STATE PLANE COORDINATES BY AUTOMATIC DATA PROCESSING

PUBLICATION 62-4

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Foreword

The plane coordinate system used in this State is based on the Lambert conformal conic projection with two standard parallels for each zone. The tables in this publication are to be used for the conversion of geographic positions to plane coordinates or plane coordinates to geographic positions. The constants of the projection are listed with the tables.

These tables can be used with machine or hand-held electronic calculators. For more sophisticated hand-held electronic calculators and large computers, formulas given in Coast and Geodetic Survey Publication 62-4, State plane coordinates by automatic data processing, revised 1973, should be used. This document is sold by the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402, stock no. 003-002-00109-4.

The following formulas and sample computations show the general methods for computing either type of coordinates:

Plane Coordinates from Geographic Positions

$$x = R \sin \theta + C$$

$$y = R_b - R \cos \theta$$

Grid azimuth = geodetic azimuth - θ + second term

where

R is the radius for the latitude of the station,

R_b is a constant for a zone,

θ is the mapping angle for the longitude of the station,

C is the value of x assigned to the central meridian for a zone.

PREFACE

This publication has been prepared to meet continuing requests from public and private organizations for information for computing state plane coordinates on electronic data processing equipment. The necessary equations and constants for the various states and territories are included in this publication. However, no programming information is included. since there are many different types of equipment now in use and new models are constantly being developed. Anyone familiar with the basic trigonometric functions and his particular type of equipment should be able to program the computation with the information contained in this publication.

For valuable technical assistance in the preparation of this publication. the author wishes to thank Lansing G. Simmons. Chief Geodesist, Geodesy Division: Buford K. Meade. Chief. Triangulation Branch. Geodesy Division: Rosemary E. Riordan. Chief. Programming Branch. Electronic Computing Division: and other members of the Geodesy Division.

CONTENTS

INTRODUCTION

This publication contains tables and formulas for computing state plane coordinates by electronic data processing equipment. There are four different types of projection used in the plane coordinate systems of the United States and its territories. They are the Lambert conformal conic, transverse Mercator, oblique Mercator, and approximate azimuthal equidistant projections. These are all based on the Clarke Spheroid of 1866. The plane coordinates in all the projections, except the one used for Guam, are in U.S. Survey Feet (one U.S. Survey Foot equals 1200/3937 meter exactly).

For convenience in computing coordinates by desk calculator, projection tables were computed and published for the different state systems except Alaska. These tables contain the factors most difficult to compute, namely the radii of the parallels for the Lambert projection and values of y on the central meridian for the parallels on the transverse Mercator projection. These are tabulated at intervals of one minute of latitude. In Alaska, the published plane coordinates for 2½-minute intersections of meridians and parallels are used for converting geographic positions to plane coordinates and vice versa.

The formulas in this publication are the same as the ones used in computing the projection tables. This will insure that coordinates computed by means of the formulas will agree as closely as possible with coordinates computed with the use of the plane coordinate projection tables.

The theory of the various projections is not included in this publication. The essentials of the different projections are discussed separately in the text which follows. The equations for computing plane coordinates from geographic positions and the inverse (computing geographic positions from plane coordinates) are given under the appropriate heading. An explanation of the constants and their derivations are given in the appendix. The constants for the various zones are listed in tabular form at the end of this publication.

Although zones 2 through 9 in the Alaska plane coordinate system are on the transverse Mercator projection, the formulas for these zones are different from those used in the other states and are listed separately from them. Zone 1 of the Alaska plane coordinate system covers southeast Alaska and is on an oblique Mercator projection. The plane coordinate systems for Puerto Rico, Virgin Islands, St. Croix, and American

Samoa are computed on the Lambert conformal conic projection and use the same equations that are used for the states using that type of projection. The plane coordinate system for Guam is based on an approximate azimuthal equidistant projection. A separate projection has not been designed specifically for the District of Columbia. Within the District the plane coordinate projections for Maryland or Virginia may be used.

An arbitrary constant is added to all x' values (x' being the distance from the central meridian, + to the east, − to the west) to keep all x coordinates positive. If this arbitrary constant is not large enough, a point too far west of the central meridian can have a negative x value which is perfectly all right. Cases of this type actually exist in Florida on Dry Tortugas, and in Puerto Rico on Mona Island. In Maine, Monhegan Island is south of the design latitude of the east zone in which the island is located. Its y value is therefore negative. Here again, the fact that y is negative does not mean that it is incorrect. If any points are computed which are too far to the west of the central meridian or south of the lowest parallel of latitude for which the coordinate zone was intended, valid negative coordinates may be obtained, and can be used as long as the algebraic sign is taken into consideration.

Since 1957, an electronic computer has been used for mass computation of plane coordinates, using the formulas in this publication. The plane coordinates of the intersections of meridians and parallels in the United States, excluding Alaska and Hawaii, at intervals of 2½ minutes were computed previous to 1957 on more primitive data processing equipment, using the basic projection tables and essentially the same procedures as used with desk calculators.

State coordinates were originally computed from geographic positions rounded to three decimal places in seconds—equivalent to about 0.1 foot. Thus, azimuths and lengths, either geodetic or grid, could not be precisely determined by inverse computation between points close together.

This situation has been rectified by rounding geographic positions to 4, or even 5, decimal places in seconds which permits much greater precision in azimuth and length determination by inverse computation using either geographic positions or plane coordinates.

Owing to the limited number of terms used in converting geographic positions to plane coordinates, the values of the plane coordinates are not defined as precisely as they might be in an *absolute* sense. But no significant harm is done since the effect of the omitted terms is virtually the same on all coordinates in a limited area: thus, in a *relative* sense, the plane coordinates are well defined.

LAMBERT PROJECTION

The Lambert projections used in the plane coordinate systems of the United States are Lambert conformal conic projections with two standard parallels. The projection for American Samoa is a Lambert conformal conic projection with one standard parallel. The formulas given are adaptations of those used originally in computing the projection tables by which all of the plane coordinates were computed by desk calculator before 1957.

These equations are not the only ones that can be used in computing Lambert coordinates. They may not even be the best. However, they yield coordinates which check values computed using the projection tables within a hundredth of a foot (or at most two hundredths).

FORWARD COMPUTATION

Formulas.—The formulas for computing coordinates from geographic positions follow:

ϕ = latitude of station
λ = longitude of station

$$s \quad 101.27940\,65\{60(L_7 \quad \phi') + L_8 \quad \phi'' + [1{,}052.89388\,2$$

$$(4.48334\,4 \quad 0.02352\,0 \cos^2\phi)\cos^2\phi]\sin\phi\cos\phi\}$$

(ϕ' is the degrees and minutes of ϕ expressed in whole minutes)
(ϕ'' is the remainder of ϕ in seconds)

$$R \quad L_3 + sL_5\left\{1 + \left(\frac{s}{10^8}\right)^2\left[L_9 \quad \left(\frac{s}{10^8}\right)L_{10} + \left(\frac{s}{10^8}\right)^2 L_{11}\right]\right\}$$

$\theta^* = L_6(L_2 - \lambda)$
(θ and λ are in seconds)

*In Alaska, zone 10, for stations in east longitude $\theta = L_6(\lambda - 662.400.000000)$ (east λ entered with positive sign).

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$$x = L_1 + R \sin \theta$$

$$y = L_4 - R + 2R \sin^2 \frac{\theta}{2}$$

Convergence $= \theta$
Scale factor $= k$

$$k = \frac{L_6 R (1 - 0.00676\ 86580 \sin^2 \phi)^{1/2}}{20{,}925{,}832.16 \cos \phi}$$

The value of θ will not be greater than 5° 07′ except in Alaska, zone 10, where it will not be greater than 9° 34′.

The constants (L's) are listed for the various states and zones at the end of this publication. The rectifying latitude of the central parallel is divided into two parts, L_7 and L_8. The degrees and minutes in whole minutes are represented by L_7. The remainder in seconds is represented by L_8. This separation is made because the equations were designed for an electronic computer which had a fixed word capacity of only 10 figures and the rectifying latitude expressed in seconds would be 11 figures.

Sample computations to help clarify the equations and to indicate the number of decimal places to carry in the different terms follow.

Example.—The computation of the plane coordinates of a point in zone 10 of the Alaska plane coordinate system is given as an example.

Given: Latitude, ϕ = 54° 27′ 30″.00000
Longitude, λ = 164° 02′ 30″.00000 W (= 590,550″.00000)

Problem is to compute:

$\left.\begin{matrix} x \\ y \end{matrix}\right\}$ State plane coordinates

Convergence, θ
Scale factor, k

From the table of constants:

$L_1 = 3{,}000{,}000.000$
$L_2 = 633{,}600.00000$
$L_3 = 15{,}893{,}950.36$
$L_4 = 16{,}564{,}628.77$
$L_5 = .99984\ 80641$
$L_6 = .79692\ 23940$

$L_7 = 3{,}161$
$L_8 = 47.87068$
$L_9 = 3.79919$
$L_{10} = 5.91550$
$L_{11} = 44$

$$\phi' = 60 \times 54 + 27$$

$$= 3{,}267$$

$$\phi'' = 30.00000$$

$\sin\phi = 0.81369\ 30040$

$\cos\phi = 0.58129\ 48436$

$$s = 101.27940\ 65\{60(3{,}161 - 3{,}267) + 47.87068$$
$$- 30.00000 + [1{,}052.89388\ 2 - (4.48334\ 4$$
$$- 0.02352\ 0 \times 0.58129\ 5^2) \times 0.58129\ 48436^2] \times 0.81369\ 30040$$
$$\times 0.58129\ 48436\}$$
$$= -591{,}960.9632$$

$$R = 15{,}893{,}950.36 + (-591{,}960.9632)(0.99984\ 80641)\Bigg\{1$$
$$+ \left(\frac{-591{,}960.9632}{10^8}\right)^2 \left[3.79919 - \left(\frac{-591{,}961}{10^8}\right) \times 5.91550\right.$$
$$\left. + \left(\frac{-591{,}961}{10^8}\right)^2 \times 44\right]\Bigg\}$$
$$= 15{,}893{,}950.36 - 591{,}871.0231\ [1$$
$$+ 0.00003\ 50417\ 8(3.79919 + 0.03501\ 7 + 0.00154\ 2)]$$
$$= 15{,}301{,}999.7826$$

$$\theta = 0.79692\ 23940(633{,}600.00000 - 590{,}550.00000)$$
$$= +34{,}307''.50906\ 17$$
$$= +9^\circ\ 31'47''.50906\ 17$$

$\sin\theta = +0.16556\ 16538\ 58$ $\quad\quad$ $\sin\dfrac{\theta}{2} = 0.08306\ 79189\ 55$

$$x = 3{,}000{,}000.0000 + 15{,}301{,}999.7826(+0.16556\ 16538\ 58)$$
$$= 5{,}533{,}424.3913 \text{ feet}$$

$$y = 16{,}564{,}628.77 - 15{,}301{,}999.7826$$
$$+ 2 \times 15{,}301{,}999.7826 \times 0.08306\ 79189\ 55^2$$
$$= 1{,}473{,}805.1278 \text{ feet}$$

Convergence $= \theta$

$$= +9^\circ\ 31'47''.50906$$

Scale factor $= k$

$$k=\frac{0.79692\,23940\times 15{,}301{,}999.78(1-0.00676\,86580\times 0.81369\,30040^2)^{1/2}}{20{,}925{,}832.16\times 0.58129\,48436}$$

$$=\frac{12{,}167{,}150.75}{12{,}164{,}078.33}$$

$$=1.00025\,26$$

INVERSE COMPUTATION

Formulas.—Formulas for computing geographic positions from plane coordinates follow:

$$\theta=\text{arc tan}\,\frac{x-L_1}{L_4-y}$$

$$\lambda^{**}=L_2-\frac{\theta}{L_6}$$

(θ and λ are in seconds)

$$R=\frac{L_4-y}{\cos\theta}$$

$$s_1=\frac{L_4-L_3-y+2R\sin^2\dfrac{\theta}{2}}{L_5}$$

$$s_2=\frac{s_1}{1+\left(\dfrac{s_1}{10^8}\right)^2L_9-\left(\dfrac{s_1}{10^8}\right)^3L_{10}+\left(\dfrac{s_1}{10^8}\right)^4L_{11}}$$

$$s_3=\frac{s_1}{1+\left(\dfrac{s_2}{10^8}\right)^2L_9-\left(\dfrac{s_2}{10^8}\right)^3L_{10}+\left(\dfrac{s_2}{10^8}\right)^4L_{11}}$$

$$s=\frac{s_1}{1+\left(\dfrac{s_3}{10^8}\right)^2L_9-\left(\dfrac{s_3}{10^8}\right)^3L_{10}+\left(\dfrac{s_3}{10^8}\right)^4L_{11}}$$

$\omega'=L_7-600$ (degrees and minutes of ω in whole minutes)

$\omega''=36{,}000+L_8-0.00987\,36755\,53\,s$ (remainder of ω in seconds)

$\omega=\omega'+\omega'$

**In Alaska, zone 10, if λ is greater than 180° it should be subtracted from 360° to give the proper east longitude.

$\phi' = L_7 - 600$ (degrees and minutes of ϕ in whole minutes)

$\phi'' = \omega'' + [1{,}047.54671\ 0 + (6.19276\ 0$

$+ 0.05091\ 2 \cos^2 \omega) \cos^2 \omega] \sin \omega \cos \omega$

(remainder of ϕ in seconds)

$\phi = \phi' + \phi''$

Example.—An example of the computation of latitude and longitude from plane coordinates follows. The constants of the equations are the same as the ones listed for the computation of plane coordinates from a geographic position.

Given: $x = 5{,}533{,}424.3913$ feet

$y = 1{,}473{,}805.1278$ feet

To compute: Latitude $(= \phi)$

Longitude $(= \lambda)$

$$\theta = \text{arc tan} \frac{5{,}533{,}424.3913 - 3{,}000{,}000.0000}{16{,}564{,}628.77 - 1{,}473{,}805.1278}$$

$$= \text{arc tan} \quad +0.16787\ 84704\ 78)$$

$$= +9°\ 31'47''.50906\ 13$$

$$= +34{,}307''.50906\ 13$$

$$\cos \theta = 0.98619\ 94416\ 81$$

$$\sin \frac{\theta}{2} = 0.08306\ 79189\ 54$$

$$\lambda = 633{,}600.00000 - \frac{+34{,}307.50906\ 13}{0.79692\ 23940}$$

$$= 590{,}550''.00000$$

$$= 164°\ 02'30''.00000$$

$$R = \frac{16{,}564{,}628.77 - 1{,}473{,}805.1278}{0.98619\ 94416\ 81}$$

$$= 15{,}301{,}999.7826$$

$$s_1 = \frac{16{,}564{,}628.77 - 15{,}893{,}950.36 - 1{,}473{,}805.1278 + 2 \times 15{,}301{,}999.7826 \times 0.08306\,79189\,54}{0.99984\,80641}$$

$$= -592{,}040.5296$$

$$s_2 = \frac{-592{,}040.5296}{1 + \left(\frac{-592{,}040.53}{10^8}\right)^2 \times 3.79919 - \left(\frac{-592{,}040.53}{10^8}\right)^3 \times 5.91550 + \left(\frac{-592{,}040.53}{10^8}\right)^4 \times 44}$$

$$= -591{,}960.94$$

$$s_3 = \frac{-592{,}040.5296}{1 + \left(\frac{-591{,}960.94}{10^8}\right)^2 \times 3.79919 - \left(\frac{-591{,}960.94}{10^8}\right)^3 \times 5.91550 + \left(\frac{-591{,}960.94}{10^8}\right)^4 \times 44}$$

$$= -591{,}960.96$$

$$s = \frac{-592{,}040.5296}{1 + \left(\frac{-591{,}960.96}{10^8}\right)^2 \times 3.79919 - \left(\frac{-591{,}960.96}{10^8}\right)^3 \times 5.91550 + \left(\frac{-591{,}960.96}{10^8}\right)^4 \times 44}$$

$$= -591{,}960.9633$$

$$\omega' = 3{,}161 - 600$$
$$= 2{,}561'$$

$$\omega'' = 36{,}000.00000 + 47.87068 - 0.00987\,36755\,53(-591{,}960.9633)$$
$$= 41{,}892''.70117$$

$$\omega = 2{,}561' + 41{,}892''.70117$$
$$= 42°\ 41' + 11°\ 38'12''.70117$$
$$= 54°\ 19'12''.70117$$

$$\sin\omega = 0.81228\,91544$$

$$\cos\omega = 0.58325\,49439$$

$$\phi' = 3{,}161 - 600$$
$$= 2{,}561'$$

$$\phi'' = 41{,}892.70117 + [1{,}047.54671\,0 + (6.19276\,0 + 0.05091\,2 \times 0.58325\,494^2) \times 0.58325\,494^2] \times 0.81228\,91544 \times 0.58325\,49439$$
$$= 42{,}390''.00000$$

$$\phi = 2{,}561' + 42{,}390''.00000$$
$$= 42°\ 41' + 11°\ 46'30''.00000$$
$$54°\ 27'30''.00000$$

TRANSVERSE MERCATOR PROJECTION

The formulas used for the transverse Mercator projections for the state plane coordinate systems, excepting those for Alaska, are not strictly exact. However, they are sufficiently exact for the purpose intended. The absolute values of the coordinates of a point may not be quite correct, but the differences between stations in any particular area will be sufficiently correct for all practical purposes. The computation of plane coordinates of all points computed before the appearance of the electronic computer was by a logarithmic method and later a machine method based on the logarithmic method.

