FINAL REPORT

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THE FLORIDA DEPARTMENT OF TRANSPORTATION
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on Project

"Effective and Efficient Deployment of Dynamic Message Signs to Display Travel Time Information"

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Transportation Research Center
The University of Florida

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METRIC CONVERSION CHART

U.S. UNITS TO METRIC (SI) UNITS

LENGTH

SYMBOL	WHEN YOU KNOW	MULTIPLY BY	TO FIND	SYMBOL
in	inches	25.4	millimeters	mm
ft	feet	0.305	meters	m
yd	yards	0.914	meters	m
mi	miles	1.61	kilometers	km

METRIC (SI) UNITS TO U.S. UNITS

LENGTH

SYMBOL	WHEN YOU KNOW	MULTIPLY BY	TO FIND	SYMBOL
mm	millimeters	0.039	inches	in
m	meters	3.28	feet	ft
m	meters	1.09	yards	yd
km	kilometers	0.621	miles	mi

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16. Abstract

Many transportation agencies have recommended travel times as the default messages on dynamic message signs (DMSs). This report presents analytical models to assist traffic engineers with effective operations of existing DMSs and deployment of new DMSs to display travel times on major freeways and arterials. More specifically, given the locations of existing DMSs, one of the proposed models determines the destinations for which to display travel times to maximize the benefit to motorists. Three extended models further improve the benefit by relocating DMSs, adding new ones to the current system or deploying a DMS system to a new area. The case study on the I-95/595 corridor in FDOT District 4 demonstrates the applicability of the proposed models. These models can be used in conjunction with other general guidelines provided by the federal and state agencies for optimal deployment and operations of DMS to display travel times.

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EXECUTIVE SUMMARY

In the literature, there are several studies describing results from surveys designed to assess the public's perception of dynamic message signs[1] (DMSs) and their impacts on motorists' route choice, e.g., after traffic incidents. Analytically, there are also articles that propose and (approximately) solve optimization models for deploying or locating DMSs in order to maximize their benefits, e.g., in the management of traffic after an incident. On the other hand, DMSs are often used to display travel times to various destinations during normal operation when there is no traffic incident, conditions, or events requiring any warning to be issued to motorists. According to our survey of the literature, there is no study that evaluates the benefit associated with the practice of displaying travel times on DMSs or offers an analytical model for determining optimal destinations for which to display the travel times on DMSs.

In this report, we use the (relative) variability in travel time as a measure for the benefit associated with displaying a travel time on a DMS. We surmise that this variability causes anxiety and stress when the motorists have to arrive at their destinations by certain times or as intended. In particular, we hypothesize that displaying the travel time to a destination with more variability offers more benefit (i.e., providing the travel time reduces more stress or anxiety) to motorists than displaying the time to one with less variability. We calculate the average and the standard deviation of travel time on each highway segment and use the ratio of the standard deviation over the average as a measure of variability. Mathematically, we use μ_a and σ_a to represent the average and the standard deviation of the travel time for highway segment labeled as "a." Thus, the variability of highway segment a is σ_a/μ_a. In our study, the benefit a motorist gained from knowing the travel time for highway segment a is exactly this ratio. (Note that σ_a/μ_a is a ratio of two numbers in units of time, i.e., in minutes. Thus, the ratio, i.e., our measure of benefit, has no unit, or is dimensionless.) If 100 motorists see the travel time, then the benefit associated with displaying the travel time for segment a is $100(\sigma_a/\mu_a)$.

Using the traffic information from STEWARD[2] (a data warehouse hosted by the Center for Multimodal Solutions for Congestion Mitigation at the University of Florida) and the

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1 Also known as changeable or variable message signs.

2 See http://cce-trc-cdwserv.ce.ufl.edu/steward/index.html for more information.

FSUTMS[3] statewide model, we calculated the variability of each of 60 highway segments along I-95 and I-595 in District 4. At approximately 5:30 PM on May 17, 2011, 26 DMSs displayed travel times to destinations such as Commercial Blvd., I-595, Golden Gates, Hillsboro Blvd., Turnpike, and I-75. However, six of these DMSs displayed travel times that are less than 5 minutes. These times do not form an interval with a width of at least two minutes, thus not meeting one of the FDOT's requirements. Based on our estimates of travel demands and the variabilities we calculated, the benefit from the remaining 20 DMSs on May 17 is 4,804.56. (For details, see Table 5.1 in Section 5.)

When the objective is to maximize the benefit, this report offers analytical models for (1) determining destinations to which to display travel times (or the display selection model), (2) locating additional DMSs (or the DMS location model), and (3) deploying a system of new DMSs (or the DMS deployment model). Using data from I-95 and I-595 as described above, the display selection model suggests that the maximum benefit from the 26 DMSs used on May 17 is 6,961.79. Compared to those on May17, the displays from the model increase the benefit by approximately 44%. In parts, this increase comes from displaying acceptable travel times (greater than 5 minutes) on the six DMSs excluded from the earlier benefit calculation.

Our survey of the literature indicates that most, if not all, factors useful in selecting travel time displays are not quantifiable. Because they rely on quantifiable measures of benefits, models in this report cannot consider these useful factors when selecting the displays. To alleviate this limitation, Tables 5.3 and 5.4 list top five destinations in term of their benefits to motorists for each DMS along I-95 and 595. These tables offer alternatives when the destination with the highest benefit may not be as advantageous when other factors are considered or is unsuitable for reasons not addressed in our models. In fact, most of the destinations displayed on May 17 are on these top-five lists. Although not yielding the highest benefit based on the variability, the displays on May 17 may be efficient when other factors are taken into account.

When the DMS location model is used to locate ten new (or additional) DMSs, the model suggests that eight new ones should be located between Sheridan Rd. and Highway 824 on I-95, four in each direction, and two should be located on I-595 between Pine island and Highway 411 only in the west-bound direction. (For details, see Section 7.) The benefit from the system of 72

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[3] Florida Standard Urban Transportation Model Structure.

DMSs (62 existing and 10 new) is 31,953.07. When compared the benefit (25,047.92) from the existing 62 DMSs, adding 10 new DMSs increases the benefit by approximately 28%. However, DMSs also provide benefits other than reducing travel anxiety, which are not captured in this report.

In Section 8, we use the robust DMS deployment model to deploy 30 DMSs along I-95 and I-595 assuming that there is no existing DMS initially. The model is developed to proactively address the uncertainty associated with travel demand in deploying DMS. When compared with a nominal plan obtained against the average OD demand, the robust plan is able to increase the benefit associated with the worst-case scenarios by 4.23%. The case study demonstrates that the robust model is able to improve the worst-case benefit without comprising much the average benefit of DMSs.

To summarize, this report proposes a method for measuring the benefit associated with displaying travel times on DMSs and offers analytical models useful for determining optimal locations for which to display travel times, locating additional DMSs, and deploying a new system of DMSs. It should be cautioned that the results stated above are based on our estimated travel demands and data from STEWARD and the FSUTMS statewide model. In general, decisions to change display messages, relocating existing, and adding new DMSs should also involve objectives other than maximizing the benefit as measured above (or, equivalently, reducing travel anxiety) and be based on data that have been verified and approved by responsible agencies.

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1 Introduction

Many states and cities currently post travel time messages via dynamic message signs (DMSs), e.g., to allow drivers to make informed travel decisions while en route to their destinations. In our state, Florida Department of Transportation (FDOT) District 4 recently deployed such a system on I-95 and I-595. However, the benefits or effectiveness of a DMS system depend on factors such as the accuracy of travel time estimate or forecast, the driving public's knowledge of the prevailing traffic conditions and their ability to infer travel times from these conditions. Among cites in the U.S., the level of user satisfaction with existing deployments of DMS varies significantly. A systematic approach to planning, deploying, and operating a DMS system is key to its success and maintaining a high level of user satisfaction. Although Florida currently has a guideline for displaying travel time messages, analytical decision-making models can improve the effectiveness and usefulness of displayed messages.

This report presents such models to assist traffic engineers with effective operations of existing DMSs and deployment of future DMSs to display travel times on major freeways and arterials. More specifically, given the locations of existing DMSs, the developed models will determine (1) the destinations for which to display travel times and (2) the locations of new DMS deployments along major freeways and arterials, at ramps and freeway interchanges in order to maximize the system benefits and user satisfaction.

The results from this report should assist in establishing the guidelines for optimal use of DMSs to display travel times. Moreover, providing useful real-time information on DMSs should lead to a reduction in system-wide travel time, traffic congestion, and uncertainty and anxiety associated with vehicular travel. These three factors also help in promoting a safer travel environment.

The remainder of the report is organized as follows. Section 2 presents a literature review on displaying travel time messages on DMSs. Section 3 discusses how to quantify the benefit that a motorist receives from the displayed travel time information. Based on the proposed measure of benefit, Section 4 develops a model to select the destinations to which the travel times should be displayed on DMSs while Section 5 demonstrates the model in a case study on the I-95/595 corridor. Section 6 presents two extensions to the model, concerning optimal relocation or deployment of DMSs on a road network. The models are also demonstrated in the same I-95/595 network in Section 7. Section 8 presents and demonstrates a robust approach that proactively addresses the travel demand uncertainty in deploying DMSs. Finally, Section 9 concludes the report.

2 Literature Survey

DMSs[4] are programmable electronic control devices that can display messages composed of alphanumeric characters, symbols, or both. They are capable of displaying short messages to drivers related to upcoming traffic conditions or in some jurisdictions, safety campaign related messages such as "Click It or Ticket". Because of their effectiveness in disseminating information to motorists, they have emerged as a key component of intelligent transportation system (ITS) implementations.

This section reviews the current usage of permanently affixed DMSs for travel time messages. The messages display the current estimated time to reach some specific destination ahead, often including the distance to that destination. Although the public may not be aware of this, the time is automatically estimated based on current conditions, typically from automatic vehicle identification reader or loop detector data. The literature review below primarily covers three aspects of a DMS system: usage guidelines, impacts of the displayed information, and analytical models and algorithms for DMS deployment and operations.

2.1 Guidelines

While DMSs are relatively new traffic control and information devices, state and federal agencies have been swift in providing guidelines and recommendations regarding DMS usage.

The 2009 Manual on Uniform Traffic Control Devices (MUTCD) contains a chapter of guidance regarding DMS (FHWA, 2009). It recommends the signs to be used for supporting the following applications:

- Incident management and route diversion
- Warning of adverse weather conditions
- Special event applications associated with traffic control or conditions
- Control at crossing situations
- Lane, ramp, and roadway control
- Priced or other types of managed lanes
- Travel times
- Warning situations

[4] In the literature, dynamic message signs are also referred to as *changeable message signs* (CMS) or *variable message signs* (VMS).

- Traffic regulations
- Speed control
- Destination guidance

MUTCD also specifies the size of the characters, the number of characters per line, the number of lines per sign, and the length of time to display the messages.

In a policy memorandum (FHWA, 2004), FHWA strongly suggests that dynamic, credible travel time messages should be displayed on DMS. It states that successful practices from Atlanta, GA, have shown that these messages can benefit both local commuters and unfamiliar travelers (by the inclusion of the distance with the travel time).

FDOT views DMSs as a way to provide traveler information en route to motorists to improve safety, help motorists make more educated decisions regarding route choice, reduce trip time, fuel consumption and emissions, and improve the public's perception of the usefulness of DMS (FDOT, 2008). The agency adopted a policy effective on September 17, 2009, that requires travel time display as the default display on DMSs (FDOT, 2009). The agency further prioritizes other messages in the following order:

- Conditions that require motorists to take action or alter their driving
- Traffic incidents, hazardous and/or uncommon road conditions, work zone activities, and severe weather conditions
- Florida Department of Law Enforcement Alerts such as America's Missing: Broadcast Emergency Response (AMBER) Alerts, Law Enforcement Officer (LEO) Alerts and Silver Alerts
- Traveler information related to special events, emergencies, and incidents impacting mobility and safety
- In the absence of accurate travel time information, at locations where travel time information would not be useful, or when not being preempted with other messages listed above, the default display shall be a blank sign

In summary, both the federal and state agencies recognize the benefits of displaying travel time messages on DMS and thus recommend doing so if the information is available.

2.2 Impacts of Displaying Travel Time Information

This section summarizes results from surveys regarding the perception, diversion decisions, and values travelers/motorists place on travel times and their reliabilities.

2.2.1 Overall Perception

The public's perception plays an important role in determining the effectiveness of a DMS system. Several studies have been conducted to gauge the public's perception. Table 2.1 summarizes the findings of these studies.

Table 2.1: Percentage of travelers who found DMS to be useful

Reference	Location	% Found Useful
Chatterjee et al. (2002)	London, UK	13%, 27%, and 40% find DMS to be "very", "quite" and "occasionally useful"
Peng et al. (2004)	Milwaukee, WI	75% find DMS either "useful" or "somewhat useful"
Richards and McDonald (2007)	Southampton, UK	"high" (% not specified)
Tay and deBarros (2008)	Alberta, Canada	82.5% saw and remembered message

In general, the public have found DMSs to be useful. Additionally, Tay and deBarros (2008) and Edwards and Young (2009) found that accuracy, timeliness, and credibility are important regarding the information displayed. Furthermore, studies have shown that a blank DMS can be misinterpreted and implies under-utilization of resources (e.g., Wardman et al., 1997; FHWA, 2004; Richards and McDonald, 2007; and Tay and deBarros, 2008).

2.2.2 Diversion

One of the main goals of a DMS is to inform drivers of upcoming traffic conditions. If the situation warrants (e.g., an accident, congestion, or roadwork ahead) and an alternative exists, the driver can choose to alter his route or stay on the current route. Numerous studies have shown that the diversion decision is influenced by the content of the displayed message, which can be typically classified into four categories: qualitative information, qualitative guidance, quantitative information, and quantitative guidance.

Table 2.2 contains a brief summary of surveyed diversion rates in response to the displayed messages on DMS.

Table 2.2: Diversion rates from various surveys

Reference	Location	Diversion %
Khattak et al. (1991)	Chicago, IL	42% diverted over a duration of 6 months in response to en-route delays
FHWA (1996)	Long Island, NY	5-10% of the time when passive information is displayed; 10-20% when specific recommendations are displayed
Peng et al. (2004)	Milwaukee, WI	66% diverted at least once a month based on the DMS information
Richards and McDonald (2007)	Southhampton, UK	59% of commuters and 70% of non-commuters would change routes in response to a DMS message warning of delay
Foo et al. (2008)	Toronto, Canada Express to collector	18% before a DMS message changes; 24.5% in the first 4 minutes after the change and 20.6% ten minutes after the change
	Toronto, Canada Collector to express	32.7% before message change; 37% in the first 4 minutes and 32.8% ten minutes after the change

The above table shows that the public pay attention and are likely to respond to the displayed messages. However, the diversion rates in the table should be interpreted with caution. They were mostly obtained from stated-preference (SP) surveys, in which people are commonly known to overstate their diversion propensity.

2.2.3 Predictability of Travel Time

When placed at key decision points where drivers can decide to take alternative routes, the travel time information provided by DMSs can be critical to drivers' route choices. However, if there is little diversion opportunity, the provided travel time information will not significantly influence route choices. On the other hand, the primary value of displayed travel times on DMSs is to reduce the negative effects of travel time uncertainty.

Results from travel diaries in the Puget Sound area show that 66% of people use traffic information as anticipation of traffic congestion, and only 6.5% use the information to minimize travel time (Tsirimpa et al., 2007). This empirical result reveals that drivers still feel the displayed travel time on a DMS to be of value even when no diversion opportunity exists. The information may reduce the anxiety related to the uncertainties associated with traveling. For example, the displayed travel time allows them to predict their arrival time and make appropriate arrangements to mitigate the negative effects of potential late arrivals. However, no study has quantified such benefit because it is very difficult to be singled out and cannot be captured by conventional system performance measures.

Below we review three studies that measure the value of travel time reliability, a concept relevant to the travel time predictability.

Lam and Small (2001) analyzed the results of a revealed preference (RP) survey of drivers on a section of SR91 in Orange County, CA. They developed three models with different considerations and used the standard deviation of travel time as a measure of travel time reliability. The first model estimates the value of travel time (VOT) and the value of reliability (VOR) to be $16.37 per hour and $22.72 per hour, respectively. For the second model, the estimated VOT is $19.22 per hour. On the other hand, the second model separates the estimated VOR into two categories, $11.90 per hour for men and $28.72 per hour for women. Similar to the second model, the estimates from the third model are $22.87 per hour for VOT, $15.12 per hour for men's VOR, and $31.91 per hour for women's VOR.

Small et al. (2005) combined data from two RP surveys and a SP survey of the drivers on SR91 and estimated the distributions of VOT and VOR. For the two RP surveys, the median VOT and VOR are $21.46 and $19.56 per hour, respectively. The medians from the SP survey are $11.92 for VOT and $5.40 for VOR. Brownstone and Small (2005) compiled and analyzed several datasets from the RP and SP surveys conducted along SR91 and I-15 express toll lanes in California. The results from these two corridors yield VOT's between $20 and $30 per hour and VOR's between $12 and $15 for men and between $30 and $32 for women.

These studies have found that drivers, particularly women, value the travel time reliability in a comparable level as they value the travel time itself. Although the travel time predictability and reliability are two different concepts, these findings imply that drivers may place a significant value on the travel time messages displayed on DMSs as well.

2.3 Models for DMS Deployment and Operations

Many have realized that a systematic approach to planning, deploying, and operating a DMS system is key to its success and maintaining a high level of user satisfaction. The 2009 MUTCD (FHWA, 2009) provides specific guidance on the locations of DMSs as follows:

- DMS should be located sufficiently upstream of known bottlenecks and high crash locations to enable road users to select an alternative route or take other appropriate action in response to a recurring condition
- DMS should be located sufficiently upstream of major diversion decision points, such as interchanges, to provide adequate distance over which road users can change lanes to reach one destination or the other
- DMS should not be located within an interchange
- DMS should not be positioned at locations where the information load on drivers is already high or drivers frequently perform lane-changing maneuvers in response to static guide sign information, or because of merging or weaving conditions

The above guidance can be applied to identify potential sites to locate DMSs. Given a limited number of DMSs, analytical models are still needed to prioritize the potential sites and optimally locate the available signs.

