Land Use Forecasting Techniques For Use In Small Urban Areas Volume 2

DRAFT

ACTIVITY WEIGHTED TECHNIQUE	MULTIPLE LINEAR REGRESSION TECHNIQUE	DENSITY SATURATION GRADIENT METHOD
ACCESSIBILITY MODEL	STOUFFER'S INTERVENING OPPORTUNITIES MODEL	SCHNEIDER'S INTERVENING OPPORTUNITY MODEL
DELPHI TECHNIQUE	LAND USE ALLOCATION MODEL	OPPORTUNITY ACCESSIBILITY MODEL

U.S. DEPARTMENT OF TRANSPORTATION
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LAND USE FORECASTING TECHNIQUES FOR USE IN SMALL URBAN AREAS

Volume 2

Prepared by

Will Terry Moore

August 1977

U.S. Department of Transportation
Federal Highway Administration
Office of Planning
Urban Planning Division

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PREFACE

This report consists of two volumes and is intended to provide information on simple land use forecasting techniques that are considered most practical for use in transportation planning for smaller urban areas of less than 200,000 ~~approximately 50,000 to 500,000~~ population. These simpler forecasting techniques are considered more practical for areas of this size due to their lesser staff, time, cost, and data requirements. Volume 1 consists of the main report and Volume 2 contains the appendices.

Chapter I presents a brief discussion of land use forecasting in the urban transportation planning process. Several selected forecasting techniques that can be applied without the use of a computer are described and evaluated in Chapter II. Also, the models are applied to the UTOWN urban area. UTOWN is a hypothetical area used in the Federal Highway Administration and Urban Mass Transportation Administration transportation planning courses. Chapter III compares the characteristics of selected non-computerized models and their forecasting performance.

Chapter IV describes and evaluates selected forecasting techniques that are appropriate for small areas which are specifically designed for use on a computer. The descriptions touch on the background, theory, capabilities, input and output requirements, calibration, application considerations, and software of each model. The evaluations include a discussion of the potential usefulness of each technique in urban transportation studies.

Appendix A describes a comparative test of two intervening opportunity - accessibility land use models using Boston, Massachusetts data. Appendix B describes a comparative test of five simple land use models using data from Greensboro, North Carolina. Appendix C presents a methodology for developing activity distribution models by linear regression analysis. Appendix D contains detailed information on UTOWN including a geographical description of the area, socio-economic, travel, transit network, and highway network data.

This report is not to be interpreted as an endorsement of any particular procedure described as opposed to any other procedure not included in this report.

APPENDIX A

A Test of Two Intervening Opportunity - Accessibility Land Use Models

A TEST OF TWO INTERVENING OPPORTUNITY-
ACCESSIBILITY LAND USE MODELS

by

Will Terry Moore

Urban Planning Division

and

Robert C. Sword

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August 5, 1977

ABSTRACT

A TEST OF TWO INTERVENING OPPORTUNITY-ACCESSIBILITY LAND USE MODELS

by

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This paper presents the results of a comparative test of two operational land use forecasting techniques. The two forecasting techniques are Stouffer's Intervening Opportunities Model (Stouffer's Model) and Schneider's Intervening Opportunities Model (Schneider Model). Both models are based upon the general concept that the probability of an opportunity being accepted decreases as some function of the number of opportunities ranked closer to some central distribution point (CDP). 1950 and 1963 zonal dwelling unit data for the Boston, Massachusetts region were used in this comparison test. The Boston region had been

previously structured into 626 traffic analysis zones for transportation planning purposes. Only 453 zones were used to calibrate the Boston travel models. The dwelling unit data used in this test were for the 453 calibration zones. The Boston region was structured into twenty time rings, and zones were assigned to the rings according to their traveltime from one CDP and then from five additional CDP's. Stouffer's model was used to make an uncalibrated and calibrated forecast of dwelling unit growth from 1950 to 1963 using one CDP. Schneider's model was then used to make uncalibrated and calibrated forecasts using one CDP, and then similar forecasts using six CDP's. The proportion of total dwelling unit growth to be distributed from each of the six CDP's was determined by the relative size of each CDP's employment base. To evaluate the forecasting accuracy of both models the 1963 forecasts were compared with the 1963 actual data. The error from the "one CDP" calibrated forecast for both models (zonal level) was approximately one half the error from the similar uncalibrated forecast. The forecasting accuracy of Schneider's model was increased by approximately a factor of two when growth was allocated from six CDP's, rather than from a single CDP.

A TEST OF TWO INTERVENING OPPORTUNITY-ACCESSIBILITY LAND USE MODELS

This paper presents the results of a comparative test of two land use forecasting techniques. The two forecasting techniques are Stouffer's Intervening Opportunities Model (Stouffer's Model) and Schneider's Intervening Opportunities Model (Schneider's Model). Both models are based upon the general concept that the probability of an opportunity being accepted decreases as some function of the number of opportunities ranked closer to some central distribution point (CDP).

Two major objectives of this research were (1) to compare the relative accuracy of the two forecasting techniques through a series of ex post facto tests, holding all conditions constant except the interrelationships among variables, so that any differences in "forecast" would be a function only of inherent differences in the models. And (2) to confirm several of the findings and conclusions of a similar study by Swerdloff and Stowers (1) which were:

> "Although the two intervening opportunity models performed satisfactorily as used in this study, some evidence pointed to the possibility of improvement by allocating growth from all major centers of employment rather than from just a single point, the CBD. In addition, each of the two models implies a different straight line plot on different semilogarithmic coordinates which did not hold true for Greensboro over the entire study area. Apparently the hypotheses are valid, but

separate functions may be necessary for the built up, inner city area, and the developing suburban area."

"Both the plot of total allocated dwellings (linear-ordinate) vs. total accumulated opportunities (logarithmic-abscissca) [Stouffer Calibration] and the plot of dwellings to be located (logarithmic-ordinate) vs. total accumulated opportunities from the central point (linear-abscissa) [Schneider Calibration] for the Greensboro data appear to exhibit two distinct straight line segments, rather than one, as required by the initial model formulation. Also, the zones comprising the transition area between the two straight line segments for the Schneider calibration are the same ones as those at the juncture of the two line segments for the Stouffer calibration."

STUDY AREA AND DATA

The study area used in this test was the Boston, Massachusetts region. Boston was chosen because of the excellent historical data that was available for each traffic analysis zone. Also, Boston's study area was significantly different in physical structure (many large employment centers and non-concentric in shape) than the Greensboro, North Carolina study area (one major employment center and concentric in shape) which Swerdloff and Stowers used.

The Boston region shown in figure 1 had been previously structured into 626 traffic analysis zones for transportation planning purposes. Only 453 zones were used to calibrate the Boston travel models. This research used 1950 and 1963 dwelling unit data for the 453 calibration zones. For this research the Boston study area was further structured into twenty time rings, each of which was composed of a whole number of zones and an approximately equal number of opportunities. Traffic zones were assigned to rings according to their ranking in travel-time from one CDP and then from five additional CDP's.

MODEL DESCRIPTION AND METHODOLOGY

Stouffer's Intervening Opportunities Model (2)

The basic premise of Stouffer's Intervening Opportunities Model (Stouffer's IOM) is that the number of persons or jobs locating at a given distance (from some central point) is directly proportional to the number of opportunities (units of residential or nonresidential capacity) at that distance and inversely proportional to the number of intervening opportunities encountered up to that distance. The formulation of the model is:

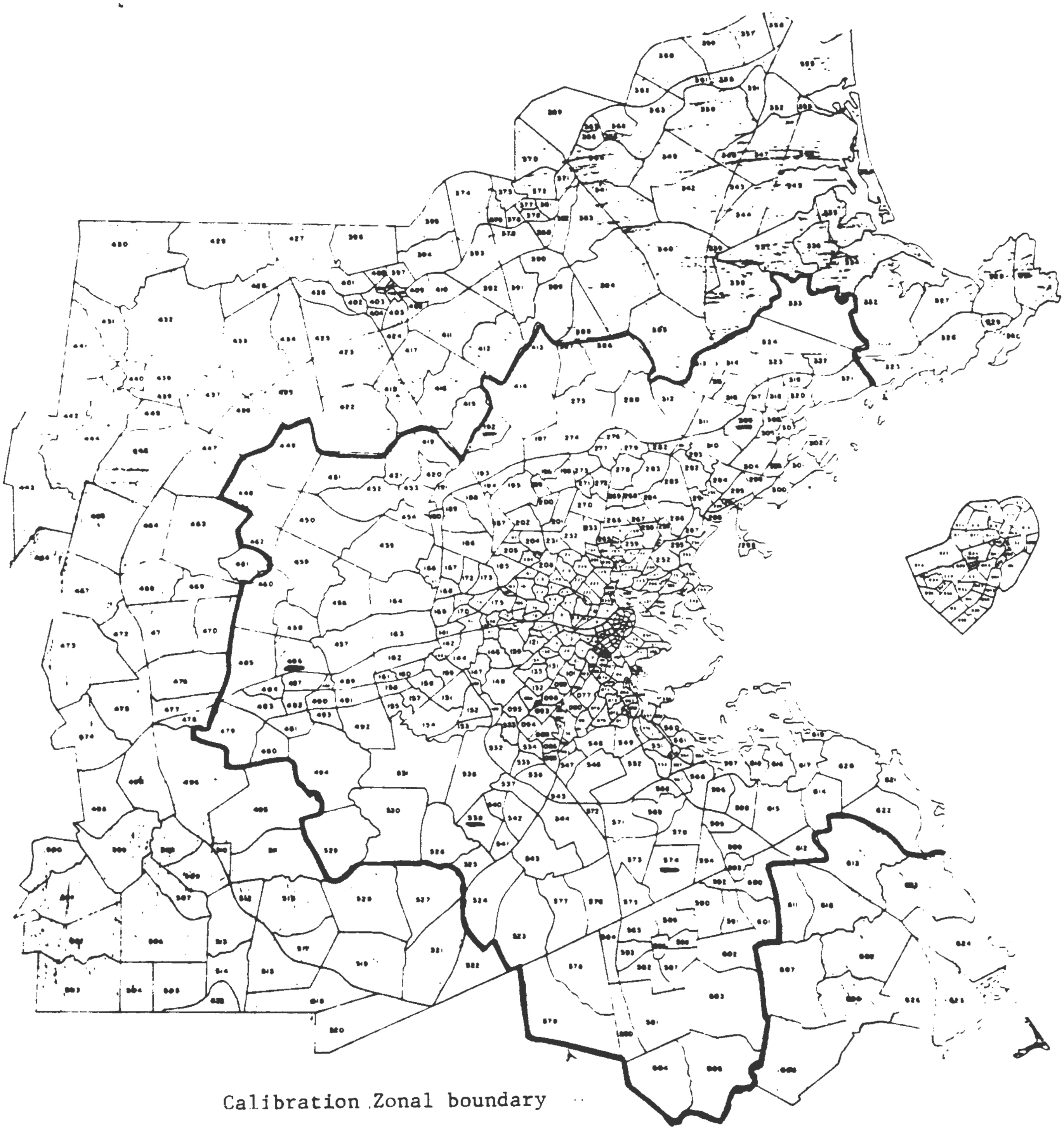
$$g_p = k \frac{O_p}{O}$$

where:

g_p = number of activities (population, households, jobs, etc.) forecast to be located in an analysis interval p

Figure 1

Boston Massachusetts Region Zonal Structure
(The shaded zones are the six major employment centers CDP's)



O_p = total opportunities (available residences or jobs) in interval p

O = total number of opportunities (sum of all opportunities) from the point of origination through and including interval p

k = proportionality constant to assure that activities allocated (located) equals the actual number of activities (total growth) that is being allocated

Schneider's Intervening Opportunities Model (2)

The basic premise of Schneider's Intervening Opportunities Model (Schneider's Model) is that the probability of an activity (person, jobs, etc.) finding a suitable opportunity (a unit of available residential or nonresidential capacity) for location at a given distance is hypothesized to be a monotonically decreasing function of the number of intervening opportunities (number of opportunities encountered up to that distance), opportunities being ranked by time from some central distribution point. The formulation of the model is:

$$d(G_p) = g_t \; [e^{-\ell O} \; - \; e^{-\ell(O+O_p)}]$$

where:

G_p = total number of locations in opportunity interval from the central distribution point up to interval p

g_t = total growth to be allocated

ℓ = model parameter expressing the probability of an opportunity being accepted for location

O = total number of opportunities ranked from the central distribution point up to interval p

O_p = Opportunities in interval p

e = base of natural logarithms = 2.71828

Schneider's Model has a negative exponential formulation. The formulation produces a number which ranges from zero to one. The logic of this formulation is as follows: g_t represents the total amount of activity which is forecast to take place within the study area within a given time period. If the term $[e^{-\ell O} - e^{-\ell(O+O_p)}]$ generates a number that is greater than one, this would imply that more activity is being allocated to zone one than has been allocated or predicted for the entire region. The other limit of this term would be zero. Thus, the entire term ranges from zero to one.

