U.S. Department
of Transportation
**National Highway
Traffic Safety
Administration**

Research Note

September 2001

Crash Prevention Boundary for Road Departure Crashes---Derivation

August Burgett and Kirsten Gunderson

Background

The concept of a Crash Prevention Boundary (CPB) was introduced in Ref. 1. In that case, the relationships that describe the CPB were developed for rear-end crashes. The crash prevention boundary is an analytically derived deterministic expression that separates driver performance into successful crash avoidance and unsuccessful crash avoidance. The underlying idea behind the analytical framework of a crash prevention boundary (CPB) is that for any given set of initial dynamic conditions, there is a subset of values of driver brake response time, t_b, and level of deceleration, d_F, which will result in crash avoidance. The corollary is that there is also a subset of values of these two variables that produce a crash. The CPB is a deterministic relationship that separates these two subsets of possibilities. This note develops the CPB equations for one family of road departure crashes. The family of road departure crashes addressed here consists of those crashes where the vehicle is traveling at constant speed on a straight road or a curve.

Context: Road Departure on a straight road

The first situation addressed in this development is pictured in Figure 1. In this situation, a driver is traveling at constant speed a straight road, but at an angle ?, with respect to the centerline of the road. At some point the driver applies a steering force to prevent the vehicle from leaving the roadway. Note that for this development, the roadway includes the shoulder, if one is present. For convenience in developing the relationships, the time at which the vehicle crosses the lane edge of the outermost lane of travel is defined as the zero time reference, t=0. The crash prevention performance of the driver, i.e. action that prevents the vehicle from leaving the roadway, is described by the time after crossing the lane edge at which the driver begins a steering maneuver, t_s, and the level of lateral acceleration, a_L, that is generated by the steering maneuver. The CPB describes the relationship between these two performance parameters for the limiting case between road departure and no road departure. For the purpose of this development, a_L is considered to be a constant

Derivation of CPB

As suggested in the CPB background, the CPB is a relationship between driver response time and level of effort that separates crash avoidance performance from unsuccessful crash avoidance performance. Thus the derivation follows a path that seeks the relationships that occur for performance on their boundary. Before the driver begins to steer, i.e. $t<t_s$, $a_L=0$. After the driver begins to steer, i.e. for $t>t_s$, it is assumed that the vehicle experiences a constant level of lateral acceleration.

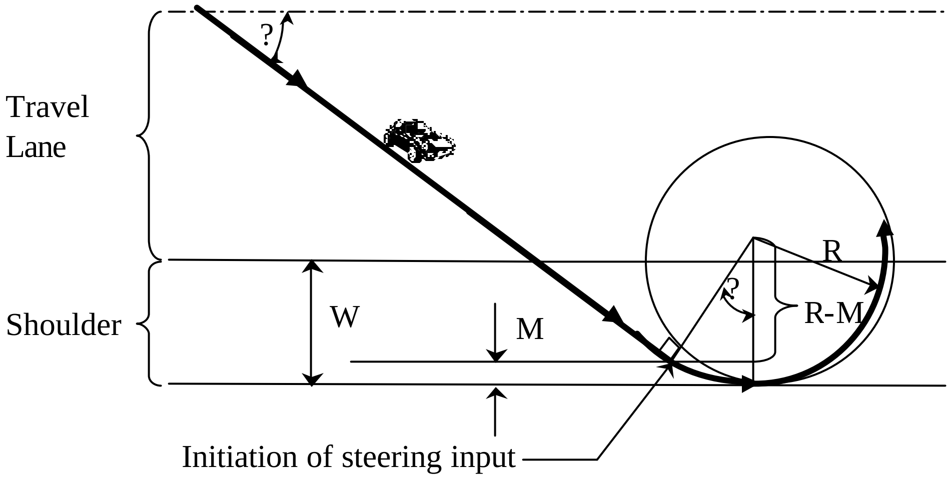

Figure 1. Road Departure Prevention Control and Trajectory

If the combination of t_s and a_L satisfies the CPB relationship, the vehicle will travel in a circular (approximately) trajectory that will take the outside front wheel to the edge of the roadway. Better performance, i.e. shorter t_s or larger a_L, will result in the vehicle staying further away from the edge of the roadway. Poorer performance, i.e. larger t_s or smaller a_L, will result in the vehicle leaving the roadway.