All analytical models reported in the literature focus on locating DMSs for the benefits incurred in the incident scenarios. During our survey, we found no model that addresses how to select destinations for which to display travel times during a normal operation when there is no traffic incident. Below are summaries of several journal articles on DMS locations.

Abbas and McCoy (1999) considered maximizing the potential benefits realized by traffic diversion in response to the DMS display of an incident. When a DMS displays information about an incident downstream, drivers have the opportunity to divert to alternative routes. Traffic volume on the freeway after the diversion generally decreases, resulting in a reduction of delay and accidents. However, the volume on the alternative routes would increase, resulting in an increase of delay and accidents. Lastly, delay and accidents on the freeway downstream of the incident could either increase or decrease, depending on whether the traffic volume increases or decreases as a result of the diversion (e.g., drivers could re-enter the freeway after passing the incident location). Using these relationships, the authors defined the potential benefit of a DMS at a diversion point that includes the reduction in delay and accidents. The benefit is then maximized by optimally locating the DMS via a genetic algorithm. Fu et al. (2007) extended the above study by introducing a multi-period benefit model (considering time-of-day variation in travel demand distribution), incorporating a logit route choice model to determine the time-dependent division rate under incident conditions, and taking into account variations in incident characteristics such as rate, duration, and capacity reduction. The objective is to maximize the travel time saving for all time periods and links. The travel time saving for a specific link during a specific time period is defined as the number of incidents in that link multiplied by the difference between the total vehicle delay caused by an incident without a DMS being present and the total vehicle delay caused by an incident with a DMS being present. A path-based user equilibrium model is used to replicate the network traffic.

Chiu et al. (2001) investigated how to locate DMSs in order to maximize the long-run expected benefits of DMS when incidents occur in a stochastic manner. There are two levels, upper and lower, of decision-making in this problem. The upper level identifies a strategy for optimally locating DMSs (given the associated benefits and costs) and the lower level predicts how the traffic responds to the DMSs. The problem is formulated as a bi-level, stochastic integer programming model. At the upper level, the objective is to minimize the expected total travel time subject to the number of signs installed. At the lower level, DYNASMART-P is incorporated to simulate the flow of the traffic through the network. A Tabu search algorithm is used to solve the bi-level problem. Chiu and Huynh (2007) extended this model with the addition of simulating the effects of pre-trip information via advanced traveler information systems.

2.4 Summary

While a modest amount of research has been performed on assessing the benefits and efficient operations of DMSs, little research has been done on quantifying the benefits associated with displaying the travel time, particularly during normal operations when there is no traffic incident. To our knowledge, there is no analytical model for locating DMSs with the objective of maximizing the benefits associated with displaying travel time messages. Research to date has focused on placing DMSs to maximize their benefit relieving congestion after a traffic incident. Moreover, none has investigated how to determine the destinations for which to display travel times on DMS.

Survey results show that the public perceives DMSs to be useful. Generally, drivers are seeing and understanding the signs. However, a blank DMS is often misinterpreted as being broken or no traffic incident ahead. Timeliness, accuracy, and credibility are frequently identified by motorists as important factors in influencing their reception of the messages. Furthermore, motorists have expressed a desire for more signs and more frequent message updates on the existing signs.

The rate at which drivers divert to alternative routes varies with the content of the information displayed on the DMS. Drivers tend to divert more often when a quantitative message is displayed than when qualitative information is provided. Furthermore, guidance messages tend to induce higher diversion than just the information on traffic conditions. However, the content of the message is not the only factor contributing to diversions. Studies have shown that driver characteristics also play a role in the diversion decision. The driver's gender, age, trip length, and familiarity with the road network are some of the common factors. Male and younger drivers, familiar with the network and on a long journey, generally lead to higher diversions.

3 Measuring Benefits from Travel Time Displays

As discussed in Section 2, our survey reveals that little or no research in the literature addresses the benefits of displaying travel time message on DMSs, particularly when there is little diversion opportunity. In this situation, the primary value of the displayed travel times is to reduce the negative effects of travel time uncertainty or unpredictability. To fill this gap in the literature, this section therefore discusses how we quantify the benefit that a motorist receives from knowing the travel times to locations along the route to his or her destination without diverting. Later, we use such a benefit in selecting destinations to which the travel times should be displayed and deciding the locations of new DMSs or those to be relocated.

Herein, we assume that motorists travel because they are required to be at a certain place by a certain time to conduct a certain activity. This is particularly true during the peak travel period in the morning when most motorists have to be at work by 8 or 9 AM. It is our premise that knowing accurate travel times in such a situation reduces the anxiety or stress associated with the desire to arrive at the final destination on time or as intended. Under this premise, we make the following assumptions:

1) When the travel time to a location is predictable or can be estimated accurately, e.g., from a posted speed limit, displaying such time on a DMS offers little or no additional information to motorists because they can estimate the time themselves. Therefore, displaying travel times that are predictable or have little variability offers little or no benefit to motorists

2) While other methods and/or formulas exist, we use the ratio of the standard deviation over the average as our measure of variability in travel time. The ratio is a measure of variability relative to the average travel time and a large ratio indicates a high variability Mathematically, it is common to let μ and σ denote the average (or mean) and standard deviation of values, such as travel times, that can vary in an unpredictable manner. In the transportation literatures, variances (or σ^2) and standard deviations are often used interchangeably. Many choose one over the other mostly for convenience or conciseness

3) We assume that the travel times on different links or highway segments are independent, i.e., they do not influence each other. Although not realistic, this is a common and acceptable assumption in the literature because it simplifies the calculation of the variability of a path

travel time. Under this assumption, the average and variance of a path travel time are simply the sum of the averages and variances of travel times of the highway segments on the path

4) When reaching destination A requiring passing through point B, travelers to destination A also benefit from the travel time information to point B. In our model, the benefit to a motorist traveling to A is proportional to the ratio of the average travel time to B over the time to A

To develop mathematical expressions for our models, we represent a collection of highways for DMS deployment as a network consisting of nodes and links. Nodes in the network are denoted as i or j and represent intersections, highway accesses, or other prominent landmarks. A link denotes a road or highway segment and is typically represented as a pair of nodes, e.g., (i, j), where i denotes the start of the segment and j does the end. For convenience, we also refer to a link (or arc) in the network simply as a instead of (i, j). The mean and variance of the travel time for link a are μ_a and σ_a^2, respectively. We represent a path from node s to node t in a network as a sequence of connected links. For the network in Figure 3.1, the sequence (1, 3), (3, 4), (4, 5) forms a path from 1 to 5. In general, we let P_{st} denote the sequence of links that forms a path from s to t.

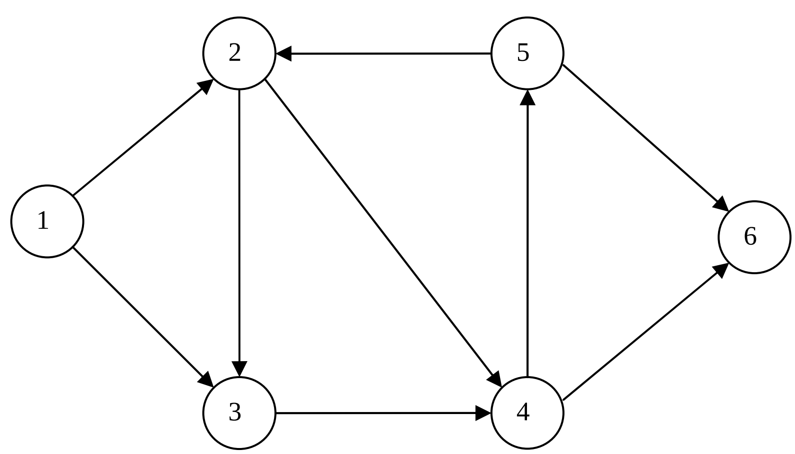

Figure 3.1: An example of a network

Given the independent travel time assumption (Assumption 3) above, the variability of path P_{st} is given by the following expression:

$$v_{st} = \frac{\sqrt{\sum_{a \in P_{st}} \sigma_a^2}}{\sum_{a \in P_{st}} \mu_a}.$$

To be more realistic, the benefit associated with knowing the travel time from s to t should be proportional to or a function of v_{st}. Moreover, this proportion or form of the benefit function should be different for different motorists as well. However, estimating the proportion and determining the benefit function for each motorist is beyond the scope of this project. In this report, we assume the benefit of knowing the travel time from s to t is simply the variability associated with the path's travel time or v_{st}.

To calculate the (partial) benefit associated with partial travel time information for the situation described in Assumption 4 above, consider a path consisting of the sequence: (1, 2), (2, 3), (3, 4), (4, 5), (5, 6) in the network of Figure 3.1 and there is a DMS locating on link (2, 3) that displays the travel time to node 5. A motorist traveling from 1 to 6 along this path would partially benefit from knowing the travel time for the path segment from 2 to 5, i.e., along the segments (2, 3), (3, 4), and (4, 5). In this report, the partial benefit is based on the average travel time of this path segment as a percentage of the segment starting at node 2 (the "designated" location of the DMS on link (2, 3)) to node 6, the destination of the original path. In particular, the benefit to a motorist traveling from 1 to 6 in our example is αv_{26}, where v_{26} is the variability of the path segment from 2 to 6 and α is the ratio of the average travel times of the two path segments previously described, i.e.,

$$\alpha = \frac{\mu_{23} + \mu_{34} + \mu_{35}}{\mu_{23} + \mu_{34} + \mu_{35} + \mu_{56}}.$$

On the other hand, we assume that a motorist traveling from 1 to 4 using the path (1, 2), (2, 3), (3, 4) receives no benefit from knowing the travel time from 2 to 5. We surmise that a motorist cannot accurately estimate the travel time from 2 to 4 from that between 2 and 5, especially when link travel times have high variabilities.

In general, let $P_{st} = (s, i_1), (i_1, i_2), \cdots, (i_n, t)$ and assume that there is a DMS on link (i_k, i_{k+1}) displaying the travel time to an intermediate node j, where $k \in \{1, \cdots, n-1\}$ and

$i_{k+1} \leq j$. For a motorist traveling from s to t, the benefit associated with knowing the travel time to node j provided by the DMS located on link (i_k, i_{k+1}) is

$$B_{i_k,i_{k+1}}^{j,(s,t)} = \alpha v_{i_k,j}$$

where $v_{i_k,j}$ is the variability of the path segment from i_k to j and α is the following ratio of average travel times, i.e.,

$$\alpha = \frac{\sum_{a \in P_{i_k,j}} \mu_a}{\sum_{a \in P_{i_k,t}} \mu_a}.$$

As defined above, $\alpha \leq 1$.

The benefit as calculated above applies to each motorist who notices the travel time displayed on a DMS. In practice, only a fraction of motorists pays attention to DMSs, a fact well documented in the literature. (See Table 2.1 in Section 2.) Thus, if d_{st} is the number of motorists traveling from s to t, then only $f d_{st}$ would benefit from the display when f is the fraction of those paying attention to DMSs while driving. In addition, there is also a probability that a motorist may unintentionally miss seeing the display. Let p denote this probability. Then, among the $f d_{st}$ motorists, $(1 - p) f d_{st}$ is expected to see and benefit from the display. In the case where there are two DMSs located on the same link and displaying a travel time to the same location, the number of motorists expected to see and benefit from the display increases to $(1 - p^2) f d_{st}$. (Observe that $(1 - p^2) > (1 - p)$ when $0 < p < 1$.) The expression is similar for the case with more than two DMSs.

4 Selecting Destinations from Travel Time Displays

This section discusses the problem or model for determining the destinations to which the travel times should be displayed on DMSs. We assume that the location of every DMS is known and propose an integer programming problem for determining the destinations whose travel times yield the most benefit to motorists as measured by the variability discussed in the previous section.

We assume that there are Q DMSs already deployed on the network and they are numbered from 1 to Q. For each link a, the set $\Omega_a \subseteq \{1, 2, \cdots, Q\}$ contains the indices of DMSs located on the link. For those links with no DMS, $\Omega_a = \emptyset$. Below, the decision variable $y_q^j = 1$ if DMS q displays the travel time to node j. Otherwise, $y_q^j = 0$. To be more concise, we let w denote an OD pair instead of (s, t). (Henceforth, we use w instead of (s, t) to denote an OD pair when it is clear and more concise to do so.) Then, the problem for determining the values of y_q^j that maximize the benefit or, more concisely, the display selection (DS) problem can be formulated as follows:

$$
\begin{aligned}
DS: \quad \max \quad & \sum_a \sum_j \sum_w \left(1 - p^{z_a^j}\right) f_w d_w B_a^{j,w} \\
s.t. \quad & z_a^j = \sum\nolimits_{q \in \Omega_a} y_q^j, \forall a, j, \\
& \sum\nolimits_j y_q^j \leq 1, \forall q = 1, \cdots, Q, \\
& y_q^j \in \{0,1\}, \forall j; q = 1, \cdots, Q.
\end{aligned}
$$

The expression in the objective calculates the benefit from the travel times displayed on Q DMSs. As before, $B_a^{j,w}$ represents the benefit that a motorist for OD pair w receives from knowing the travel time to node j from the DMS located on link a, d_w is the number of motorists who travel between OD pair w, f_w denotes the fraction of those who pay attention to DMSs, and p represents the probability that a motorist will not see a DMS. For the constraints, the first set calculates the value of the variable z_a^j, the number of DMSs on link a that display the travel time to node j. The second set ensures that at most one travel time is displayed on a DMS. (When the equipment permits the possibility, it is relatively simple to allow for two or more travel times to be displayed on a DMS.) The last constraint forces y_q^k to be binary.

As presented above, the function describing the objective of the DS problem is nonlinear. This makes the problem more difficult to solve. By introducing auxiliary variables, we can equivalently replace the objective of the DS problem with a linear function. Let $h_a^{j,n} = 1$ if $z_a^j = n$, where $n = 1, 2, \cdots, N$ and N is the maximum number of DMSs on a given link. Then, the above objective function can be equivalently written as follows:

$$\max \sum_a \sum_j \sum_w \left(\sum_{n=1}^{N} (1 - p^n) h_a^{j,n} \right) f_w d_w B_a^{j,w}.$$

Similarly, the first set of constraints can be replaced by the following set of equations:

$$\sum_{n=1}^{N} n h_a^{j,n} = \sum_{q \in \Omega_a} y_q^j, \quad \forall a, j, \qquad (4.1)$$

$$\sum_{n=1}^{N} h_a^{j,n} \leq 1, \qquad \forall a, j, \qquad (4.2)$$

$$h_a^{j,n} \in \{0,1\}, \qquad \forall a, j, n. \qquad (4.3)$$

As before, equation (4.1) calculates the number of DMSs on link a that display the travel time to node j. Equation (4.2) ensures that at most one $h_a^{j,n}$ can have a value "1" and (4.3) requires $h_a^{j,n}$ to be binary. Instead of solving the DS problem as stated earlier, we solve, in the next section, the linear version of the DS problem instead, i.e., one in which the objective function and the first set of constraints are replaced by the above function and equations (4.1) – (4.3).

In summary, the full model for the display selection problem is as follows:

$$DS: \quad \max \quad \sum_a \sum_j \sum_w \left(\sum_{n=1}^{N} (1 - p^n) h_a^{j,n} \right) f_w d_w B_a^{j,w}$$

$$s.t \qquad \sum_{n=1}^{N} n h_a^{j,n} = \sum_{q \in \Omega_a} y_q^j, \forall a, j,$$

$$\sum_{n=1}^{N} h_a^{j,n} \leq 1, \forall a, j,$$

$$\sum_j y_q^j \leq 1, \forall q = 1, \cdots, Q,$$

$$h_a^{j,n} \in \{0,1\}, \forall a, j, n,$$

$$y_q^j \in \{0,1\}, \forall j; q = 1, \cdots, Q.$$

The above model is a binary integer linear programming model and can be solved effectively by algorithms such as the branch and bound and the cutting-plane algorithm. The solution to the problem will specify for each DMS an optimal destination for to display the travel time. The inputs for the model include locations of DMSs, the OD demands, i.e., d_w, benefits that a motorist receives from knowing the travel time to each downstream node from each existing DMS location, i.e., $B_a^{j,w}$, the fraction of motorists paying attention to DMSs while driving, i.e., f_w, and the probability that a motorist may unintentionally miss seeing the display, i.e., p.

5 Selecting Destinations for Travel Time Displays along I-95 & I-595

To illustrate, this section applies the display selection problem to the DMSs along two freeway segments, one from I-95 and the other from I-595. The I-95 segment begins at West Indiantown Road and ends south of US 395. For I-595, the segment begins at Highway 27 and ends just east Highway A1A. Along these two highway segments, there are 62 DMSs. We do not consider the DMSs on other highway segments in our study.