When being used for forecasting purposes a major requirement of both models is the need to estimate the future distribution of opportunities (future activity capacity) for the particular activity being allocated. Opportunity is usually defined as the product of available land for a given activity and the density of the activity (unit activity per unit land). In this research the 1963 dwelling unit opportunities were already known.

STOUFFER'S MODEL APPLICATIONS

Uncalibrated Forecast (using one CDP)

The Stouffer formulation $g_p = k \frac{0_p}{0}$ can be applied without the need to assume parameter values. Using the 1950 and 1963 dwelling unit data as structured into the previously discussed twenty time and opportunity rings the forecast number of dwellings in ring p, was determined by direct substitution in the formula. The ring forecasts were then proportioned among the constituent zones on the basis of opportunities.

Calibrated Forecast (using one CDP)

For an explanation of the fitting of Stouffer's model to 1950-1963 Boston data the model must be converted into its continuous differential form as follows:

$$d(G_p) = \frac{K\ d(0)}{0}$$

by integrating

$$G_p = K \text{ Ln } 0 + C$$

where:

G_p = The total number of dwellings allocated to all opportunities from the central point up to and including opportunity interval p

$d(G_p)$ = Dwellings allocated to opportunity interval p

$d(0)$ = Opportunities in interval p

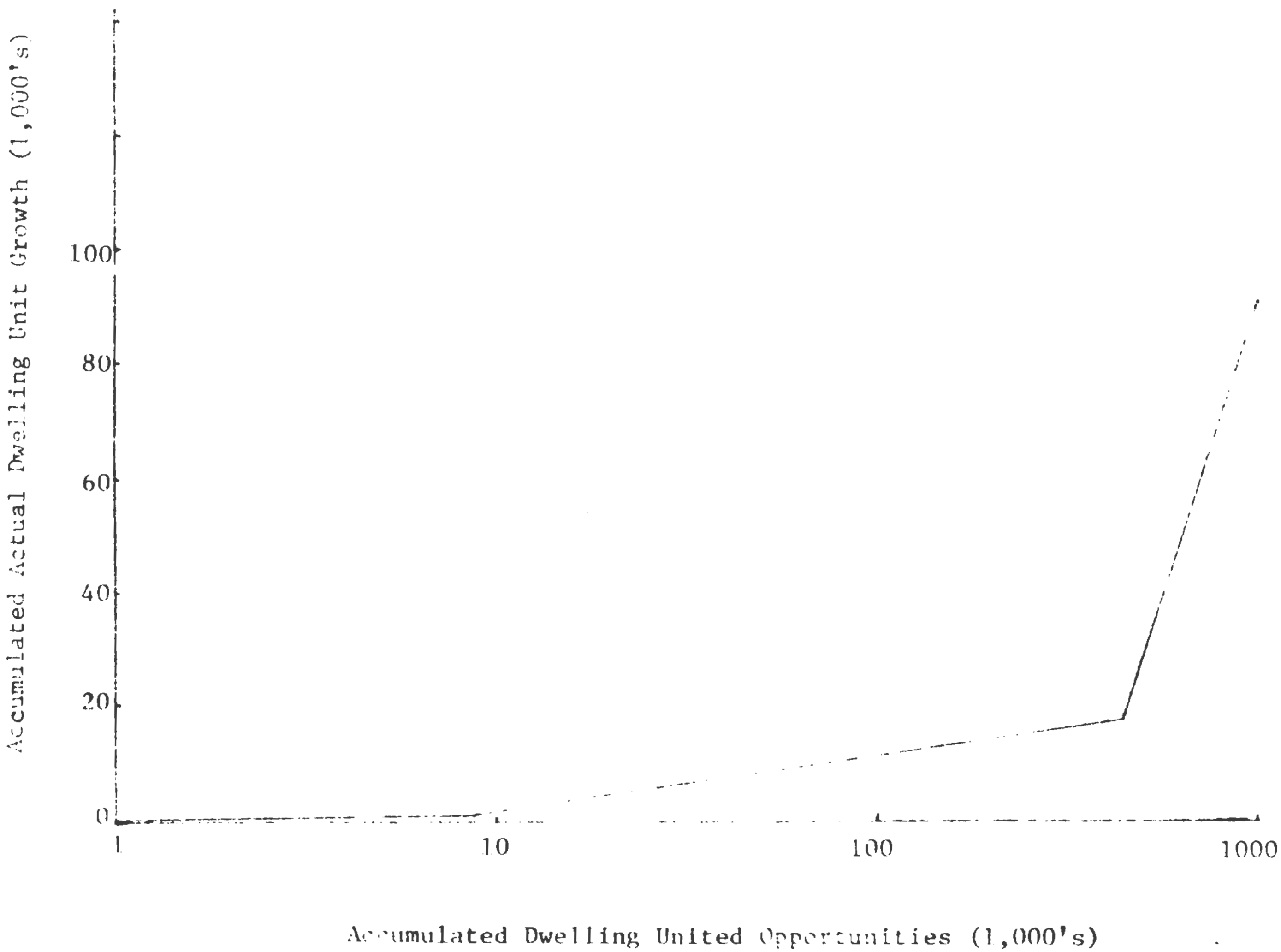
C = Constant of integration

In theory this equation plots as a straight line of slope K where the ordinate, total allocated dwellings, is a linear scale and the abscissa, total accumulated opportunities, is a logarithmic scale. However, since a similar plot for Greensboro, North Carolina (1), resulted in two straight lines with different slopes, it was decided to predict where (if at all) this change of slope would occur for the Boston plot. Dwelling unit density was thus calculated for each ring and the location where a significant change in density occurred was selected as the predicted change of slope location. There were two rings where a large change in density occurred. They were located approximately 2 miles and 7.5 miles from the CDP.

A plot was then made using the actual accumulated zonal dwelling unit growth (1950-1963) as the ordinate and accumulated 1950 opportunities as the abscissa, the zones being ranked by traveltime to the CDP as shown in Figure 2 (the three lines were hand fitted). The actual plot had changes of slope at approximately the same distances as predicted above. From further analysis of the Boston region it appeared that at approximately 2 miles from the CDP the old central city ended and the old suburbs began. The second change in slope at about 7.5 miles appeared to be the beginning of the newly developing suburbs. This results tends to confirm Swerdloff's and Stower's hypotheses that separate functions (i.e., "K" values) may be necessary for built up, inner city areas, and the developing suburban area. The three "K" values were used to make 1963 growth estimates for the individual zones and the ring forecasts were then determined by summing the forecasts for the constituent zones.

Figure 2

Plot for Derivation of "K" in Calibrated Stouffer's Model (using one CDP)



SCHNEIDER'S MODEL APPLICATIONS

Uncalibrated Forecast (using one CDP)

As a necessary condition for applying Schneider's model the parameter "ℓ" must be stipulated. For the first trial of the model for a 1963 forecast without benefit of the 1950-1963 Boston data, "ℓ" was estimated from the assumption that the actual dwelling unit increase within the study boundaries was 99 percent of the aggregate Boston area oriented growth. (The theoretical model is based on a distribution to an unbounded area; application to a definite area required specification of the number of accepted opportunities being outside the boundary or equivalently, the percentage accepted up to the boundary.) The "ℓ" resulting from this assumption was 2.31×10^{-6}. Once the "ℓ" value was determined the dwelling unit growth was distributed to each zone and the ring forecasts were then determined by summing the forecasts for the constituent zones.

Calibrated Forecast (using one CDP)

For an explanation of the fitting of Schneider's model to 1950-1963 Boston data, the formula can be restated after integration as $G_p = G_t[1 - e^{-\ell O}]$ subtracting G_t from both sides and rearranging,

$$G_t - G_p = G_t e^{-\ell O}$$

or

$$\ln (G_t - G_p) = \ln G_t - \ell O$$

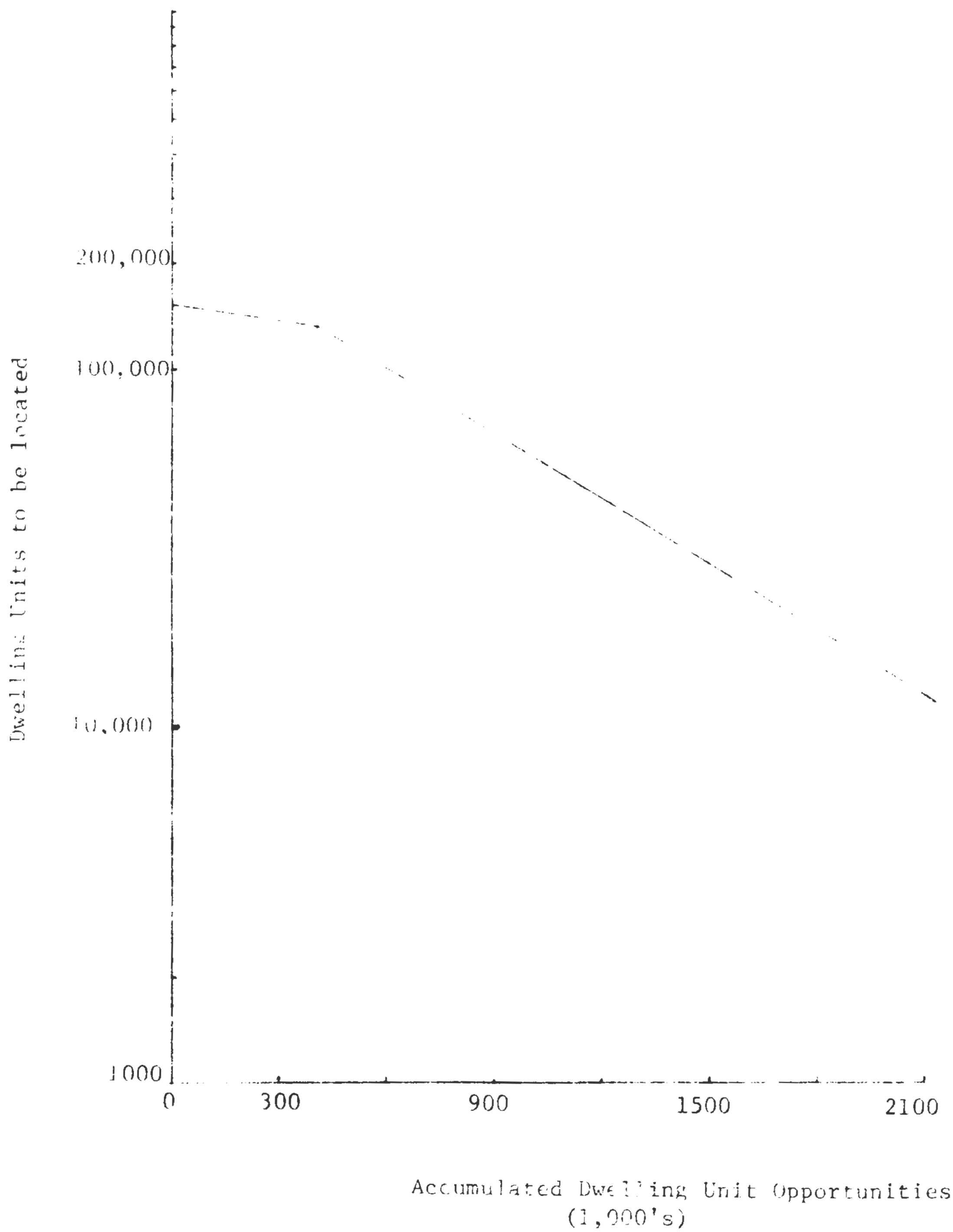
In theory this relationship plots as a straight line where the ordinate $(G_t - G_p)$ is in logarithmic scale and the abscissa (total accumulated opportunities from the CDP (O)) is in linear scale. The slope is "ℓ" and the intercept G_t. As previously discussed in the Stouffer model applications, a prediction was made that changes in slope would occur at approximately 2 miles and 7.5 miles from the CDP. The actual plot of the quantity $(G_t - G_p)$ versus accumulated opportunities (O) in semilogarithmic form for the Boston region reflected a minor change in slope at about 2 miles and a major change at about 7.5 miles from the CDP as shown in figure 3. Two lines were hand fitted to the plot. Their slopes were calculated to be 4.2×10^{-7} for the inner city segment and 13.7×10^{-7} for the newly developing suburbs segment. The slopes of these fitted lines can be loosely compared to the short and long trip "ℓ's" which have become standard practice in applying Schneider's model to trip distribution. The two "ℓ" values were then used to distribute dwelling unit growth to each zone and the ring forecasts were determined by summing the forecasts for the constituent zones.