Therefore, the following equations are based on the logarithmic method of computation, so that the original plane coordinate computations can be reproduced as closely as possible.

FORWARD COMPUTATION

Formulas.—The following formulas are used for computing plane coordinates from geographic positions.

ϕ = latitude of station

λ = longitude of station

$$S_1=\frac{30.92241\ 724\cos\phi}{(1-0.00676\ 86580\sin^2\phi)^{1/2}}\left[T_2-\lambda-3.9174\left(\frac{T_2-\lambda}{10^4}\right)^3\right]\qquad(\lambda\text{ in seconds})$$

$$S_m=S_1+4.0831\left(\frac{S_1}{10^5}\right)^3$$

$$x=T_1+3.28083\ 333\ S_mT_5+\left(\frac{3.28083\ 333\ S_mT_5}{10^5}\right)^3T_6$$

(T_1 and x will be in the millions in New Jersey; x may be in the millions in certain other states.)

$$\phi_1=\phi+\frac{25.52381}{10^{10}}S_m^2(1-0.00676\ 86580\sin^2\phi)^2\tan\phi\ (\phi\text{ in seconds})$$

$$\phi_2 = \phi + \frac{25.52381}{10^{10}} S_m^2 \, (1 - 0.00676\,86580 \sin^2 \phi_1)^2 \tan \phi_1$$

$$y = 101.27940\,65\, T_5 \, \{60(\phi' - T_3) + \phi'' - T_4 - [1{,}052.89388\,2$$
$$- (4.48334\,4 - 0.02352\,0 \cos^2 \phi_2) \cos^2 \phi_2] \sin \phi_2 \cos \phi_2\}$$

$$\left\{\begin{array}{l} \phi' \text{ is degrees and minutes of } \phi_2 \text{ in whole minutes} \\ \phi'' \text{ is the remainder of } \phi_2 \text{ in seconds} \end{array}\right\}$$

$$\Delta\alpha = (T_2 - \lambda) \left[\sin \frac{\phi + \phi_2}{2} + \frac{1.9587}{10^{12}} (T_2 - \lambda)^2 \sin \frac{\phi + \phi_2}{2} \cos^2 \frac{\phi + \phi_2}{2} \right] \qquad (\lambda \text{ in seconds})$$

$$k = T_5 \left[\quad + \frac{(1 + 0.00681\,47849 \cos^2 \phi)^2}{881.74916\,2\, T_5^2} \left(\frac{x - T_1}{10^6} \right)^2 \right]$$

The constants (T's) are listed for the various states and zones at the end of this publication. The rectifying latitude of the latitude of the origin is divided into two parts, T_3 and T_4. The degrees and minutes in whole minutes are represented by T_3. The remainder in seconds is represented by T_4. This separation is made because the equations were designed for an electronic computer which had a fixed word capacity of only 10 figures and the rectifying latitude expressed in seconds would be 11 figures.

Sample computations to help clarify the equations and to indicate the number of decimal places to carry in the different terms follow.

Example.—The computation of the plane coordinates of triangulation station *Indian* 1947 in the west zone of Idaho is given as an example.

Given: Latitude, ϕ = 48° 07′50.″94100
Longitude, λ = 116° 22′02.″59200 (= 418,922.″59200)

Problem is to compute:

x, y } State plane coordinates
Convergence, $\Delta\alpha$
Scale factor, k

From the table of constants:

T_1 = 500.000.000	T_4 = 18.35156
T_2 = 416.700.00000	T_5 = .99993 33333
T_3 = 2.491	T_6 = .38062 27

$\sin\phi = 0.74467\ 0637$

$\cos\phi = 0.66743\ 21255$

$\tan\phi = .11572\ 489$

$$\frac{30.92241\ 724 \times 0.66743\ 21255}{-0.00676\ 86580 \times 0.74467\ 06371^2)^{1/2}} \left[416.700.00000 \quad 418.922.59200 - 3.9174\left(\frac{416.700.000 - 418.922.592}{10^4}\right)^3\right]$$

$$= \frac{20.63861\ 466}{0.99812\ 15089}\left[-2.222.59200 - 3.9174\left(\frac{-2.222.592}{10^4}\right)^3\right]$$

$$= -45.956.6613$$

$$-45.956.6613 + 4.0831\left(\frac{-45.956.6613}{10^5}\right)^3$$

$$= -45.957.0576$$

$$x = 500.000.000 + 3.28083\ 333(-45{,}957.0576)(0.99993\ 33333)$$

$$+\left(\frac{3.28083\ 333(-45.957.0576) \times 0.99993\ 33333}{10^5}\right)^3 \times 0.38062\ 27$$

$$= 500{,}000.000 - 150{,}767.3945 - 1.3044$$

$$= 349{,}231.301 \text{ feet}$$

$$\phi_1 = 173{,}270''\!.94100 + \frac{25.52381}{10^{10}}(-45{,}957.0576)^2(1$$

$$-0.00676\ 86580 \times 0.74467\ 064^2)^2 \times 1.11572\ 489$$

$$= 173{,}270''\!.94100 + 5.39075\ 9 \times 1.10736\ 50$$

$$= 173{,}276''\!.91054$$

$\sin\phi_1 = 0.74468\ 995$

$\tan\phi_1 = 1.11578\ 987$

$$\phi_2 = 173{,}270''\!.94100 + \frac{25.52381}{10^{10}}(-45{,}957.0576)^2(1$$

$$-0.00676\ 86580 \times 0.74468\ 995^2)^2 \times 1.11578\ 987$$

$$173.270''\!.94100 + 5.39075\ 9 \times 1.10742\ 90$$

$$= 173.276''\!.91088$$

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$\sin\phi_2 = 0.74468\ 99541$

$\cos\phi_2 = 0.66741\ 05725$

$$y = 101.27940\ 65(0.99993\ 33333)\{60(2{,}887 - 2{,}491) + 56.91088 - 18.35156 - [1{,}052.89388\ 2 - (4.48334\ 4 - 0.02352\ 0 \times 0.66741\ 06^2) \times 0.66741\ 057^2] \times 0.74468\ 99541 \times 0.66741\ 05725\}$$

$$= 101.27265\ 45[23{,}760.00000 + 38.55932 - (1{,}052.89388\ 2 - 4.47286\ 7 \times 0.44543\ 687) \times 0.49701\ 39486]$$

$$= 2{,}357{,}247.281 \text{ feet}$$

Computation of convergence, $\Delta\alpha$

$\phi = 48°\ 07'50''.941$

$\phi_2 = 48°\ 07'56''.911$

$\dfrac{\phi+\phi_2}{2} = 48°\ 07'53''.926$

$\sin\dfrac{\phi+\phi_2}{2} = 0.74468\ 030$

$\cos\dfrac{\phi+\phi_2}{2} = 0.66742\ 135$

$$\Delta\alpha = (416{,}700.000 - 418{,}922.592)\left[0.74468\ 030 + \frac{1.9587}{10^{12}}(416{,}700.000 - 418{,}922.592)^2 \times 0.74468\ 030 \times 0.66742\ 135^2\right]$$

$$= -2{,}222.592(0.74468\ 351)$$

$$= -1{,}655''.13$$

$$= -0°\ 27'35''.13$$

Computation of scale factor, k

$$k = 0.99993\ 33333\left[1 + \frac{(1 + 0.00681\ 47849 \times 0.66743\ 213^2)^2}{881.74916\ 2 \times 0.99993\ 33333^2}\left(\frac{349{,}231.301 - 500{,}000.000}{10^6}\right)^2\right]$$

$$= 0.99993\ 33333\left[1 + \frac{1.00608\ 0720}{881.63159\ 9}(-0.15076\ 8699)^2\right]$$

$= 0.99993\ 33333(1 + 0.00002\ 5940)$

$= 0.99995\ 927$

INVERSE COMPUTATION

Formulas.—The formulas for computing geographic positions from e coordinates follow:

$$S_{g_1} = x - T_1 - T_6\left(\frac{x - T_1}{10^5}\right)^3$$

$$S_m = \frac{0.30480\ 06099}{T_5}\left[x - T_1 - T_6\left(\frac{S_{g_1}}{10^5}\right)^3\right]$$

$$\omega' = T_3$$

$$\frac{\qquad\qquad}{T_5}\ y \qquad \text{(remainder of } \omega \text{ in seconds)}$$

$$\omega = \omega' + \omega'$$

$$(\phi')' = T_3 \qquad \text{(degrees and minutes of } \phi \text{ and } \phi' \text{ in whole minutes)}$$

$$= \omega'' + [1{,}047.5467\ 0 + (6.19276\ 0$$

$$+ 0.05091\ 2\ \cos^2\omega)\cos^2\omega]\ \sin\omega\cos\omega \qquad \text{(remainder of } \phi' \text{ in seconds)}$$

$$\phi' = (\phi')' + (\phi')''$$

$$(\phi)'' = (\phi')'' - 25.52381(1 - 0.00676\ 86580\ \sin^2\phi')^2\left(\frac{S_m}{10^5}\right)^2 \tan\phi'$$

$$\phi = (\phi')' + (\phi)''$$

$$S_a = S_m - 4.0831\left(\frac{S_m}{10^5}\right)^3$$

$$S_1 = S_m - 4.0831\left(\frac{S_a}{10^5}\right)^3$$

$$\Delta\lambda_1 = \frac{S_1(1 - 0.00676\ 86580\ \sin^2\phi)^{1/2}}{30.92241\ 724\ \cos\phi}$$

$$\Delta\lambda_a = \Delta\lambda_1 + 3.9174\left(\frac{\Delta\lambda_1}{10^4}\right)^3$$

$$\lambda'' = T_2 - \Delta\lambda_1 - 3.9174\left(\frac{\Delta\lambda_a}{10^4}\right)^3$$

Example.—An example of the computation of latitude and longitude from plane coordinates follows. The constants of the equations are obtained from the list used in the computation of plane coordinates from a geographic position.

Given: $x = 349.231.301$ feet

$y = 2.357.247.281$ feet

To compute: Latitude $(=\phi)$

Longitude $(=\lambda)$

$$S_{g_1} = 349.231.301 - 500.000.000 - 0.38062\,27\left(\frac{349.231.301 - 500.000.000}{10^5}\right)^3$$

$$= -150.768.699 - 0.38062\,27(-3.42715\,4)$$

$$= -150.767.395$$

$$\frac{0.30480\,06099}{0.99993\,33333}\left[349.231.301 - 500.000.000 - 0.38062\,27\left(\frac{-150.767.395}{10^5}\right)^3\right]$$

$$= 0.30482\,09313(-150.768.699 + \ .304)$$

$$-45.957.0578$$

$$\omega' = 2.491'$$

$$18.35156 + \frac{0.00987\,36755\,53}{0.99993\,33333} \times 2.357.247.281$$

$$18.35156 + 23.276.24660$$

$$= 23.294''59816$$

$$2,491' + 23.294''59816$$

$$= 41°\,31' + 6°\,28'14''59816$$

$$= 47°\,59'14''59816$$

$$\sin\omega = 0.74299\,75222$$

$$\cos\omega = 0.66929\,41670$$

$(\phi')' = 2.491$

$$(\phi')'' = 23.294.59816 + [1.047.54671\ 0 + (6.19276\ 0 + 0.05091\ 2 \times 0.66929\ 42^2) \times 0.66929\ 4167^2] \times 0.74299\ 75222 \times 0.66929\ 41670$$

$$= 23.294.59816 + [1.047.5467\ \ 0 + (6.21556\ 6)(0.44795\ 468)] \times 0.49728\ 39077$$

$$= 23.294.59816 + 522.31271$$

$$= 23.816''91087$$

$$\phi' = 2.491' + 23.816''91087$$

$$= 41°\ 31' + 6°\ 36'56''91087$$

$$= 48°\ 07'56''91087$$

$$\sin\phi' = 0.74468\ 995$$

$$\tan\phi' = 1.11578\ 987$$

$$(\phi)'' = 23.816.91087 - 25.52381(1 - 0.00676\ 86580 \times 0.74468\ 995^2)^2\left(\frac{-45.957.0578}{10^5}\right)^2 \times 1.11578\ 987$$

$$= 23.816.91087 - 28.47921(1 - 0.00375\ 36481)^2(+0.21120\ 51)$$

$$= 23.816.91087 - 5.96988$$

$$= 23.810''94099$$

$$\phi = 2.491' + 23.810''94099$$

$$= 41°\ 31' + 6°\ 36'50''94099$$

$$= 48°\ 07'50''94099$$

$$S_a = -45.957.0578 - 4.0831\left(\frac{-45.957.0578}{10^5}\right)^3$$

$$= -45.956.6615$$

$$S_1 = -45.957.0578 - 4.0831\left(\frac{-45.956.6615}{10^5}\right)^3$$

$$= -45.956.6615$$

$$\sin\phi = 0.74467\ 06371$$

$$\cos\phi = 0.66743\ 21255$$

$$\Delta\lambda_1 = \frac{-45.956.6615(1 - 0.00676\ 86580 \times 0.74467\ 06371^2)^{1/2}}{30.92241\ 724 \times 0.66743\ 21255}$$

$$= \frac{-45.956.6615 \times 0.99812\ 15089}{20.63861\ 466}$$

$$= -2.222.54900$$

$$\Delta\lambda_{ii} = -2.222.54900 + 3.9174\left(\frac{-2.222.549}{10^4}\right)^3$$

$$= -2.222.54900 - 0.04301$$

$$-2.222.59201$$

$$\lambda'' = 416.700.00000 - (-2.222.54900) - 3.9174\left(\frac{-2.222.592}{10^4}\right)^3$$

$$= 416.700.00000 + 2.222.54900 + 0.04301$$

$$= 418.922''.59201$$

$$\lambda = 116^\circ\ 22'02''.59201$$

ALASKA PROJECTION

Zones 2 to 9

Constants and formulas for computing state plane coordinates in zones 2 to 9 of the Alaska plane coordinate system follow:

CONSTANTS

Zone	Code	C (feet)	Central Meridian (CM)
2	5002	500,000.000	511,200.00000 (= 142°)
3	5003	500,000.000	525,600.00000 (= 146°)
4	5004	500,000.000	540,000.00000 (= 150°)
5	5005	500,000.000	554,400.00000 (= 154°)
6	5006	500,000.000	568,800.00000 (= 158°)
7	5007	700,000.000	583,200.00000 (= 162°)
8	5008	500,000.000	597,600.00000 (= 166°)
9	5009	600,000.000	612,000.00000 (= 170°)

FORWARD COMPUTATION

Formulas.—The following formulas are used for computing plane coordinates from geographic positions.

$$x = C + \frac{1{,}017{,}862.150 \cos \phi}{(1 + 0.00681\,47849 \cos^2 \phi)^{1/2}} \left(\frac{CM'' - \lambda''}{10^4}\right) \Bigg[1$$

$$\frac{3.91740\,509}{10^4} \left(\frac{CM'' - \lambda''}{10^4}\right)^2 \left(-2 \cos^2 \phi - \frac{0.68147\,8}{10^2} \cos^4 \phi \right)$$

$$+ \frac{4.60382}{10^8} \left(\frac{CM'' - \lambda''}{10^4}\right)^4 (1 - 20 \cos^2 \phi + 23.6047 \cos^4 \phi$$

$$+ 0.4907 \cos^6 \phi) + \ldots \Bigg]$$

(It is possible for x to be in the millions.)

$$y = 101.26927\,8503 \Bigg[\phi'' - 193{,}900.05442\,0 - \Big(\;{,}052.89394\,3$$

$$- 4.48338\,6 \cos^2 \phi + \frac{2.3559}{10^2} \cos^4 \phi \Big) (1 - \cos^2 \phi)^{1/2} \cos \phi \Bigg]$$

(Continued)

$$+\frac{24{,}673.67480(1-\cos^2\phi)^{1/2}\cos\phi}{(1+0.00681\ 47849\cos^2\phi)^{1/2}}\left(\frac{CM''-\lambda''}{10^4}\right)^2\left[1\right.$$

$$+\frac{1.95870\ 3}{10^4}\left(\frac{CM''-\lambda''}{10^4}\right)^2\left(-1+6\cos^2\phi+\frac{6.13330\ 6}{10^2}\cos^4\phi\right.$$

$$\left.+\frac{1.8577}{10^4}\cos^6\phi\right)+\frac{1.5346}{10^8}\left(\frac{CM''-\lambda''}{10^4}\right)^4(1-60\cos^2\phi$$

$$\left.+117.75\cos^4\phi+4.089\cos^6\phi)+\ldots\right]$$

ϕ = latitude of station

ϕ'' = latitude expressed in seconds

λ'' = longitude of station in seconds

$$\Delta\alpha''=(1-\cos^2\phi)^{1/2}\left(\frac{CM''-\lambda''}{10^4}\right)\left[10{,}000.0000\right.$$

$$+7.83481\left(\frac{CM''-\lambda''}{10^4}\right)^2\left(\cos^2\phi+\frac{2.044}{10^2}\cos^4\phi\right.$$

$$\left.\left.+\frac{0.9}{10^4}\cos^6\phi\right)+\frac{0.3683}{10^2}\left(\frac{CM''-\lambda''}{10^4}\right)^4(3\cos^4\phi-\cos^2\phi)+\right]$$

$$k=0.9999\left[1+\frac{(1+0.00681\ 47849\cos^2\phi)^2}{881.57282\ 1}\left(\frac{x-C}{10^6}\right)^2+\right]$$

For approximate plane coordinate computations, the terms containing $\left(\frac{CM''-\lambda''}{10^4}\right)^4$ in the formulas for both x and y may be omitted. The omission of these terms will result in differences of not more than 0.125 foot in the computed x coordinate and/or 0.002 foot in the computed y coordinate when $(CM-\lambda)$ is $\pm 3°$. When $(CM-\lambda)$ is $\pm 2°$ the differences will be 0.017 foot in x and/or 0.0002 foot in y.

Example.—The computation of the plane coordinates of a point in zone 5 is given as an example.

✓ Given: Latitude, ϕ = 71° 00′00.″00000 (= 255,600.″00000)

5 is Longitude, λ = 155° 00′00.″00000 (= 558,000.″00000)

Problem is to compute:

$\left.\begin{matrix} x \\ y \end{matrix}\right\}$ State plane coordinates

Convergence, $\Delta\alpha$

Scale factor, k

From the table of constants:

$$C = 500.000.000$$
$$CM = 158° = 568,800''00000$$

$\cos\phi = 0.32556\ 81545$

$\cos^2\phi$ 0.10599 46232

$\cos^4\phi = 0.01123\ 5$

$\cos^6\phi$ 0.00119

$\dfrac{CM''-\lambda''}{10^4}$ 56.88000 0000 55.80000 0000

1.08000 0000

$\left(\dfrac{CM''-\lambda''}{10^4}\right)^2$ .16640 000

$\left(\dfrac{CM''-\lambda''}{10^4}\right)^4 = 1.36049$

$$x\ 500,000.000 + \frac{1,017.862.150 \times 0.32556\ 81545}{(1 + 0.00681\ 47849 \times 0.10599\ 46232)^{1/2}} (1.08000\ 0000) \Big[- \frac{3.91740\ 509}{10^4} (1.16640\ 000) \Big(1 - 2 \times 0.10599\ 4623$$
$$\frac{0.68147\ 8}{10^2} \times 0.01123\ 5\Big) + \frac{4.60382}{10^8} (1.36049)(1 - 20 \times 0.10599\ 5 + 23.6047 \times 0.01123\ 5 + 0.4907 \times 0.00119)\Big]$$

$$= 500.000.000 + \frac{357.894.1818}{1.00036\ 1100} \Big[\frac{4.56926\ 13}{10^4} (1 - 0.21198\ 925$$
$$0.00007\ 656) + \frac{6.2635}{10^8} (1 - 2.1199 + 0.2652 + 0.0006)\Big]$$

$$= 500{,}000.000 + 357{,}764.9929(1 - 0.00036\ 00277 - 0.00000\ 00535)$$

$$= 857{,}636.168 \text{ feet}$$

$$y = 101.26927\ 8503 \Big[255{,}600.00000 - 193{,}900.05442\ 0 \Big(\ ,052.89394\ 3 - 4.48338\ 6 \times 0.10599\ 462$$

$$+ \frac{2.3559}{10^2} \times 0.01123\ 5 \Big) (1 - 0.10599\ 46232)^{1/2} \times 0.32556\ 81545 \Big]$$

$$+ \frac{24{,}673.67480(1 - 0.10599\ 46232)^{1/2} \times 0.32556\ 81545}{(1 + 0.00681\ 47849 \times 0.10599\ 46232)^{1/2}} (1.16640\ 000) \Big[1$$

$$+ \frac{\ .95870\ 3}{10^4} (1.16640\ 0) \Big(-1 + 6 \times 0.10599\ 462 + \frac{6.13330\ 6}{10^2} \times 0.01123\ 5$$

$$+ \frac{1.8577}{10^4} \times 0.00119 \Big) \cdot \frac{.5346}{10^8} (1.3605)(1 - 60 \times 0.10599$$

$$+ \quad 7.75 \times 0.01123\ 5 + 4.089 \times 0.0012) \Big]$$

$$= 101.26927\ 8503(61{,}375.97866\ 5) + 8{,}855.97812 \Big[1$$

$$+ \frac{2.28463}{10^4} \qquad + 0.63596\ 77 + 0.00068\ 91 + 0.00000\ 02)$$

$$+ \frac{2.0878}{10^8} (1 - 6.359 + \quad .323 + 0.005) \Big]$$

$$= 6{,}215{,}501.0768 + 8{,}855.97812(1 - 0.00008\ 3010 - 0.00000\ 0084)$$

$$= 6{,}224{,}356.319 \text{ feet}$$

$$\Delta\alpha'' = (1 - 0.10599\ 46232)^{1/2} \times 1.08000\ 0000 \Big[10{,}000.0000$$

$$+ 7.83481 \times 1.16640 \Big(0.10599\ 5 + \frac{2.044}{10^2} \times 0.0112 + \frac{0.9}{10^4} \times 0.00119 \Big)$$

$$+ \frac{0.3683}{10^2} \times \quad .3605(3 \times 0.01124 - 0.10599) \Big]$$

$$= + \quad 02116\ 006 \times 10{,}000.97037$$

$$= +\ 10.212''592$$

$\Delta\alpha = +2°\ 50'12''.592$

$$k = 0.9999\Bigg[1 + \frac{(1+0.00681\ 47849\times 0.10599\ 46232)^2}{881.57282\ 1}\left(\frac{857.636.168-500.000.000}{10^6}\right)^2\Bigg]$$

$$= 0.9999(1+0.00014\ 5295)$$

$$= 1.00004\ 53$$

INVERSE COMPUTATION

Formulas.—The formulas for computing geographic positions from plane coordinates follow:

$$(\omega')'' = 193{,}900.05442\ 0 + 0.00987\ 46630\ 2498\ y$$

$$(\phi')'' = (\omega')'' + \left(1{,}047.54669\ 1 + 6.19301\ 1\cos^2\omega' + \frac{5.0699}{10^2}\cos^4\omega'\right)(1-\cos^2\omega')^{1/2}\cos\omega'$$

$$\phi'' = (\phi')'' - 233.97364\ 50\left(\frac{x-C}{10^6}\right)^2(1+0.00681\ 47849\cos^2\phi')^2\left(\frac{1}{\cos^2\phi'} - 1\right)^{1/2}\Bigg[1 - \frac{1.89056\ 040}{10^4}\left(\frac{x-C}{10^6}\right)^2\left(1.95911\ 13 + \frac{3}{\cos^2\phi'} + \frac{8.1359}{10^2}\cos^2\phi' + \frac{2.79}{10^4}\cos^4\phi'\right) + \frac{1.42969}{10^8}\left(\frac{x-C}{10^6}\right)^4(1 + 0.00681\ 47849\cos^2\phi')\left(15.5 + \frac{45}{\cos^4\phi'} - \frac{0.307}{\cos^2\phi'} + 1.53\cos^2\phi'\right)\Bigg]$$

$$\lambda'' = CM'' - 9{,}824.51307\ 2\,\frac{(1+0.00681\ 47849\cos^2\phi')^{1/2}}{\cos\phi'}\left(\frac{x-C}{10^6}\right)\Bigg[1 - \frac{3.78112\ 080}{10^4}(1+0.00681\ 47849\cos^2\phi')\left(\frac{x-C}{10^6}\right)^2\left(-1 + \frac{2}{\cos^2\phi'} + 0.00681\ 47849\cos^2\phi'\right) + \frac{4.28906\ 24}{10^8}(1$$

$$+0.00681\ 47849\cos^2\phi')^2\left(\frac{x-C}{10^6}\right)^4\left(1.054+\frac{24}{\cos^4\phi'}-\frac{20}{\cos^2\phi'}-\frac{1.36}{10^2}\cos^2\phi'\right)\Bigg]$$

For approximate computation of geodetic positions from plane coordinates, the terms containing $\left(\frac{x-C}{10^6}\right)^4$ in the formulas for both ϕ and λ may be omitted. The omission of these terms will result in differences of not more than 0.00014 second in the computed ϕ and/or 0.01492 second in the computed λ when $(CM-\lambda)$ is $\pm 3°$. When $(CM-\lambda)$ is $\pm 2°$ the differences will be 0.00002 second in ϕ and/or 0.00196 second in λ.

Example.—For an example of the computation of latitude and longitude from plane coordinates, the same point is used as in the forward computation. The constants are the same as in the forward computation, namely:

$$C=500{,}000.000$$

$$CM=158°=568{,}800''.00000$$

Given: $x=\ \ 857{,}636.168$ feet

$y=6{,}224{,}356.319$ feet

To compute: Latitude $(=\phi)$

Longitude $(=\lambda)$

$$(\omega')''=193{,}900.05442\ 0+0.00987\ 46630\ 2498\times 6{,}224{,}356.319$$

$$=255{,}363''.47561\ 8$$

$$\cos\omega'=0.32665\ 21687$$

$$\cos^2\omega'=0.10670\ 16393$$

$$\cos^4\omega'=0.01138\ 5$$

$$(\phi')''=255{,}363.47561\ 8+\left(1{,}047.54669\ 1+6.19301\ 1\times 0.10670\ 16+\frac{5.0699}{10^2}\times 0.01138\ 5\right)(1-0.10670\ 16393)^{1/2}\times 0.32665\ 21687$$

$$=255{,}363.47561\ 8+1{,}048.20807\ 2\times 0.94514\ 46242\times 0.32665\ 21687$$

$$=255{,}687''.09260\ 8$$

$\cos\phi' = 0.32516\ 88926$

$\cos^2\phi' = 0.10573\ 48087$

$\cos^4\phi' = 0.01117\ 9850$

$$\left(\frac{x-C}{10^6}\right) = \frac{857{,}636.168 - 500{,}000.000}{10^6}$$

$$= +0.35763\ 6168$$

$$\left(\frac{x-C}{10^6}\right)^2 = 0.12790\ 36287$$

$$\left(\frac{x-C}{10^6}\right)^4 = 0.01635\ 9338$$

$$\phi'' = 255{,}687.09260\ 8 - 233.97364\ 50\ (0.12790\ 36287)\ (1 + 0.00072\ 05600)^2 \left(\frac{1}{0.10573\ 48087} - 1\right)^{1/2} \Big[1 - \frac{1.89056\ 040}{10^4} \times 0.12790\ 3629 \left(1.95911\ 13 + \frac{3}{0.10573\ 48087} + \frac{8.1359}{10^2} \times 0.10573 + \frac{2.79}{10^4} \times 0.011\right) + \frac{1.42969}{10^8}\ (0.01635\ 93)\ (1 + 0.00072) \left(15.5 + \frac{45}{0.01117\ 98} - \frac{0.307}{0.1057} + 1.53 \times 0.106\right)\Big]$$

$$= 255{,}687.09260\ 8 - 29.92607\ 822 \times 1.00144\ 1639 \times 2.90819\ 9338 \Big[1 - \frac{0.24180\ 954}{10^4}\ (30.33198\ 1 + 0.00860\ 2 + 0.00000\ 3) + \frac{0.02340\ 6}{10^8}\ (15.5 + 4{,}025.1 - 2.9 + 0.2)\Big]$$

$$= 255{,}687.09260\ 8 - 87.15646\ 815\ (1 - 0.00073\ 36643 + 0.00000\ 09451)$$

$$= 255.687.09260\ 8 - 87.09260\ 7$$

$$= 255{,}600''00000\ 1$$

$$\phi = 71^\circ\ 00'00''00000$$

$$\lambda'' = 568{,}800.00000$$

$$-9{,}824.51307\ 2\,\frac{(1+0.00681\ 47849\times 0.10573\ 48087)^{1/2}}{0.32516\ 88926}(+0.35763\ 6168)\Bigg[1$$

$$-\frac{3.78112\ 080}{10^4}\times 1.00072\ 0560\times 0.12790\ 36287\Bigg(-1$$

$$+\frac{2}{0.10573\ 48087}+0.00072\ 06\Bigg)$$

$$+\frac{4.28906\ 24}{10^8}(1.00072\ 0560)^2\times 0.01635\ 9338\Bigg(1.054$$

$$+\frac{24}{0.01117\ 9850}-\frac{20}{0.10573\ 48}-\frac{1.36}{10^2}\times 0.106\Bigg)\Bigg]$$

$$=568{,}800.00000-10{,}809.35765\ 8\Bigg[1-\frac{0.48396\ 7547}{10^4}(-1$$

$$+18.91524\ 68+0.00072\ 06)+\frac{0.07026\ 738}{10^8}(1.054$$

$$+2{,}146.719-189.152-0.001)\Bigg]$$

$$=568{,}800.00000-10{,}809.35765\ 8\times 0.99913\ 43016$$

$$=557{,}999''.99998\ 6$$

$$\lambda = 154^\circ\ 59'59''.99999$$

ALASKA PROJECTION
Zone 1 (Code 5001)

The following constants are used in the formulas for computing in zone 1 of the Alaska plane coordinate system.