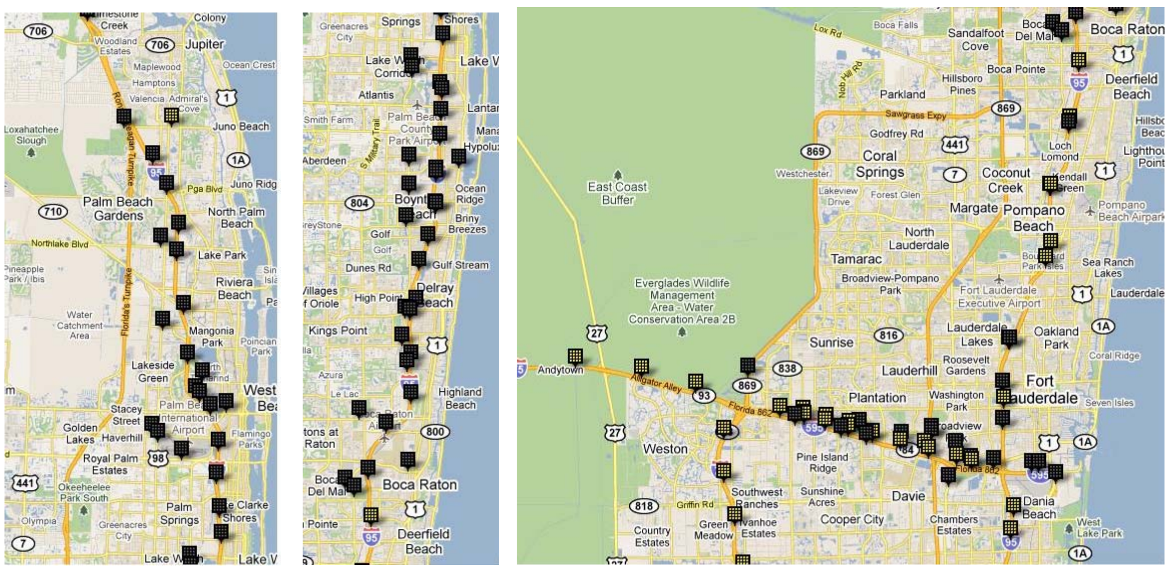

Figure 5.1: I-95 and I-595 segments for our study

Appendix A provides details regarding how we represent I-95 and I-595 segments in Figure 5.1 as a network of nodes and links, and constructed the data inputs for the model. To summarize, the network we created contains 60 nodes and 120 links. (See Figure A.2.). The data for traffic volumes and link travel times and their variability are from two sources: STEWARD[5] and the FSUTMS[6] statewide model. Specifically, the data are from the period between 3:30 and 6:30 PM (our afternoon peak period) and during the workdays between November 1, 2009 to December 31, 2009. Using the ramp data from the FSUTMS statement model, we also computed the number of vehicles that enter and leave each node. We refer these two numbers,

5 See http://cce-trc-cdwserv.ce.ufl.edu/steward/index.html for more information.
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entering and leaving, as the trip production and attraction for each node respectively. Using the traffic volume on each link and the node production and attraction data, we estimated the travel demand for each OD pair by solving the OD demand estimation problem in Appendix A with $(\omega_1, \omega_2, \omega_3) = (0, 1000, 1000)$ and $\beta = 15$. The latter requires that motorists travel 15 miles on average. An optimal solution from the OD demand estimation problem yields approximately 343,729 motorists traveling between approximately 300 OD pairs between 3:30 and 6:30 PM. Figure 5.2 displays the distributions of the number of motorists per each travel distance.

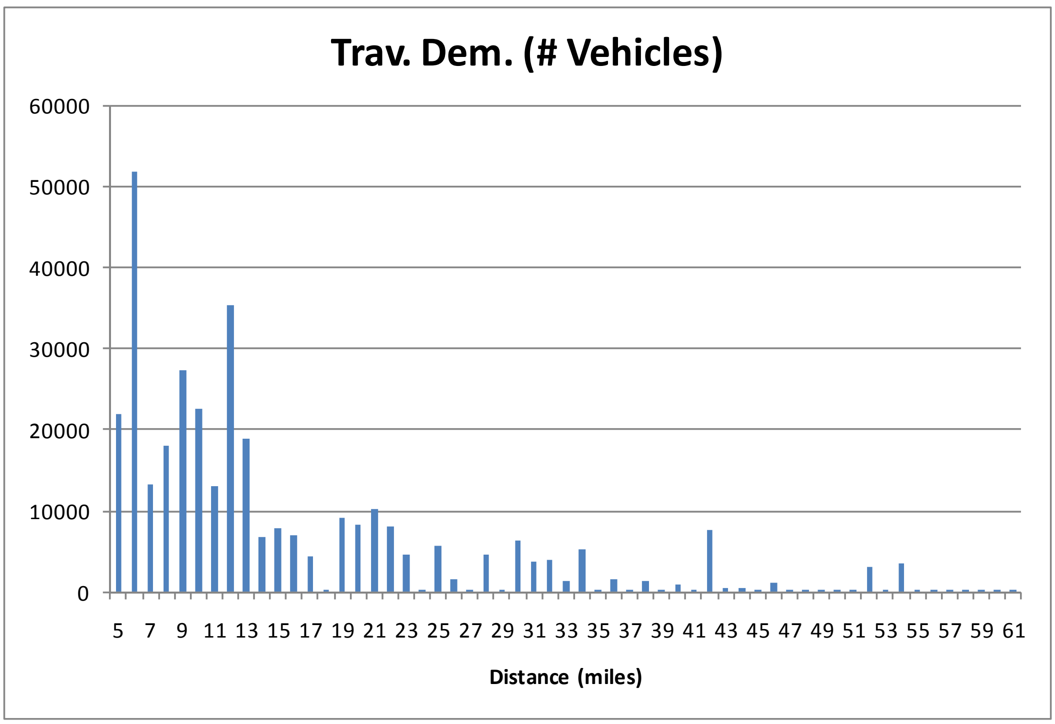

Figure 5.2: Travel demand estimates for each travel distance

For comparison and later discussion, we examine the display messages on the 62 DMSs along I-95 and I-595 shown in Figure 5.1. At approximate 5:30PM on May 17, 2011, 26 of these DMSs displayed messages concerning travel times. Below, Table 5.1 shows the travel time messages on the 26 DMSs. The DMS numbers in the table are not official. They are mainly for references. Columns labeled "Location Description" are the descriptions shown on www.smartsunguide.com. The remaining two columns display the destinations and their estimated travel times at the date and time given above. Some destination names listed in the table are not the same as the names actually displayed. Instead, they are our more descriptive node names listed in Table A.1 in Appendix A. Observe also that the county line is outside of our network in Figure 5.1.

Using the technique for evaluating benefits in Section 4 and the travel demands from the OD demand estimation problem in Appendix A, the benefit from the travel time displays in Tables 5.1 is 7128.58 units when we set $f = 1$ and $p = 0.5$. (In the benefit calculation, we excluded DMS 41 and 55 because the former is no longer used for travel time display and the destination, “County Line”, displayed on the latter is outside our network in Appendix A.) FDOT also requires an estimate for travel time to be in form of an interval with a width of at least two minutes. Six DMSs (49 – 51 and 56 – 58) do not satisfy this two-minute requirement and, when excluded from the calculations, the benefit reduces to 4804.56, the number listed in the caption of Table 5.1. On the other hand, replacing destinations such as Commercial Blvd., Hillsboro Blvd., and I-95 & I-595 interchange with 62nd St., Oakland Park Blvd., and Yamato Rd. as shown in Table 5.2 increases the benefit from 4804.56 to 5494.59 or, approximately, by 14%. (In Table 5.2, the new destinations are in bold face.) Like those in Table 5.1, new destinations in Table 5.2 attract at least 6,000 trips during our peak travel period between 3:30 and 6:30 PM, our criterion for determining destinations that are commonly known among motorists.

Observe that destinations are not necessarily located along on the highways on which the DMSs are located. In Table 5.1, DMS 45 located on I-595 displays the travel time to Griffin Road, a destination on I-95. In Table 5.2, DMS 44 and 45, both located on I-595, display travel times to Hollywood Blvd., another destination along I-95. However, the benefits associated with the latter DMSs and their displays suggest that, based on our estimates in Appendix A, a significant number of motorists on I-595 travel to destinations along I-95. The similar is also true with DMS 42 in Table 5.2.

Table 5.1: Display messages from www.smartsunguide.com at 5:30 PM on May 17, 2011 (Benefit = 4804.56)

	I-95 South		
DMS	Location	Display Destination	Time (min)
16	Before Hillsboro Blvd.	Commercial Blvd	10 – 12
17	Before Sample Rd	Commercial Blvd	7 – 9
18	Beyond Copans Rd	Commercial Blvd	5 – 7
19	Before Cypress Creek Rd	I-95&I-595 Interchange	8 – 10
23	Before Sheridan	Turnpike/167th St (Golden Glades)	10 – 12
24	Pembroke Rd./ Hwy 824	Turnpike/167th St (Golden Glades)	10 – 12
	I-95 North		
DMS	Location	Display Destination	Time (min)
38	Before Copans Rd	Hillsboro Blvd	5 – 7
39	At Cypress Creek Rd	Hillsboro Blvd	8 – 10
41	*At Broward Blvd**	*Commercial Blvd*	*5 – 7*
42	Before Griffin Rd	Commercial Blvd	9 – 11
43	Before Hallandale Beach Blvd	I-95&I-595 Interchange	6 – 8
	I-595 East		
DMS	Location	Display Destination	Time (min)
44	I-75 South before Hwy 27	I-75/I-595/Sawgrass(SR 869)	7 – 9
45	I-75 South Before I-595	Griffin Rd/10th St (FLL Airport)	12 – 14
46	Beyond SW 136th St	Turnpike/Hwy 441	7 – 9
47	Beyond Flamingo Rd	Turnpike/Hwy 441	6 – 8
48	Beyond Hiatus Rd	Turnpike/Hwy 441	5 – 7
49	*Beyond Nob Hill Rd*	*Turnpike/Hwy 441*	*< 5****
50	*Beyond Pine Island Rd*	*Turnpike/Hwy 441*	*< 5****
51	*Beyond University Dr*	*Turnpike/Hwy 441*	*< 5****
	I-595 West		
DMS	Location	Display Destination	Time (min)
55	*I-75 North before Hwy 27*	*County Line***	*29 – 32*
56	*Beyond Hiatus Rd*	*I-75/I-595/Sawgrass(SR 869)*	*< 5****
57	*Beyond Nob Hill Rd*	*I-75/I-595/Sawgrass(SR 869)*	*< 5****
58	*Beyond Pine Island Rd*	*I-75/I-595/Sawgrass(SR 869)*	*< 5****
59	Beyond Florida Turnpike	I-75/I-595/Sawgrass(SR 869)	7 – 9
60	Before SR 7/ 441	I-75/I-595/Sawgrass(SR 869)	8 – 10
61	Before Turnpike/SR 7	I-75/I-595/Sawgrass(SR 869)	11 – 13

*This DMS is no longer used for travel time display.
**County line is not in our model and its estimated time is a shown on Smartsunguide.com.
***Not an interval with a width of two or more minutes.

Table 5.2: Improved display messages (Benefit = 5494.59)

DMS	Location	Display Destination	Time (min)
	I-95 South		
16	Before Hillsboro Blvd.	**62nd St/Cypress Creek Rd**	8 – 10
17	Before Sample Rd	**62nd St/Cypress Creek Rd**	7 – 9
18	Beyond Copans Rd	**Oakland Park Blvd**	7 – 9
19	Before Cypress Creek Rd	**Oakland Park Blvd**	5 – 7
23	Before Sheridan	Turnpike/167th St (Golden Glades)	10 – 12
24	Pembroke Rd./ Hwy 824	Turnpike/167th St (Golden Glades)	10 – 12
	I-95 North		
DMS	Location	Display Destination	Time (min)
38	Before Copans Rd	**51st St/Yamato Rd**	11 – 13
39	At Cypress Creek Rd	**51st St/Yamato Rd**	13 – 15
41	*At Broward Blvd**	*Commercial Blvd*	*5 – 7*
42	Before Griffin Rd	**I-75/I-595/Sawgrass(SR 869)**	11 – 13
43	Before Hallandale Beach Blvd	**Sterling Rd.**	5 – 7
	I-595 East		
DMS	Location	Display Destination	Time (min)
44	I-75 South before Hwy 27	**Hollywood Blvd**	20 - 25
45	I-75 South Before I-595	**Hollywood Blvd**	18 – 20
46	Beyond SW 136th St	Turnpike/Hwy 441	7 – 9
47	Beyond Flamingo Rd	Turnpike/Hwy 441	6 – 8
48	Beyond Hiatus Rd	Turnpike/Hwy 441	5 – 7
49	*Beyond Nob Hill Rd*	*Turnpike/Hwy 441*	*< 5 ******
50	*Beyond Pine Island Rd*	*Turnpike/Hwy 441*	*< 5 ******
51	*Beyond University Dr*	*Turnpike/Hwy 441*	*< 5******
	I-595 West		
DMS	Location	Display Destination	Time (min)
55	*I-75 North before Hwy 27*	*County Line***	*29 – 32*
56	*Beyond Hiatus Rd*	*I-75/I-595/Sawgrass(SR 869)*	*< 5 ******
57	*Beyond Nob Hill Rd*	*I-75/I-595/Sawgrass(SR 869)*	*< 5 ******
58	*Beyond Pine Island Rd*	*I-75/I-595/Sawgrass(SR 869)*	*< 5 ******
59	Beyond Florida Turnpike	I-75/I-595/Sawgrass(SR 869)	7 – 9
60	Before SR 7/ 441	I-75/I-595/Sawgrass(SR 869)	8 – 10
61	Before Turnpike/SR 7	I-75/I-595/Sawgrass(SR 869)	11 – 13

*This DMS is no longer used for travel time display.
**County line is not in our model and its estimated time is a shown on Smartsunguide.com.
***Not an interval with a width of two or more minutes.

As an alternative to the above analysis, we solved the display selection problem described in Section 5 using the same 26 DMSs. In the problem, we limited the destinations to only important interchanges and those that attract 6000 or more trips according to our estimates in Appendix A. (We surmise that motorists are more familiar with and thus prefer these destinations.) Table 5.3 shows the optimal displays from the display selection problem with the two-minute requirement incorporated. The displays yield 6961.79 in benefit. When compared to the displays in Table 5.1, those in Table 5.3 provide a 44% increase in benefit. In parts, this increase comes from displaying travel times meeting the two-minute requirement on the six DMSs excluded from the earlier benefit calculation. The similar also holds when comparing Tables 5.2 and 5.3.

We caution that the above comparison is based on the OD demand pattern we obtained from solving the OD estimation problem in Appendix A. Although it represents our best estimate based on the information available, it may not reflect the actual condition on I-95/595. More importantly, in addition to the benefit described in Section 3, the selection of travel times to display on DMSs often involves other factors such as the motorists' familiarity with the various destinations and the availability of travel times. (For example, the current system at District 4 does not support displaying travel time from I-75 to I-95 when traveling along 1-595.) Unfortunately, our literature survey indicates that most, if not all, of these factors are not quantifiable. To address this issue manually, Tables 5.4 and 5.5 list the top five destinations in term of their benefits to motorists for each DMS along I-95 and 595. The tables offer alternatives when the destination with the highest benefit is unsuitable for reasons not addressed in our model.

Table 5.3: Optimal display messages with the two-minute requirement (Benefit = 6961.79)

DMS	Location	Display Destination	Time (min)
	I-95 South		
DMS	Location	Display Destination	Time (min)
16	Before Hillsboro Blvd.	62nd St/Cypress Creek Rd	8 – 10
17	Before Sample Rd	62nd St/Cypress Creek Rd	7 – 9
18	Beyond Copans Rd	Oakland Park Blvd	7 – 9
19	Before Cypress Creek Rd	Oakland Park Blvd	5 – 7
23	Before Sheridan	Turnpike/167th St (Golden Glades)	9 – 11
24	Pembroke Rd./ Hwy 824	Turnpike/167th St (Golden Glades)	5 – 7
	I-95 North		
DMS	Location	Display Destination	Time (min)
38	Before Copans Rd	51st St/Yamato Rd	11 – 13
39	At Cypress Creek Rd	51st St/Yamato Rd	13 – 15
41	At Broward Blvd	No longer in use	
42	Before Griffin Rd	I-75/I-595/Sawgrass(SR 869)	11 – 13
43	Before Hallandale Beach Blvd	Sterling Rd	5 – 7
	I-595 East		
DMS	Location	Display Destination	Time (min)
44	I-75 South before Hwy 27	Hollywood Blvd	20 – 24
45	I-75 South Before I-595	I-95&I-595 Interchange**	13 – 15
46	Beyond SW 136th St	I-95&I-595 Interchange	8 – 10
47	Beyond Flamingo Rd	I-95&I-595 Interchange	7 – 9
48	Beyond Hiatus Rd	I-95&I-595 Interchange	6 – 8
49	Beyond Nob Hill Rd	I-95&I-595 Interchange	5 – 7
50	Beyond Pine Island Rd	Griffin Rd/10th St (FLL Airport)	6 – 8
51	Beyond University Dr	Griffin Rd/10th St (FLL Airport)	5 – 7
	I-595 West		
DMS	Location	Display Destination	Time (min)
55	I-75 North before Hwy 27	County Line*	29 – 32
56	Beyond Hiatus Rd	Dark	
57	Beyond Nob Hill Rd	Dark	
58	Beyond Pine Island Rd	Dark	
59	Beyond Florida Turnpike	I-75/I-595/Sawgrass(SR 869)	7 – 9
60	Before SR 7/ 441	I-75/I-595/Sawgrass(SR 869)	8 – 10
61	Before Turnpike/SR 7	I-75/I-595/Sawgrass(SR 869)	11 – 13

*County line is not in our model and its estimated time is a shown on Smartsunguide.com.
** The current system does not provide the travel time from I-75 to I-95/I-959 interchange.

