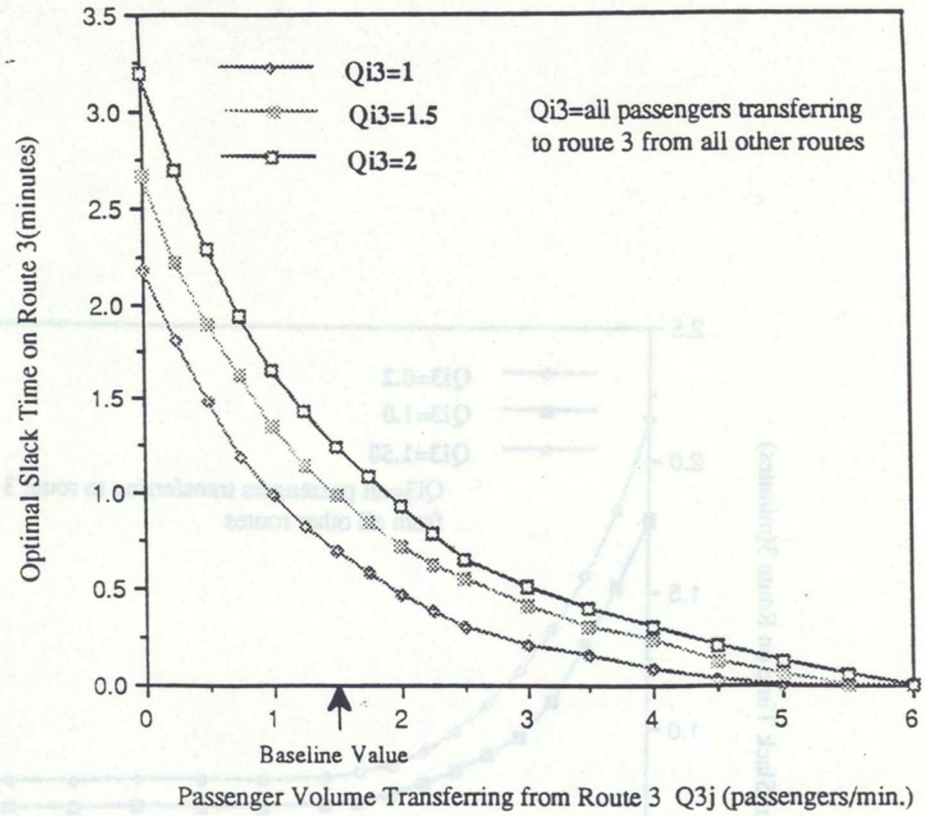

Figure 10a : Effect of the Transfer Passenger Volume on the Optimal Slack Time

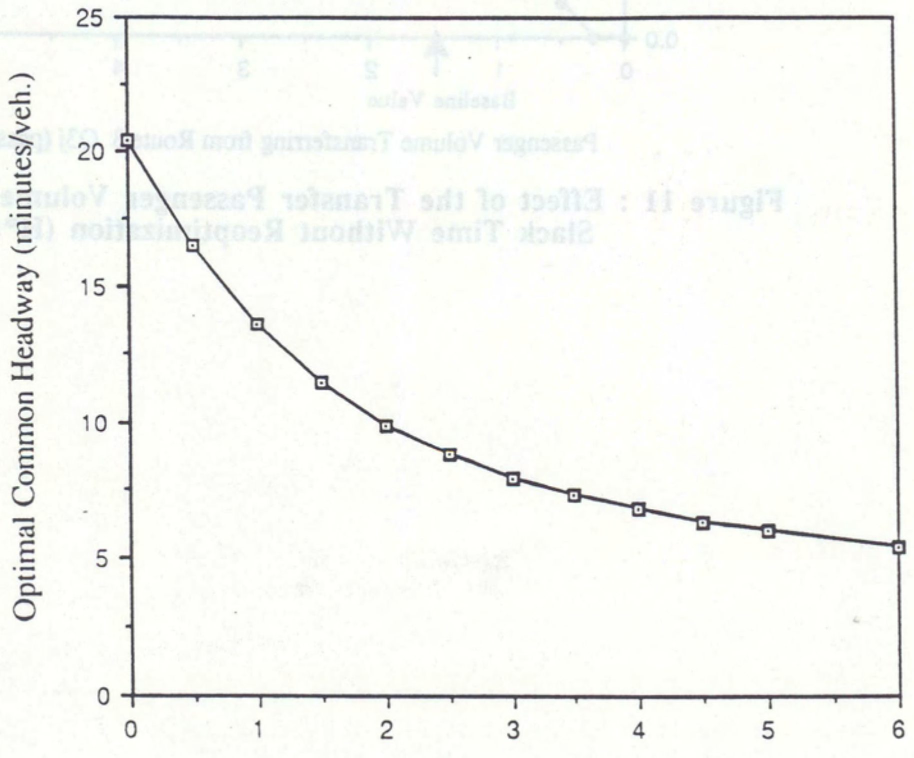

Figure 10b : Effect of the Transfer Passenger