$A = 6{,}388{,}906.01513$

$B = 1.00029\ 97727\ 3$

$C = 0.00447\ 59913\ 1$

$D = \frac{Ak_c}{B} = 6{,}386{,}352.67013$

$F = -\sin \alpha_0 = 0.32701\ 55171\ 76$

$G = \cos \alpha_0 = 0.94501\ 89688\ 71$

$H = -\tan \alpha_0 = 0.34604\ 12203$

$I = \frac{Ak_c}{a} = 1.00157\ 73595$

$e = 0.08227\ 18542\ 23003$

$e^2 = 0.00676\ 86579\ 97291$

$\pi = 3.14159\ 26535\ 89793$

$\epsilon = 2.71828\ 18284\ 59045$

$k_c = 0.99990\ 00000\ 00$

$a = 6{,}378{,}206.40000$

$\lambda_0 = 101°\ 30'50''.51319 (= 365{,}450''.51319)$

FORWARD COMPUTATION

Formulas.—The following formulas are used for computing plane coordinates from geographic positions.

$$\mu = \log_\epsilon \tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right) - \frac{e}{2} \log_\epsilon \left(\frac{1 + e \sin \phi}{1 - e \sin \phi}\right)$$

$$P = \frac{\epsilon^{(B\mu - C)} - \epsilon^{-(B\mu - C)}}{2}$$

$$Q = \frac{\epsilon^{(B\mu - C)} + \epsilon^{-(B\mu - C)}}{2}$$

$$u = D \text{ arc tan} \frac{GP + F \sin B(\lambda - \lambda_0)}{\cos B(\lambda - \lambda_0)}$$ (factor multiplied by D is in radians)

$$v = \frac{D}{2} \log_\epsilon \frac{Q + FP - G \sin B(\lambda - \lambda_0)}{Q - FP + G \sin B(\lambda - \lambda_0)}$$

$$x = 3.28083\ 33333\ 3\ (-0.6u + 0.8v + 5{,}000{,}000.0000) \text{ feet}$$

$$y = 3.28083\ 33333\ 3\ (0.8u + 0.6v - 5{,}000{,}000.0000) \text{ feet}$$

Convergence:

$$\gamma = \arctan \frac{3Q \cos B(\lambda - \lambda_o) - 4P \sin B(\lambda - \lambda_o) - 4H}{4Q \cos B(\lambda - \lambda_o) + 3P \sin B(\lambda - \lambda_o) + 3H}$$

Scale factor:

$$k = \frac{I(1 - e^2 \sin^2 \phi)^{1/2} \cos (u/D)}{\cos \phi \cos B(\lambda - \lambda_o)} \qquad (u/D \text{ is in radians})$$

Example.—The computation of the plane coordinates of an arbitrary position in zone 1 is given as an example.

Given: Latitude, $\phi = 55°\ 00'00''.00000$
Longitude, $\lambda = 134°\ 00'00''.00000 (= 482{,}400''.00000)$

Problem is to compute:

$\left.\begin{matrix} x \\ y \end{matrix}\right\}$ State plane coordinates

Convergence, γ

Scale factor, k

$$\frac{\pi}{4} + \frac{\phi}{2} = 45° + 27°\ 30'00''.00000\ 0$$

$$= 72°\ 30'00''.00000\ 0$$

$$\log_e \tan 72°\ 30'00''.00000\ 0 = 1.15423\ 45536\ 1$$

$$\frac{1 + e \sin \phi}{1 - e \sin \phi} = 1.14452\ 64060\ 0$$

$$\log_e \left(\frac{1 + e \sin \phi}{1 - e \sin \phi}\right) = 0.13499\ 09322\ 30$$

$$\frac{e}{2} \log_e \left(\frac{1 + e \sin \phi}{1 - e \sin \phi}\right) = 0.00555\ 29771\ 49$$

$$\mu = 1.15423\ 45536\ 1 - 0.00555\ 29771\ 5$$

$$= 1.14868\ 15764\ 6$$

$$B\mu + C = 1.15350\ 19111\ 8$$

$$\epsilon^{(B\mu + C)} = 3.16927\ 20084\ 36$$

$$\epsilon^{-(B\mu+C)} = 0.31552\ 98747\ 91$$

$$P = 1.42687\ 10668\ 2$$

$$Q = 1.74240\ 09416\ 1$$

$$\lambda - \lambda_o = 482{,}400.00000\ 0 - 365{,}450.51319$$

$$= 116{,}949''.48681\ 0$$

$$B(\lambda - \lambda_o) = 116{,}984''.54507\ 7$$

$$= 32^\circ\ 29'44''.54507\ 7$$

$$\sin B(\lambda - \lambda_o) = 0.53723\ 64135\ 57$$

$$\cos B(\lambda - \lambda_o) = 0.84343\ 17020\ 06$$

$$u = 6{,}386{,}352.67013 \text{ arc tan } \frac{1.34842\ 02242\ 8 + 0.17568\ 46436\ 3}{0.84343\ 17020\ 06}$$

$$= 6{,}386{,}352.67013 \text{ arc tan } 1.80702\ 81971\ 7$$

$$= 6{,}386{,}352.67013 \times 1.06535\ 04835\ 7$$

$$= 6{,}803{,}703.90537$$

$$v = 3{,}193{,}176.33506\ 5 \log_\epsilon \frac{1.70131\ 13198\ 9}{1.78349\ 05633\ 3}$$

$$= 3{,}193{,}176.33506\ 5(-0.04717\ 31163\ 517)$$

$$= -150{,}632.07879$$

$$x = 3.28083\ 33333\ 3(-4{,}082{,}222.34322 - 120{,}505.66303 + 5{,}000{,}000.00000)$$

$$= 2{,}615{,}716.5328 \text{ feet}$$

$$y = 3.28083\ 33333\ 3(5{,}442{,}963.12430 - 90{,}379.24727 - 5{,}000{,}000.00000)$$

$$= 1{,}156{,}768.9366 \text{ feet}$$

Convergence:

$$\gamma = \text{arc tan}\ \frac{4.40878\ 8575 - 3.06626\ 8378 - 1.38416\ 4881}{5.87838\ 4767 + 2.29970\ 1284 + 1.03812\ 3661}$$

$$= \text{arc tan}\ (-0.00451\ 86346)$$

$$= -0°\ 15'32''.0$$

Scale factor:

$$u/D = 1.06535\ 0484 \text{ radians}$$

$$= 61°\ 02'24''.3112$$

$$k = \frac{1.00157\ 7360(1 - 0.00454\ 18376)^{1/2} \times 0.48419\ 758}{0.57357\ 644 \times 0.84343\ 17020}$$

$$= \frac{0.48385\ 877}{0.48377\ 255}$$

$$= 1.00017\ 82$$

INVERSE COMPUTATION

Formulas.—The formulas for computing geographic positions from plane coordinates follow:

$$u = -0.18288\ 03657\ 61x + 0.24384\ 04876\ 81y + 7{,}000{,}000.0000$$

$$v = 0.24384\ 04876\ 81x + 0.18288\ 03657\ 61y - 1{,}000{,}000.0000$$

$$R = \frac{\epsilon^{(v/D)} - \epsilon^{-(v/D)}}{2}$$

$$S = \frac{\epsilon^{(v/D)} + \epsilon^{-(v/D)}}{2}$$

$$\mu = \frac{1}{2B} \log_\epsilon \frac{S + FR + G \sin (u/D)}{S - FR - G \sin (u/D)} - \frac{C}{B} \qquad (u/D \text{ is in radians})$$

$$\chi_r = 2 \text{ arc tan } \epsilon^{\mu} - \frac{\pi}{2} \qquad (\chi_r \text{ and arc tan } \epsilon^{\mu} \text{ are in radians})$$

$$\phi_r = \chi_r + (0.00676\ 10325\ 71 + 0.00005\ 31722\ 05 \cos^2 \chi$$
$$+ 0.00000\ 05730\ 27 \cos^4 \chi$$
$$+ 0.00000\ 00071\ 28 \cos^6 \chi) \sin \chi \cos \chi \qquad (\phi_r \text{ is in radians})$$

$\phi'' = 206{,}264.80624\ 7\ \phi_r$

$$\lambda = \lambda_o + \frac{1}{B} \text{ arc tan } \frac{F \sin(u/D) - GR}{\cos(u/D)} \qquad (u/D \text{ is in radians})$$

Example.—An example of the computation of latitude and longitude from plane coordinates follows. The constants of the equations are obtained from the list at the beginning of this section dealing with the Alaska projection, zone 1.

Given: $x = 2{,}615{,}716.5328$ feet

$y = 1{,}156{,}768.9366$ feet

To compute: Latitude $(=\phi)$

Longitude $(=\lambda)$

$$u = -0.18288\ 03657\ 61 \times 2{,}615{,}716.5328$$

$$+ 0.24384\ 04876\ 81 \times 1{,}156{,}768.9366 + 7{,}000{,}000.0000$$

$$= 6{,}803{,}703.90539$$

$$v = 0.24384\ 04876\ 81 \times 2{,}615{,}716.5328$$

$$+ 0.18288\ 03657\ 61 \times 1{,}156{,}768.9366 - 1{,}000{,}000.0000$$

$$= -150{,}632.07878$$

$$v/D = \frac{-150{,}632.07878}{6{,}386{,}352.67013}$$

$$= -0.02358\ 65581\ 750$$

$$\epsilon^{(v/D)} = 0.97668\ 94305\ 55$$

$$\epsilon^{-(v/D)} = 1.02386\ 69209\ 6$$

$$R = -0.02358\ 87452\ 025$$

$$S = 1.00027\ 81757\ 575$$

$$u/D = \frac{6{,}803{,}703.90539}{6{,}386{,}352.67013}$$

$= 1.06535\ 04835\ 7$ radians

$= 61°\ 02'24''.31107\ 9$

$\sin(u/D) = 0.87495\ 86851\ 82$

$\cos(u/D) = 0.48419\ 75828\ 36$

$$\mu = 0.49985\ 01585\ 53 \log_e \frac{1.81941\ 68445\ 21}{0.18113\ 95069\ 940} - 0.00447\ 46499\ 32$$

$$= 0.49985\ 01585\ 53 \log_e 10.04428\ 50636 - 0.00447\ 46499\ 32$$

$$= 0.49985\ 01585\ 53 \times 2.30700\ 38223\ 7 - 0.00447\ 46499\ 32$$

$$= 1.14868\ 15764\ 6$$

$$\chi_r = 2 \text{ arc tan } \epsilon^{1.14868\ 15764\ 6} - \frac{\pi}{2}$$

$$= 2 \text{ arc tan } 3.15403\ 18174\ 3 - 1.57079\ 63267\ 949$$

$$= 0.95673\ 87821\ 85 \text{ radian } (= 54°\ 49'01''.53953\ 6)$$

$\sin\chi = 0.81731\ 68417\ 50$

$\cos\chi = 0.57618\ 84936\ 30$

$$\phi_r = 0.95673\ 87821\ 85 + (0.00676\ 10325\ 71 + 0.00001\ 76528\ 09$$
$$+ 0.00000\ 00631\ 59 + 0.00000\ 00002\ 61) \times 0.47092\ 85598\ 66$$
$$= 0.95993\ 10885\ 95 \text{ radian}$$

$$\phi'' = 206.264.80624\ 7 \times 0.95993\ 10885\ 95$$
$$= 198.000''.00000$$

$\phi = 55°\ 00'00''.00000$

$$\lambda = 101°\ 30'50''.51319$$
$$+ 0.99970\ 03171\ 07 \text{ arc tan } \left[\frac{0.32701\ 55171\ 76(0.87495\ 86851\ 82)}{0.48419\ 75828\ 36} - \frac{0.94501\ 89688\ 71(-0.02358\ 87452\ 025)}{0.48419\ 75828\ 36}\right]$$

$= 101° \ 30'50''.51319 + 0.99970\ 03171\ 07 \text{ arc tan } 0.63696\ 49282\ 501$

$= 101° \ 30'50''.51319 + 116{,}949''.48681$

$= 134° \ 00'00''.00000$

PUERTO RICO, VIRGIN ISLANDS, AND ST. CROIX PROJECTION

This is a Lambert projection in which the same formulas are used as in the state projections. The necessary constants are as follows:

$L_1 = 500.000.00$	$L_7 = 1088$
$L_2 = 239.160.0000$	$L_8 = 48.44933$
$L_3 = 63.542.221.66$	$L_9 = 3.82699$
$L_4 = 63.687.479.44$*	$L_{10} = 1.51030$
$L_5 = .99999\ 39449$	$L_{11} = 0$
$L_6 = .31288\ 82281$	

*63.787.479.44 for St. Croix

	Code
Puerto Rico and Virgin Islands	5201
St. Croix	5202

SAMOA PROJECTION

The plane coordinate projection for American Samoa is a Lambert projection and the same formulas are used as in the state projections. The necessary constants are as follows:

Code 5300

$L_1 = 500.000.00$	$L_7 = -851$
$L_2 = 612.000.0000$	$L_8 = -49.53291$
$L_3 = -82.312.234.65$	$L_9 = 3.82892$
$L_4 = -82.000.000.00$	$L_{10} = -1.16664$
$L_5 = .99999\ 99999$	$L_{11} = 0$
$L_6 = -.24643\ 52205$	

Enter latitudes with a negative sign.

Constant L_5 is actually exactly one, but it is entered this way to avoid a digit in the units place, thus conforming to the number of places carried in all the other projections.

GUAM PROJECTION

The plane coordinate projection for Guam approximates an azimuthal equidistant projection, one in which the azimuth at the origin to any point and the distance from the origin to that point are the same on the projection as they are on the ellipsoid. This is often called a tangent plane projection. Due to the limited size of Guam, this projection may, for all practical purposes, be considered a true azimuthal equidistant projection.

The origin of the projection is station *Agana Monument* 1945, the position of which is: latitude 13° 28′20″.87887 N., longitude 144° 44′55″.50254 E. The plane coordinates of the origin have been assigned the values, x equals 50,000 meters, y equals 50,000 meters so that the plane coordinates of all points on the island will be positive in sign.

The geographic position of *Agana Monument* given above is based on the Guam Datum of 1963 and Clarke Spheroid of 1866.