After the driver begins to steer, the value of lateral acceleration is given by

$$a_L = \frac{V^2}{R} \qquad t>t_s \qquad (1)$$

As can be seen in Figure 1, for the limiting conditions that result in the front wheel just touching the edge of the roadway, the value of R must be such that:

$$\frac{R-M}{R} = \cos y_0 \qquad (2)$$

Where, $?_0$ is the value of the angular location, ?, of the initial point of the circular motion and M is the distance from the edge of the roadway at $t = t_s$.

The relationship between the value of R and the value of t_s can be determined from the relationship that describes the value of M; where W is the width of the shoulder:

$$M = W - V \cdot t_s \cdot \sin q \qquad (3)$$

and the trigonometric equality:

$$q = y_0 \qquad (4)$$

These expressions can be combined to form the relationship between a_L and t_s for performance on the boundary between successful and unsuccessful crash avoidance. The key step in this process is finding the relationship between the radius of the circular path, velocity and other pertinent variables. From equation 2 it is seen that $R = \dfrac{M}{1-\cos y_0}$ which, when combined with equations (1), (3), and (4) produces the relationship that describes the CPB:

$$a_L = \frac{V^2 [1 - \cos q]}{W - Vt_S \cdot \sin q} \tag{5}$$

Conversely equation (5) can be rearranged to isolate t_S in which case the relationship can also be written as:

$$t_S = \frac{1}{\sin q} \left[\frac{W}{V} - \frac{V}{a_L}(1 - \cos q) \right] \tag{6}$$

<u>Extension of perspective on CPB</u>

The relationships above use the time, relative to crossing the outer lane edge, at which the driver chooses to begin steering. Another perspective is to use the time-to-road-departure (TRD) immediately before the initiation of steering as the indicator of driver performance. The extension of equation (5) or (6) to the TRD frame of reference is straight forward. At the instant before the driver begins to steer, the TRD is denoted by TRD_S and is given by:

$$TRD_S = \frac{M}{V \cdot \sin q} = \frac{W - V \cdot t_S \cdot \sin q}{V \cdot \sin q} \tag{7}$$

This expression can be rearranged to show that:

$$t_s = \frac{W}{V \cdot Sinq} - TRD_S \tag{8}$$

Substitution of equation (8) into equation (5) results in the following TRD_S-based expressions for the CPB.

$$a_L = \frac{V(1 - \cos q)}{TRD_S \cdot \sin q} \tag{9}$$

An example of the shape of the CPB is shown in Figure 2 for the t_S-based relationship and in Figure 3 for the TRD_S-based relationship.

<u>Context: Road Departure on a curve</u>

The second situation addressed in this development is pictured in Figure 4. In this situation, a driver is approaching a curve at constant speed and with no angle with respect to the centerline of the road. After passing the beginning of the curved section, coincident with the line labeled R_r, the driver's trajectory points toward the edge of the road. The curve is assumed to be circular and tangent to the approach section, which is straight. At some point along this trajectory the driver applies a steering force to prevent the vehicle from leaving the roadway. Note that for this development, the roadway includes the shoulder, if one is present. For convenience in developing the relationships, the time at which the vehicle crosses the lane edge of the outermost lane of travel is defined as the zero time reference, t=0.