Table 5.4: Top five display destinations for each DMS on I-95

DMS	Location	Destination	Ben.	Destination	Ben.	Destination	Ben.	Destination	Ben.	Destination	Ben.
	I-95 South										
1	Before Pga	Okeechobee Blvd.	417	Palm Beach Lakes Blvd	388	45th St	340	Blue Heron Blvd	293	Northlake Blvd	242
2	Before Blue Heron	Palm Beach Lakes Blvd	646	45th St	547	Okeechobee Blvd.	525	Belvedere Rd	340	10th Ave	338
3	Before Palm Beach Lake	10th Ave	628	Forest Hill Blvd	583	Belvedere Rd	551	State Rd 80/ Southern Blvd	526	Glades Rd	506
4	Before Okeechobee	10th Ave	565	Forest Hill Blvd	514	Glades Rd	489	51st St/ Yamato Rd	471	Lantana Rd.	466
5	Before Belvedere	Atlantic Ave	715	10th Ave	701	15th Ave/ Woolbright Rd	687	Lantana Rd.	656	Boynton Beach Blvd	655
6	South of Southern	Lantana Rd.	735	10th Ave	705	Atlantic Ave	681	6th Ave	653	15th Ave/ Woolbright Rd	651
7	Before Forrest Hil	Atlantic Ave	816	Lantana Rd.	779	15th Ave/ Woolbright Rd	776	Boynton Beach Blvd	731	6th Ave	667
8	Before 6th	Atlantic Ave	880	15th Ave/ Woolbright Rd	825	Boynton Beach Blvd	763	Glades Rd	697	51st St/ Yamato Rd	675
9	Before Lantana	Atlantic Ave	840	15th Ave/ Woolbright Rd	778	Boynton Beach Blvd	707	Glades Rd	699	51st St/Yamato Rd	672
10	Before Gateway	Atlantic Ave	806	Glades Rd	727	51st St/ Yamato Rd	704	Congress Ave	656	Linton Blvd	634
11	Before Boynton Beach	Glades Rd	869	51st St/ Yamato Rd	822	Atlantic Ave	819	Congress Ave	755	Co Hwy 798/ Palmetto Park Rd	743
12	Before WoolBright	Glades Rd	923	51st St/ Yamato Rd	850	Co Hwy 798/ Palmetto Park Rd	834	Congress Ave	761	Hillsboro Blvd	738
13	Before Atlantic	Linton Blvd	875	Glades Rd	845	Co Hwy 798/ Palmetto Park Rd	776	51st St/ Yamato Rd	759	Hillsboro Blvd	694
14	Before Linton	Glades Rd	764	Co Hwy 798/ Palmetto Park Rd	717	51st St/ Yamato Rd	661	Hillsboro Blvd	649	10th St	460
15	Before Glades	Hillsboro Blvd	813	10th St	619	Sample Rd	543	Atlantic Rd	353	Copans Rd	351
16	Before Hillsboro	Atlantic Rd	231	62nd St/ Cypress Creek Rd	167	Oakland Park Blvd	102	Commercial Blvd	94		
17	Before Sample	Atlantic Rd	451	62nd St/ Cypress Creek Rd	443	Oakland Park Blvd	435	Commercial Blvd	398	Sunrise Blvd	378
18	Beyond Copans	Oakland Park Blvd	605	Commercial Blvd	589	Sunrise Blvd	383				
19	Before Cypress Creek	Oakland Park Blvd	753	Sunrise Blvd	633	Broward Blvd	308				
20	Beyond Oakland Park	Turnpike/ Hwy 441	433	Nob Hill Rd	406	Pine Island Rd	377	University Dr	351	Sheridan Rd	305
21	Beyond Sunrise	Pine Island Rd	1072	University Dr	986	Nob Hill Rd	544	Sheridan Rd	284	Sterling Rd	255
22	At Davie	Nob Hill Rd	842	I-75/I-595/ Sawgrass(SR 869)	581	125th Ave	559	Hiatus Rd/ 112th Ave	539	Indian Trace	13
23	Before Sheridan	Hallandale Beach Blvd	1340	Turnpike/167th St (Golden Glades)	812	Ives Dairy Blvd	760	Dolphin Expy/ I-395	5	26th Rd/ Rickenbacker Cswy	5
24	AT Pembroke Rd/ Hwy 824	Turnpike/167th St (Golden Glades)	1376	Dolphin Expy/ I-395	235	Airport Expy/I-195	221	26th Rd/ Rickenbacker Cswy	4	Northlake Blvd	242
	I-95 North										
25	Before Pga Blvd	Indiantown Rd	238	Donald Ross Rd	235						
26	Before Northland	Pga Blvd	361	Donald Ross Rd	318	Indiantown Rd	293				
27	Before Palm Beach Lake	Blue Heron Blvd	496	Northlake Blvd	446	Pga Blvd	296	Donald Ross Rd	201	Indiantown Rd	133
28	Before Forrest Hill	45th St	847	Palm Beach Lakes Blvd	792	Blue Heron Blvd	741	Okeechobee Blvd.	721	Northlake Blvd	604
29	Before 6th Ave	State Rd 80/Southern Blvd	669	Forest Hill Blvd	661	45th St	423	Palm Beach Lakes Blvd	401	Blue Heron Blvd	390
30	Before Lantana	45th St	440	Palm Beach Lakes Blvd	419	Blue Heron Blvd	404	Okeechobee Blvd.	395	Forest Hill Blvd	386
31	Before Gateway	6th Ave	625	Forest Hill Blvd	519	45th St	495	10th Ave	487	Palm Beach Lakes Blvd	476
32	Before Boynton Beach	6th Ave	754	Lantana Rd.	683	Forest Hill Blvd	605	10th Ave	578	45th St	535
33	Before Woolbright	Boynton Beach Blvd	1376	Gateway Blvd/ 22nd Ave	1178	Hypoluxo Rd.	1099	6th Ave	846	Lantana Rd.	783
34	Before Linton	Okeechobee Blvd.	417	Palm Beach Lakes Blvd	388	45th St	340	Blue Heron Blvd	293	Northlake Blvd	242
35	Before Congress	6th Ave	978	Boynton Beach Blvd	936	Lantana Rd.	924	Hypoluxo Rd.	887	Gateway Blvd/ 22nd Ave	882
36	Before Glades	Boynton Beach Blvd	683	6th Ave	637	Gateway Blvd/ 22nd Ave	621	Lantana Rd.	609	15th Ave/ Woolbright Rd	601
37	Before 10th St	51st St/ Yamato Rd	617	Glades Rd	474	Co Hwy 798/ Palmetto Park Rd	437	Congress Ave	435	Boynton Beach Blvd	312
38	Before Copans Rd	51st St/ Yamato Rd	392	Glades Rd	322	Co Hwy 798/ Palmetto Park Rd	304	10th St	260	Hillsboro Blvd	247
39	At Cypress Cree	Sample Rd	388	51st St/ Yamato Rd	360	Glades Rd	306	Co Hwy 798/ Palmetto Park Rd	293	10th St	273
40	Before Oakland Park	Copans Rd	467	Atlantic Rd	448	51st St/Yamato Rd	170	10th St	161	Glades Rd	149
41	At Broward Blvd	Commercial Blvd	885	62nd St/Cypress Creek Rd	626	Atlantic Rd	609	Copans Rd	586	51st St/ Yamato Rd	194
42	Before Griffin Rd	Hiatus Rd/ 112th Ave	262	Nob Hill Rd	257	125th Ave	256	I-75/I-595/ Sawgrass(SR 869)	250	Pine Island Rd	233
43	Before Hallandale Beach	Sterling Rd	607	I-95&I-595 Interchange	468	Griffin Rd/10th St (FLL Airport)	441	I-75/I-595/ Sawgrass(SR 869)	291	125th Ave	284

Table 5.5: Top five display destinations for each DMS on I-595

DMS	Location	Destination	Ben.	Destination	Ben.	Destination	Ben.	Destination	Ben.	Destination	Ben.
	I-595 East										
44	75 South before Hwy 27	Hollywood Blvd	197	Sheridan Rd	189	Sterling Rd	180	Griffin Rd/10th St (FLL Airport)	171	I-95&I-595 Interchange	162
45	75 South Before I-595	I-95&I-595 Interchange	195	Hollywood Blvd	194	Sheridan Rd	186	Turnpike/ Hwy 441	186	Sterling Rd	176
46	Beyond SW 136th St	I-95&I-595 Interchange	886	Turnpike/Hwy 441	836	University Dr	777	Commercial Blvd	682	Oakland Park Blvd	666
47	Beyond Flamigo Rd	I-95&I-595 Interchange	916	Turnpike/Hwy 441	854	Commercial Blvd	757	Oakland Park Blvd	737	Sunrise Blvd	717
48	Beyond Hiatus Rd	I-95&I-595 Interchange	813	Turnpike/Hwy 441	743	Commercial Blvd	709	Oakland Park Blvd	687	Sunrise Blvd	663
49	Beyond Nob Hill Rd	I-95&I-595 Interchange	979	Commercial Blvd	670	Oakland Park Blvd	648	Sunrise Blvd	620	Broward Blvd	568
50	Beyond Pine Island Rd	Griffin Rd/10th St (FLL Airport)	672	Commercial Blvd	600	Oakland Park Blvd	575	Sunrise Blvd	540	Broward Blvd	479
51	Beyond University Dr	Griffin Rd/10th St (FLL Airport)	918	Sheridan Rd	645	Sterling Rd	634	Commercial Blvd	575	Oakland Park Blvd	548
52	Beyond Davie Rd	Sheridan Rd	906	Sterling Rd	828	Commercial Blvd	531	Oakland Park Blvd	502	Hollywood Blvd	486
53	Beyond SR 84	Sheridan Rd	906	Sterling Rd	828	Commercial Blvd	531	Oakland Park Blvd	502	Hollywood Blvd	486
	I-595 West										
55	75 North before Hwy 27										
56	Beyond Hiatus Rd	Indian Trace	307	Hwy 27	268						
57	Beyond Nob Hill Rd	Indian Trace	324	Hwy 27	280						
58	Beyond Pine Island Rd	Indian Trace	381	Hwy 27	322						
59	Beyond Florida Turnpike	125th Ave	1229	Hiatus Rd/ 112th Ave	1226	I-75/I-595/ Sawgrass(SR 869)	1069	Indian Trace	468	Hwy 27	389
60	Before SR 7/ 441	Nob Hill Rd	1410	Hiatus Rd/ 112th Ave	1160	125th Ave	1132	I-75/I-595/ Sawgrass(SR 869)	1008	Indian Trace	329
61	Before Turnpike/ SR 7	Nob Hill Rd	1410	Hiatus Rd/ 112th Ave	1160	125th Ave	1132	I-75/I-595/ Sawgrass(SR 869)	1008	Indian Trace	329
62	Before I-95 South	University Dr	893	Hiatus Rd/112th Ave	461	Nob Hill Rd	448	125th Ave	411	Pine Island Rd	392

6 Deploying, Relocating, and Adding DMSs

The previous sections discuss the problem of how to select destinations for travel time displays in order to improve the benefit to motorists. This section offers another method for increasing the benefit. They involve relocating DMSs and adding new ones to the current system of DMSs. In our view, relocating decisions typically involves no more than 20% of the existing DMSs. On the other hand, it is also possible to relocate all DMSs or 100%. Doing so is extreme and akin to deploying DMSs to a new area. (In other words, the model in this section can be used for deploying DMSs in a new area.) Below, we formulate the DMS relocation problem and show afterward how to modify it to do the following:

- Add new signs to the current system of DMSs
- Deploy a system of DMSs to a new area or network

6.1 DMS Relocation Problem

We view the decision to relocate an existing DMS as consisting of two steps. The first step is to disallow the DMS to display any travel time and the second is to install a "replacement" DMS at a new location. As before, Q denotes the number of existing DMSs and $\Omega_a \subseteq \{1, 2, \cdots, Q\}$ is a set containing the indices of the DMSs currently located on link a. Recall from Section 4 that $y_q^j = 1$ if DMS q displays the travel time to node j. If the decision is to relocate M existing DMS, then the first step in our scheme is only allow $(Q - M)$ existing DMSs to display travel times, i.e., $\sum_q \sum_j y_q^j = (Q - M)$.

Our optimization problem below assumes that the replacement DMSs are indexed from 1 to M and let $x_{a,m}^j = 1$ if we locate replacement DMS m on link a and let it display the travel time to node j. Otherwise, $x_{a,m}^j = 0$. As mentioned previously, the objective of the relocation is to maximize the benefit to travelers and the DMS relocation problem can be stated as follows:

$$
\begin{aligned}
\max \quad & \sum_a \sum_j \sum_w \left(1 - p^{z_a^j}\right) f_w d_w B_a^{j,w} \\
s.t. \quad & \sum_{q=1}^{Q} \sum_j y_q^j = (Q - M) \\
& \sum_{q \in \Omega_a} \sum_j y_q^j + \sum_j \sum_{m=1}^{M} x_{a,m}^j \le mx_a, && \forall a \\
& z_a^j = \sum_{q \in \Omega_a} y_q^j + \sum_{m=1}^{M} x_{a,m}^j, && \forall a, j \\
& \sum_j \sum_a x_{a,m}^j = 1, && \forall m = 1, \cdots, M \\
& \sum_j y_q^j \le 1, && \forall q = 1, \cdots, Q \\
& y_q^j, x_{a,m}^j \in \{0,1\}, && \forall a, j, q = 1, \cdots, Q, m = 1, \cdots, M
\end{aligned}
$$

Similar to the display selection problem in Section 4, the objective of the above problem is to maximize the benefit. As previously explained, the first constraint allows $(Q - M)$ existing DMSs to display travel times. The second set of constraints ensures that the number of DMSs, existing and replacement, on link a does not exceed the maximum allowed or mx_a. The third set determines the number of DMSs on link a that display the travel time to node j. The fourth set forces each replacement DMS to locate on only one link and to display the travel time to only one node and the fifth does only the latter for existing DMS. Finally, the last set of constraints limits y_q^j and $x_{a,m}^j$ to binary values.

The objective function of the above problem is nonlinear. Similar to Section 4, we can further introduce binary auxiliary variables to linearize it, and transform the problem to be a binary integer linear programming model. The model involves more decision variables, and is thus more difficult to solve than the destination selection problem in Section 4.

6.2 Adding New DMSs and Deploying to a New Area

We now consider the decision to add new DMSs to a current system of DMS. In view of our previous scheme for handling DMS relocation, adding M new DMSs to current system is the same as allowing all of Q existing DMSs to display travel times and adding "replacement" DMSs to the system. To transform the above DMS relocation problem into one that adds new

DMSs, replace $(Q - M)$ on the right hand side of the first constraint in the problem in Section 6.1with Q.

When deploying DMSs to a new area, there is no existing DMS. In terms of the relocation problem, this corresponds to relocating **all** Q DMSs or setting $M = Q$. By doing so, the right hand side of the first constraint in the relocation problem reduces to zero. Consequently, all y's must be zero, become irrelevant, and can be removed along with any terms of constraints involving them. In other words, the DMS deployment problem can be written as follows:

$$
\begin{aligned}
\max \quad & \sum_a \sum_j \sum_w \left(1 - p^{z_a^j}\right) f_w d_w B_a^{j,w} & \\
s.t. \quad & \sum_j \sum_{m=1}^{M} x_{a,m}^j \le m x_a, & \forall a \\
& z_a^j = \sum_{m=1}^{M} x_{a,m}^j\,, & \forall a, j \\
& \sum_j \sum_a x_{a,m}^j = 1, & \forall m = 1, \cdots, M \\
& x_{a,m}^j \in \{0,1\}, & \forall a, j, m = 1, \cdots, M
\end{aligned}
$$

The above model can be applied to deploy a new DMS system to maximize the benefit of displaying travel time messages. Because a DMS serves other usage purposes other than displaying travel time, it is wise to select the potential locations based on the general rules and guidance of the DMS locations from the federal and state agencies, and then apply the model to determine a deployment plan from the potential locations.

7 Relocating and Adding New DMSs along I-95 & I-595

In this section, we use the data from Section 5 to illustrate the use of the DMS relocation model in relocating and adding new DMSs. We assume that the maximum number of DMSs allowed on a link or highway segment is as specified below:

$$\text{maximum number of DMS} = \begin{cases} 0 & \text{if segment length} < 0.5 \text{ mile} \\ 1 & \text{if } 0.5 \text{ mile} \leq \text{segment length} < 1.0 \text{ mile} \\ 2 & \text{if segment length} \geq 1 \text{ mile} \end{cases}$$

We solved the relocation problem with M varied from 0 to 40 and graphically display the benefits from the relocations in Figure 7.1. For $M > 40$, the benefit is the same as that at $M = 40$ or 34,833.25. This implies that 22 DMSs out of the 62 in the I-95/595 corridor are at their optimal locations (for the purpose of travel time displays) and relocating the other 40 DMSs increases the benefit by approximately 39% or from 25,047.92 to 34,833.25. At the other extreme, relocating only 5 DMSs yields an approximately 14% increase in benefit from 25,567.02 to 28,456.30.