Uncalibrated Forecast (using six CDP's)

Swerdloff and Stowers (1) inferred that possible improvements in accuracy might be obtained from the two opportunity models by allocating growth from several major centers of employment rather than from a single point. To determine what the improvement, if any, would be using Schneider's model, six major employment centers were selected (see figure 1) and a portion of the total regional dwelling unit growth

Figure 3

Plot for Derivation of "ℓ" in Calibrated Schneider's Model
(using one CDP)



was allocated to each of the six CDP's according to their share of the total regional employment. The "ℓ" value used to allocate the portions of total dwelling unit growth from each CDP to all zones of the Boston region was 2.31 x 10^{-6}. Each of the six forecasts of zonal dwelling unit growth were summed for each zone to determine the total zonal forecast growth.

Calibrated Forecast (using six CDP's)

To make a calibrated forecast using six CDP's, it was necessary to determine "ℓ" values associated with each CDP. This was accomplished by ranking the zones by traveltime for each CDP and then plotting (G_t - G_p) as the ordinate and total accumulated opportunities from the CDP as the abscissa for each center, and then fitting lines to these plots as shown in figures 4 through 9. Again two lines were obtained. The slopes ("ℓ" values) of the first and second lines respectively for each plot were approximately equal, therefore to facilitate calculation it was decided to use "ℓ" values of 7.25 x 10^{-7} and 2.31 x 10^{-6}. The respective portions of total dwelling unit growth was distributed from each CDP to all zones. Each of the six forecasts of zonal dwelling unit growth were then summed for each zone to determine the total zonal forecast growth.