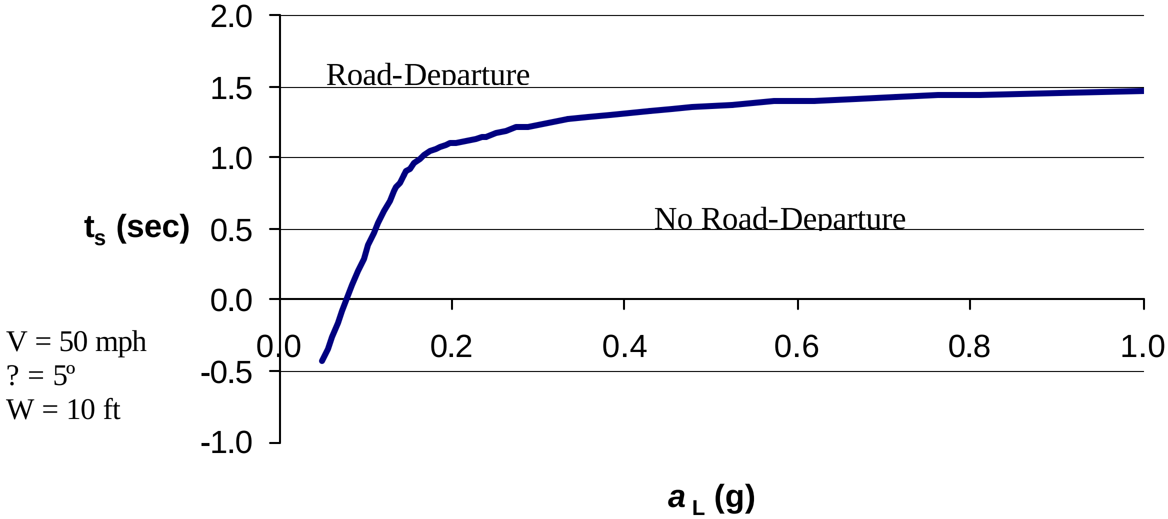

Figure 2, Example of t_s-based Crash Prevention Boundary for Road-Departure Crashes on a straight road

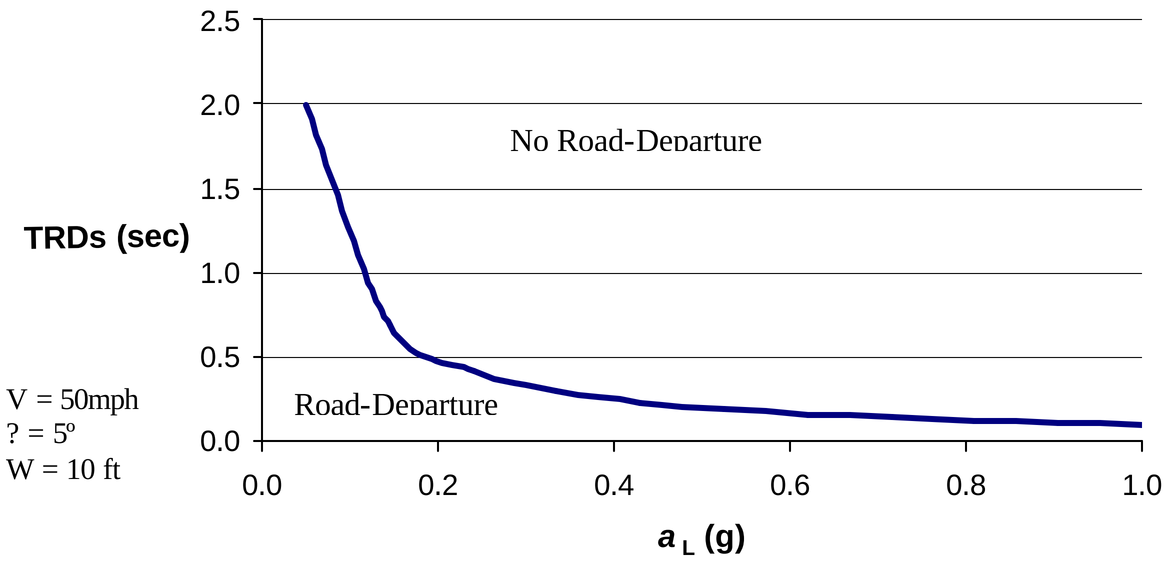

Figure 3, Example of TRD_s-based Crash Prevention Boundary for Road-Departure Crashes on a straight road

The location of the vehicle at t=0 is denoted as point 1 in Figure 4. The crash prevention performance of the driver, i.e. action that prevents the vehicle from leaving the roadway, is described by the time after crossing the lane edge at which the driver begins a steering maneuver, t_s, and the level of lateral acceleration, a_L, that is generated by the steering maneuver. The lateral acceleration is assumed to be constant. The CPB describes the relationship between these two performance parameters for the limiting case between road departure and no road departure.