FORWARD COMPUTATION

Formulas.—The following formulas are used for computing plane coordinates from geographic positions.

$$x = 50{,}000.0000 + \frac{30.92241\ 724(\lambda'' - 521{,}095.50254)\cos\phi}{(1 - 0.00676\ 86580\sin^2\phi)^{1/2}}\ \text{meters}$$

$$\begin{aligned} y = 50{,}000.0000 + 30.87002\ 482[\phi'' - 48{,}263.28375\ 78 \\ - (1{,}052.89394\ 3 - 4.48338\ 6\cos^2\phi + 0.02355\ 9\cos^4\phi)\sin\phi\cos\phi] \\ + \left(\frac{x - 50{,}000.0000}{10^4}\right)^2 \frac{(\tan\phi)(1 - 0.00676\ 86580\sin^2\phi)^{1/2}}{0.12756\ 4128}\ \text{meters} \end{aligned}$$

Example.—To compute the plane coordinates of the position:

$$\phi = 13^\circ\ 20'20''.53846\ \text{N.}$$

$$\lambda = 144^\circ\ 38'07''.19265\ \text{E.}$$

$x = 50{,}000.0000$

(Continued)

$$+\frac{30.92241\ 724(520.687.19265-521.095.50254)\times 0.97302\ 19026}{(1-0.00676\ 86580\times 0.23071\ 27589^2)^{1/2}}$$

$$=50.000.0000+\frac{30.08818\ 926(-408.30989)}{0.99981\ 98414}$$

$$=50.000.0000-12.287.5189$$

$$=37.712.4811 \text{ meters}$$

$$y=50.000.0000+30.87002\ 482\ [48.020.53846$$

$$-48.263.28375\ 78-(1.052.89394\ 3-4.48338\ 6\times 0.97302\ 19^2$$

$$+0.02355\ 9\times 0.97302\ 19^4)\times 0.23071\ 27589\times 0.97302\ 19026]$$

$$+\left(\frac{-12.287.5189}{10^4}\right)^2\frac{0.23710\ 95226\times 0.99981\ 98414}{0.12756\ 4128}$$

$$=50.000.0000+30.87002\ 482\ [-242.74529\ 8-(1.052.89394\ 3$$

$$-4.24474\ 3+0.02111\ 8)\times 0.22448\ 85676]+2.80589$$

$$=50.000.0000-14,760.80476+2.80589$$

$$=35.242.0011 \text{ meters}$$

INVERSE COMPUTATION

Formulas.—The following formulas are used for computing geographic positions from plane coordinates.

$$\omega_1''=48.263.28377\ 02+0.03239\ 38839\ 0\left[y-50.000.0000-\left(\frac{x-50.000.0000}{10^4}\right)^2\times 1.87770\right]$$

$$\phi_1''=\omega_1''+(1.047.54669\ 1+6.19301\ 1\cos^2\omega_1+0.05069\ 9\cos^4\omega_1)\sin\omega_1\cos\omega_1$$

$$\omega_2''=48.263.28377\ 02+0.03239\ 38839\ 0\left[y-50.000.0000-\left(\frac{x-50.000.0000}{10^4}\right)^2\frac{(\tan\phi_1)(1-0.00676\ 86580\sin^2\phi_1)^{1/2}}{0.12756\ 4128}\right]$$

$$\phi_2'' = \omega_2'' + (1.047.54669\ 1 + 6.19301\ 1\ \cos^2 \omega_2 + 0.05069\ 9\ \cos^4 \omega_2) \sin \omega_2 \cos \omega_2$$

$$\omega_3'' = 48.263.28377\ 02 + 0.03239\ 38839\ 0 \left[y - 50.000.0000 - \left(\frac{x - 50.000.0000}{10^4} \right)^2 \frac{(\tan \phi_2)(1 - 0.00676\ 86580\ \sin^2 \phi_2)^{1/2}}{0.12756\ 4128} \right]$$

$$\phi'' = \omega_3'' + (1.047.54669\ 1 + 6.19301\ 1\ \cos^2 \omega_3 + 0.05069\ 9\ \cos^4 \omega_3) \sin \omega_3 \cos \omega_3$$

$$\lambda'' = 521.095.50254 + \frac{(x - 50.000.0000)(1 - 0.00676\ 86580\ \sin^2 \phi)^{1/2}}{30.92241\ 724\ \cos \phi}$$

Example.—To compute the geodetic position of the point the plane coordinates of which are:

$$x = 37.712.4811 \text{ meters}$$

$$y = 35.242.0011 \text{ meters}$$

$$\omega_1'' = 48.263.28377\ 02 + 0.03239\ 38839\ 0 \left[35.242.0011 - 50.000.0000 - \left(\frac{37.712.4811 - 50.000.0000}{10^4} \right)^2 \times 1.87770 \right]$$

$$= 48.263.28377\ 02 + 0.03239\ 38839\ 0(-14.757.9989 - 2.8350)$$

$$= 47.785''12303\ 06$$

$$\phi_1'' = 47.785.12303\ 06 + (1.047.54669\ 1 + 6.19301\ 1 \times 0.97328\ 459^2 + 0.05069\ 9 \times 0.97328\ 459^4) \times 0.22960\ 20734 \times 0.97328\ 45873$$

$$= 47.785.12303\ 06 + (1.047.54669\ 1 + 5.86653\ 3 + 0.04549\ 4) \times 0.22346\ 81593$$

$$= 47.785.12303\ 06 + 1.053.45871\ 8 \times 0.22346\ 81593$$

$$= 48.020''53751\ 12$$

$$\omega_2'' = 48{,}263.28377\ 02 + 0.03239\ 38839\ 0 \left[35{,}242.0011 - 50{,}000.0000 - \left(\frac{37{,}712.4811 - 50{,}000.0000}{10^4}\right)^2 \times \frac{0.23710\ 95177(1 - 0.00676\ 86580 \times 0.23071\ 27543^2)^{1/2}}{0.12756\ 4128}\right]$$

$$= 48{,}263.28377\ 02 + 0.03239\ 38839\ 0(-14{,}757.9989 - 11.83586\ 0 \times 0.23710\ 95177 \times 0.99981\ 98414)$$

$$= 48{,}263.28377\ 02 + 0.03239\ 38839\ 0(-14{,}760.80479)$$

$$= 47{,}785''.12397\ 36$$

$$\phi_2'' = 47{,}785.12397\ 36 + (1{,}047.54669\ 1 + 6.19301\ 1 \times 0.97328\ 459^2 + 0.05069\ 9 \times 0.97328\ 459^4) \times 0.22960\ 20779 \times 0.97328\ 45862$$

$$= 47{,}785.12397\ 36 + (1{,}047.54669\ 1 + 5.86653\ 3 + 0.04549\ 4) \times 0.22346\ 81634$$

$$= 47{,}785.12397\ 36 + 1{,}053.45871\ 8 \times 0.22346\ 81634$$

$$= 48{,}020''.53845\ 85$$

$$\omega_3'' = 48{,}263.28377\ 02 + 0.03239\ 38839\ 0 \left[35{,}242.0011 - 50{,}000.0000 - \left(\frac{37{,}712.4811 - 50{,}000.0000}{10^4}\right)^2 \times \frac{0.23710\ 95226(1 - 0.00676\ 86580 \times 0.23071\ 27589^2)^{1/2}}{0.12756\ 4128}\right]$$

$$= 48{,}263.28377\ 02 + 0.03239\ 38839\ 0(-14{,}757.9989 - 11.83586\ 0 \times 0.23710\ 95226 \times 0.99981\ 98414)$$

$$= 48{,}263.28377\ 02 + 0.03239\ 38839\ 0(-14{,}760.80479)$$

$$= 47{,}785''.12397\ 36$$

$$\phi'' = 47{,}785.12397\ 36 + (1{,}047.54669\ 1 + 6.19301\ 1 \times 0.97328\ 459^2 + 0.05069\ 9 \times 0.97328\ 459^4) \times 0.22960\ 20779 \times 0.97328\ 45862$$

$$= 47{,}785.12397\,36 + (1{,}047.54669\,1 + 5.86653\,3 + 0.04549\,4) \times 0.22346\,81634$$

$$= 47{,}785.12397\,36 + 1{,}053.45871\,8 \times 0.22346\,81634$$

$$= 48{,}020''.53845\,85$$

$$\phi = 13^\circ\,20'20''.53846 \text{ N.}$$

$$\lambda'' = 521{,}095.50254 + \frac{(37{,}712.4811 - 50{,}000.0000)(1 - 0.00676\,86580 \times 0.23071\,27589^2)^{1/2}}{30.92241\,724 \times 0.97302\,19026}$$

$$= 521{,}095.50254 + \frac{(-12{,}287.5189) \times 0.99981\,98414}{30.08818\,926}$$

$$= 521{,}095.50254 - 408.30989$$

$$= 520{,}687''.19265$$

$$\lambda = 144^\circ\,38'07''.19265 \text{ E.}$$

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APPENDIX

The constants in the tables of constants and in the formulas for the various projections are explained in the following section.

The code number at the beginning of each list of constants in the tables is used to identify the zone in which a computation is to be made. It is used when the constants for all of the state plane coordinate zones in the United States along with the computation formulas are loaded onto the computer simultaneously, as is the practice in the Coast Survey. This code number is also entered as input data with each geodetic position, or plane coordinate, which is to be converted. In addition, some symbol or sign must be entered with the code number, either at the time of entering individual geodetic positions or plane coordinates or when the constants and formulas are loaded, to direct the machine to the Lambert or transverse Mercator equations, whichever are to be used. Of course, if the constants for one zone only are loaded at any one time the code number need not be used.

Convergence is the angle between the meridian through a point and a line through the point parallel to the y axis. It may be used in the following formula for computing geodetic azimuths from grid azimuths, and vice versa.

$$\text{geodetic azimuth} = \text{grid azimuth} + \text{convergence} + \text{second term}$$

Azimuths are measured clockwise from the south. The formula for the second term, also called the $(t-T)$ correction, may be found in the references listed in the bibliography or in the various state plane coordinate projection tables. This term may be significant when azimuths of lines longer than a few miles are to be computed.

The scale factor, k, as computed by the formulas in the foregoing text is the value at a point. For a short line, the scale factor at its midpoint may be used. If the scale factor for a long line is to be computed, or greater precision for a short line is desired, the following formula may be used:

$$k = \frac{k_1 + 4k_m + k_2}{6},$$

in which k_1 and k_2 are the scale factors at the ends of the line and k_m is the scale factor at the midpoint of the line. For converting geodetic distance to grid distance, and vice versa, the following formula is used.

$$\text{grid distance} = \text{geodetic distance} \times \text{scale factor}$$

To convert plane coordinates in one system (or zone) to plane coordinates in another system (or zone), it is best to compute the geographic position from the plane coordinates in the first system and with this position compute the plane coordinates in the desired system (or zone). The same geodetic datum must be used for both coordinate systems.

For limited areas it is possible to convert directly from one plane coordinate system to another. If the plane coordinates on both systems of four or more stations are available, transformation equations can be derived empirically for converting the coordinates directly from one system to another. This may be the most practicable method when the surveys, on which the two sets of plane coordinates are based, are not on the same geodetic datum.

For further details regarding the special publications or other references mentioned in the following discussions, please consult the bibliography.

LAMBERT PROJECTION

Forward Computation.—L_1 is the false easting, or x coordinate, of the central meridian.

L_2 is the central meridian expressed in seconds.

L_3 is the map radius of the central parallel (ϕ_o).

L_4 is the map radius of the lowest parallel of the projection table plus the y value on the central meridian at this parallel. This y value is zero in most, but not all, cases.

L_5 is the scale (m) of the projection along the central parallel (ϕ_o).

L_6 is the l computed from the basic equations for the Lambert projection with two standard parallels.

L_7 is the degrees and minutes portion, in minutes, of the rectifying latitude for ϕ_o, where $\phi_o = \text{arc sin } l$.

Rectifying latitude, ω, is defined by the equation:

$$\omega'' = \phi'' - 525.32978\,0 \sin 2\phi + 0.55747\,8 \sin 4\phi - 0.00073\,5 \sin 6\phi.$$

(*See* SPECIAL PUBLICATION NO. 67, pp. 122–128.)

By substitution of trigonometric identities, this can be shown to be equivalent to the following equation:

$$\omega'' = \phi'' - [1{,}052.89388\,2 - (4.48334\,4 - 0.02352\,0 \cos^2 \phi) \cos^2 \phi] \sin \phi \cos \phi.$$

The significance of rectifying latitude is that the meridional arc on the earth ellipsoid from 0° to $\phi°$ equals the length of a great circle from 0° to $\omega°$ on a sphere whose circumference is equal to the meridional ellipse of the earth ellipsoid, ω being defined by the foregoing equation. It follows that the meridional arc on the earth ellipsoid from ϕ_1 to ϕ_2 equals the length of a great circle on the sphere from ω_1 to ω_2.

This property is also used, in the inverse computation of a geodetic position from plane coordinates, for finding the difference of geodetic latitude corresponding to a computed meridional arc on the ellipsoid.

L_8 is the remainder of ω_o, i.e., the seconds,

$L_9 = (1/6R_oN_o) \times 10^{16}$,

$L_{10} = [\tan \phi_o/24(R_oN_o)^{3/2}] \times 10^{24}$.

$L_{11} = [(5 + 3 \tan^2 \phi_o)/120R_oN_o^3] \times 10^{32}$,

$$\text{where } R_o = \frac{a(1-e^2)}{(1-e^2 \sin^2 \phi_o)^{3/2}} \text{ and } N_o = \frac{a}{(1-e^2 \sin^2 \phi_o)^{1/2}}.$$

In the formula for computing R, the map radius, s is the length of the meridional arc from ϕ_o to ϕ. The constant 101.27940 65 is the length, in feet, of one second of arc on a sphere whose circumference equals the meridional arc of the Clarke 1866 ellipsoid. Although the last figure is truly nearer 4, the figure 5 is used in the equation because it was used in the original computation of the plane coordinate projection tables.

In the formula for the scale factor, k, the constant 0.00676 86580 is e^2, and the constant 20,925,832.16 is a in feet.

The exact formula for factor L_{10} is

$$\frac{(5R_o - 4N_o) \tan \phi_o}{24R_o^2N_o^2} \times 10^{24}$$

but the approximate form

$$\frac{\tan \phi_o}{24(R_oN_o)^{3/2}} \times 10^{24}$$

was used in the original computations except Alaska, zone 10. This approximate form is not significantly different from the exact formula since R_o and N_o are nearly the same value.

The term containing L_{11} was not used in most of the state plane coordinate projections. That is why the L_{11} values in the table of constants are listed as zero even though they would seem significant if computed from the equation for L_{11}. This term was used in Alaska, zone 10, and in the offshore zone of Louisiana because these zones have a greater latitude spread than the other projections and the term becomes significant. It is also used in the projection for Michigan, because the projection tables there were computed by electronic data processing equipment to greater precision than those of other states. For this reason the term containing L_{11} is significant and it is included in the computation.

The formulas used in this projection were obtained from SPECIAL PUBLICATION NO. 193. Their derivation may be found in SPECIAL PUBLICATION NO. 251.

Inverse Computation.—In the inverse computation the L's are the same as in the forward computation.

The arc of the meridian from ϕ_0 to ϕ is s as before. This should really be computed by an iterative process but it seemed that the method used here would be easier to understand and follow. The value of s is converted to a corresponding rectifying latitude (actually a latitude difference) which in turn is transformed to the geodetic latitude (again a latitude difference) which is desired.

The constant 600 is introduced in the equation for ω' and cancelled by the constant 36,000 in the equation for ω'' to keep the sign of ω'' positive.

The constant 0.00987 36755 53 is the reciprocal of 101.27940 65, explained in the forward computation.