Figure 4

Plot for Derivation of "ℓ" in Calibrated Schneider's Model
for CDP No. 1

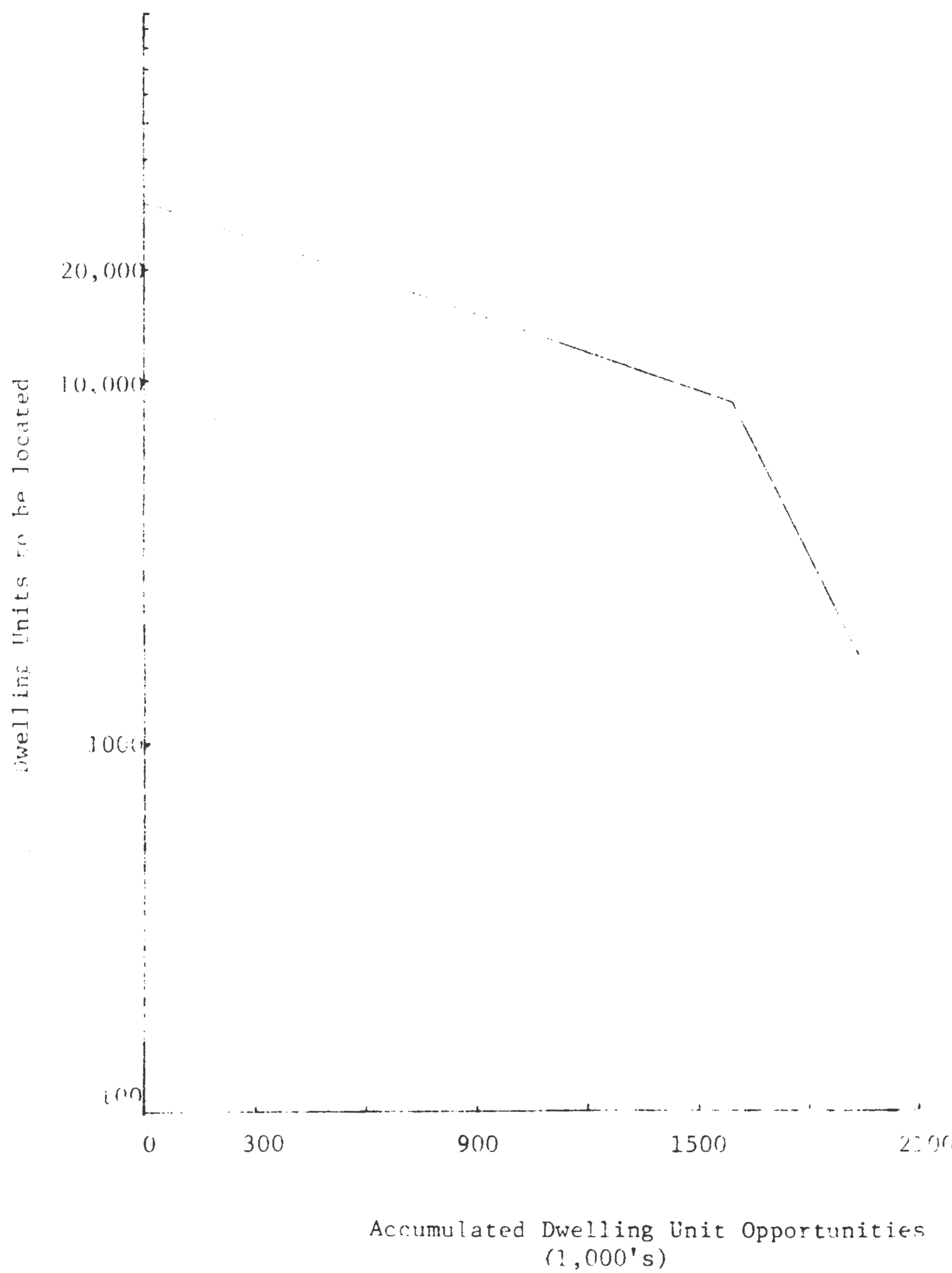


Figure 5

Plot for Derivation of "ℓ" in Calibrated Schneider's Model
for CDP No. 2

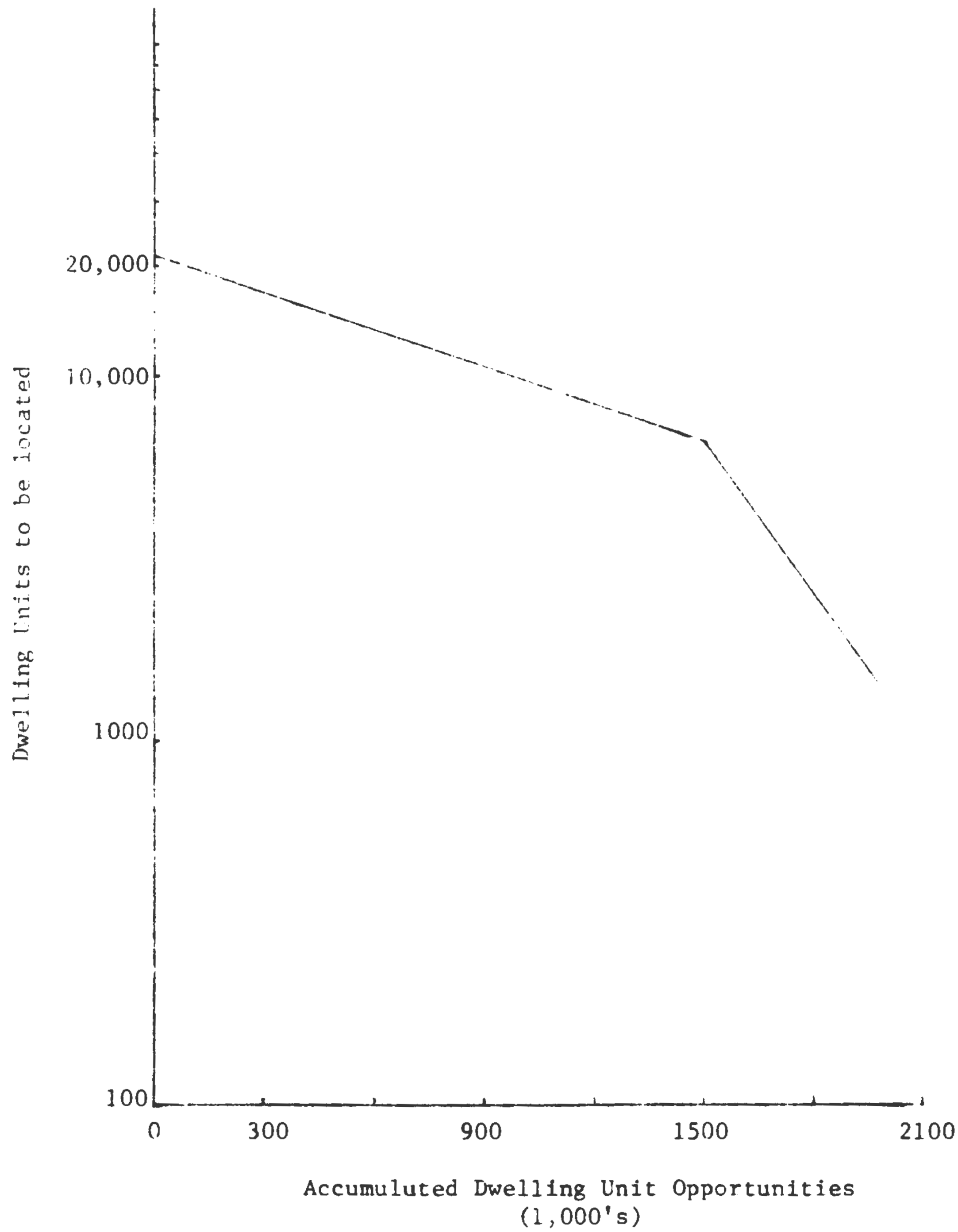


Figure 6

Plot for Derivation of "ℓ" in Calibrated Schneider's Model
for CDP No. 3

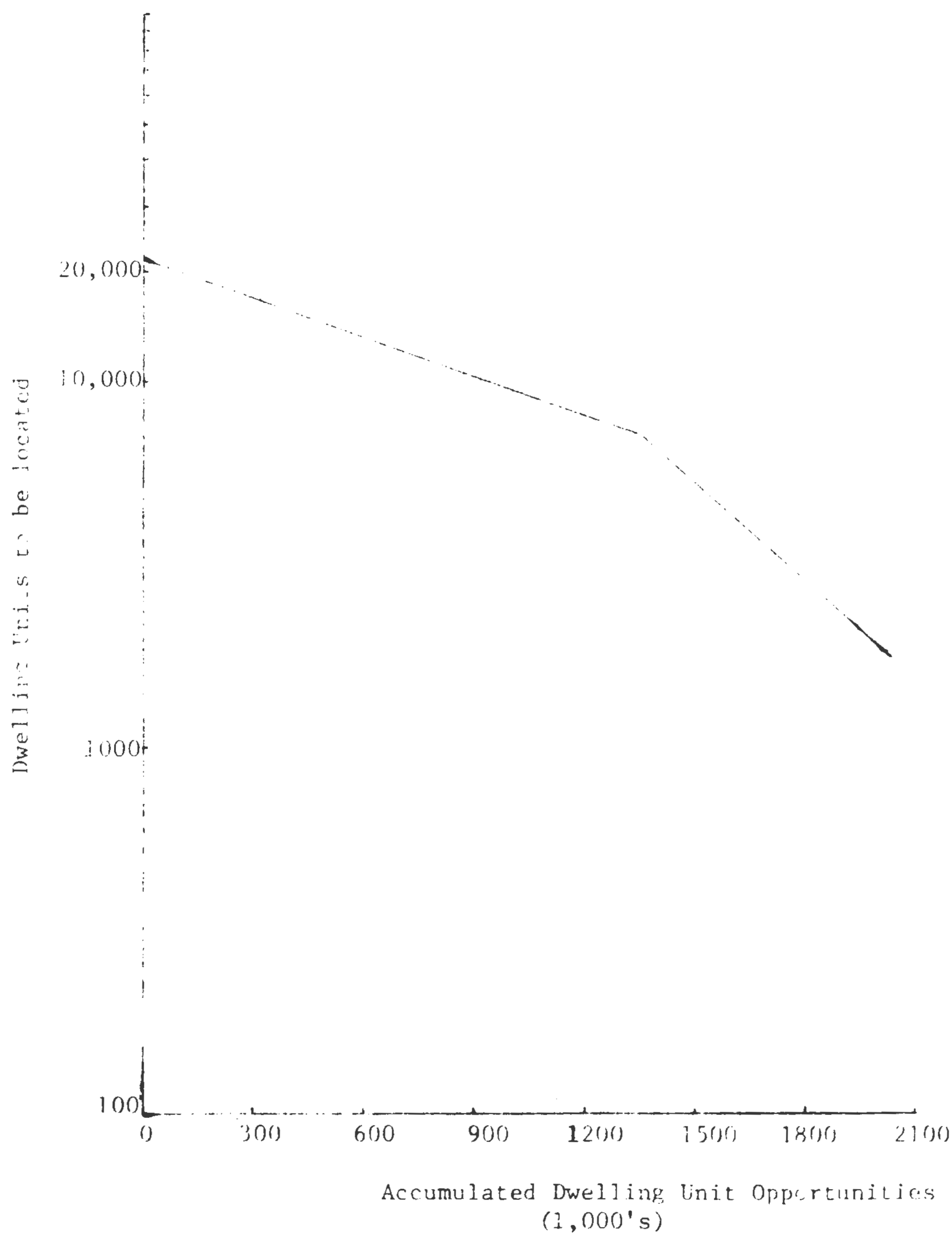


Figure 7

Plot for Derivation of "ℓ" in Calibrated Schneider's Model
for CDP No. 4

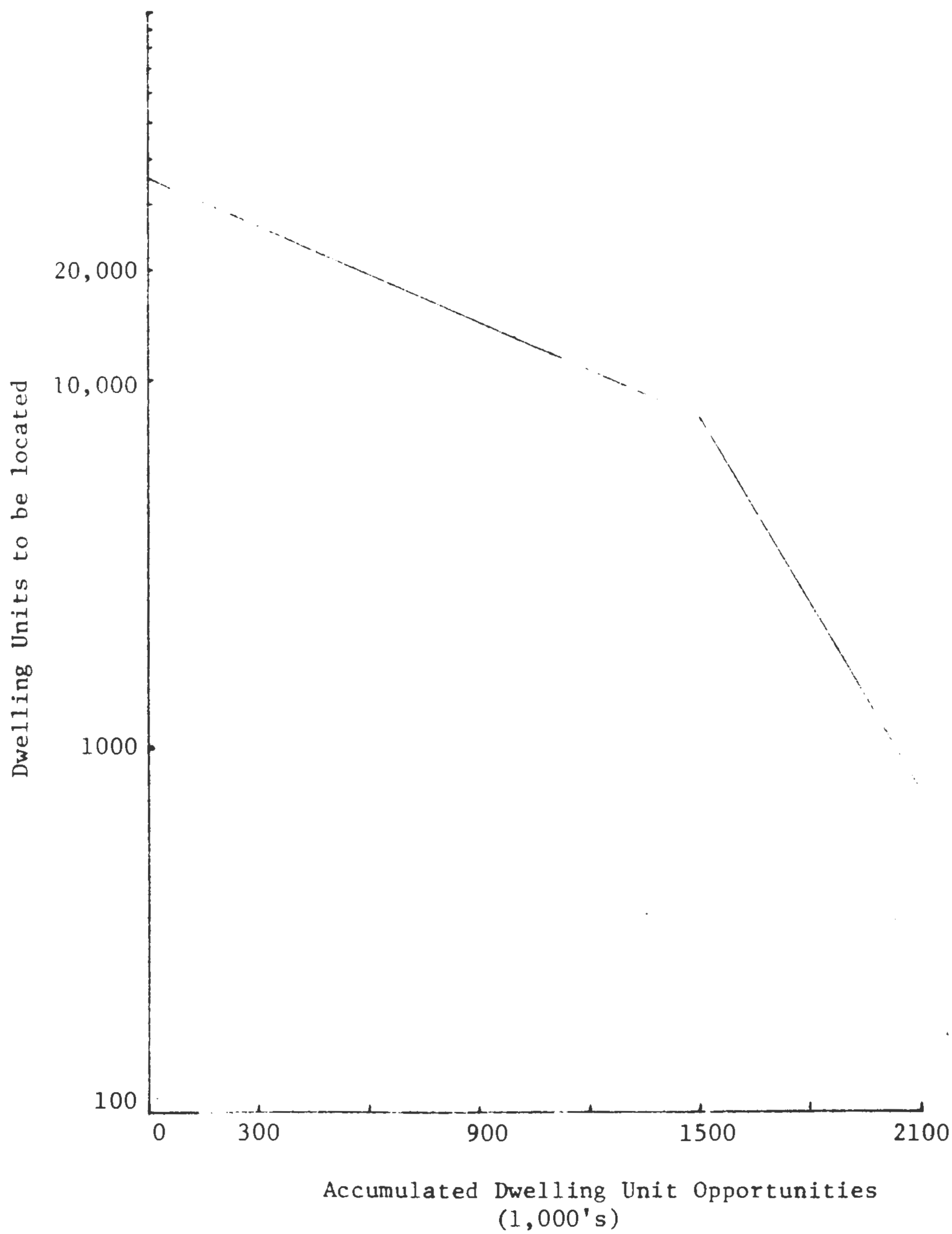


Figure 8

Plot for Derivation of "ℓ" in Calibrated Schneider's Model
for CDP No. 5

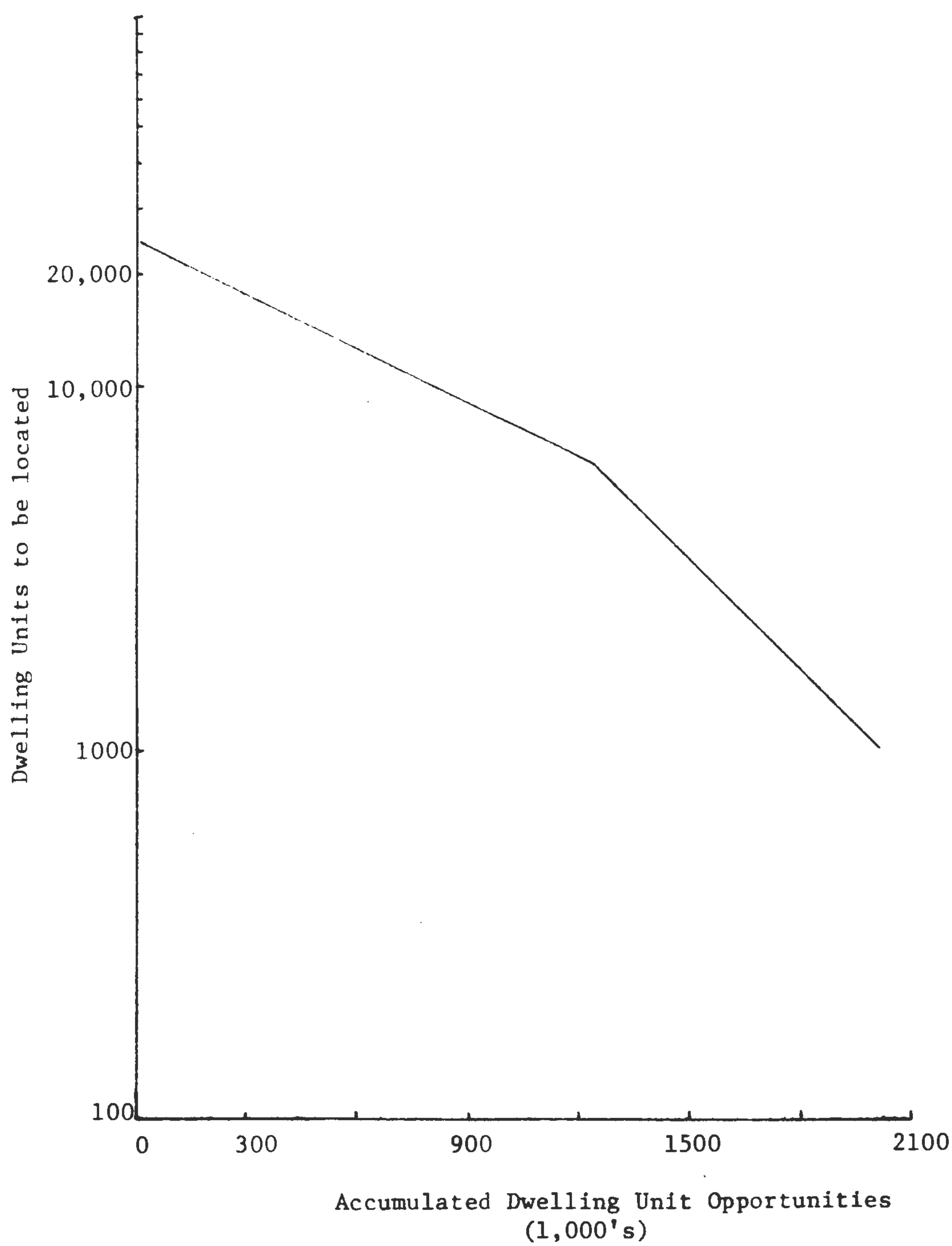
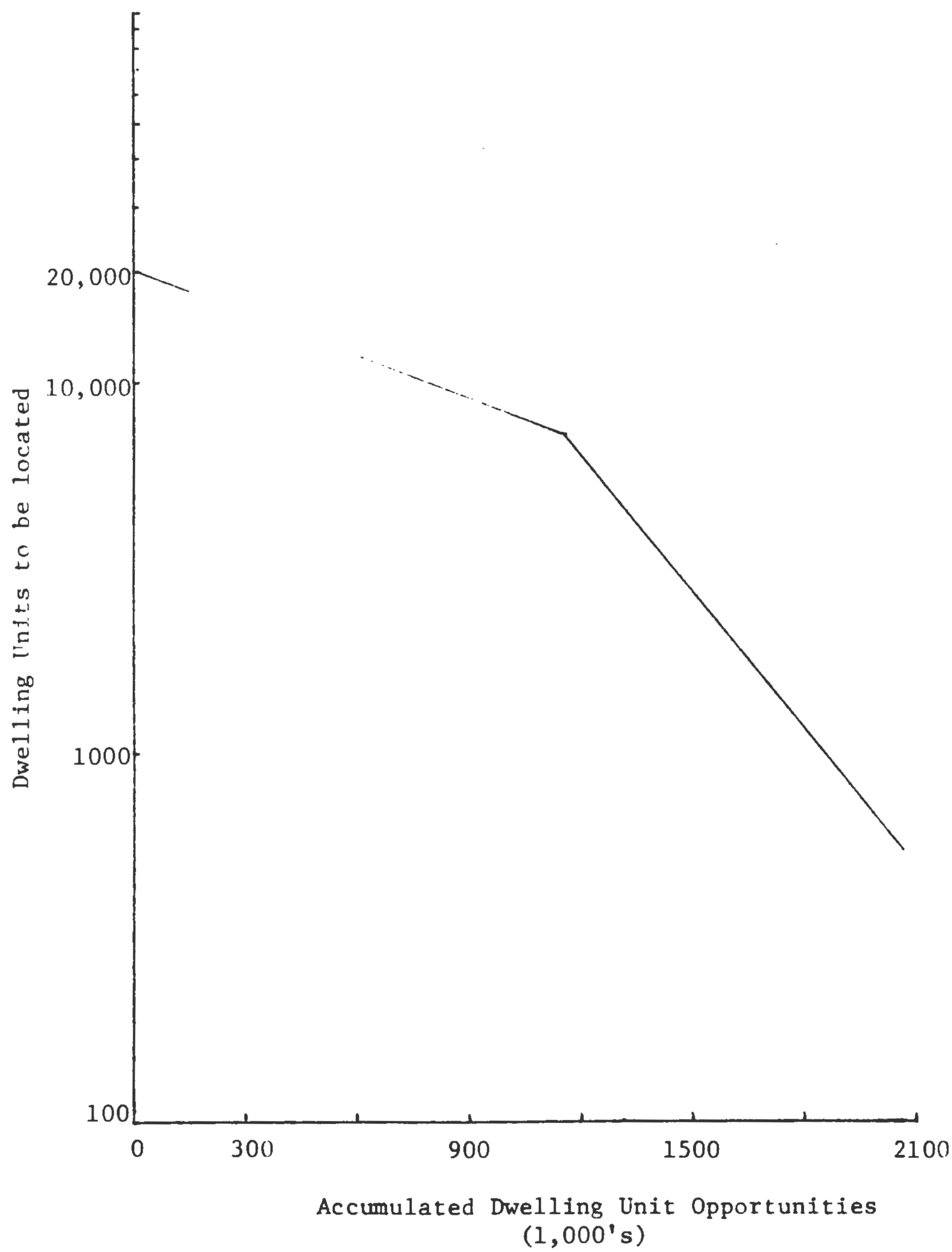


Figure 9

Plot for Derivation of "ℓ" in Calibrated Schneider's Model for CDP No. 6



PERFORMANCE AND INTERPRETATION OF RESULTS

The results obtained from applying the two opportunity techniques to the Boston region are analyzed and compared with the results obtained by Swerdloff and Stowers (1) and are presented in the following paragraphs.