Volume on the Optimal Common Headway

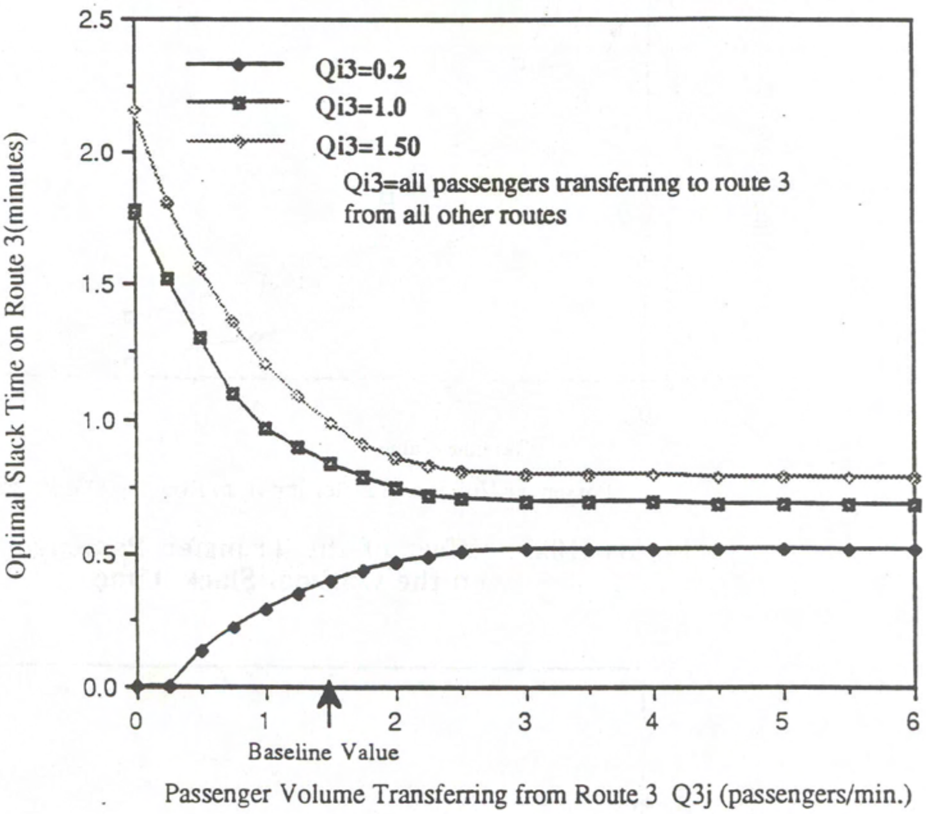

Figure 11 : Effect of the Transfer Passenger Volume on the Optimal Slack Time Without Reoptimization (H*=10.6 mins/veh.)