The rectifying latitude, ω, is transformed to geodetic latitude, ϕ, by an equation of the basic form: $\phi = \omega + A \sin 2\omega + B \sin 4\omega + C \sin 6\omega$. Rather than use the coefficients A, B, and C as derived in SPECIAL PUBLICATION NO. 67 for this equation, better consistency is obtained by reversing the equation used in the forward computation. This was done empirically by computing the rectifying latitudes for geodetic latitudes 30°, 40°, and 50° using the equation in the forward computation. These values were substituted in the above equation one at a time giving three equations in terms of A, B, and C. Solving these equations simultaneously gives the following results: $A = +525.32950\ 0$; $B = +0.78045\ 9$; and $C = +0.00159\ 1$. Therefore, $\phi'' = \omega'' + 525.32950\ 0 \sin 2\omega + 0.78045\ 9 \sin 4\omega + 0.00159\ 1 \sin 6\omega$. This equation was changed, by the substitution of the appropriate trigonometric identities, to the form:

$$\phi'' = \omega'' + [1{,}047.54671\ 0 + (6.19276\ 0 + 0.05091\ 2 \cos^2 \omega) \cos^2 \omega] \sin \omega \cos \omega.$$

TRANSVERSE MERCATOR PROJECTION

Forward Computation.—T_1 is the false easting, or x coordinate. of the central meridian.

T_2 is the central meridian expressed in seconds.

T_3 is the degrees and minutes portion, in minutes, of the rectifying latitude, ω_o, for ϕ_o, the latitude of the origin. (See the derivation and explanation of ω'' under L_7 of the Lambert projection.)

T_4 is the remainder of ω_o, i.e., the seconds.

T_5 is the scale along the central meridian.

$T_6 = (1/6R_m N_m T_5^2) \times 10^{15}$. ($R_m$ and N_m are computed for the mean latitude of the area in the zone, or projection:

$$R = \frac{a(1-e^2)}{(1-e^2 \sin^2 \phi)^{3/2}}, \qquad N = \frac{a}{(1-e^2 \sin^2 \phi)^{1/2}}.)$$

In the formula for S_1 the constant 0.00676 86580 is e^2. The constant 30.92241 724 is $a \times \text{arc } 1''$, in meters.

The constant 3.9174 is an arc-sin correction derived as follows:

$$\log \Delta\lambda = 5.88460\,85 + \tfrac{1}{2} \log c$$ (*See* Sp. Pub. No. 8, p. 18.)

$$\Delta\lambda = 766{,}670\,(c)^{1/2}$$

$$c_{\lambda(\log)} = \frac{(\Delta\lambda)^2}{766{,}670^2} = 1.70130\,85 \times 10^{-12}\,(\Delta\lambda)^2$$

$$c_{\lambda(\text{factor})} = \frac{1.70130\,85 \times 10^{-12}\,(\Delta\lambda)^2}{0.43429\,448} = 3.9174 \times 10^{-12}\,(\Delta\lambda)^2$$

In the formula for S_m the constant 4.0831 is also an arc-sin correction derived as follows:

$$\log s = 5.88461 - \bar{8}.50900 + \tfrac{1}{2} \log c$$ (*See* Sp. Pub. No. 8, p. 18.)

$$= 7.37561 + \tfrac{1}{2} \log c$$

$$s = 23{,}747{,}100\,(c)^{1/2}$$

$$c_{s(\log)} = \frac{s^2}{23{,}747{,}100^2} = 1.77328\,6 \times 10^{-15} s^2$$

$$c_{s(\text{factor})} = \frac{1.77328\,6 \times 10^{-15}}{0.43429\,448}\, s^2 = 4.0831 \times 10^{-15} s^2$$

The reduction factor used to convert meters to feet is 3.28083 333. This value was used so that the plane coordinates computed originally by logarithms would be duplicated.

From SPECIAL PUBLICATION NO. 8,

$$C=\frac{(1-e^2 \sin^2 \phi)^2 \tan \phi}{2a^2 (1-e^2) \sin 1''}$$

in which the constant coefficient

$$\frac{1}{2a^2 (1-e^2) \sin 1''}=\frac{1}{0.03917\,8916 \times 10^{10}}=\frac{25.52393}{10^{10}}.$$

In the original tables for computing coordinates by logarithms, C was computed using the equation:

$$\log C_{(\text{Clarke } 1866)}=\log C_{(\text{International})}+22.65+20.11 \cos 2\phi$$

(in units of the sixth decimal place). A more nearly exact equation has been determined since then, namely:

$$\log C_{(\text{Clarke } 1866)}=\log C_{(\text{International})}+24.76+20.07 \cos 2\phi.$$

Again in order to duplicate the original computations, even though not quite exact, the constant factor, C, was "corrected" by the difference between these two equations for C. Disregarding the small difference between the coefficients of cos 2ϕ, the main difference is $24.76-22.65$ which in the sixth decimal place is 0.00000 211, the logarithm of 1.00000 486. The coefficient of C, $\frac{25.52393}{10^{10}}$, is divided by this factor to give $\frac{25.52381}{10^{10}}$, the coefficient used in the equations for ϕ_1 and ϕ_2.

For a description of the constants in the formula for y, see the discussion for the *Lambert Projection* in this appendix.

In the formula for $\Delta\alpha$ the factor $1.9587/10^{12}$ equals $\sin^2 1''/12$ which comes from the factor F of SPECIAL PUBLICATION NO. 8 in which

$$F=\frac{1}{12} \sin \phi' \cos^2 \phi' \sin^2 1''.$$

In the formula for k, the factor 0.00681 47849 is e'^2 which equals $(a^2-b^2)/b^2$. The factor 881.74916 2 equals $2a^2/(1-e^2)\times 10^{12}$ where a is in feet.

Inverse Computation.—In the inverse computation the *T*'s are the same as in the forward computation.

0.30480 06099 is the reciprocal of 3.28083 333.
0.00987 36755 53 is the reciprocal of 101.27940 65.

For an explanation of the constants in the equation for $(\phi')''$ see the discussion for the *Lambert Projection* (Inverse Computation) in this appendix.

The formulas used in this projection were obtained from SPECIAL PUBLICATION NO. 193. These are approximate in that terms in the series of higher order than the third power are omitted. Their derivation may be found in SPECIAL PUBLICATION NO. 251.

ALASKA PROJECTION, ZONES 2 TO 9

The formulas for this projection are given here in general terms. except for the first term of the formula for y, and the equations for $(\omega')''$ and $(\phi')''$ in the inverse computation. The coefficients there are computed from the parameters of the Clarke Spheroid of 1866. The basic equations were obtained from DEPARTMENT OF THE ARMY TECHNICAL MANUAL NO. TM 5-241-8, and their derivation may be found in SPECIAL PUBLICATION NO. 251. The equations have been modified so that the only trigonometric function appearing in them is the cosine. As in all state projections, the foot is equal to 1200/3937 meter exactly.

Forward Computation.—The following formulas are used for computing plane coordinates from geographic positions.

$$x = C + \frac{ak_0 \text{ arc } 1'' \times 10^4}{(1-e^2)^{1/2}} \cdot \frac{\cos\phi}{(1+e'^2\cos^2\phi)^{1/2}} \cdot \frac{CM''-\lambda''}{10^4}\left\{1 - \frac{\text{arc}^2\, 1'' \times 10^8}{6}\left(\frac{CM''-\lambda''}{10^4}\right)^2 (1 - 2\cos^2\phi - e'^2\cos^4\phi) + \frac{\text{arc}^4\, 1''\times 10^{16}}{120}\left(\frac{CM''-\lambda''}{10^4}\right)^4 [1 - 20\cos^2\phi + (24 - 58e'^2)\cos^4\phi + 72e'^2\cos^6\phi] + \ldots\right\}$$

$$y = 101.26927\,8503\,[\phi'' - \omega_0'' - (1.052.89394\,3 - 4.48338\,6\cos^2\phi + 0.02355\,9\cos^4\phi)(1-\cos^2\phi)^{1/2}\cos\phi]$$

(Continued)

$$+\frac{ak_0 \operatorname{arc}^2 1'' \times 10^8}{2(1-e^2)^{1/2}} \cdot \frac{(1-\cos^2\phi)^{1/2}\cos\phi}{(1+e'^2\cos^2\phi)^{1/2}}\left(\frac{CM''-\lambda''}{10^4}\right)^2\Bigg\{1$$

$$+\frac{\operatorname{arc}^2 1''\times 10^8}{12}\left(\frac{CM''-\lambda''}{10^4}\right)^2(-1+6\cos^2\phi+9e'^2\cos^4\phi$$

$$+4e'^4\cos^6\phi)+\frac{\operatorname{arc}^4 1''\times 10^{16}}{360}\left(\frac{CM''-\lambda''}{10^4}\right)^4[1-60\cos^2\phi$$

$$+(120-330e'^2)\cos^4\phi+600e'^2\cos^6\phi]+\ldots\Bigg\}$$

$$\Delta\alpha''=(1-\cos^2\phi)^{1/2}\frac{CM''-\lambda''}{10^4}\Bigg[10{,}000.0000$$

$$+\frac{\operatorname{arc}^2 1''\times 10^{12}}{3}\left(\frac{CM''-\lambda''}{10^4}\right)^2(\cos^2\phi+3e'^2\cos^4\phi+2e'^4\cos^6\phi)$$

$$+\frac{\operatorname{arc}^4 1''\times 10^{20}}{15}\left(\frac{CM''-\lambda''}{10^4}\right)^4(3\cos^4\phi-\cos^2\phi)+\ldots\Bigg]$$

$$k=k_0\left[1+\frac{(1-e^2)\times 10^{12}}{2a^2k_0^2}(1+e'^2\cos^2\phi)^2\left(\frac{x-C}{10^6}\right)^2+\ldots\right]$$

C is the false easting or x coordinate of the central meridian. k_0 is the scale factor along the central meridian, e^2 equals $(a^2-b^2)/a^2$ and e'^2 equals $(a^2-b^2)/b^2$, where a and b are the semimajor and semiminor axes of the ellipsoid used. CM'' is the longitude of the central meridian expressed in seconds, ϕ'' is the latitude, and λ'' the longitude, both in seconds, of the point being computed.

The first term in the formula for y is the length of the meridional arc, reduced by the scale factor k_0, from the origin to the latitude, ϕ, of the point being computed. The coefficient, 101.26927 8503, is k_0 multiplied by the length in feet of a second of arc on a sphere whose circumference equals the meridional ellipse of the Clarke Spheroid of 1866. Its formula is

$$ak_0 \operatorname{arc} 1''(1-n)\left(1+\frac{5}{4}n^2+\frac{81}{64}n^4+\frac{325}{256}n^6+\ldots\right)$$

where n equals $(a-b)/(a+b)$. (*See* SPECIAL PUBLICATION NO. 67, p. 125.) In the projection as applied to zones 2 to 9 of the Alaska plane coordinate system, a is expressed in feet in all formulas, and k_0 is 0.9999. ω'' is defined by the formula:

$$\omega''=\phi''-\left(\frac{3}{2}n-\frac{9}{16}n^3+\frac{3}{32}n^5+\ldots\right)\frac{1}{\text{arc }1''}\sin 2\phi$$

$$+\left(\frac{15}{16}n^2-\frac{15}{32}n^4+\frac{135}{2048}n^6+\ldots\right)\frac{1}{\text{arc }1''}\sin 4\phi$$

$$-\left(\frac{35}{48}n^3-\frac{105}{256}n^5+\ldots\right)\frac{1}{\text{arc }1''}\sin 6\phi+\ldots.$$

(*See* SPECIAL PUBLICATION NO. 67, pp. 122–128, for the derivation.)

By substitution of the appropriate trigonometric identities, this equation reduces to the following equation in terms of the cosine function of ϕ.

$$\omega''=\phi''-\left[\left(3n+\frac{15}{4}n^2+\frac{13}{4}n^3-\frac{15}{8}n^4-\frac{291}{128}n^5+\frac{135}{512}n^6+\cdots\right)\frac{1}{\text{arc }1''}\right.$$

$$-\left(\frac{15}{2}n^2+\frac{70}{3}n^3-\frac{15}{4}n^4-\frac{105}{8}n^5+\frac{135}{256}n^6+\ldots\right)\frac{1}{\text{arc }1''}\cos^2\phi$$

$$\left.+\left(\frac{70}{3}n^3-\frac{105}{8}n^5+\ldots\right)\frac{1}{\text{arc }1''}\cos^4\phi\right](1-\cos^2\phi)^{1/2}\cos\phi$$

The three expressions involving n, divided by arc 1″, reduce to: 1,052.89394 3; 4.48338 6; and 0.02355 9 for computations based on the Clarke Spheroid of 1866. Arc 1″ is introduced here because the original derivation is in radians. In computing ω''_0, ϕ is 54° since that is the latitude of the origin of this projection.

Inverse Computation.—The following formulas are used for computing geographic positions from plane coordinates.

$$(\omega')''=\omega''_0+0.00987\ 46630\ 2498\ y$$

$$(\phi')''=(\omega')''+(1{,}047.54669\ 1+6.19301\ 1\cos^2\omega'$$

$$+0.05069\ 9\cos^4\omega')(1-\cos^2\omega')^{1/2}\cos\omega'$$

$$\phi''=(\phi')''-\frac{(1-e^2)\times 10^{12}}{2\,a^2k_0^2\text{ arc }1''}\left(\frac{x-C}{10^6}\right)^2(1+e'^2\cos^2\phi')^2\left(\frac{1}{\cos^2\phi'}\right.$$

$$-1\Big)^{1/2}\left\{1-\frac{(1-e^2)\times 10^{12}}{12a^2k_o^2}\left(\frac{x-C}{10^6}\right)^2\left[2-6e'^2+\frac{3}{\cos^2\phi'}\right.\right.$$

$$\left.+(12e'^2-9e'^4)\cos^2\phi'+6e'^4\cos^4\phi'\right]$$

$$+\frac{(1-e^2)^2\times 10^{24}}{360a^4k_o^4}\left(\frac{x-C}{10^6}\right)^4(1+e'^2\cos^2\phi')\left(16-72e'^2+\frac{45}{\cos^4\phi'}\right.$$

$$\left.\left.-\frac{45e'^2}{\cos^2\phi'}+224e'^2\cos^2\phi'\right)+\ldots\right\}$$

$$\lambda''=CM''-\frac{(1-e^2)^{1/2}\times 10^6}{ak_o\ \text{arc}\ 1''}\cdot\frac{(1+e'^2\cos^2\phi')^{1/2}}{\cos\phi'}\left(\frac{x-C}{10^6}\right)\left[1\right.$$

$$-\frac{(1-e^2)\times 10^{12}}{6a^2k_o^2}(1+e'^2\cos^2\phi')\left(\frac{x-C}{10^6}\right)^2\left(-1+\frac{2}{\cos^2\phi'}\right.$$

$$\left.+e'^2\cos^2\phi'\right)+\frac{(1-e^2)^2\times 10^{24}}{120a^4k_o^4}(1$$

$$+e'^2\cos^2\phi')^2\left(\frac{x-C}{10^6}\right)^4\left(1+8e'^2+\frac{24}{\cos^4\phi'}\right.$$

$$\left.\left.-\frac{20}{\cos^2\phi'}-2e'^2\cos^2\phi'\right)+\ldots\right]$$

The constant 0.00987 46630 2498 is the reciprocal of 101.26927 8503. the derivation of which is explained in the discussion of the forward computation. The constants in the equation for $(\phi')''$ were derived from the general equation

$$\phi''=\omega''+(A+B\cos^2\omega+C\cos^4\omega)(1-\cos^2\omega)^{1/2}\cos\omega.$$

The constants A, B, and C were computed empirically as follows. Using the equation in the forward computation. the values of ω'' were computed for the following values of ϕ: 30°. 40°. 50°. 60°. and 70°. These values substituted in the general equation gave five equations in three unknowns (A, B, and C). These equations were solved by a least squares solution to obtain the constants used in the equations for $(\phi')''$.