The sum of squares of dwelling unit forecasting error was used as the single accuracy measure in the Swerdloff and Stowers (1) study and therefore it was also used in this research. Forecasting error is simply the difference between the actual 1963 zonal dwelling unit growth and the forecast 1963 growth from the models. The sum of squares of differences between estimated and actual are analogous to "unexplained" variances of a statistical model, however, since valid statistical inference cannot be drawn, this terminology should not be used.

The sum of squares of dwelling unit forecasting error was calculated for two levels of geographic aggregation, the traffic analysis zone and time ring, for all trails as shown in Table 1. Also shown are selected results from the Swerdloff and Stowers study (1).

The error measurements of Table 1 provide an index which can be used to compare results in any specific column, that is, for the same level of aggregation. However, any attempt to make a comparison between columns will be meaningless, since different numbers of units and different variances from mean growth rates are involved at different levels of aggregation.

TABLE 1

SUM OF SQUARES OF DWELLING
UNIT FORECASTING ERROR

METHOD	LEVELS OF AGGREGATION			
	BOSTON		GREENSBORO	
	Traffic Zone	Time Ring	District	Time Ring
STOUFFER'S MODEL				
One CDP				
Uncalibrated Forecast	17.50	45.30	42.20	34.80
Calibrated Forecast	8.00	14.00	30.70	14.60
SCHNEIDER'S MODEL				
One CDP				
Uncalibrated Forecast	17.80	46.20	41.30	33.80
Calibrated Forecast	8.40	16.00	30.10	16.50
Six CDP's				
Uncalibrated Forecast	8.52	11.30	-	-
Calibrated Forecast	7.70	10.40	-	-

All Values Multiplied By 10^{-7}

The results from the Boston study indicates an increase in error as the analysis units become larger, while error was found to decrease in the Greensboro study. This is attributed to the difference in the development structure of the two areas. The level of error from Schneiders' and Stouffers model was reduced by selecting six major employment centers as growth distributors. Schneiders' model gave the best uncalibrated forecast when six CDP's were used to distribute growth.

In analyzing the error from the one-CDP uncalibrated distribution using Schneider's model it was found that 63% of the error was caused by 10% of the zones. Similarly, in the calibrated one-center and six-center forecasts approximately 45% of the error was caused by 5% of the analysis zones. Approximately 70% of the large-error zones are re-occuring in both models. This implies that these high error zones require special analysis. Table 2 shows this analysis in detail.

TABLE 2

ANALYSIS OF ERRORS

	Stouffer Uncal. (1 CDP)	Stouffer Cal. (1 CDP)	Schneider Uncal. (1 CDP)	Schneider Cal. (1 CDP)	Schneider Uncal. (6 CDP's)	Schneider Cal (6 CDP's)
Error	174,661,937	80,015,043	177,822,937	83,955,762	85,234,029	76,907,754
	132,379,014	42,634,998	112,752,061	34,975,098	41,405,589	30,571,396
No. of High Error Zones	35	22	39	19	19	15
% of High Error Zones	8	5	9	5	5	4
% Error attributable to H-E Zones	75	52	63	43	46	49

FINDINGS AND CONCLUSIONS

1 - An evaluation of the results obtained from applying these models to the study area of Greensboro, North Carolina, and Boston, Massachusetts, shows that the models performed very similarly in both cases.

2 - Both intervening opportunity models should plot as a straight line on different semilogarithmic coordinates. This linear relationship was found not to hold true for the entire study area of Boston as was also found for the entire study area of Greensoboro, North Carolina. Before the actual plotting of the equation using Boston area data, an attempt was made to predict where (if at all) this break would occur. The net dwelling unit density was calculated for twenty time rings, and it was predicted that the break would occur where a very large change in density occurred. The large changes in density occurred at approximately two miles and 7.5 miles from the CDP. Given this prediction, a plot of the equation was made on semilogarithmic paper and the plot had changes in slope at approximately two miles and 7.5 miles from the CDP. It appears that these changes in slope reflect the end of the old city plus the beginning of the old suburb and the end of the old suburb plus the beginning of the newly developing suburb. This is an indication that the use of different analytical approaches for the inner city and newly developing suburbs would produce better forecasts.

3 - The forecasting accuracy of Schneider's Intervening Opportunities Model was increased by approximately a factor of two when growth was allocated from several major centers of employment, rather than from a single point the CDP. This supported Swerdloff's and Stowers' similar hypotheses.

4 - The error from the one center calibrated forecast for both models (zonal level) was approximately one half the error from the similar uncalibrated forecast. (See Table 1.)

5 - Five of the six high employment zones which were selected as centers for DU distribution were consistently underforecasted. This shows the tendency of households to locate close to the employment centers in the developing suburbs, all other considerations being equal.

6 - A large portion of the error was attributed to a small number of zones. These zones should be excluded from the distribution of Stouffer's and Schneider's Models and analyzed by other means.

7 - The overall performance of these two models indicate that they are sufficiently accurate to be recommended for use in relatively large and structually complex study areas such as the Eastern Massachusetts region.

REFERENCES

1. Swerdloff, C. H., and Stowers, J. R., "A Test of Some First Generation Residential Land Use Models," Transportation Research Record Number 126, 1966.

2. Moore, Will Terry, Land Use Forecasting Techniques For Use In Small Urban Areas (Draft Report), U.S. Federal Highway Administration, Urban Planning Division, Washington, D.C., August 1977.

TABLES

FIGURES

APPENDIX B

A Test of Some First Generation Residential Land Use Models (Extracted from Transportation Research Board Record Number 126)

A Test of Some First Generation Residential Land Use Models

CARL N. SWERDLOFF and JOSEPH R. STOWERS, Highway Engineers, U.S. Bureau of Public Roads

•THIS paper reports on a comparative evaluation of five operational residential land use forecasting techniques, four of which have been previously used in urban transportation planning studies. These techniques are representative of the earliest of efforts in the development of operational urban activity simulation models and continue to serve, either in their original or in modified form, a great number of transportation planning organizations. Urban activity simulation models currently under development, while in most cases considerably more complex and, hopefully, more accurate, in many instances draw upon notions and fundamental concepts which either originated with or were adapted to these early techniques. Improvements being introduced in these later, second generation models include more complex statistical estimating procedures, the stratification of residential locators into several distinct groups, and the incorporation of behavioral relationships in the model formulation. These newer techniques may require several years of research, evaluation, and refinement before they become fully operational. Meanwhile, the less sophisticated approaches evaluated in this report should continue to be useful to smaller metropolitan areas lacking the resources for developmental research.

The primary objective of this project was to compare the relative accuracy of these approaches through a series of ex post facto tests, holding all conditions constant except the interrelationships among variables, so that differences in "forecasts" would be a function only of inherent differences in models.

There is a temptation to interpret a study of this nature as a contest of sorts and to turn to a table of results for the proclaimed "winner." Any such evaluation of the results is unwarranted for several reasons. First, the contestants are not all of the same class. Some are more truly "forecasts," and some are merely data fitting problems. The latter involve fitting different numbers of parameters. More information is used in some than in others. Perhaps most important, the results represent a sample of one, out of a rather large universe of possible test conditions. Entirely different results might occur in other cities, at other time periods, by other forecasters, working with other data problems.

GENERAL PROCEDURES

The five residential land use forecasting procedures are each variants of work done by others. The only innovations introduced here are the authors' simplifications and modifications to suit peculiar test conditions—apologies are made to the progenitors of these models for possible misrepresentations of their original work. In any realistic planning application, more care would necessarily be given to the particular forecasting tool used. Trends would be more carefully analyzed, the forecasters would be more familiar with the area, and output of models would be scrutinized in detail and modified as judgment indicated. In contrast, the authors have applied the models coldly and crudely, accepting the immediate output in an attempt to make objective comparisons.

Paper sponsored by Committee on Land Use Evaluation and presented at the 45th Annual Meeting.

The techniques used were (a) the density-saturation gradient method, (b) accessibility model, (c) regression, and (d and e) two intervening opportunity models.

The density-saturation gradient method (DSGM) is a simplification of the approach used by the Chicago Area Transportation Study (1, 2). Of the five techniques, the DSGM is least computer oriented, more demanding of subjective inputs, and therefore least suitable for objective comparison with other approaches, particularly when the forecasters are not intimately familiar with the area. The method is based essentially on the regularity of the decline in density and percent saturation with distance from the CBD, and the stability of these relationships through time.

The simple accessibility model is based upon the concept formulated by Walter Hansen (3, 4). Growth in a particular area is hypothesized to be related to two factors: the accessibility of the area to some regional activity distribution, and the amount of land available in the area for development. The accessibility of an area is an index representing the closeness of the area to all other activity in the region. All areas compete for the aggregate growth and share in proportion to their comparative accessibility positions weighted by their capability to accommodate development as measured by vacant, usable land.

The third method used in this study, multiple linear regression, is a popular approach because of its operational simplicity and ability to handle several variables (5, 6, 7). The proportion of total regional growth which locates in a particular area is assumed to be related to the magnitude of a number of variables which in some manner are measures of geographic desirability as viewed by those making the locational decision. The procedure is to determine those factors, and their weights, which in linear combination can be related to the amount of growth which has been observed to take place over a past time period. These factors (called independent variables) and their weights (regression coefficients), in linear combination (the regression equation) can then be applied to the individual analysis areas to forecast the magnitude of growth (the dependent variable).

Although more commonly applied to the problem of trip distribution, the intervening opportunities models can be used in simulating the distribution of urban activity. Two separate and distinct formulations were applied in this study, both based upon the general notion that the probability that an opportunity is accepted decreases as some function of the number of opportunities ranked closer to a central distributing point. The Stouffer formulation was originally applied to intra-urban migration (8). A related formulation has more recently been investigated as a trip distribution technique (9). Schneider's formulation was originally applied to trip distribution (10) and is currently being used in distributing urban activity (11, 12).

The test area used in this study was Greensboro, North Carolina. This city was chosen for a number of reasons. First and most important, a rather extensive information file on a small area basis for two time periods (1948 and 1960) was available. Secondly, it was felt that Greensboro was representative of the kind and size city for which forecasting techniques of the kind being examined would still be most appropriate after the development of more sophisticated models in the largest metropolitan areas.

The data for the study came from two major sources. The data obtained from the University of North Carolina contained a wide variety of information for the Greensboro area coded to 3,980 grid cells, each one 1000 ft square, for a circular area of about 7-mi radius. These data included quantitative measures of land use, population, residential density, proximity to various activities and to the CBD, and certain environmental measures (13). With certain exceptions, these data were available at the grid level for two time periods, 1948 and 1960.