The driver exits the roadway onto the shoulder, in a straight path, at point 1. The vehicle continues to travel in a straight pathway until it reaches point 2, at which time the driver applies a steering maneuver. For this development, the steering is assumed to be constant, making the path of travel circular. In the limiting case of no road departure, the outermost edge of the trajectory of vehicle travel will be tangent to the outer edge of the shoulder, denoted in Figure 4 as point 4. At this point the radius of the shoulder and the radius of the vehicle trajectory are collinear.

Definition of terms:

- R_r radius of curvature of road
- R_v radius of curvature of vehicle trajectory
- W width of the shoulder
- D_o initial offset of vehicle from edge of road
- D_1 distance from beginning of curve to point 1, along vehicle path

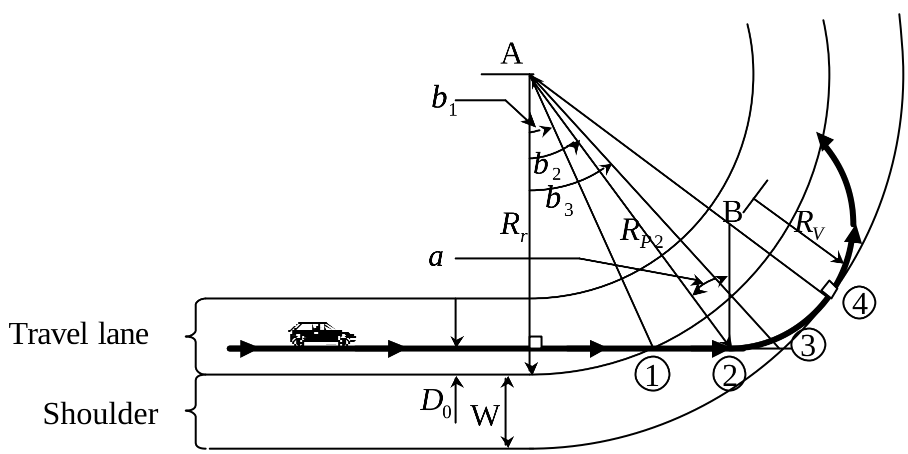

Figure 4. Geometry of road departure prevention on a curve

D_2 distance between points 1 and 2 $D_2=V*t_s$

D_3 distance from beginning of curve to outside edge of shoulder, along straight line extension of original direction of travel.

R_{P2} distance from center of curve to point 2

Angles of convenience

a

ß1: $\sin(ß_1) = D_1/R_r$

ß2: $\tan(ß_2) = (D_1 + D_2)/(R_r - D_o)$

ß3: $\sin(ß_3) = D_3/(R_r + W)$

Derivation of CPB

Before the driver begins to steer, i.e. $t<t_s$, $a_L=0$. After the driver begins to steer, i.e. for $t>t_s$, the vehicle experiences a constant level of lateral acceleration. If the combination of t_s and a_L satisfies the CPB relationship, the vehicle will travel in a circular (approximately) trajectory that will take the outside front wheel to the edge of the roadway.