With some modification. the formulas for Alaska Zones 2 to 9 may be used to compute universal transverse Mercator (UTM) coordinates. The constants containing the factor a (excepting n) must be recomputed using meters instead of feet. Constants containing k_o must be recomputed using 0.9996 instead of 0.9999 for k_o. The false easting, or x coordinate, of the central meridian will be a constant 500.000 meters. Since the origin of the UTM projection is at the Equator the values of ϕ_o and ω_o will be zero.

ALASKA PROJECTION, ZONE 1

The formulas given, in the preceding text, for this projection are variations of those appearing in PUBLICATION 65–1, PART 49 and in SPECIAL PUBLICATION No. 251 which are as follows.

Forward Computation.—The skew coordinates u, v are computed by the formulas:

$$\tan (Bu/Ak_c) = \frac{\cos \alpha_0 \sinh (B\mu + C) - \sin \alpha_0 \sin B(\lambda - \lambda_0)}{\cos B(\lambda - \lambda_0)}$$

$$\tanh (Bv/Ak_c) = \frac{-\cos \alpha_0 \sin B(\lambda - \lambda_0) - \sin \alpha_0 \sinh (B\mu + C)}{\cosh (B\mu + C)}$$

The u and v coordinates are converted to x and y coordinates by rotation and translation of the plane coordinate axes.

Convergence, γ

$$\tan \gamma = \frac{4 \tan \alpha + 3}{4 - 3 \tan \alpha}$$

where

$$\tan \alpha = \frac{\tan \alpha_0 - \sin B(\lambda - \lambda_0) \sinh (B\mu + C)}{\cos B(\lambda - \lambda_0) \cosh (B\mu + C)}$$

Scale factor, k

$$k = \frac{k_c A \cos (Bu/Ak_c)}{N \cos \phi \cos B(\lambda - \lambda_0)}$$

Inverse Computation.—Plane coordinates in x and y are converted to u and v coordinates, and the corresponding geographic position is computed by the following formulas.

$$\tanh (B\mu + C) = \frac{\cos \alpha_0 \sin (Bu/Ak_c) - \sin \alpha_0 \sinh (Bv/Ak_c)}{\cosh (Bv/Ak_c)}$$

$$\tan B(\lambda - \lambda_0) = \frac{-\cos \alpha_0 \sinh (Bv/Ak_c) - \sin \alpha_0 \sin (Bu/Ak_c)}{\cos (Bu/Ak_c)}$$

An explanation of the projection and definitions of the constants in these formulas are given in PUBLICATION 65–1, PART 49.

ST. CROIX PROJECTION

The projection for St. Croix is the same as the one used for Puerto Rico and the Virgin Islands, with one exception. The y values have been increased by 100,000.00 feet so that all y coordinates on St. Croix will be positive.

GUAM PROJECTION

The plane coordinates of the geodetic stations on Guam were obtained by first computing the geodetic distances and azimuths of all points from the origin by inverse computations. The coordinates were then computed by the equations: $\Delta x = d \sin \alpha$; $\Delta y = d \cos \alpha$. This really gives a true azimuthal equidistant projection. The equations given here are simpler, however, than those for a geodetic inverse computation, and the resulting coordinates computed using them will not be significantly different from those computed rigidly by inverse computation. This is the reason it is called an approximate azimuthal equidistant projection.

The numerical constants in the following general equations (obtained from the leaflet RELATIONSHIP BETWEEN LOCAL PLANE COORDINATES AND GEOGRAPHIC POSITIONS) are for computation on the Clarke Spheroid of 1866. For the method of their derivation, refer to the section in this appendix entitled *Alaska Projection, Zones 2 to 9.*

GENERAL EQUATIONS

CONSTANTS

ϕ = latitude of station

λ = longitude of station

ϕ_0'' = latitude of origin in seconds

λ_0'' = longitude of origin in seconds

$C_1 = x$ coordinate of origin

$C_2 = y$ coordinate of origin

a = semimajor axis of ellipsoid

b = semiminor axis of ellipsoid

$$e^2 = \frac{a^2 - b^2}{a^2}$$

$$\omega_0'' = \phi_0'' - (1{,}052.89394\,3 - 4.48338\,6 \cos^2 \phi_0 + 0.02355\,9 \cos^4 \phi_0) \sin \phi_0 \cos \phi_0$$

(North latitude, east longitude; both entered positive)

Forward Computation.—The following formulas are used for computing plane coordinates from geographic positions.

$$x = C_1 + \frac{a \text{ arc } 1'' (\lambda'' - \lambda''_o) \cos \phi}{(1 - e^2 \sin^2 \phi)^{1/2}} \text{ meters}$$

$$y = C_2 + 30.87002\ 482\ [\phi'' - \omega''_o - (1{,}052.89394\ 3 - 4.48338\ 6 \cos^2 \phi + 0.02355\ 9 \cos^4 \phi) \sin \phi \cos \phi] + \left(\frac{x - C_1}{10^4}\right)^2 \frac{(\tan \phi)(1 - e^2 \sin^2 \phi)^{1/2}}{2a \times 10^{-8}} \text{ meters}$$

Inverse Computation.—The following formulas are used for computing geographic positions from plane coordinates.

$$\omega''_1 = \omega''_o + 0.03239\ 38839\ 0 \left[y - C_2 - \left(\frac{x - C_1}{10^4}\right)^2 \frac{(\tan \phi_o)(1 - e^2 \sin^2 \phi_o)^{1/2}}{2a \times 10^{-8}}\right]$$

$$\phi''_1 = \omega''_1 + (1{,}047.54669\ 1 + 6.19301\ 1 \cos^2 \omega_1 + 0.05069\ 9 \cos^4 \omega_1) \sin \omega_1 \cos \omega_1$$

$$\omega''_2 = \omega''_o + 0.03239\ 38839\ 0 \left[y - C_2 - \left(\frac{x - C_1}{10^4}\right)^2 \frac{(\tan \phi_1)(1 - e^2 \sin^2 \phi_1)^{1/2}}{2a \times 10^{-8}}\right]$$

$$\phi''_2 = \omega''_2 + (1{,}047.54669\ 1 + 6.19301\ 1 \cos^2 \omega_2 + 0.05069\ 9 \cos^4 \omega_2) \sin \omega_2 \cos \omega_2$$

$$\omega''_3 = \omega''_o + 0.03239\ 38839\ 0 \left[y - C_2 - \left(\frac{x - C_1}{10^4}\right)^2 \frac{(\tan \phi_2)(1 - e^2 \sin^2 \phi_2)^{1/2}}{2a \times 10^{-8}}\right]$$

$$\phi'' = \omega''_3 + (1{,}047.54669\ 1 + 6.19301\ 1 \cos^2 \omega_3 + 0.05069\ 9 \cos^4 \omega_3) \sin \omega_3 \cos \omega_3$$

$$\lambda'' = \lambda''_o + \frac{(x - C_1)(1 - e^2 \sin^2 \phi)^{1/2}}{a \text{ arc } 1'' \cos \phi}$$

In the equations for the inverse computation used in the Guam projection, ω''_o is slightly different from the value used in the forward computation. This value was changed so that an inverse computation of the origin will give its exact position. The reason that the inverse computation will not duplicate the position of the origin, exactly, when the ω_0'' in the forward computation is used, is that the numerical coefficients in the equation for ϕ_1'' are not quite exact at this low latitude, since they were computed in the latitude range from 30° to 70°. This will not affect the results, to any significant degree, within the limits of the island of Guam.

The scale factor of this projection is one (scale exact) at the origin. It is also exact along any geodetic line, actual or extended, which passes through the origin. The larger scale factors will be at the extreme edges of the island, in the direction of the circles centered on the origin. The maximum to be expected will be about 1 part in 100,000, the grid distance being greater than the geodetic.

CONSTANTS FOR LAMBERT PROJECTION

	ALASKA	ARKANSAS	ARKANSAS	CALIFORNIA
ZONE CODE	10 5010	North 0301	South 0302	I 0401
L_1 L_2	3,000,000.00 633,600.00	2,000,000.00 331,200.00	2,000,000.00 331,200.00	2,000,000.00 439,200.00
L_3 L_4	15,893,950.36 16,564,628.77	29,277,593.61 29,732,882.87	31,014,039.23 31,511,724.20	24,245,358.05 24,792,436.23
L_5 L_6	.99984 80641 .79692 23940	.99993 59370 .58189 91407	.99991 84698 .55969 06871	.99989 46358 .65388 43192
L_7 L_8 L_9 L_{10} L_{11}	3161 47.87068 3.79919 5.91550 44	2126 46.35656 3.81452 3.26432 0	2033 56.94711 3.81550 3.08256 0	2441 26.75847 3.80992 3.93575 0

	CALIFORNIA	CALIFORNIA	CALIFORNIA	CALIFORNIA
ZONE CODE	II 0402	III 0403	IV 0404	V 0405
L_1 L_2	2,000,000.00 439,200.00	2,000,000.00 433,800.00	2,000,000.00 428,400.00	2,000,000.00 424,800.00
L_3 L_4	25,795,850.31 26,312,257.65	27,057,475.85 27,512,992.04	28,182,405.33 28,652,931.96	30,194,145.54 30,649,424.27
L_5 L_6	.99991 46793 .63046 79732	.99992 91792 .61223 20427	.99994 07628 .59658 71443	.99992 21277 .57001 19219
L_7 L_8 L_9 L_{10} L_{11}	2336 30.81964 3.81147 3.70114 0	2256 35.52018 3.81265 3.52998 0	2189 10.35494 3.81362 3.39020 0	2076 52.10305 3.81523 3.16593 0

CONSTANTS FOR LAMBERT PROJECTION

	CALIFORNIA	CALIFORNIA	COLORADO	COLORADO
ZONE CODE	VI 0406	VII 0407	North 0501	Central 0502
L1	2,000,000.00	4,186,692.58	2,000,000.00	2,000,000.00
L2	418,500.00	426,000.00	379,800.00	379,800.00
L3	31,846,570.92	30,891,382.10	24,751,897.68	25,781,376.91
L4	32,271,267.72	35,055,396.31	25,086,068.20	26,243,052.74
L5	.99995 41438	.99998 85350	.99995 68475	.99993 59117
L6	.54951 75982	.56124 32071	.64613 34829	.63068 95773
L7	1992	2040	2406	2337
L8	00.16335	22.88096	24.62308	29.65162
L9	3.81642	3.81572	3.81044	3.81146
L10	3.00292	3.09520	3.85610	3.70326
L11	0	0	0	0

	COLORADO	CONNECTICUT	FLORIDA	IOWA
ZONE CODE	South 0503	0600	North 0903	North 1401
L1	2,000,000.00	600,000.00	2,000,000.00	2,000,000.00
L2	379,800.00	261,900.00	304,200.00	336,600.00
L3	26,977,133.89	23,659,233.56	36,030,443.05	22,736,950.34
L4	27,402,231.82	23,914,389.02	36,454,924.53	23,162,461.59
L5	.99994 53995	.99998 31405	.99994 84343	.99994 53686
L6	.61337 80528	.66305 94147	.50252 59000	.67774 45518
L7	2261	2483	1802	2551
L8	34.26662	19.67980	26.11701	20.02265
L9	3.81257	3.80929	3.81898	3.80827
L10	3.54046	4.03278	2.65643	4.19479
L11	0	0	0	0

CONSTANTS FOR LAMBERT PROJECTION

ZONE CODE	IOWA South 1402	KANSAS North 1501	KANSAS South 1502	KENTUCKY North 1601
L1	2,000,000.00	2,000,000.00	2,000,000.00	2,000,000.00
L2	336,600.00	352,800.00	354,600.00	303,300.00
L3	23,936,585.11	25,644,959.12	26,896,024.48	26,371,820.68
L4	24,374,096.67	25,979,068.57	27,351,521.50	26,724,051.82
L5	.99994 83705	.99995 68556	.99993 59200	.99996 20817
L6	.65870 10213	.63271 48646	.61452 81068	.62206 72671
L7	2463	2346	2266	2299
L8	22.59905	27.97215	34.41020	30.63364
L9	3.80959	3.81133	3.81250	3.81202
L10	3.98630	3.72376	3.55102	3.62113
L11	0	0	0	0

ZONE CODE	KENTUCKY South 1602	LOUISIANA North 1701	LOUISIANA South 1702	LOUISIANA Offshore 1703
L1	2,000,000.00	2,000,000.00	2,000,000.00	2,000,000.00
L2	308,700.00	333,000.00	328,800.00	328,800.00
L3	27,467,860.75	33,624,568.36	36,271,389.35	41,091,749.54
L4	27,832,235.64	34,079,629.33	36,756,553.45	41,576,762.39
L5	.99994 53808	.99991 47417	.99992 57458	.99989 47956
L6	.60646 23718	.52870 06734	.50001 26971	.45400 68519
L7	2231	1907	1792	1612
L8	36.57874	12.68515	28.55026	59.30342
L9	3.81301	3.81758	3.81911	3.82138
L10	3.47771	2.84511	2.63885	2.27436
L11	0	0	0	25

CONSTANTS FOR LAMBERT PROJECTION

	MARYLAND	MASSACHUSETTS	MASSACHUSETTS	MICHIGAN
ZONE CODE	1900	Mainland 2001	Island 2002	North 2111
L_1	800,000.00	600,000.00	200,000.00	2,000,000.00
L_2	277,200.00	257,400.00	253,800.00	313,200.00
L_3	25,989,474.99	23,111,975.14	23,784,678.44	20,041,716.18
L_4	26,369,112.76	23,549,477.32	23,924,398.02	20,589,420.09
L_5	.99994 98485	.99996 45506	.99999 84844	.99994 10344
L_6	.62763 41196	.67172 86561	.66109 53994	.72278 99381
L_7	2323	2523	2474	2768
L_8	59.69369	19.53138	19.47463	22.25085
L_9	3.81166	3.80870	3.80943	3.80501
L_{10}	3.67392	4.12738	4.01174	4.68430
L_{11}	0	0	0	36

	MICHIGAN	MICHIGAN	MINNESOTA	MINNESOTA
ZONE CODE	Central 2112	South 2113	North 2201	Central 2202
L_1	2,000,000.00	2,000,000.00	2,000,000.00	2,000,000.00
L_2	303,600.00	303,600.00	335,160.00	339,300.00
L_3	21,001,715.22	22,564,848.51	18,984,319.62	20,006,679.72
L_4	21,594,768.40	23,069,597.22	19,471,398.75	20,493,457.15
L_5	.99995 09058	.99994 50783	.99990 28166	.99992 20223
L_6	.70640 74100	.68052 92633	.74121 96637	.72338 80702
L_7	2687	2564	2861	2771
L_8	50.76661	22.23938	24.63011	20.89747
L_9	3.80622	3.80808	3.80362	3.80497
L_{10}	4.46875	4.15706	5.01609	4.76197
L_{11}	35	33	0	0

CONSTANTS FOR LAMBERT PROJECTION

	MINNESOTA	MONTANA	MONTANA	MONTANA
ZONE CODE	South 2203	North 2501	Central 2502	South 2503
L 1 L 2	2,000,000.00 338,400.00	2,000,000.00 394,200.00	2,000,000.00 394,200.00	2,000,000.00 394,200.00
L 3 L 4	21,327,006.06 21,874,349.14	18,689,498.40 19,157,874.26	19,432,939.76 19,919,806.36	20,500,650.51 21,096,820.93
L 5 L 6	.99992 20448 .70092 77824	.99997 14855 .74645 18080	.99992 20151 .73335 38278	.99991 07701 .71490 12442
L 7 L 8 L 9 L 10 L 11	2661 20.12517 3.80662 4.46959 0	2888 20.21285 3.80322 5.09490 0	2821 21.96779 3.80422 4.90135 0	2729 21.15820 3.80560 4.64814 0