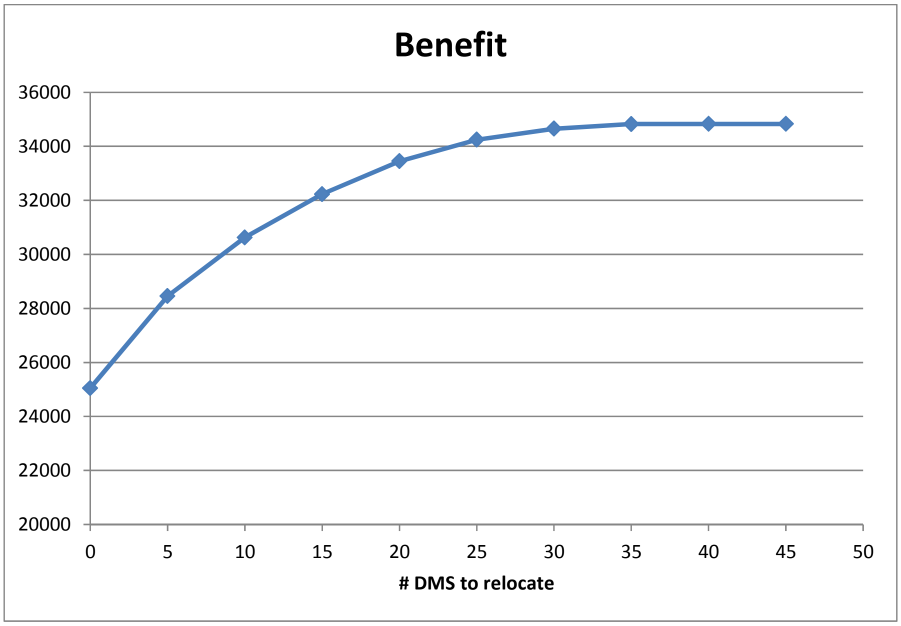

Figure 7.1: Benefit associated with DMS relocation

Tables 7.1 and 7.2 display the results from solving the relocation problem with $M = 10$. In Table 7.1, six DMSs from I-95 are relocated to the new locations indicated at the bottom of the table. Note that there are four new locations on I-95, each one receives two replacement DMSs. Similarly, four DMSs from I-595 are relocated, two to I-95 and the remaining to two different locations along I-595 West (see the bottom of Table 7.2). As shown in Figure 7.1, relocating these 10 DMS increases the benefit from 25,047.92 to 30,630.15, an approximately 22% increase.

Table 7.1: Relocation plan for I-95

DMS	I-95 South		DMS	I-95 North	
1	Before PGA Blvd	Relocate	25	Before PGA Blvd	Relocate
2	Before Blue Heron Blvd				
3	before Palm Beach Lake Blvd		26	Before Northland Blvd	Relocate
4	Before Okeechobee Blvd		27	Before Palm Beach Lakes Blvd	
5	Before Belvedere Blvd				
6	South of Southern Blvd				
7	Before Forrest Hill Blvd		28	Before Forrest Hill Blvd	
8	Before 6th Ave		29	Before 6th Ave	
9	Before Lantana Rd		30	Before Lantana Rd	
10	Before Gateway Blvd		31	Before Gateway Blvd	
11	Before Boynton Beach Blvd				
12	Before Woolbright Rd		32	Before Boynton Beach Blvd	
13	Before Atlantic Ave		33	Before Woolbright Rd	
14	Before Linton Blvd		34	Before Linton Blvd	
15	Before Glades Rd		35	Before Congress Ave	
16	Before Hillsboro Blvd.	Relocate	36	Before Glades Rd	
17	Before Sample Rd	Relocate	37	Before 10th St	
18	Beyond Copans Rd		38	Before Copans Rd	Relocate
19	Before Cypress Creek Rd		39	At Cypress Creek Rd	
20	Beyond Oakland Park Blvd		40	Before Oakland Park Blvd.	
21	Beyond Sunrise Blvd		41	At Broward Blvd	
22	At Davie Blvd	Relocate	42	Before Griffin Rd	
23	Before Sheridan				
24	At Pembroke Rd./ Hwy 824		43	Before Hallandale Beach Blvd	
new	Btw Sheridan & Hollywood	2 DMS	new	Btw Sheridan & Hollywood	2 DMS
new	Btw Hollywood & Hwy 824	2 DMS	new	Btw Hollywood & Hwy 824	2 DMS

Table 7.2: Relocation plan for I-595

DMS	I-595 East		DMS	I-595 West	
44	I-75 South before Hwy 27	Relocate			
45	I-75 South Before I-595	Relocate	55	I-75 North before Hwy 27	Relocate
46	Beyond SW 136th St				
47	Beyond Flamingo Rd		56	Beyond Hiatus Rd	
48	Beyond Hiatus Rd		57	Beyond Nob Hill Rd	
49	Beyond Nob Hill Rd		58	Beyond Pine Island Rd	
50	Beyond Pine Island Rd				
51	Beyond University Dr		59	Beyond Florida Turnpike	
52	Beyond Davie Rd		60	Before SR 7/ 441	
53	Beyond SR 84		61	Before Turnpike/ SR 7	
54	Before US 1	Relocate	62	Before I-95 South	
			new	Btw. University & Pine Island	1 DMS
			new	Btw. Hwy 411 & University	1 DMS

We also modified the relocation problem to optimally add 10 new DMSs without relocating any of the existing 62 DMSs. The optimal locations of the 10 DMSs are the same as shown at the bottom of Tables 7.1 and 7.2. However, the benefit associated with having 72 DMSs is 31,953.07. This is only 4.3% higher than the benefit from just relocating 10 DMSs and suggests that relocating 10 existing DMSs may be more cost-effective than adding 10 new ones.

Note that the results discussed above are for illustration only. They are based on the data we collected and generated as described in Appendix A. Decisions to change display messages, relocating existing, and adding new DMSs should be based on data that have been verified and approved by responsible agencies. Moreover, the models in this and previous sections only maximize the benefit from the travel time displays. However, there are other benefits from displaying information such as those for incident management, road condition advisements, and law enforcement alerts. (See Section 2.) These benefits are not in the scope of our study. However, they should be included in decisions regarding the operation and deployment of DMSs.

8 Robust DMS Deployment Model

Section 6 presents two deterministic formulations for deploying, adding, or relocating DMSs to maximize the benefit associated with displaying travel times. The OD demands, one set of inputs essential to the two models, are not known with certainty. In practice, the demands can fluctuate or vary significantly from day to day. This section presents robust counterparts of the models developed in Section 6. They proactively address the uncertainty associated with the OD demands in the planning stage. To illustrate, this section focuses developing the robust counterpart of the deterministic DMS deployment model.

8.1 Model Formulation

We represent the demand uncertainty as a set of demand scenarios indexed by $s = 1, \cdots, S$. The probability of occurrence of demand scenario s is p_s and the demand for OD pair w under the scenario is d_s^w. In our model, $B_a^{j,w}$ still denotes the benefit that one motorist for OD pair w receive from knowing the travel time to node j displayed by the DMS on link a, which remains the same across different scenarios.

In contrast, the total benefit of a DMS deployment plan is different for different demand scenarios. Considering that most decision makers are risk averse and may be more concerned with the worst-case performance, we maximize the benefits associated with the worst-case demand scenarios. More specifically, we determine a deployment plan to maximize the mean of the benefits under the worst-case scenarios whose collective probability of occurrence is ρ, say, 5%. (Hereinafter, we call those scenarios as ρ-percent worst-case scenarios.) See Figure 8.1 for an illustration of the objective. The probability density function and the cumulative probability function of a continuous benefit are shown in the figure. The area of the shaded region under the density function is ρ, and the mean of the benefit located in the region is the objective to maximize.

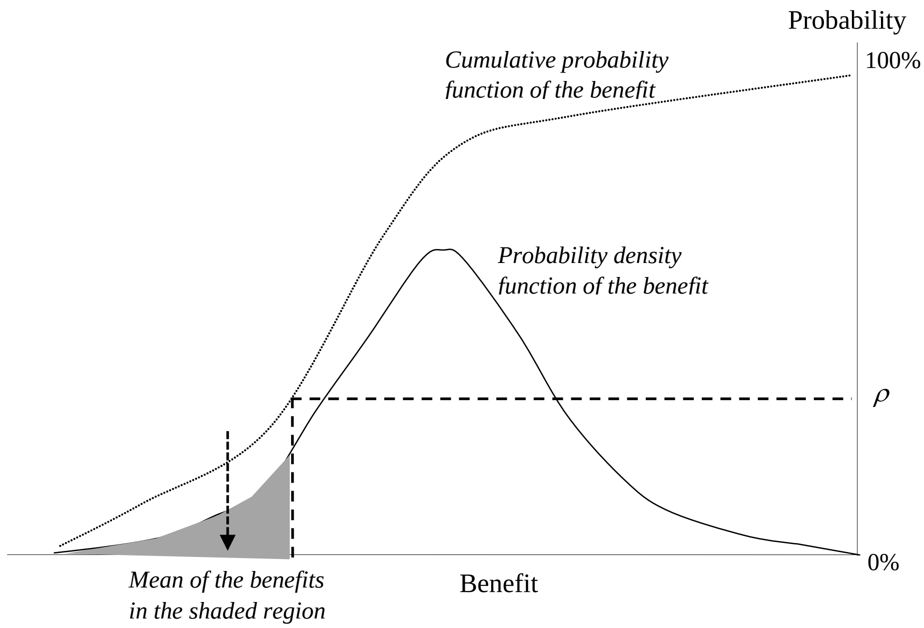

Figure 8.1: Illustration of the objective function

Note that the probability functions in Figure 8.1 are associated with one particular DMS deployment plan. Our intention is to find a plan that leads to the maximal average benefit of the ρ-percent worst-case scenarios. This is equivalent to maximizing the following equation:

$$\max \xi - \frac{1}{\rho}\sum_{s=1}^{S} p_s \cdot \left[\xi - \sum_a \sum_j \sum_w \left(1 - p^{z_a^j}\right) f_w d_s^w B_a^{j,w}\right]^+$$

where ξ is a free decision variable. The proof of the equivalency (see Appendix B) can be established by using an argument similar to those in Rockafellar and Uryasev (2002).

Therefore, the scenario-based robust DMS deployment model can be written as follows:

$$\begin{aligned}
\max \quad & \xi - \frac{1}{\rho}\sum_{s=1}^{S} p_s \cdot \left[\xi - \sum_a \sum_j \sum_w \left(1 - p^{z_a^j}\right) f_w d_s^w B_a^{j,w}\right]^+ & \\
s.t. \quad & \sum_j \sum_{m=1}^{M} x_{a,m}^j \le m x_a, & \forall a \\
& z_a^j = \sum_{m=1}^{M} x_{a,m}^j , & \forall a, j \\
& \sum_j \sum_a x_{a,m}^j = 1, & \forall m = 1, \cdots, M \\
& x_{a,m}^j \in \{0,1\}, & \forall\ a, j, m = 1, \cdots, M
\end{aligned}$$

In the above, $[t]^+ = \max\{0, t\}$. The optimal value of the objective function represents the mean of the benefits under the worst-case scenarios whose collective probability of occurrence is ρ. All the constraints remain the same as those of its deterministic counterpart.

The above robust model has a similar structure as its deterministic counterpart, and thus requires similar computational effort. However, an additional effort is needed to generate the demand scenarios and prepare data inputs accordingly to the model.

8.2 Case Study on I-95/595

To illustrate the model, we use the data from Section 5 for a case study on the I-95/595 corridor. We make the following assumptions:

- There is no existing DMS along the corridor, i.e., we are deploying a new system of DMSs
- There is a sufficient space to install at most one DMS per freeway segment or link in the network in Figure A.2
- There are 30 DMSs to deploy

Two important questions about the scenario-based robust optimization are how many scenarios should be included and how to specify these scenarios and their associated probabilities. Intuitively, the more scenarios we include, the more robust solution we are likely to obtain. However, as the number of scenarios increases, the problem may become prohibitively large. Since the distribution of the OD demand is unknown, we assumed an equal probability of occurrence for each demand scenario and obtained the scenarios by solving the OD demand estimation problem in Appendix A with varying average travel distance, i.e., β. The robust DMS deployment model was solved with the number of scenarios varying from 20 to 1050. The relative difference of the resulting plans is plotted in Figure 8.2. The difference is defined as follows:

$$u = \sum_a \sum_j \sum_m \frac{\left|\hat{x}^j_{a,m} - x^j_{a,m}\right|}{2M}$$

where $\hat{x}^j_{a,m}$ is the plan previously generated with a smaller number of scenarios.

Figure 8.2 displays the relative differences between plans from two consecutive numbers of scenarios. As expected, the difference decreases as the number of scenarios increases. However, it can be observed that relatively small number of scenarios is already enough to produce similar robust plans. When the number of scenarios is greater than 750, the plan difference is less than 10%.

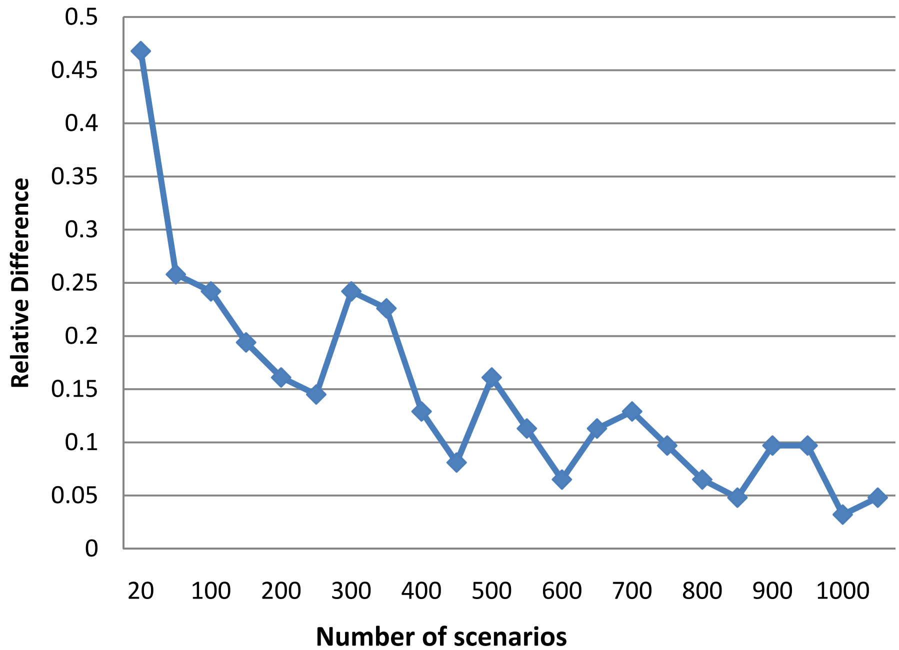

Figure 8.2: Relative difference of robust plans with different numbers of scenarios

Table 8.1 displays the deployment plan with 1050 demand scenarios. We further conducted a Monte-Carlo simulation to test its performance, and compared it with a nominal plan. The latter was obtained by solving the deterministic model with the average OD demand. The simulation shows that the robust plan is able to improve the average benefit associated with 5% worst-case scenarios from 25,376.58 to 26,448.65, a 4.23% increase. However, the average benefit across all the demand scenarios slightly deceases by 1.95%, from 34,517.64 to 33,843.53. This is acceptable as the robust model is worst-case oriented. The simulation demonstrates that the approach is able to improve the worst-case performance without comprising much the average performance.

Table 8.1: Deployment plan for I-95

DMS	I-95 South	DMS	I-95 North
1	Before Okeechobee Blvd	15	Before Lantana Rd
2	Before Southern Blvd		
3	Before Belvedere Rd	16	Before Hypoluxo Rd
4	Before Forest Hill Blvd	17	Before Gateway Blvd/22nd Ave
5	Before10th Ave		
6	Before 6th Ave South		
7	Before Lantana Rd	18	Before Boynton Beach Blvd
8	Before Hypoluxo Rd	19	Before Woolbright Rd
9	Beyond Oakland Park Blvd	20	Before Atlantic Ave
10	Beyond Sunrise Blvd	21	Before Linton Blvd
11	Before Sheridan Rd		
12	Before Hollywood Blvd	22	Before Congress Ave
13	Before Pembroke Rd	23	Before Hallandale Beach Blvd
14	At Pembroke Rd	24	At Pembroke Rd

Table 8.2: Deployment plan for I-595

DMS	I-595 East	DMS	I-595 West
44	Beyond Flamingo Rd	55	Before S Pine Island Rd
45	Beyond Hiatus Rd	56	Beyond Turnpike/ Highway 441
46	Beyond Nob Hill Rd	57	Before SR 7/441

9 Conclusions

DMSs are capable of displaying messages composed of alphanumeric characters, symbols, or both. The messages can warn motorists about downstream traffic conditions, adverse weather, traffic incidents, etc. The federal and state transportation agencies have recognized the benefits of displaying travel time on DMS, and thus recommended it as the default message.

While a modest amount of research has been performed on DMS, little has been done on quantifying the benefits associated with displaying travel times on DMS. None has investigated how to determine the destinations for which to display travel times. Moreover, no quantitative model exists for locating DMSs for the purpose of providing travel time information during normal operations.

This report has presented methodology and models to fill the above voids. Assuming that the primary value of displayed travel time on DMS is to reduce the negative effects of travel time unpredictability, we proposed a surrogate benefit measure, which is based on travel time variability. The measure reflects the considerations that, when the travel time to a location is predictable, displaying such time on a DMS offers little or no additional value; the information is much more valuable if the travel time to this destination varies substantially and there are a large number of drivers traveling to the destination.

Based on the proposed benefit measure, we developed a binary linear programing model to determine the destinations for which to display travel times on currently deployed DMSs to maximize the benefit to motorists. The model was applied to the DMSs along I-95 and I-595 in FDOT District 4 with synthesized and somewhat hypothetical input data. It has been demonstrated that the model is readily applicable in practice, and can potentially improve the current operations of DMSs.

This report has further offered alternative approach to further improve the benefit, by relocating DMSs, adding new ones to the current system or deploying a DMS system to a new area. The models are extensions to the aforementioned destination selection model. We applied them to the same network of I-95 and I-595. The models suggested optimal locations for relocating or adding new DMSs to maximize the benefit of travel time displays. Given that DMSs are used for many other purposes, it is advisable that the models should be used in conjunction with the rules and guidance provided by the federal and state transportation agencies. More specifically, the rules and guidance should be applied first to determine potential locations of DMSs for their multiple potential usages. The models can then be used to select ones among the candidate locations to maximize the benefits of displaying travel time.

The last model presented in the report applies a robust optimization approach to proactively address travel demand uncertainty in deploying DMS. The model was also demonstrated and validated in the same network of I-95 and I-595. The model can be used when uncertainty is a major concern in the planning stage of a DMS deployment. The proposed approach is general and can be extended without much difficulty to accommodate other types of uncertainty.