After the driver begins to steer, the value of lateral acceleration is given by

$$a_L = \frac{V^2}{R_v} \qquad t>t_s \qquad (10)$$

The relationship of the radius of the vehicle trajectory at point 4 to other features of the motion is shown in Figure 4. As plotted in the derivation for road departure on a straight road, the key step in this process is finding the relationship between the radius of the path of the velocity and other pertinent variables. The length of the radius of the vehicle trajectory can be determined by applying the Law of Cosines to the triangle with vertices A, 2, and B, and angle $\angle A2B = a$. This produces the relationship:

$$(R_r + W - R_v)^2 = R_{P2}^2 + R_v^2 - 2 \cdot R_{P2} \cdot R_v \cdot \cos a \qquad (11a)$$

Rearranging and simplifying this expression yields:

$$R_v = \frac{(R_r + W)^2 - R_{P2}^2}{2[(R_r + W) - R_{P2} \cdot \cos a]} \qquad (11b)$$

It can also be seen from Figure 4 that:

$$a = b_2 \qquad (12a)$$

and

$$\cos b_2 = \frac{R_r - D_0}{R_{P2}} \qquad (12b)$$

Thus,

$$R_{P2} \cdot \cos a = (R_r - D_0) \qquad (12c)$$

Similarly the expression for $(R_{P2})^2$ is:

$$R_{P2}^2 = (R_r - D_0)^2 + (D_1 + V \cdot t_s)^2 \qquad (12d)$$

where

$$D_1^2 = R_r^2 - \left(R_r - D_0\right)^2 \qquad (12e)$$

Also note that

$$D_3^2 = \left(R_r + W\right)^2 - \left(R_r - D_0\right)^2 \qquad (12f)$$

Combining these expressions, and velocity relationships between D1, D3, and the basic parameters, provides the following equation for R_v:

$$R_v = \frac{D_3^2 - \left(D_1 + V \cdot t_s\right)^2}{2\left[\left(R_r + W\right) - \left(R_r - D_0\right)\right]} \qquad (13)$$

Combining equation (13) with equation (10) produces the expression for lateral acceleration on the CPB.

<u>Extension of perspective on CPB; Use of Time to Road Departure (TRD) as measure of driver performance.</u>

The relationships above use the time, relative to crossing the outer lane edge, at which the driver chooses to begin steering. Another perspective is to use the time-to-road-departure (TRD) at the time of initiation of steering as the indicator of driver performance. The extension of equations for the CPB to the TRD frame of reference is straight forward. At the instant before the driver begins to steer, the TRD is denoted by TRD_S and is given by:

$$TRD_s = \frac{D_3 - \left(D_1 + D_2\right)}{V} \qquad (14)$$

This expression can be rearranged to show that:

$$\left(D_1 + V \cdot t_s\right) = \left(D_3 - V \cdot TRD_s\right) \qquad (15)$$

Substitution of equation (15) into equation (13) results in the following TRD_s-based expression for R_v. This can be combined with equation (10) to form the CPB.

$$R_v = \frac{2\left(D_3 \cdot V \cdot TRD_s\right) - \left(V \cdot TRD_s\right)^2}{2\left[\left(R_r + W\right) - \left(R_r - D_0\right)\right]} \qquad (18)$$

Examples of CPB for crash prevention on a curve are shown in Figures 6 and 7.

Acknowledgment:
The authors appreciate the suggestion by Dr. David Smith for the idea of extending the derivation to include a Time-to-Road-Departure based formulation. Editorial assistance by Gowrishankar Srinivasan is also appreciated.

Reference

1. Burgett & Miller, "A New Paradigm for Rear-End Crash Prevention Driving Performance," SAE Paper No. 2001-01-0463, SAE 2001 World Congress, Detroit, March 2001