The data supplied by Alan M. Voorhees and Associates included 1960 population, employment, accessibility to shopping, and accessibility to employment, for each of about 250 zones. These latter accessibility measures were computed from zone-to-zone traveltimes over the highway network.

A number of problems were encountered in combining the data from these two sources in a form suitable for testing of the models. Principal among these were the following.

1. The aggregation of grids to zones. Since it was felt desirable to work at a level of aggregation more typical of transportation studies, it was necessary to define new zone boundaries following grid lines approximating the irregular old zone boundaries. No important error was introduced since only accessibility scores from the original zone file were used in subsequent analyses—all extensive quantities used were grid aggregates.

2. Estimation of 1948 dwelling units. Consideration of all data sources and the purpose of the study led to the decision to use dwelling units as the item to be predicted. However, 1948 dwelling unit data were not directly available. Estimates were made and various checks applied by using 1948 land area, a 1948 USGS map for suburban areas, 1950 census block statistics for the central city (changes were not large for the inner area from 1948-1960), and the 1960 land area and dwelling unity densities.

3. Estimation of accessibility measures for 1960 for certain zones at the fringe. The area covered by the zone file did not extend to the boundaries of the grid coverage area in all directions. Rather than eliminate this area entirely, estimates of accessibility measures were made for about one-half of the outer ring of zones by examining contours of iso-accessibility lines, which follow fairly regular patterns in the fringe area.

MODEL DESCRIPTION AND METHODOLOGY

Density-Saturation Gradient Method

The DSGM is the least formally structured forecasting procedure of the five. No formal theoretical statements or mathematical hypotheses are required, although the staff of the Chicago Area Transportation Study have presented excellent conceptual explanations of their empirical findings and rationale for their projections (1). This theoretical development, however, is not essential to the purpose of this paper.

Before discussion of the actual application of the DSGM to the Greensboro area, mention should be made of certain reservations which existed prior to the testing. The only known previous application of this approach was for the Chicago area. There was some initial fear that the regularities in activity distribution about the central place, which is axiomatic to the method, would not be manifest for a city of the size of Greensboro. The declines in density and percent capacity result from the operation of the competitive land market, a mechanism which might not exert the dominating influence upon spatial organization in a city of Greensboro's size. It will be seen that these fears were unwarranted, and that in fact the distribution of residential activity was markedly structured about the CBD.

Two semi-independent forecasts were made using the DSGM in order to determine the sensitivity of the results to variations in the critical assumptions made. A principal distinction was that the first trial was made using air-line distance from the high value corner (HVC) as the key spatial variable, whereas traveltime to the HVC was used in the second trial. (The HVC is a point representative of the hypothetical activity center of the CBD).

Figure 1 shows the relationship between 1948 dwelling unit density and air-line distance from the HVC. Each point on this plot represents the gross residential density (street area included) for a ring around the HVC. Each ring is defined by the boundaries of all zones whose centroids fall within $\pm \frac{1}{2}$ mile of the nominal distance of the ring from the HVC with the exception of the first or CBD ring. The plot indicates a surprisingly regular decline in residential densities with distance from downtown in Greensboro in 1948. This was encouraging since the reliability of the DSGM depends greatly on the strength and stability of this relationship.

The method depends equally upon the relationship between distance and percent saturation. To compute the latter, residential capacity must be defined. Mathematically capacity is defined as existing dwelling units plus the product of vacant available, suitable land, and expected residential density. A decision had to be made at this juncture as to the density values to be used in the computation. Theoretically this should be the anticipated average density at which all future residential development will occur.

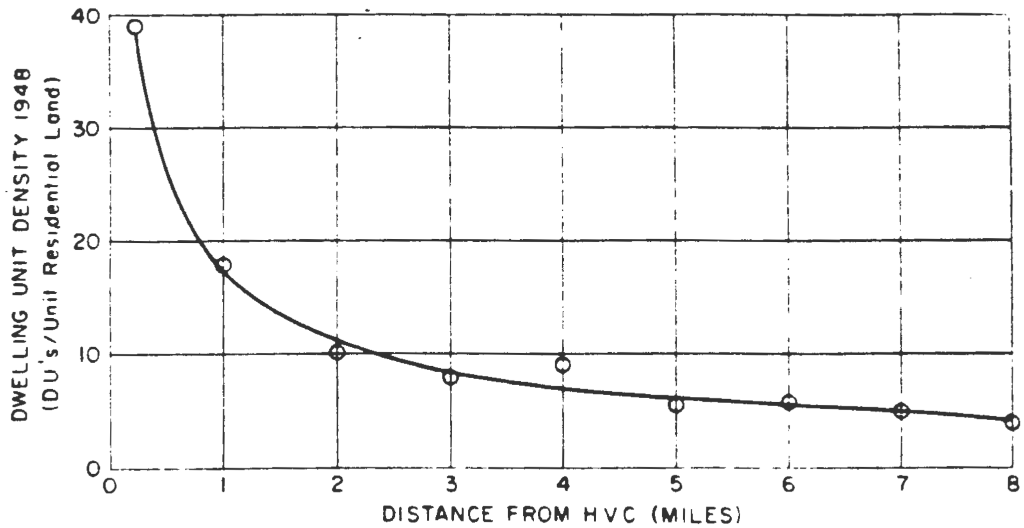


Figure 1. Dwelling unit density by distance bands—1948.

These values should be developed from an intensive analysis of trends in residential density patterns and zoning policies. For purposes of this study, however, future densities for each zone were assumed to be those given by the smooth hand-fitted curve of Figure 1. Prior to the acceptance of this single curve for the density gradient, gradients were plotted for each of five sectors. Although these plots exhibited less regular relationships, no significant variation between sectors was noted.

Vacant, suitable land for residential development was estimated by subtracting marginal land and land zoned for nonresidential uses from 1948 nonurban land. A systematic, but subjective procedure was used in the treatment of zoning: land was weighted by factors ranging from 0 for grids zoned only for industry to 1.0 for grids zoned only for residential use; land in grids zoned for mixed uses and other nonindustrial uses was weighted subjectively on a scale from zero to unity.

Having future residential development densities and vacant available land, it was possible next to compute both the residential saturations, in dwelling units and existing percent saturation, for each distance ring from the HVC. The latter values, resulting from the division of saturation into 1948 dwelling units, were then used to construct the percent saturation gradient. Figure 2 conforms very well with the plot expected for an urban area. The rather distinct and sharp transition between the $3\frac{1}{2}$- and $4\frac{1}{2}$-mi points indicates a transition from the area of urban character into the predominantly rural portions of the study region. The almost negligible slope of the curve beyond the $4\frac{1}{2}$-mi point is indicative of agricultural development and the absence of any strong competition for location with reference to central Greensboro.

The next step involved the 1960 projection of the percent saturation curve, also shown in Figure 2. (Percent saturation gradients by sector for 1948 were also plotted; however, as in the case of the density gradient, there was some additional scatteration of points, but no basis for using sector-specific gradients.) This is the most critical and subjective step in the forecasting process, the only restraint on the projected curve being that the area under the new curve must account for the projected regional growth. The number of dwelling units in the study area grew from a 1948 total of 27,191 to 41,250 in 1960 or a growth of 52 percent. One can proceed in almost an infinite number of ways insofar as establishing an acceptable projection of the percent saturation gradient. It was, however, found useful to first develop a feeling for the overall scale of the problem, that is, the area under the final curve which would be commensurate with the required final regional population. As a first approximation to the 1960 gradient each ordinate value was raised a distance equivalent to 52 percent of the 1948 value.

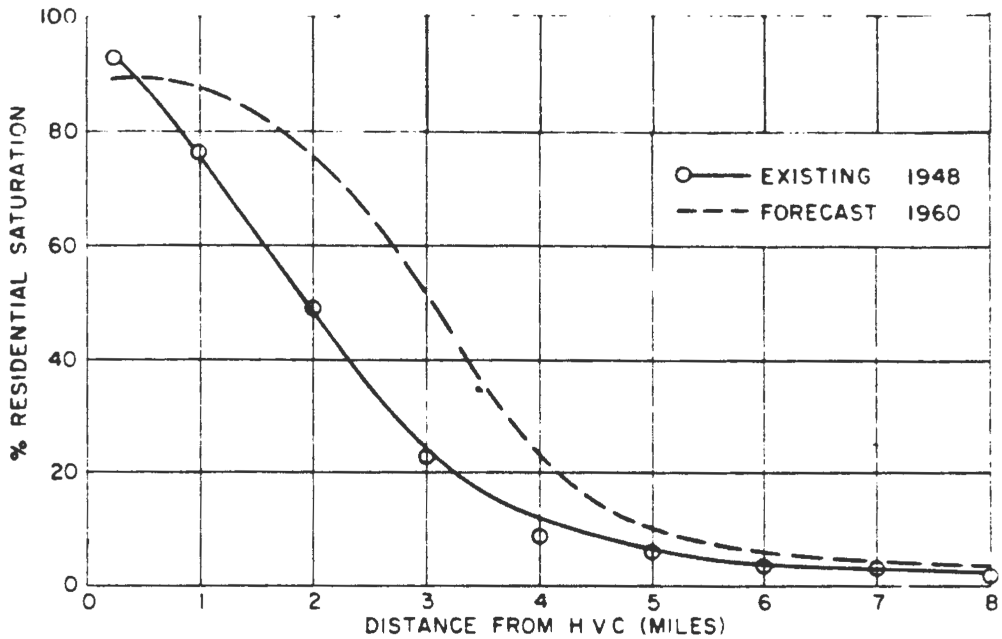


Figure 2. Residential saturation by distance bands.

The resultant curve then approximated the forecast condition under the assumption of uniform growth over the entire region. The following general criteria were then introduced to modify the naive first approximation of the shape of the gradient in 1960:

1. The bulk of the residential growth would occur in the 2-, 3-, and 4-mi rings.
2. The inner ring would suffer a slight decline.
3. The shape of the gradient would tend to bow out in the 1- to 3-mi range.
4. The sharp transition in slope of the 1948 saturation gradient observed at about the 4- to 5-mi point would become less abrupt in 1960.
5. The areas 5 miles and beyond would show some exurban growth, but the general flat slope would remain.

Relatively few attempts were necessary to arrive at a solution which was of satisfactory shape and which conformed with the actual 1948-1960 increase in total dwelling units.

Multiplying the appropriate ordinate value from the forecast percent saturation gradient (Fig. 2) by the ring saturation quantities established the forecast dwelling unit totals by analysis ring.

The projected growth of each ring was distributed to zones in a two-step process following the logic of CATS. The allocation to districts (defined by ring-sector boundaries) was handicapped by a lack of historical data. Ideally the trends in land use composition and growth rates between sectors should be studied in detail. For trial one, however, the simple assumption was made that sectors would share growth in proportion to available residential capacity.

The final distribution to zones was based on a systematic, but subjective linear weighting of the following factors:

1. Distance to convenience shopping,
2. Available residential capacity,
3. Distance to the major street system,
4. Percent of industrial development in the zone, and
5. Percent of residential development in the zone.

Trial two, which was conducted independently of trial one, differed from the above procedure in two principal ways:

1. Traveltime to the HVC was substituted for airline distance as the major independent variable. Zones were aggregated into 1-min interval rings for all analyses.
2. Ring growth was allocated to sectors (i.e., the district-level forecast) in proportion to the product of each sector's available residential capacity and the number of existing (1948) dwelling units.

Otherwise, the process followed that of trial one, including the method of estimating density and holding capacity, the sector definitions, and the allocation of growth from districts to zones.

Figure 3 shows the dwelling unit density gradient as determined from the ring analysis for trial two. As expected the same general shape is observed as for trial one. Figure 4 shows both the percent saturation curve calculated for the 1948 base period, and the forecast of the 1960 percent saturation curve. The shape of the latter gradient is quite similar to that for trial one except for a slight decrease in the growth allocated to the inner rings, resulting in a lessening of the bowing effect and a reduction in the slope of the gradient in the intermediate areas.