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Appendix A: Test Data

This appendix describes the data used in testing and illustrating the applications of the models in the main sections of this report. It consists of three sections. The first describes freeway segments in District 4 from which we extract the network for our models and analyses. The remaining sections explain how we obtain link data and estimate travel demands for every OD pair.

1. Road Network

The road network in our test data consists of two freeway segments, one from I-95 and the other from I-595, as shown in Figure A.1. The I-95 segment in the figure begins at West Indiantown Road and ends south of US 395. For I-595, the segment begins at Highway 27 and ends just east of Highway A1A.

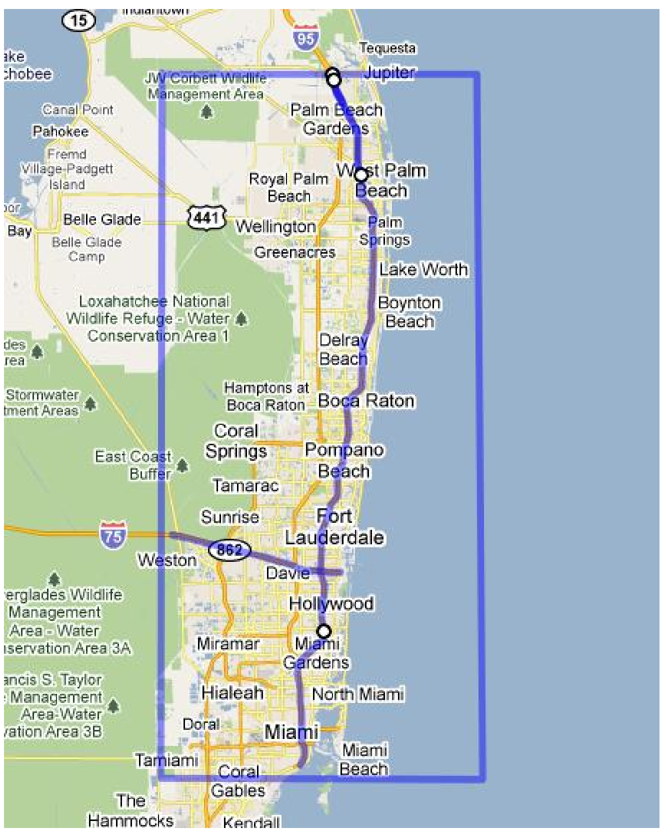


Figure A.1: The network for our test data construction.

For our models and computer implementation, the two freeway segments are transformed into a network consisting of 60 nodes (numbered from 1 to 60) and 120 links in Figure A.2. Each link in the figure is undirected and represents two anti-parallel links. For example, link (16, 17) represents two links, (16, 17) and (17, 16).

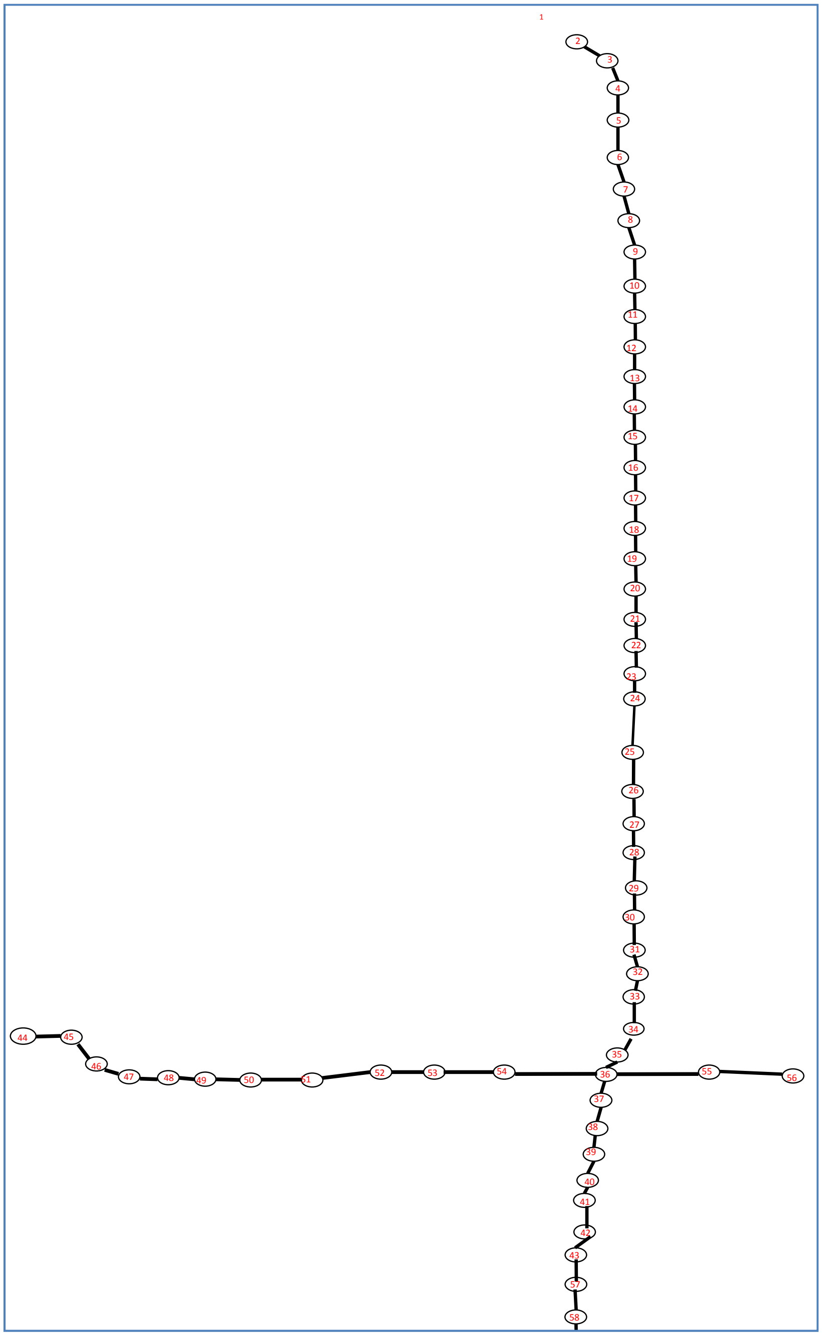


Figure A.2: A network representation of I-95 and I-595

The nodes correspond to locations of various exits on the two highway segments. Figures A.3 to A.5 are enlargements of Figure A.2 with some crossroad names indicated at each node. Square boxes in these figures represent the current locations of DMS. Only those along I-95 and I-595 are relevant to our study and they are numbered from 1 to 62. Figure A.6 (similar to Figure 5.1) displays the locations of these DMS as they appear on Florida's 511 Travel Information System website (www.smartsunguide.com).

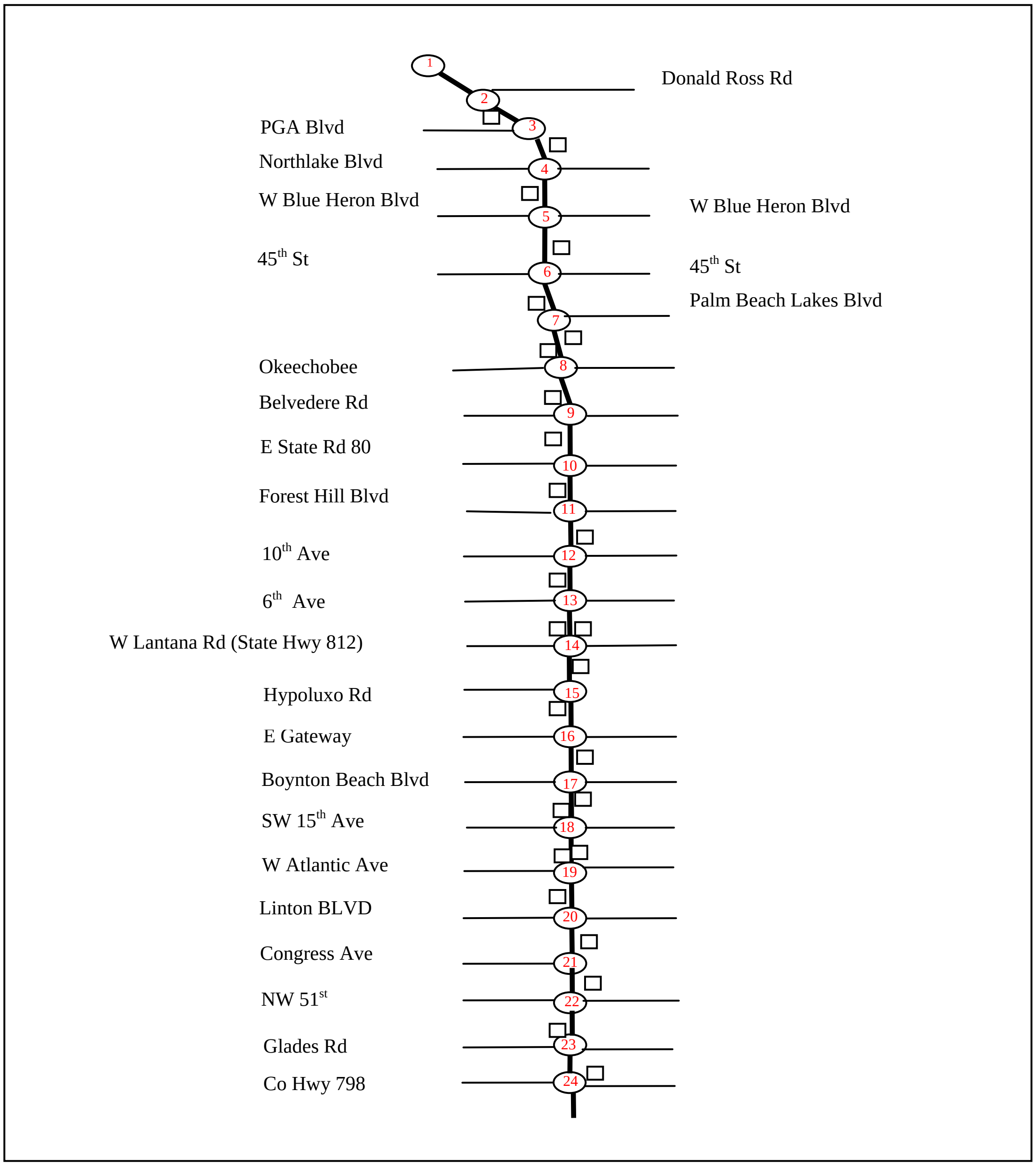


Figure A.3: An enlargement of the northern part of I-95

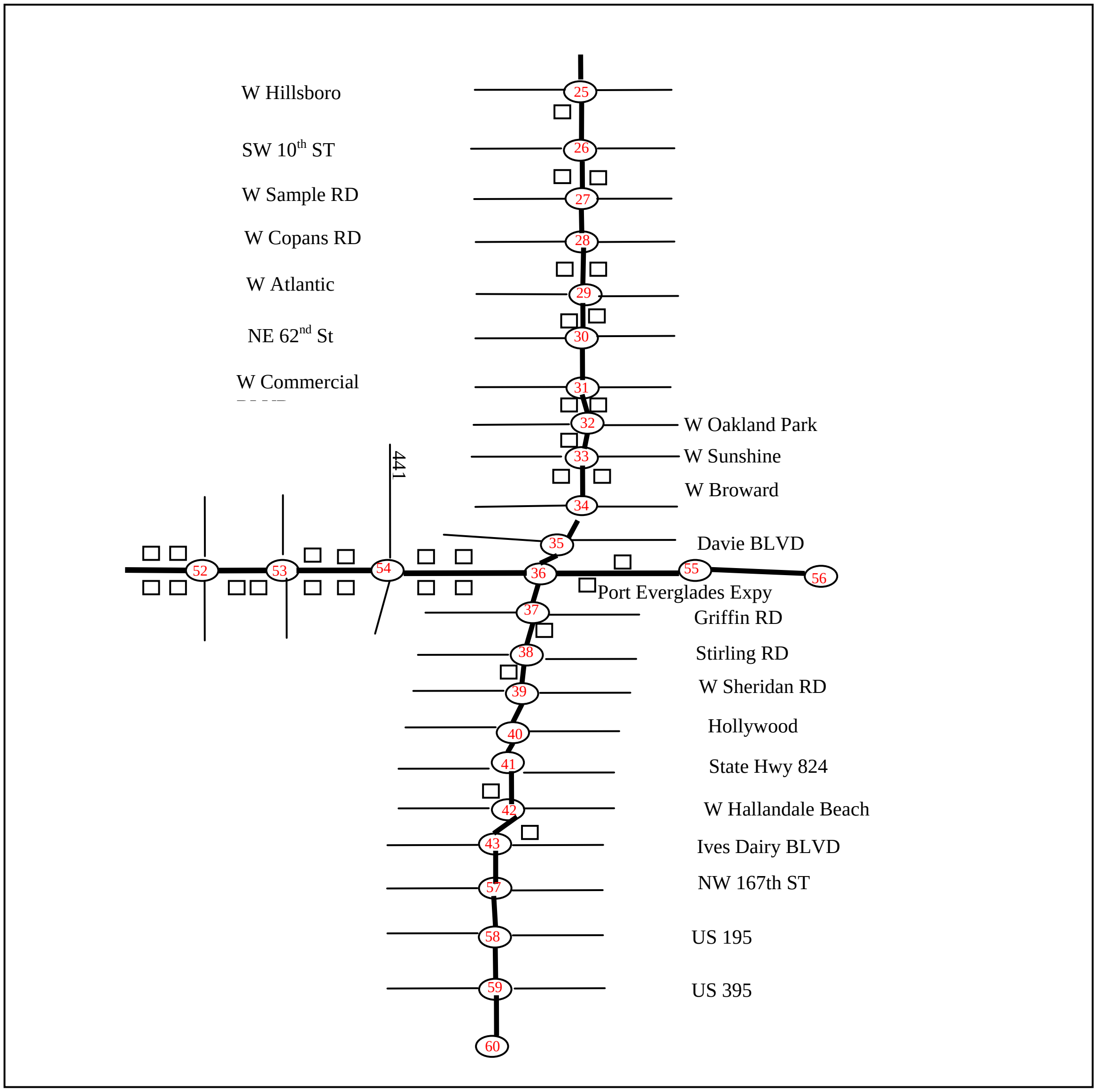


Figure A.4: An enlargement of the sorthern part of I-95

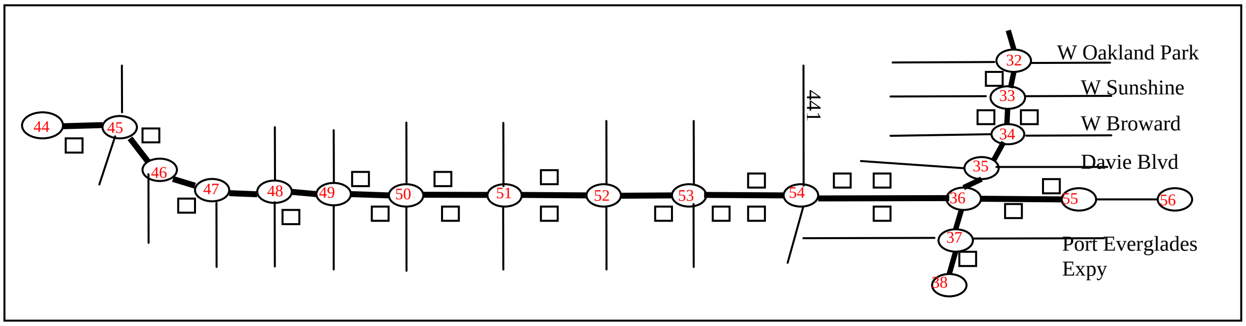


Figure A.5: An enlargement of I-595

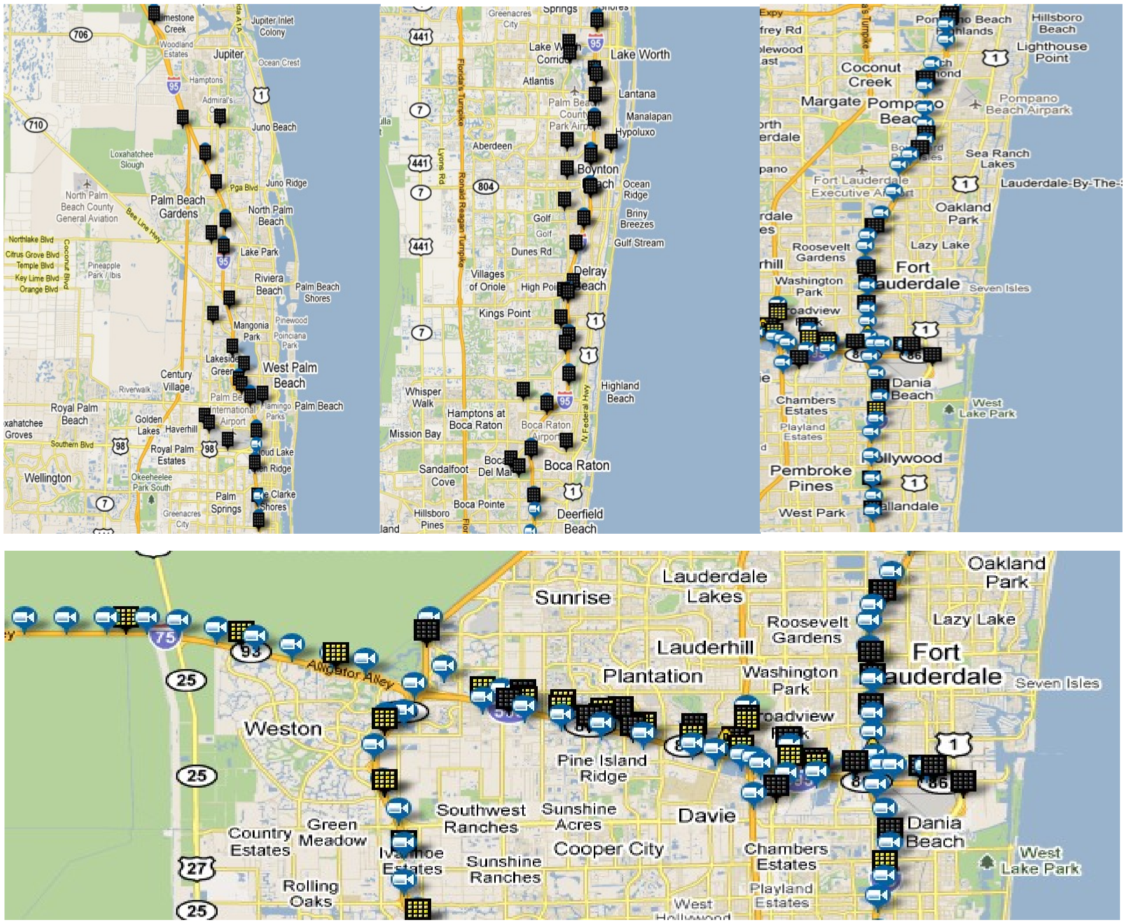


Figure A.6: Geographical locations of Dynamic Message Sign

Because the crossroad names in the last three figures are incomplete, Tables A.1 and A.2 list all crossroad names at nodes along I-595 and I-95, respectively.