	NEBRASKA	NEBRASKA	NEW YORK	NORTH CAROLINA
ZONE CODE	North 2601	South 2602	Long Island 3104	3200
L 1 L 2	2,000,000.00 360,000.00	2,000,000.00 358,200.00	2,000,000.00 266,400.00	2,000,000.00 284,400.00
L 3 L 4	23,004,346.29 23,368,977.46	24,104,561.06 24,590,781.86	24,235,000.80 24,462,545.30	29,637,059.47 30,183,611.25
L 5 L 6	.99996 45501 .67345 07906	.99992 20725 .65607 64003	.99999 49000 .65408 20950	.99987 25510 .57717 07700
L 7 L 8 L 9 L 10 L 11	2531 19.30504 3.80858 4.14653 0	2451 24.68139 3.80977 3.95865 0	2442 20.64240 3.80990 3.93780 0	2106 51.60353 3.81480 3.22483 0

CONSTANTS FOR LAMBERT PROJECTION

ZONE CODE	NORTH DAKOTA North 3301	NORTH DAKOTA South 3302	OHIO North 3401	OHIO South 3402
L_1	2,000,000.00	2,000,000.00	2,000,000.00	2,000,000.00
L_2	361,800.00	361,800.00	297,000.00	297,000.00
L_3	18,819,849.05	19,661,027.79	24,048,738.51	25,522,875.81
L_4	19,215,516.01	20,086,977.18	24,559,158.47	26,027,071.12
L_5	.99993 58426	.99993 58523	.99993 91411	.99993 59346
L_6	.74413 33961	.72938 26040	.65695 03193	.63451 95439
L_7	2876	2801	2455	2354
L_8	22.57950	20.45445	23.48125	28.63705
L_9	3.80339	3.80452	3.80971	3.81121
L_{10}	5.05972	4.84504	3.96783	3.74048
L_{11}	0	0	0	0

ZONE CODE	OKLAHOMA North 3501	OKLAHOMA South 3502	OREGON North 3601	OREGON South 3602
L_1	2,000,000.00	2,000,000.00	2,000,000.00	2,000,000.00
L_2	352,800.00	352,800.00	433,800.00	433,800.00
L_3	28,657,871.66	30,382,831.06	20,836,250.94	22,341,309.43
L_4	29,082,831.70	30,838,032.96	21,383,852.48	22,888,667.15
L_5	.99994 54101	.99993 59432	.99989 45810	.99989 46058
L_6	.59014 70744	.56761 66827	.70918 60222	.68414 73833
L_7	2161	2066	2701	2581
L_8	42.56887	52.48935	22.08858	22.74104
L_9	3.81402	3.81537	3.80602	3.80782
L_{10}	3.33440	3.14645	4.57382	4.26823
L_{11}	0	0	0	0

CONSTANTS FOR LAMBERT PROJECTION

	PENNSYLVANIA	PENNSYLVANIA	SOUTH CAROLINA	SOUTH CAROLINA
ZONE CODE	North 3701	South 3702	North 3901	South 3902
L_1	2,000,000.00	2,000,000.00	2,000,000.00	2,000,000.00
L_2	279,900.00	279,900.00	291,600.00	291,600.00
L_3	23,755,351.27	24,577,800.67	30,630,125.53	32,252,126.30
L_4	24,211,050.37	24,984,826.43	31,127,724.75	32,676,887.65
L_5	.99995 68410	.99995 95012	.99994 54207	.99993 26284
L_6	.66153 97363	.64879 31668	.56449 73800	.54465 15700
L_7	2476	2418	2053	1972
L_8	21.57953	23.87979	53.44099	3.57839
L_9	3.80940	3.81026	3.81555	3.81669
L_{10}	4.01753	3.88319	3.12127	2.94381
L_{11}	0	0	0	0

	SOUTH DAKOTA	SOUTH DAKOTA	TENNESSEE	TEXAS
ZONE CODE	North 4001	South 4002	4100	North 4201
L_1	2,000,000.00	2,000,000.00	2,000,000.00	2,000,000.00
L_2	360,000.00	361,200.00	309,600.00	365,400.00
L_3	20,922,704.09	21,993,575.61	29,010,231.09	29,456,907.29
L_4	21,366,697.03	22,461,937.05	29,535,149.91	29,972,959.94
L_5	.99993 91116	.99990 68931	.99994 84030	.99991 08771
L_6	.70773 81841	.68985 19579	.58543 97296	.57953 58654
L_7	2694	2608	2141	2116
L_8	18.93392	21.54370	44.28313	48.58548
L_9	3.80612	3.80742	3.81431	3.81466
L_{10}	4.55529	4.33519	3.29422	3.24452
L_{11}	0	0	0	0

CONSTANTS FOR LAMBERT PROJECTION

	TEXAS	TEXAS	TEXAS	TEXAS
ZONE CODE	North Central 4202	Central 4203	South Central 4204	South 4205
L_1 L_2	2,000,000.00 351,000.00	2,000,000.00 361,200.00	2,000,000.00 356,400.00	2,000,000.00 354,600.00
L_3 L_4	32,187,809.58 32,691,654.54	34,851,703.46 35,337,121.23	37,261,509.20 37,807,440.38	41,091,749.54 41,576,762.39
L_5 L_6	.99987 26224 .54539 44146	.99988 17443 .51505 88857	.99986 32433 .48991 26408	.99989 47956 .45400 68519
L_7 L_8 L_9 L_{10} L_{11}	1975 5.95074 3.81665 2.97107 0	1852 21.62181 3.81832 2.74550 0	1752 37.19059 3.81962 2.56899 0	1612 59.30342 3.82138 2.33094 0

	UTAH	UTAH	UTAH	VIRGINIA
ZONE CODE	North 4301	Central 4302	South 4303	North 4501
L_1 L_2	2,000,000.00 401,400.00	2,000,000.00 401,400.00	2,000,000.00 401,400.00	2,000,000.00 282,600.00
L_3 L_4	23,894,872.45 24,229,110.29	25,117,176.75 25,664,114.42	27,025,955.35 27,432,812.88	26,230,200.09 26,576,444.45
L_5 L_6	.99995 68422 .65935 54910	.99989 88207 .64057 85926	.99995 12939 .61268 73424	.99994 83859 .62411 78597
L_7 L_8 L_9 L_{10} L_{11}	2466 21.96231 3.80955 3.99323 0	2381 29.30066 3.81081 3.80024 0	2258 34.16878 3.81262 3.53414 0	2308 30.78682 3.81189 3.64047 0

CONSTANTS FOR LAMBERT PROJECTION

	VIRGINIA	WASHINGTON	WASHINGTON	WEST VIRGINIA
ZONE CODE	South 4502	North 4601	South 4602	North 4701
L_1	2,000,000.00	2,000,000.00	2,000,000.00	2,000,000.00
L_2	282,600.00	435,000.00	433,800.00	286,200.00
L_3	27,434,800.06	18,798,081.67	19,832,653.52	25,305,029.12
L_4	27,811,312.71	19,205,863.43	20,289,119.60	25,715,126.55
L_5	.99994 54027	.99994 22551	.99991 45875	.99994 07460
L_6	.60692 48249	.74452 03390	.72639 57947	.63777 29696
L_7	2233	2878	2786	2368
L_8	36.41072	22.15711	21.72121	57.52979
L_9	3.81298	3.80336	3.80474	3.81099
L_{10}	3.48187	5.06556	4.80336	3.77244
L_{11}	0	0	0	0

	WEST VIRGINIA	WISCONSIN	WISCONSIN	WISCONSIN
ZONE CODE	South 4702	North 4801	Central 4802	South 4803
L_1	2,000,000.00	2,000,000.00	2,000,000.00	2,000,000.00
L_2	291,600.00	324,000.00	324,000.00	324,000.00
L_3	26,639,323.45	20,124,133.05	21,050,746.99	22,161,432.25
L_4	27,070,620.78	20,489,179.67	21,430,913.91	22,672,134.66
L_5	.99992 56928	.99994 53461	.99994 07059	.99993 25474
L_6	.61819 53936	.72137 07913	.70557 66312	.68710 32423
L_7	2282	2761	2683	2595
L_8	33.82207	19.04034	48.81363	20.01691
L_9	3.81227	3.80511	3.80628	3.80761
L_{10}	3.58491	4.73451	4.52782	4.30274
L_{11}	0	0	0	0

CONSTANTS FOR TRANSVERSE MERCATOR PROJECTION

	ALABAMA	ALABAMA	ALASKA	ARIZONA
ZONE CODE	East 0101	West 0102	2 to 9 5002 to 5009	East 0201
T_1 T_2	500,000.00 309,000.00	500,000.00 315,000.00	*	500,000.00 396,600.00
T_3 T_4	1822 21.00903	1792 25.53386	*	1852 16.62358
T_5 T_6	.99996 00000 .38170 65	.99993 33333 .38174 77	*	.99990 00000 .38164 85

* Constants are listed with formulas in the text.

	ARIZONA	ARIZONA	DELAWARE	FLORIDA
ZONE CODE	Central 0202	West 0203	0700	East 0901
T_1 T_2	500,000.00 402,900.00	500,000.00 409,500.00	500,000.00 271,500.00	500,000.00 291,600.00
T_3 T_4	1852 16.62358	1852 16.62358	2271 30.53702	1453 26.09287
T_5 T_6	.99990 00000 .38164 85	.99993 33333 .38159 48	.99999 50281 .38114 54	.99994 11765 .38210 90

	FLORIDA	GEORGIA	GEORGIA	HAWAII
ZONE CODE	West 0902	East 1001	West 1002	1 5101
T_1 T_2	500,000.00 295,200.00	500,000.00 295,800.00	500,000.00 303,000.00	500,000.00 559,800.00
T_3 T_4	1453 26.09287	1792 25.53386	1792 25.53386	1124 39.52714
T_5 T_6	.99994 11765 .38210 90	.99990 00000 .38175 93	.99990 00000 .38175 93	.99996 66667 .38264 96

CONSTANTS FOR TRANSVERSE MERCATOR PROJECTION

	HAWAII	HAWAII	HAWAII	HAWAII
ZONE CODE	2 5102	3 5103	4 5104	5 5105
T_1 T_2	500,000.00 564,000.00	500,000.00 568,800.00	500,000.00 574,200.00	500,000.00 576,600.00
T_3 T_4	1214 18.21554	1264 6.77497	1303 57.83623	1294 0.05280
T_5 T_6	.99996 66667 .38257 62	.99999 00000 .38251 76	.99999 00000 .38248 12	.99999 99999 .38248 67

	IDAHO	IDAHO	IDAHO	ILLINOIS
ZONE CODE	East 1101	Central 1102	West 1103	East 1201
T_1 T_2	500,000.00 403,800.00	500,000.00 410,400.00	500,000.00 416,700.00	500,000.00 318,000.00
T_3 T_4	2491 18.35156	2491 18.35156	2491 18.35156	2191 37.04639
T_5 T_6	.99994 73684 .38076 24	.99994 73684 .38076 24	.99993 33333 .38062 27	.99997 50000 .38110 74

	ILLINOIS	INDIANA	INDIANA	MAINE
ZONE CODE	West 1202	East 1301	West 1302	East 1801
T_1 T_2	500,000.00 324,600.00	500,000.00 308,400.00	500,000.00 313,500.00	500,000.00 246,600.00
T_3 T_4	2191 37.04639	2241 32.84965	2241 32.84965	2621 15.15187
T_5 T_6	.99994 11765 .38113 32	.99996 66667 .38110 64	.99996 66667 .38110 64	.99990 00000 .38061 80

CONSTANTS FOR TRANSVERSE MERCATOR PROJECTION

	MAINE	MICHIGAN	MICHIGAN	MICHIGAN
ZONE CODE	West 1802	East 2101	Central 2102	West 2103
T_1 T_2	500,000.00 252,600.00	500,000.00 301,200.00	500,000.00 308,700.00	500,000.00 319,500.00
T_3 T_4	2561 16.25668	2481 18.72150	2481 18.72150	2481 18.72150
T_5 T_6	.99996 66667 .38065 75	.99994 28571 .38072 83	.99990 90909 .38075 41	.99990 90909 .38053 61

	MISSISSIPPI	MISSISSIPPI	MISSOURI	MISSOURI
ZONE CODE	East 2301	West 2302	East 2401	Central 2402
T_1 T_2	500,000.00 319,800.00	500,000.00 325,200.00	500,000.00 325,800.00	500,000.00 333,000.00
T_3 T_4	1772 28.62716	1822 21.00903	2141 41.66790	2141 41.66790
T_5 T_6	.99996 00000 .38172 57	.99994 11765 .38169 86	.99993 33333 .38126 43	.99993 33333 .38124 22

	MISSOURI	NEVADA	NEVADA	NEVADA
ZONE CODE	West 2403	East 2701	Central 2702	West 2703
T_1 T_2	500,000.00 340,200.00	500,000.00 416,100.00	500,000.00 420,000.00	500,000.00 426,900.00
T_3 T_4	2161 39.76857	2076 48.30429	2076 48.30429	2076 48.30429
T_5 T_6	.99994 11765 .38123 62	.99990 00000 .38123 11	.99990 00000 .38123 11	.99990 00000 .38123 11

CONSTANTS FOR TRANSVERSE MERCATOR PROJECTION

	NEW HAMPSHIRE	NEW JERSEY	NEW MEXICO	NEW MEXICO
ZONE CODE	2800	2900	East 3001	Central 3002
T_1 T_2	500,000.00 258,000.00	2,000,000.00 268,800.00	500,000.00 375,600.00	500,000.00 382,500.00
T_3 T_4	2541 16.76677	2321 27.02745	1852 16.62358	1852 16.62358
T_5 T_6	.99996 66667 .38073 27	.99997 50295 .38108 45	.99990 90909 .38161 35	.99990 00000 .38162 04

	NEW MEXICO	NEW YORK	NEW YORK	NEW YORK
ZONE CODE	West 3003	East 3101	Central 3102	West 3103
T_1 T_2	500,000.00 388,200.00	500,000.00 267,600.00	500,000.00 275,700.00	500,000.00 282,900.00
T_3 T_4	1852 16.62358	2391 22.84247	2391 22.84247	2391 22.84247
T_5 T_6	.99991 66667 .38162 88	.99996 66667 .38083 77	.99993 75000 .38084 50	.99993 75000 .38087 50

	RHODE ISLAND	VERMONT	WYOMING	WYOMING
ZONE CODE	3800	4400	East 4901	East Central 4902
T_1 T_2	500,000.00 257,400.00	500,000.00 261,000.00	500,000.00 378,600.00	500,000.00 386,400.00
T_3 T_4	2456 19.72344	2541 16.76677	2431 20.83533	2431 20.83533
T_5 T_6	.99999 37500 .38092 20	.99996 42857 .38074 20	.99994 11765 .38084 22	.99994 11765 .38084 22

CONSTANTS FOR TRANSVERSE MERCATOR PROJECTION

ZONE CODE	WYOMING West Central 4903	WYOMING West 4904		
T_1 T_2	500,000.00 391,500.00	500,000.00 396,300.00		
T_3 T_4	2431 20.83533	2431 20.83533		
T_5 T_6	.99994 11765 .38084 22	.99994 11765 .38084 22		

ZONE CODE				
T_1 T_2				
T_3 T_4				
T_5 T_6				

ZONE CODE				
T_1 T_2				
T_3 T_4				
T_5 T_6				