Accessibility Model

The generalized form of the accessibility model is as follows:

$$G_i = G_t \frac{A_i^a V_i}{\sum_i A_i^a V_i}$$

where

G_i = the forecast growth for zone i;
G_t = total regional growth $= \sum_i G_i$;

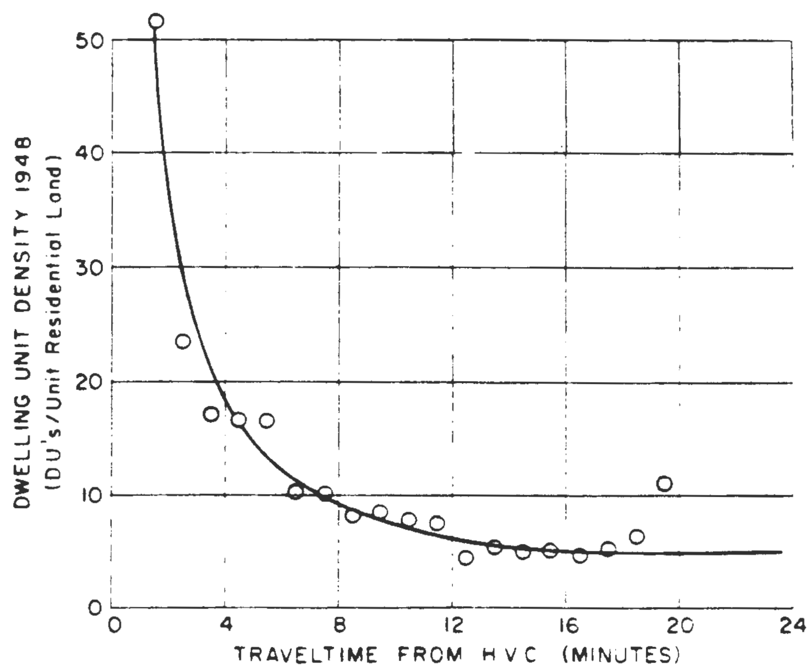


Figure 3. Dwelling unit density by time bands—1948.

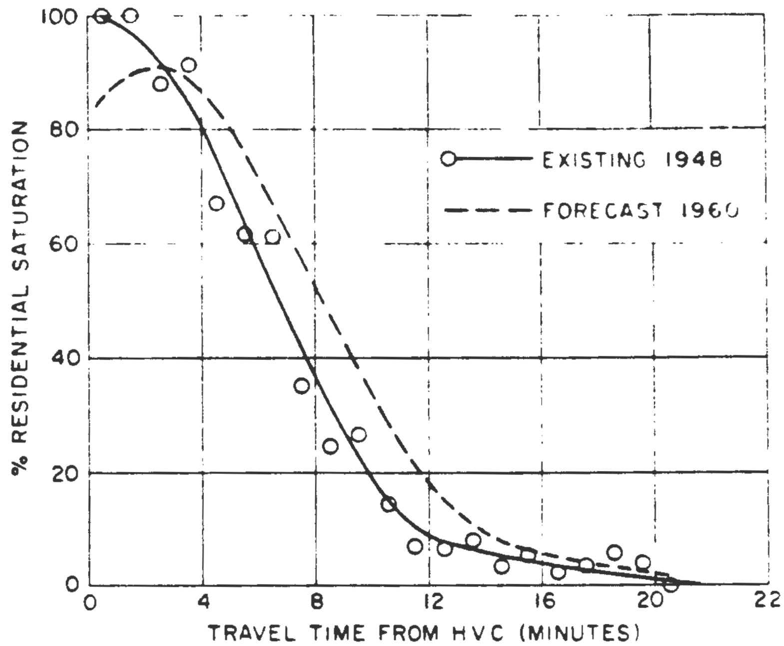


Figure 4. Residential saturation by time bands.

A_i = accessibility index for zone i;
V_i = vacant available land in zone i; and
a = empirically determined exponent.

The computation of the accessibility index traditionally is as follows:

$$A_i = \sum_j \frac{E_j}{T_{ij}^b}$$

where

E_j = a measure of activity in zone j (total employment used in this study);
T_{ij} = traveltime from zone i to zone j; and
b = an empirically determined exponent.

However, "friction factors" developed in the gravity model calibration by Alan M. Voorhees and Associates were actually used in the computation of accessibility:

$$A_i = \sum_j E_j F_{ij}$$

where F_{ij} is the friction of time separation of zones T_{ij} minutes apart. The F_{ij} values are approximately proportional to the actual number of trips T_{ij} minutes long per trip-end in each pair of zones T_{ij} minutes apart. In practice the computation of F_{ij} is considerably complicated by a desire to have the F_{ij} values form a smooth monotonic relation to T_{ij} yet maintain approximate equality between the resulting mean trip length and the actual mean trip length.

With the above definition of the model only one parameter, a, need be estimated to make the forecast. Two options were open:

1. Make a judgment of the value of a from previous work in other cities, and forecast 1960 zonal growth to have an independent test of the model; or
2. Fit a "best" value for a using the actual 1948-1960 changes in dwelling units.

Both options were actually used. For option 1 a value of 2 was assumed for a. (Hansen found that a value of about 2.7 was optimal for Washington, D.C.; the presumption that accessibility would have less influence in shaping growth in a smaller city is substantiated by the subsequent results in fitting values for a.) Methods used in fitting a to the 1948-1960 data are described in the Appendix.

Regression

For several reasons it was felt desirable to express the dependent variable of the multiple regression formulation as some function of the 1948-1960 growth rather than as some function of the absolute amount of cumulative development at a single point in time. The latter option was open, and has been used by others (13, 14); however, it was rejected to maintain comparability with the dependent variables of the other models, as well as to conform to standard practice in transportation planning models. As has been pointed out by the Traffic Research Corporation (15), there is good reason to expect greater accuracy for relatively short-range forecasts when predicting increments of growth.

Using change in dwelling units, or some function thereof, as the dependent variable, it was not possible with the available data to produce an independent forecast to check against the 1960 data. The equation parameters had to be estimated from the full 1948-1960 data files. Hence, accuracy results are shown in the next section only for a fitted model, and not for a forecast, in contrast to the other 4 methods. Dwelling unit data for a third point in time would be required to examine the forecasting reliability of the calibrated regression equation.

The usual regression approach differs from the other models used in this study in two additional important ways:

1. Many, rather than one or two independent variables may be incorporated, and
2. Variables are related to growth only in linearly weighted combinations, although variables may be transformed prior to regression.

The latter restraint is imposed by the use of a standard regression program (the BIMD 34 stepwise multiple regression program developed by the UCLA Bio Medical Center for the IBM 7090/7094 was used in this work). Of course nonlinear regression equations may be developed, but different normal equations must be solved and standard regression programs may not be used.

Numerous equations were developed, each involving the testing of various hypotheses regarding the functional relationships between variables. A total of 44 independent variables plus certain selected nonlinear transformations were examined in all, including:

1. Measures of zone size and amount of land in different uses;
2. Accessibility to employment;
3. Time and distance to HVC;
4. Zonal employment, total and by major type;
5. Densities for 1948;
6. Vacant available land;
7. Zoning protection;
8. Land value; and
9. Proportions of total land and developed land in each major use.

Four definitions of the dependent variable were tested:

1. Increase in dwelling units (DU);
2. Log DU;
3. DU per unit of available land (DU/L); and
4. Log [DU/L].

The logarithmic transformations were employed to test certain hypotheses regarding exponential relationships, as for example, are expressed in the accessibility model. The growth-per-unit-of-available-land transformations were employed in an attempt to remove all measures of zone size from the equations, and thereby, to avoid the possibility of distorted relationships due to the pecularities of area definitions.

The final equation accepted after comparing the accuracy and reasonableness of all trials was

$$Y = -2.3 + 0.061 X_1 + 0.00066 X_2 + 1.1 X_3 - 0.11 X_4 - 0.0073 X_5$$

where

Y = logarithm of growth dwelling units 1948-1960 per unit vacant land;

X_1 = zoning protection, 1948;
X_2 = percent of total land area in residential use, 1948;
X_3 = logarithm of accessibility to employment, 1960;
X_4 = dwelling unit density, 1948; and
X_5 = percent of total use land in industrial use, 1948.

The coefficient of correlation is 0.61. Table 1 contains the t and beta (β) values (standardized regression coefficient) for each of the independent variables in the equation. All regression coefficients are significantly different from zero with 95 percent confidence. Having the greatest β value, the transformed accessibility variable is shown to exhibit the most influence upon the estimate of the dependent variable. Percent of urban land which is in industrial use has the lowest β values and, therefore, contributes least to the total equation estimate.

The zoning code was a value from 0 to 9, where a higher value indicated zoning control closer to single family residential only, and lower value marginal-to-no zoning control. The positive relationship then indicates the positive environmental influence of strict residential zoning policy. The positive contribution of accessibility to work areas is self-explanatory. Also, the positive contribution of percent of total area devoted to residential development is interpreted as a measure of residential clustering. The tendency for slow growth or even decline in the residential stock of the close in, old city areas, coupled with the rapid increase in the fringe and newly settled locations accounts for the negative coefficient for dwelling unit density. The negative contribution of percent industrial land is indicative of the restraint on new residential development in areas immediately adjacent to industrial areas.

Because the estimation was couched in both logarithmic and intensity units, several operational difficulties were introduced. The estimating equation was incapable of either accepting negative values for the dependent variable or estimating decline in any zone. All zones which suffered dwelling unit decline over the calibration period were approximated to have shown no change. An additional problem was encountered for several zones which experienced dwelling unit growth, but which had no vacant land available in 1948. Without some adjustment the growth intensity value becomes infinite. These few cases were handled by substituting large arbitrary values of growth intensity. Finally, there is no built-in provision, as there is for other models, to assure that the accumulated zonal estimates obtained from the regression equation solution will equal the actual total regional growth. All regression estimates had to be factored up to sum to the actual regional growth.

TABLE 1

RELATIVE SIGNIFICANCE AND EXPLANATORY POWER OF VARIABLES IN REGRESSION EQUATION

Independent Variable	t	β
Log accessibility to employment, 1960	4.30	0.321
Zoning code, 1948	2.89	0.213
Percent of total land residential, 1948	2.70	0.187
Dwelling unit density, 1948	3.28	0.177
Percent of urban land industrial, 1948	2.98	0.159

Two Intervening Opportunity Models

Although the two opportunity models tested are based on quite different initial assumptions and take on dissimilar mathematical form, nevertheless, both can be reduced to a simple general hypothesis. In the context of this problem, the probability that a suitable residential opportunity (a unit of available capacity) is accepted for development is hypothesized to be a monotonically decreasing function of the number of intervening opportunities, opportunities being ranked by time from the HVC.

Some improvement in these models could undoubtedly be made by allocating increments of growth from more than one point, perhaps from all major centers of employment in proportion to the amount of employment in each center. This would make the test of the intervening opportunities models more comparable to the accessibility model procedure.

Stouffer Formulation. The Stouffer model may be defined in the following manner:

$$g_p = \frac{k\,O_p}{O}$$

where

g_p = number of dwelling units forecast to be located in a particular area p;
O_p = opportunities in interval p;
O = total number of opportunities from central distribution point through interval p; and
k = constant of proportionality to assure that the total number of dwellings located equals the actual total growth.

As stated, the Stouffer formulation can be applied without the need for assuming any parameter values. However, it is an operational requirement that the study area be structured into a number of discrete geographic units which are then ranked from a central distribution point, the HVC in this case. One method of aggregating areas, which Strodtbeck has shown to have some appealing properties, is to delineate a small number of rings containing approximately equal numbers of opportunities (16). For the initial application of the Stouffer model to the allocation of residential growth, the Greensboro study area was divided into 10 rings, each of which was composed of a whole number of zones and an approximately equal number of opportunities. Zones were assigned to rings according to their ranking in time from the HVC.

It was then possible to determine g_p, the forecast number of dwellings in ring p by direct substitution in the formula. The ring forecasts were then proportioned among the constituent zones on the basis of opportunities.

For an explanation of the fitting of the Stouffer equation to 1948-1960 data the equation must be converted into its continuous differential form as follows:

$$d(G_p) = \frac{k\,d(O)}{O}$$

By integrating

$$G_p = k \ln O + C$$

where

G_p = the total number of dwellings allocated to all opportunities from the central point up to and including opportunity interval p;
$d(G_p)$ = dwellings allocated to opportunity interval p;
$d(O)$ = opportunities in interval p; and
C = constant of integration.