Table A.1: Names of crossroads on I-95

Nodes	Street Name	Nodes	Street Name
1	Indiantown Rd	25	Hillsboro Blvd
2	Donald Ross Rd	26	10th St
3	PGA Blvd	27	Sample Rd
4	Northlake Blvd	28	Copans Rd
5	Blue Heron Blvd	29	Atlantic Rd
6	45th St	30	62nd St/Cypress Creek Rd
7	Palm Beach Lakes Blvd	31	Commercial Blvd
8	Okeechobee Blvd.	32	Oakland Park Blvd
9	Belvedere Rd	33	Sunrise Blvd
10	State Rd 80/Southern Blvd	34	Broward Blvd
11	Forest Hill Blvd	35	Davie Blvd
12	10th Ave	36	I-95&I-595 Interchange
13	6th Ave	37	Griffin Rd/10th St (FLL Airport*)
14	Lantana Rd.	38	Sterling Rd
15	Hypoluxo Rd.	39	Sheridan Rd
16	Gateway Blvd/22nd Ave	40	Hollywood Blvd
17	Boynton Beach Blvd	41	Pembroke Rd/Hwy 824
18	15th Ave/Woolbright Rd	42	Hallandale Beach Blvd
19	Atlantic Ave	43	Ives Dairy Blvd
20	Linton Blvd	57	Turnpike/167th St (Golden Glades)
21	Congress Ave	58	Airport Expy/I-195
22	51st St/Yamato Rd	59	Dolphin Expy/I-395
23	Glades Rd	60	26th Rd/Rickenbacker Cswy
24	Co Hwy 798/Palmetto Park Rd		

*FLL Airport = Fort Lauderdale Hollywood International Airport

Table A.2: Names of crossroads on I 595

Nodes	Street Name
44	Just west of Hwy 27 on Everglade Pkwy
45	Hwy 27
46	Indian Trace
47	Bonaventure Blvd
48	I-75/I-595/Sawgrass(SR 869)
49	125^{th} Ave
50	Hiatus Rd/112^{th} Ave
51	Nob Hill Rd
52	Pine Island Rd
53	University Dr
54	Turnpike/Hwy 441
55	Hwy A1A/Federal Hwy (FLL Airport)
56	East of Highway A1A

*FLL Airport = Fort Lauderdale Hollywood International Airport

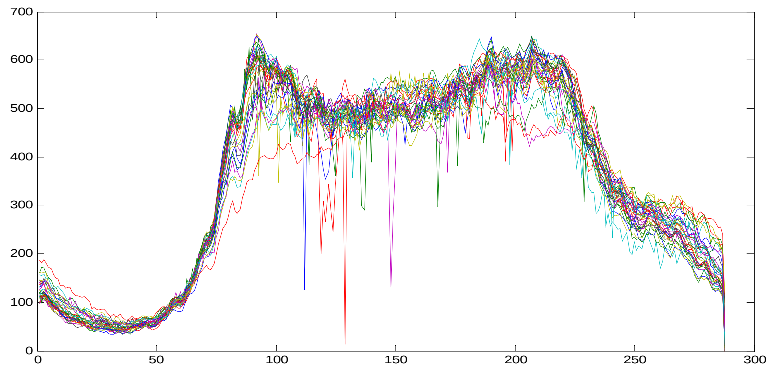
2. Link Data

Information regarding link travel times (more precisely, their means and standard deviations) and traffic volumes used in the benefit calculation and travel demand estimates come from two sources, the Statewide Transportation Engineering Warehouse for Archived Regional Data (STEWARD)[7] and the statewide model in Florida Standard Urban Transportation Model Structure (FSUTMS). STEWARD provides speeds and traffic volumes from detectors along the segment of I-95 shown in Figure A.4 at every five minutes interval during an entire day (24 hours). We used the data from STEWARD for three purposes. The first is to determine a peak travel period for our study. The travel times during peak periods have high variability or are most unreliable. Thus, displaying accurate travel times on DMS would be most beneficial to travelers. Second, we use data from STEWARD to determine the average and standard deviation of travel times for links along I-95 in Figure A.4. Because it is still under development, STEWARD does not provide any data for links along I-95 in Figure A.3 and those along I-595 in Figure A.5. For these links, their traffic volumes are from the FSUTMS statewide model, their mean or average travel times are the times based on the posted speed limits, and the variances of their travel times are random numbers that vary uniformly between the lowest and highest variance of the travel times from STEWARD.

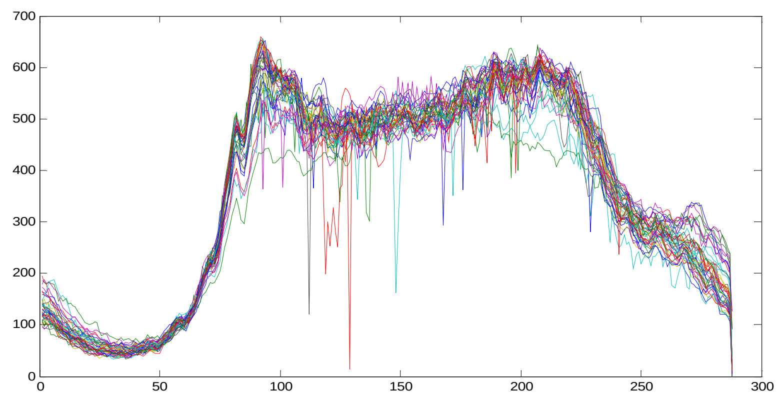
[7] See http://cce-trc-cdwserv.ce.ufl.edu/steward/index.html for more information.

2.1 Peak Period Selection

Figure A.4 displays daily data on speed and volume from 104 detectors along a segment of I-95. Each line or graph in the figure represents the traffic volume averaged over the 104 detectors at every five-minute interval in both northbound and southbound directions for each workday from November 1, 2009 until December 31, 2009. The numbers along the x-axis denotes the indices of the intervals and there are 288 five-minute intervals in 24 hours.



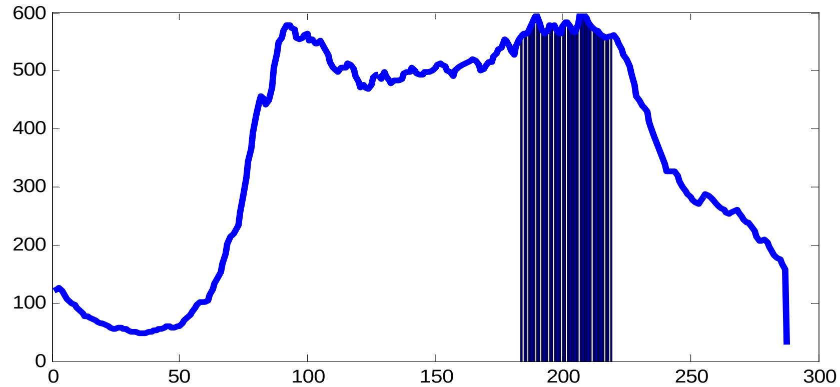
Northbound



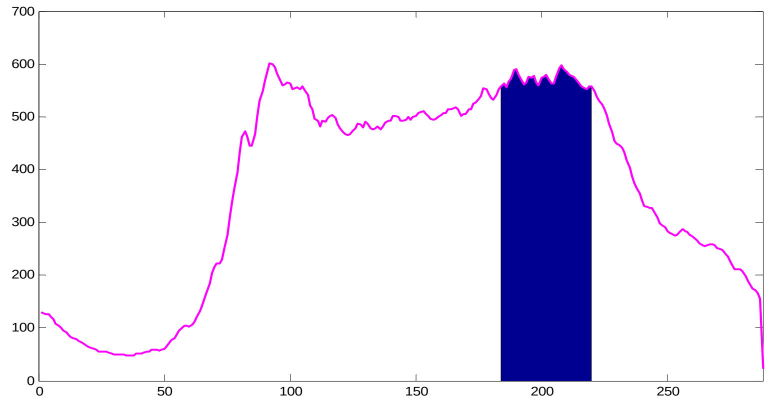
Southbound

Figure A.7: Average traffic volumes on I-95 from 01-Nov-09 to 31-Dec-09

In our study, we use a three-hour period with the highest average traffic volume as our peak period. Figure A.8 displays the average traffic volume over all the weekdays from November 1, 2009 until December 31, 2009. Based on our calculations, the period between 3:30 PM and 6:30 PM (which is shaded in Figure A.8) has the highest average traffic volume. The traffic volumes during this period are 19.82% and 19.90% of the average daily total for north and southbound, respectively.



Northbound



Southbound

Figure A.8: Average traffic volume on I-95

2.2 Link Travel Times and Traffic Volumes

Table A.3 displays the averages (or means) and standard deviations of travel time (in minutes) for links along I-95 from STEWARD. Note the standard deviation in this table varies from 0.15 to 0.50.

Table A.3: Averages and standard deviations of travel times (in minutes) from STEWARD

Link	Ave.	Std.	Link	Ave.	Std.	Link	Ave.	Std.
(25, 26)	1.18	0.17	(40, 41)	1.57	0.46	(37, 36)	1.64	0.40
(26, 27)	1.99	0.32	(41, 42)	1.09	0.49	(38, 37)	1.19	0.41
(27, 28)	1.04	0.22	(42, 43)	0.93	0.50	(39, 38)	0.99	0.42
(28, 29)	1.84	0.32	(25, 24)	2.24	0.37	(40, 39)	0.95	0.39
(29, 30)	2.27	0.36	(26, 25)	1.00	0.15	(41, 40)	1.61	0.42
(30, 31)	1.36	0.27	(27, 26)	2.10	0.32	(42, 41)	1.01	0.42
(31, 32)	1.59	0.30	(28, 27)	1.07	0.21	(43, 42)	0.78	0.40
(32, 33)	1.89	0.38	(29, 28)	1.71	0.30	(43, 57)	3.54	0.38
(33, 34)	1.15	0.41	(30, 29)	2.02	0.34	(57, 58)	8.19	0.32
(34, 35)	0.79	0.36	(31, 30)	1.29	0.25	(58, 59)	1.70	0.32
(35, 36)	0.37	0.36	(32, 31)	1.69	0.28	(59, 60)	3.01	0.41
(36, 37)	1.68	0.41	(33, 32)	1.97	0.37	(57, 43)	3.79	0.26
(37, 38)	1.15	0.41	(34, 33)	1.14	0.39	(58, 57)	7.91	0.43
(38, 39)	0.97	0.42	(35, 34)	0.94	0.36	(59, 58)	1.82	0.36
(39, 40)	1.00	0.39	(36, 35)	0.39	0.35	(60, 59)	2.69	0.28

Table A.4 displays the travel time information for the remaining links in our network. The average travel times below are based on the distance between adjacent nodes and the posted speed limits. The standard deviations are Uniform random numbers between 0.15 and 0.50, the minimum and maximum standard deviation in Table A.3.

Table A.4: Averages and standard deviations of travel times (in minutes)

Link	Ave.	Std.	Link	Ave.	Std.	Link	Ave.	Std.
(1, 2)	4.02	0.37	(13, 14)	1.54	0.35	(44, 45)	1.59	0.29
(2, 3)	3.46	0.32	(14, 15)	1.11	0.32	(45, 46)	1.16	0.30
(3, 4)	2.22	0.34	(15, 16)	1.42	0.35	(46, 47)	3.80	0.33
(4, 5)	2.08	0.32	(16, 17)	1.61	0.37	(47, 48)	0.90	0.32
(5, 6)	1.87	0.33	(17, 18)	0.99	0.32	(48, 49)	1.06	0.40
(6, 7)	3.00	0.36	(18, 19)	4.28	0.31	(49, 50)	1.04	0.43
(7, 8)	1.23	0.33	(19, 20)	1.59	0.34	(50, 51)	1.03	0.37
(8, 9)	1.19	0.34	(14, 13)	1.40	0.34	(51, 52)	1.11	0.48
(9, 10)	1.11	0.35	(15, 14)	1.18	0.28	(52, 53)	0.95	0.28
(2, 1)	3.57	0.36	(16, 15)	1.37	0.44	(53, 54)	1.56	0.35
(3, 2)	3.20	0.31	(17, 16)	1.44	0.25	(55, 56)	1.84	0.27
(4, 3)	2.14	0.34	(18, 17)	0.98	0.45	(45, 44)	1.45	0.34
(5, 4)	1.78	0.29	(19, 18)	3.75	0.41	(46, 45)	1.08	0.28
(6, 5)	1.57	0.30	(20, 19)	1.66	0.32	(47, 46)	3.58	0.25
(7, 6)	2.89	0.30	(20, 21)	1.37	0.31	(48, 47)	1.04	0.37
(8, 7)	1.21	0.34	(21, 22)	1.85	0.32	(49, 48)	1.02	0.30
(9, 8)	1.16	0.46	(22, 23)	2.61	0.36	(50, 49)	1.00	0.29
(10, 9)	1.01	0.37	(23, 24)	1.20	0.37	(51, 50)	1.08	0.20
(10, 11)	1.54	0.33	(24, 25)	2.31	0.34	(52, 51)	1.10	0.40
(11, 12)	1.91	0.31	(21, 20)	1.29	0.34	(53, 52)	0.95	0.36
(12, 13)	1.38	0.35	(22, 21)	1.77	0.25	(54, 53)	1.42	0.42
(11, 10)	1.50	0.30	(23, 22)	2.48	0.46	(55, 36)	2.06	0.30
(12, 11)	1.84	0.26	(24, 23)	1.13	0.21	(56, 55)	1.82	0.41
(13, 12)	1.39	0.23	(36, 54)	1.59	0.35			
			(36, 55)	2.11	0.30			
			(54, 36)	1.70	0.34			

Table A.5 displays the traffic volumes for links along I-95 in Figure A.4 during the peak period from 3:30 PM to 6:30 PM. (See Section A.2.1 above.) Note that the volume on link (i, j) is not necessarily the same as that on the anti-parallel link, i.e., (j, i).

Table A.5: Traffic volumes during the peak period from STEWARD

Link	Volume	Link	Volume
(25, 26)	18733	(26, 25)	18777
(26, 27)	19530	(27, 26)	19279
(27, 28)	19062	(28, 27)	17738
(28, 29)	19276	(29, 28)	19881
(29, 30)	19009	(30, 29)	19291
(30, 31)	20366	(31, 30)	17751
(31, 32)	24159	(32, 31)	23076
(32, 33)	24799	(33, 32)	23506
(33, 34)	24061	(34, 33)	23745
(34, 35)	21072	(35, 34)	14442
(35, 36)	16476	(36, 35)	12200
(36, 37)	17862	(37, 36)	16371
(37, 38)	18457	(38, 37)	18932
(38, 39)	23933	(39, 38)	23095
(39, 40)	22510	(40, 39)	22546
(40, 41)	22543	(41, 40)	22857
(41, 42)	21334	(42, 41)	21488
(42, 43)	19101	(43, 42)	20559
(43, 57)	21650	(57, 43)	27441
(57, 58)	22300	(58, 57)	18818
(58, 59)	26836	(59, 58)	18146
(59, 60)	29038	(60, 59)	25751

Table A.6 displays the traffic volumes not available from STEWARD. These volumes are from the FSUTMS Statewide Model. Because the volume on link (i, j) is the same as the one on link (j, i), only one is displayed in the table below.

Table A.6: Traffic volumes during the peak period from FSUTMS

Link	Volume	Link	Volume	Link	Volume
(1, 2)	8679	(13, 14)	13240	(36, 55)	8378
(2, 3)	8722	(14, 15)	12491	(44, 45)	2201
(3, 4)	10213	(15, 16)	11784	(45, 46)	4325
(4, 5)	10704	(16, 17)	11397	(46, 47)	4992
(5, 6)	11270	(17, 18)	11737	(47, 48)	6970
(6, 7)	11726	(18, 19)	12645	(48, 49)	8842
(7, 8)	10281	(19, 20)	14312	(49, 50)	9504
(8, 9)	12269	(20, 21)	14621	(50, 51)	9398
(9, 10)	12606	(21, 22)	15651	(51, 52)	9608
(10, 11)	11613	(22, 23)	17536	(52, 53)	9099
(11, 12)	11254	(23, 24)	17989	(53, 54)	14118
(12, 13)	12710	(24, 25)	18726	(54, 36)	12543

3. Travel Demand Estimation

Estimating travel demands for every possible pair of nodes in our network requires solving an optimization model to determine the set of travel demands for every pair of nodes that best fit the data in Section A.2 as well as the “production” and “attraction” data (defined below) at each node in our network. The latter can be obtained from the FSUTMS Statewide Model and are summarized in Section A.3.1 below. The subsequence section describes the OD demand estimation problem.