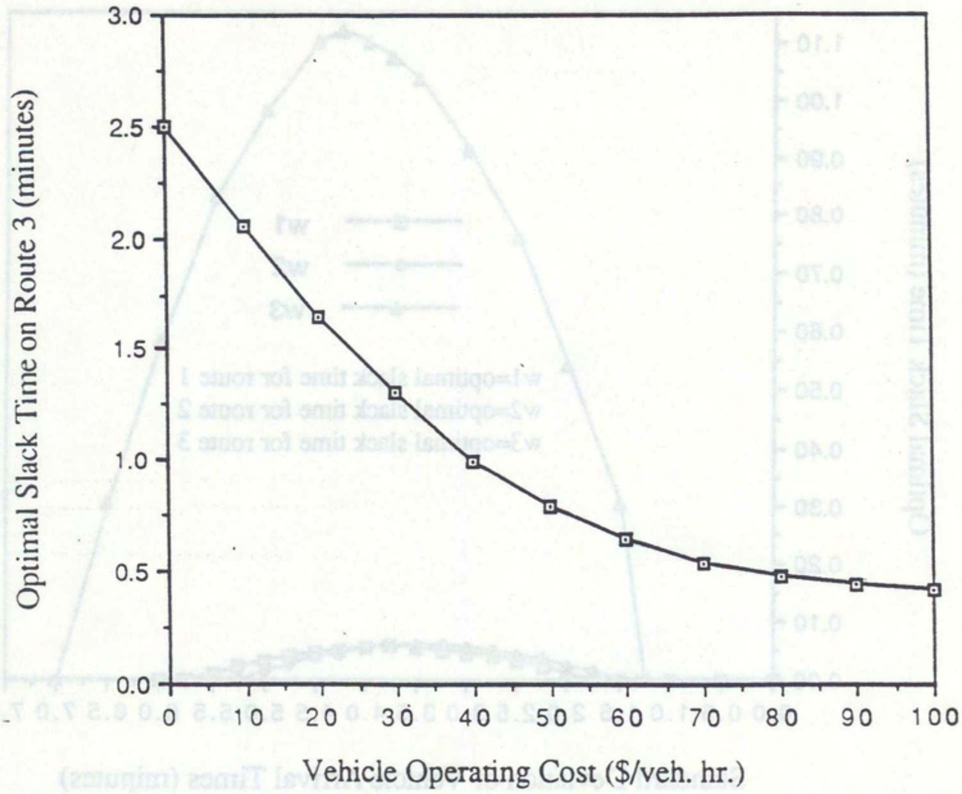

Figure 12 : Effect of the Vehicle Operating Cost on the Optimal Slack Time

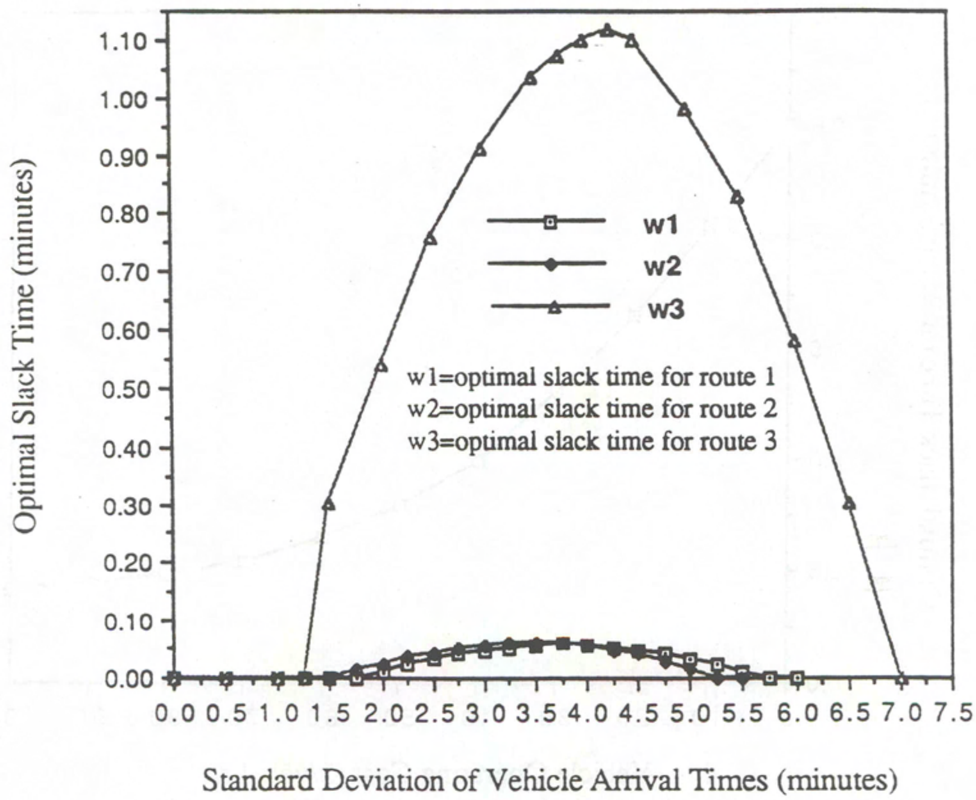

Figure 13 : Relations Between the Optimal Slack Time and the Standard Deviation of Vehicle Arrival Times

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