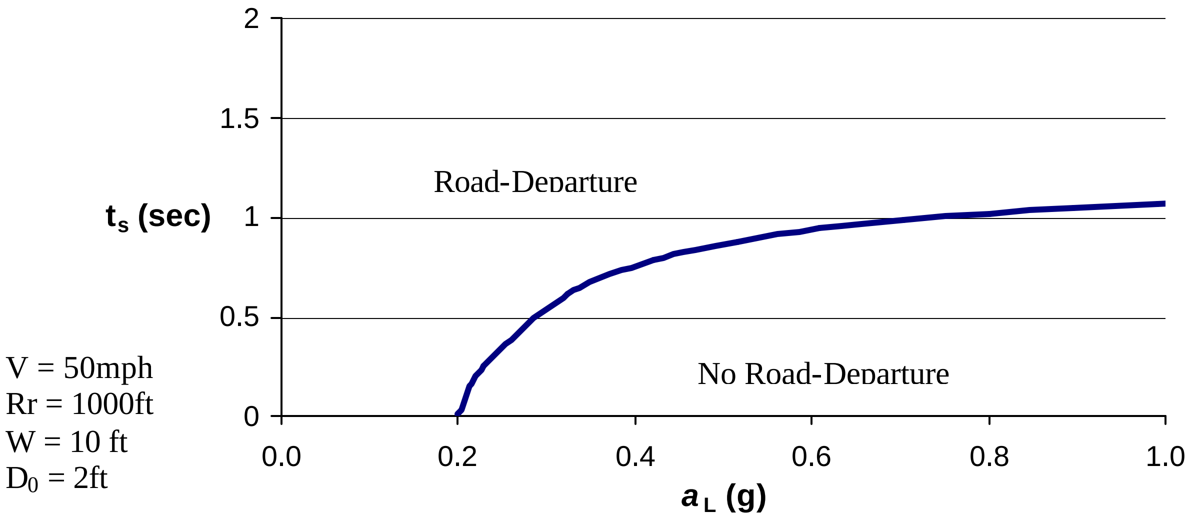

Figure 6 Example of t_s-based Crash Prevention Boundary for Road-Departure on a curve

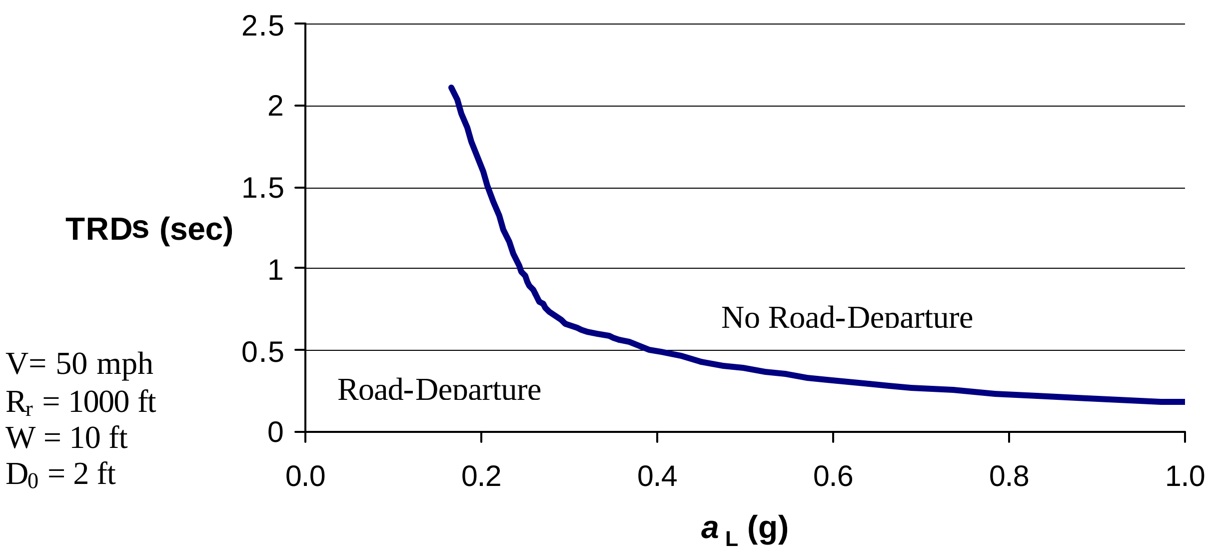

Figure 7 Example of TRDs-based Crash Prevention Boundary for Road-Departure on a curve

U.S. Department
of Transportation
**National Highway
Traffic Safety
Administration**
400 Seventh Street, S.W. , NRD-10
Washington, D.C. 20590

People Saving People
http://www.nhtsa.dot.gov