3.1 Trip Production and Attraction

Below, Table A.7 lists two numbers for each node along I-95, except for node 36. They are the “productions” at each node, i.e., the numbers of vehicles departing from the node, one due north and the other due south. Similarly, Table A.8 lists the “attraction” data for each node or the number of vehicle arriving at the node, one from north and the other from south. Table A.9 lists both production and attraction data for nodes along 1-595. Because node 36 represents the I-95 & I-595 interchange, it has no production or attraction.

Table A.7: Node production data along I-95

Node	Due North	Due South	Node	Due North	Due South
1	0	10701	24	2380	3287
2	733	798	25	3516	2502
3	1545	3596	26	2803	2322
4	1627	2252	27	3053	2296
5	2101	2837	28	2621	3059
6	2173	2406	29	2879	2838
7	2760	1133	30	3175	2825
8	1523	3503	31	2999	5218
9	2195	2082	32	2773	4464
10	2620	1695	33	2322	3469
11	2214	1970	34	3246	4126
12	1385	2904	35	2601	4286
13	1987	2499	37	2904	2692
14	2441	1730	38	3457	2630
15	2003	1456	39	3260	3367
16	2152	1868	40	3737	2981
17	1663	2207	41	3613	1568
18	1490	2569	42	3854	1819
19	2096	4447	43	4479	1245
20	2582	3276	57	10623	13304
21	1478	1809	58	2781	2622
22	3105	5053	59	4767	2909
23	3362	3999	60	17148	0

Table A.8: Node attraction data along I-95

Node	From North	From South	Node	From North	From South
1	0	10636	24	2657	3224
2	808	827	25	3528	3965
3	1315	4020	26	3439	2237
4	1861	2218	27	3067	2628
5	2234	2630	28	3017	2510
6	1969	2649	29	2616	2921
7	2652	1388	30	3125	3064
8	1759	3753	31	3022	3437
9	1453	2241	32	2864	4371
10	2635	1574	33	2044	4550
11	2346	1873	34	2187	3547
12	1403	2796	35	3475	3969
13	1998	2546	37	2702	3368
14	2434	1647	38	3805	2570
15	2213	1344	39	3179	3199
16	2254	1764	40	3817	2951
17	1881	2017	41	3650	1725
18	1657	2394	42	3830	1937
19	2532	3518	43	4136	1520
20	3335	3257	57	11078	13115
21	872	2600	58	2121	2770
22	3076	4898	59	4635	3077
23	3365	3634	60	16752	0

Table A.9: Node production and attraction data along I-595

Node	Due West	Due East	From West	From East
44	0	3044	0	2420
45	167	932	361	2811
46	0	0	0	1337
47	0	0	0	0
48	2872	7094	2960	6436
49	1462	581	68	2272
50	40	111	172	677
51	444	2570	880	2482
52	2419	2077	1676	2115
53	2433	2561	2776	1753
54	2902	2464	3926	2019
55	11516	0	6460	0

3.2 OD Demand Estimation

The following optimization problem determines OD demands that best fit the production and attraction data above and those in Section A.2. In Section 4 of the report body, we refer to the travel demand from node s to node t as d_{st}. In this section, s and t are nodes in our network in Figure A.2.

To formulate the problem, let Φ^S and Φ^F denote the set of links for which we obtain data from STEWARD and FSUTMS, respectively. In addition, $\Phi = \Phi^S \cup \Phi^F$, i.e., Φ is the set of all links in our network. We refer to the estimated traffic volume on link $(i,j) \in \Phi$ as $\hat{v}_{ij}$. For every $(i,j) \in \Phi^S$, $\hat{v}_{ij}$ is the traffic volume from STEWARD and $\hat{v}_{ij}$ is the data from FSUMTS if $(i,j) \in \Phi^F$. Additionally, $\hat{p}_s$ and $\hat{a}_t$ denote the production and attraction data at node s and t, respectively, where s and t are nodes in our network and vary from 1 to 60. The model below does not distinguish the direction of the production and attraction data. Thus, $\hat{p}_s$ is the sum of the production data due north and south if s is a node along I-95. The same also holds for nodes along I-595 and all attraction data. For node 36, $\hat{p}_{36}$ is the sum of its production data in all four directions. The same is true for $\hat{a}_{36}$. Then, the following problem (or the OD demand estimation problem) finds OD demands, d_{st}, that induce link volumes, production, and attraction data with the least deviation from $\hat{v}_{ij}, \hat{p}_s$, and $\hat{a}_t$.

$$\min \quad \omega_1 \sum_{(i,j)\in\Phi^F} \left(v_{ij} - \hat{v}_{ij}\right)^2 + \omega_2 \sum_{s=1}^{60} \left(\hat{p}_s - \sum_{t=1,t\neq s}^{60} d_{st}\right)^2 + \omega_3 \sum_{t=1}^{60} \left(\hat{a}_t - \sum_{s=1,s\neq t}^{60} d_{st}\right)^2$$

$$s.t \quad d_{st} = \sum_{p\in\Pi^{st}} u_p \,, \forall s,t = 1,\cdots,60; s \neq t$$

$$v_{ij} = \sum_{(s,t)} \sum_{p\in\Pi^{st}} \delta_{(i,j),p} u_p \,, \forall (i,j) \in \Phi$$

$$v_{ij} = \hat{v}_{ij}, \forall (i,j) \in \Phi^S$$

$$\sum_{s=1}^{60} \sum_{t=1,t\neq s}^{60} \Delta(s,t) d_{st} \geq \beta \sum_{s=1}^{60} \sum_{t=1,t\neq s}^{60} d_{st}$$

$$u_p \geq 0, \forall u \in \bigcup_{(s,t)} \Pi^{st}$$

In the above objective, ω_1, ω_2, and ω_3 are nonnegative weights and the problem seeks to minimize the weighted squared deviations from the estimated traffic volumes and the production and attraction data. In the first set of constraints, u_p is the amount of flow on path p and the sum of u_p over all possible paths from node s to t (denoted as Π^{st}) yields the corresponding travel demand for node s to t or d_{st}. In the second set of constraints, $\delta_{(i,j),p}$ is 1 if link (i, j) belongs to path p and it is zero otherwise. Thus, the expression in this set simply determines the traffic volume on link $(i, j) \in \Phi$ induced by u_p. The third set of constraints forces the induced traffic volume, v_{ij}, to be the same as $\hat{v}_{ij}$ on those links with data from STEWARD, because the data in STEWARD are believed to be much more accurate than those from FSUTMS. The fourth constraint ensures that the average travel distance of the OD demands is at least β miles. In the summation on the left, $\Delta(s, t)$ denotes the distance between node s and t. Finally, the last set of constraints requires the flow on every path to nonnegative. Consequently, d_{st} is also nonnegative via the first set of constraints.

To illustrate, consider the OD demand estimation problem with $(\omega_1, \omega_2, \omega_3) = (0, 1000, 1000)$ and $\beta = 10$. With a 60-node network, there are 3540 possible OD pairs. Solving the OD demand estimation problem with the parameters as stated yields only 159 OD pairs with nonzero demands and the travel distances between the OD pairs with positive demands vary from 6 to 28 miles. Figure A.9 displays the distribution of travel demands (in number of vehicles) as a function of the distance between OD pairs.

When we increase β from 10 to 15, there are approximately 296 OD pairs with instead of 159. The distances between these OD pairs vary from 6 to 60 miles. Figure A.10 displays the distribution of these demands.

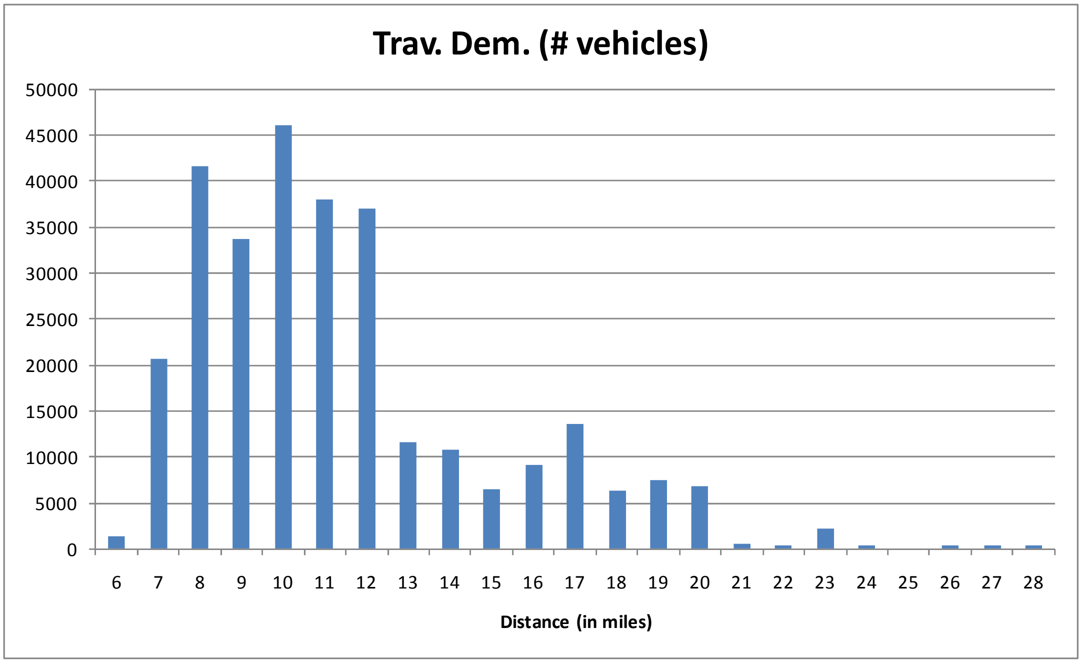


Figure A.9: Distribution of OD demands with $\beta = 10$

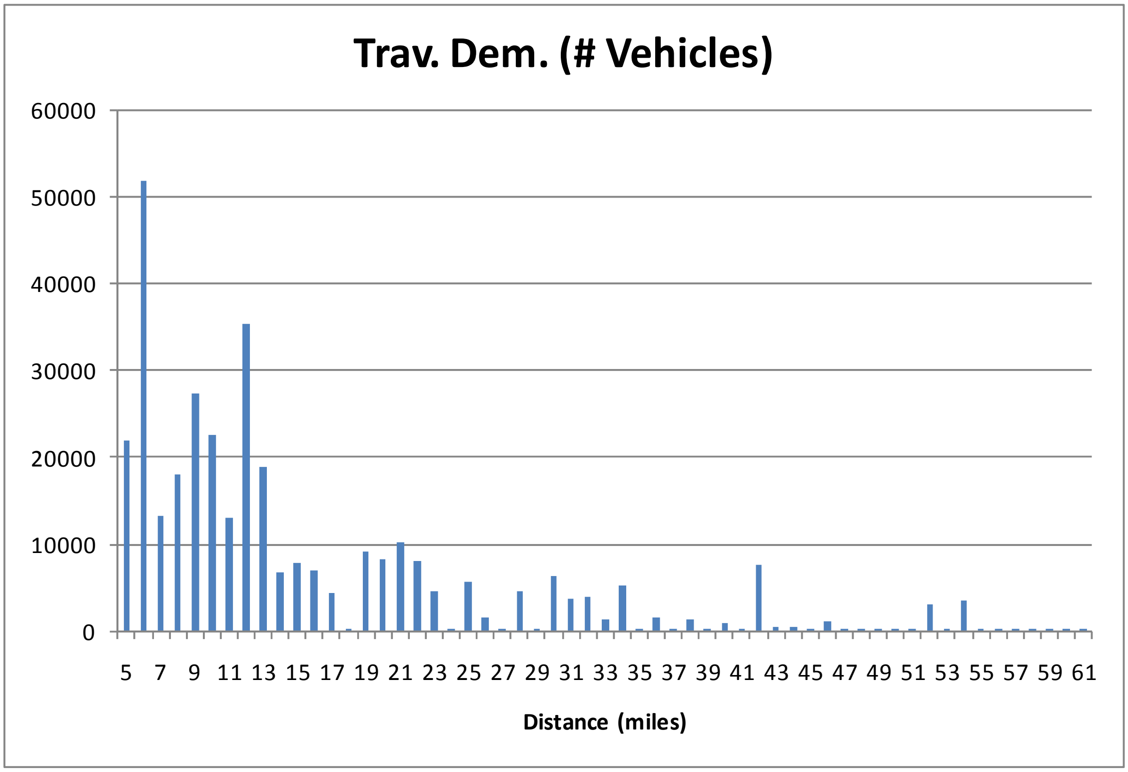


Figure A.10: Distribution of OD demands with $\beta = 15$

Appendix B: Proof of Equivalency

This appendix provides a proof of the objective function used in the robust DMS deployment model. It is assumed that the OD demand can be represented as a random vector d whose probability distribution is independent of the DMS deployment plan x. For each x, define the total benefit of travel time displays as follows:

$$B(x) = \sum_a \sum_j \sum_w \left(1 - p^{\sum_{m=1}^{M} x_{a,m}^j}\right) f_w d_w B_a^{j,w}$$

Apparently, $B(x)$ is a random variable whose distribution function is defined as follows:

$$\varphi(x, \xi) = P\{d | B(x) \leq \xi\}$$

Denote $\varphi(x, \xi^-) = P\{d | B(x) < \xi\}$.

We further denote $\phi_\rho(x)$ is the mean of the ρ-percent left tail, i.e., the mean of the benefits associated with the worst-case scenarios whose collective probability of occurrence is ρ. We have the following theorem:

Theorem B.1: $\phi_\rho(x) = \max_\xi F_\rho(x, \xi)$, where $F_\rho(x, \xi) = \xi - \frac{1}{\rho} E\{[\xi - B(x)]^+\}$, $[t]^+ = \max\{0, t\}$.

Proof: by definition:

$$\frac{F_\rho(x, \xi') - F_\rho(x, \xi)}{\xi' - \xi} = 1 + \frac{1}{\rho} E\left\{\frac{[\xi - B(x)]^+ - [\xi' - B(x)]^+}{\xi' - \xi}\right\}$$

When $\xi' \geq \xi$, we have:

$$\frac{[\xi - B(x)]^+ - [\xi' - B(x)]^+}{\xi' - \xi} = \begin{cases} 0 & B(x) \geq \xi' \geq \xi \\ -1 & \xi' \geq \xi \geq B(x) \\ \epsilon(-1,0) & \xi' > B(x) > \xi \end{cases}$$

Because $P\{d | \ \xi' \geq B(x) > \xi\} = \varphi(x, \xi') - \varphi(x, \xi)$, as $\xi' \to \xi$, $\varphi(x, \xi') \to \varphi(x, \xi)$. It follows that:

$$\lim_{\xi' \to \xi} E\left\{\frac{[\xi - B(x)]^+ - [\xi' - B(x)]^+}{\xi' - \xi}\right\} = -\varphi(x, \xi)$$

$$\frac{\partial^+ F_\rho(x,\xi)}{\partial \xi} = \lim_{\xi'\to\xi} \frac{F_\rho(x,\xi') - F_\rho(x,\xi)}{\xi' - \xi} = 1 - \frac{1}{\rho}\varphi(x,\xi) = \frac{\rho - \varphi(x,\xi)}{\rho}$$

When $\xi' \leq \xi$, we have:

$$\frac{[\xi - B\,(x)]^+ - [\xi' - B\,(x)]^+}{\xi' - \xi} = \begin{cases} 0 & B(x) \geq \xi \geq \xi' \\ -1 & \xi \geq \xi' \geq B(x) \\ \epsilon(-1,0) & \xi > B(x) > \xi' \end{cases}$$

Since

$$P\{d|\xi \geq B(x) > \xi'\} = -\varphi(x,\xi') + \varphi(x,\xi)$$

as $\xi' \to \xi$, $\varphi(x,\xi') \to \varphi(x,\xi)$. We have:

$$\lim_{\xi'\to\xi} E\left\{\frac{[\xi - B\,(x)]^+ - [\xi' - B\,(x)]^+}{\xi' - \xi}\right\} = -\varphi(x,\xi^-)$$

$$\frac{\partial^- F_\rho(x,\xi)}{\partial \xi} = \lim_{\xi'\to\xi} \frac{F_\rho(x,\xi') - F_\rho(x,\xi)}{\xi' - \xi} = 1 - \frac{1}{\rho}\varphi(x,\xi^-) = \frac{\rho - \varphi(x,\xi^-)}{\rho}$$

Because $F_\rho(x,\xi)$ is a concave function with respect to ξ, the one-sided derivatives are non-increasing. We also have:

$$\lim_{\xi\to+\infty} \frac{\partial^+ F_\rho((x,\xi)}{\partial \xi} = \lim_{\xi\to+\infty} \frac{\partial^- F_\rho((x,\xi)}{\partial \xi} = \frac{\rho - 1}{\rho}$$

$$\lim_{\xi\to-\infty} \frac{\partial^+ F_\rho((x,\xi)}{\partial \xi} = \lim_{\xi\to-\infty} \frac{\partial^- F_\rho((x,\xi)}{\partial \xi} = 1$$

Therefore, the maximum of $F_\rho(x,\xi)$ is attained with the argmax set being a closed bounded interval. The values of ξ in that set are characterized as the ones such that

$$\frac{\partial^+ F_\rho((x,\xi)}{\partial \xi} \leq 0 \leq \frac{\partial^- F_\rho((x,\xi)}{\partial \xi}$$

It implies that the values of ξ satisfy $\varphi(x,\xi^-) \leq \rho \leq \varphi(x,\xi)$, and thus $\phi_\rho(x)$ is the mean of the ρ-percent left tail.