This equation plots as a straight line of slope k where the ordinate, total allocated dwellings, is in linear form and the abscissa, total accumulated opportunities, is a logarithmic scale. As a test of the appropriateness of the Stouffer formulation in describing the spatial distribution of residential growth in Greensboro, the actual accumulated zonal dwelling unit growth 1948-1960 was plotted against accumulated 1948 opportunities, the zones being ranked by traveltime to the HVC. If the Stouffer model is valid the resulting plot should follow a straight line. It was immediately obvious that a single straight line could nof be adequately fitted to the points, but rather that two distinct straight lines were necessary (Fig. 5). After hand fitting the two lines, 1960 growth estimates were made to the individual zones from the straight lines and the error computed. These results and those computed from the initial, noncalibrated test of the Stouffer formula are discussed later with the results of the other four models.

Schneider Formulation. As applied to the distribution of residential activity, the Schneider model takes the following form:

$$d(G_p) = g_t \left[e^{-\ell O} - e^{-\ell (O + O_p)} \right]$$

where

- G_p = total number of locations in opportunity interval from the central point up to interval p.
- g_t = total growth to be allocated;
- ℓ = model parameter expressing the probability of an opportunity being accepted for location;
- O = total number of opportunities ranked from the central point up to interval p.

As a necessary condition for applying the model the parameter ℓ must be stipulated. For the first trial of the model for a 1960 forecast without benefit of the 1948-1960 data, ℓ was estimated from the assumption that the actual dwelling unit increase within the study boundaries was 99 percent of the aggregate Greensboro oriented growth. (The theoretical model is based on a distribution to an unbounded area; applicaton to a finite area requires specification of the number of accepted opportunities being outside the

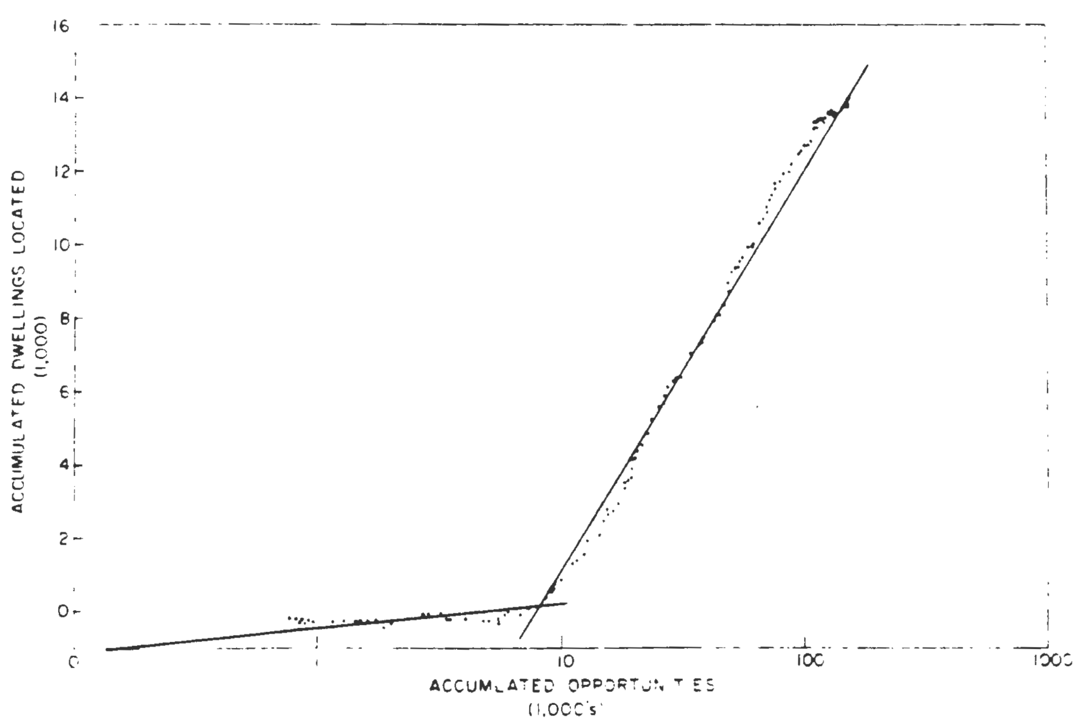


Figure 5. Test of Stouffer's formulation.

boundary, or equivalently, the percentage accepted up to the boundary.) The ℓ resulting from this assumption was 12.76×10^{-6}.

For an explanation of the fitting of the Schneider formulation to 1948-1960 data, the formula can be restated after integration as

$$G_p = g_t \left[1 - e^{-\ell O}\right]$$

Subtracting g_t from both sides and rearranging,

$$g_t - G_p = g_t e^{-\ell O}$$

or

$$\ln (g_t - G_p) = \ln g_t - \ell O$$

This relationship plots as a straight line where the ordinate, $(g_t - G_p)$, is in logarithmic scale and the abscissca, total accumulated opportunities from the central point (O), is in linear scale. The slope is ℓ and the intercept g_t.

If the Schneider formulation effectively replicates the spatial distribution of residential growth in Greensboro then plotting the actual quantity $(g_t - G_p)$ versus accumulated opportunities (O), in semilogarithmic forms, should yield a straight line (Fig. 6).

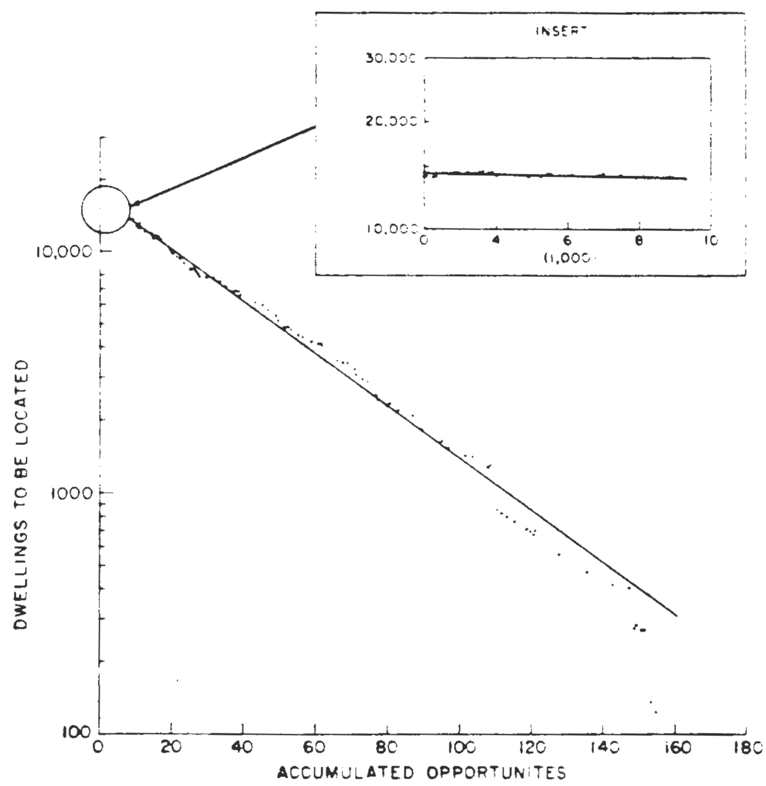


Figure 6. Test of Schneider's formulation.

As with the Stouffer formulation, the Greensboro data appear to exhibit two distinct straight line segments, rather than one, as required by the initial model formulation. The zones comprising the transition area between the two straight line segments (Fig. 6) are the same ones as those at the juncture of the two line segments for the Stouffer formulation (Fig. 5). The slopes of the fitted lines can be loosely compared to the short and long trip ℓ's which have become standard practice in applying the Schneider formula as a trip distribution model. The slope for the central city line segment is 1.707×10^{-6}, and that for the outer, suburban area is 10.9×10^{-6}.

The distribution of residential growth in Greensboro from 1948 to 1960 did not adequately conform to either of the intervening opportunities formulations over the complete range of opportunities. It is noteworthy, however, that the data plot as two straight lines in both Figures 5 and 6. It was also pointed out that the transition points in the vicinity of the intersection of the fitted straight lines in both figures were the same data points representing the same zones. Although a detailed examination of these zones has not been attempted it does appear that they approximate a transition ring in Greensboro which separates the "inner city," marginal growth area from the suburban, rapid expansion area. This band encircles the HVC at a radius of 1½ to 2 miles. For a city the size of Greensboro, which in 1948, exhibited a leveling off in the percent saturation gradient at 3½ to 4½ miles from the HVC, the area circumscribed by this transition band probably was characteristic of similar areas in most cities—old and perhaps showing signs of blight with little available residential capacity.

The inner area straight line slopes drawn to the two plots are both very close to the horizontal. In contrast, there are quite steep slopes for the plots representing suburban areas. A hypothetical locator viewing the opportunity surface from the HVC in accordance with either of the two plots apparently assesses himself a greater penalty in passing up suburban opportunities as opposed to inner-city ones. That is, the inner-city opportunities are a less desirable subset of the total as evidenced by the significantly lower slope on the plots, hence a lower probability of accepting individual opportunities. One may conjecture that location choices from the inner-city opportunity subset are responsive more to the individual living qualities of the opportunities other than its accessibility, which may be extended to the notion that the inner-city opportunities are viewed more or less as of homogeneous access in opposition to the suburban subset where opportunity access is of greater import in the locational choice.

Of interest from a purely forecasting viewpoint is the question of the stability of the handfitted lines in Figures 5 and 6. Do the slopes remain more or less constant over time and how does the transition area behave in relation to the total opportunity surface? One may speculate, for example, that the straight line relationships fitted to the data will hold over time and that the diffusion in residential location observed in the past is merely a reflection of the diffusion in the opportunity surface; that is, a physical dispersion outwards occasioned by the filling in of less distant areas, rather than of an alteration in the location function. On the other hand, it is possible that over time the slopes of the plots may be flattening out which is symptomatic of a society less restrained by the impedance of travel. Clearly, answers to speculations of this nature are required before one can estimate the applicability of the fitted lines to forecasting to a future time point.

PERFORMANCE AND INTERPRETATION OF RESULTS

Performance

The single accuracy measure which was calculated for all trial forecasts was the sum of squares of dwelling unit forecasting error. These measures were computed at four levels of geographic aggregation: sector, ring, district, and zone, for all trials. A sixth forecast was made using the naive assumption of equal growth for all zones. The error sum of squares computed under this assumption, which will be referred to as the naive model, is (n - 1) times the variance in actual zonal residential growth. It will serve as a benchmark in evaluating the results of the five techniques listed.

Table 2 gives the computed error sum of squares for all of the forecasts and calibrations at each level of aggregation. For sake of complete comparisons, the results of zone level forecasts for each of the models (not for the DSGM) have been aggregated to districts and rings defined both by time and distance from the HVC. Trial one of the DSGM was based on analysis at the level of district as defined by distance from the HVC; therefore results are not shown for districts as defined by time to HVC, and vice versa for trial two of the DSGM.

The sums of squares of differences between estimated and actual are analogous to "unexplained" variances of a statistical model. However, since valid statistical inferences obviously cannot be drawn, this terminology should not be used. The error measurements of Table 2 do provide an index which can be used to compare results in any single column, that is, for the same level of aggregation. Comparisons between columns are meaningless, since different numbers of areas and different variances from mean growth rates are involved at different levels of aggregation.

To provide some degree of comparison between levels of aggregation, as well as between forecast techniques, Table 3 gives the ratio of each error to that for the naive model.

There are rather poor results at the zone level for all five methods. In some instances the naive model, assuming equal growth for all zones, actually exceeds the level of accuracy of forecasts. The particularly discouraging results of the DSGM at the zone level are evidence of poor choice of criteria by the authors in distributing growth from districts to zones. As pointed out earlier, this method requires historical data that were not available and requires intimate familiarity with the area, which the authors lacked. The technique itself should not be blamed.

Undoubtedly, a substantial amount of the error at such a fine level of detail as the zone can be attributed to inaccuracies in data—assumptions made in certain estimates, incompatibility of merged files, differences in definitions between time periods, etc. However, other factors are contributory. The average zone contained only 109 dwelling units in 1948 and increased 56 to 165 by 1960. These values are far too small to hope for reliable predictions with any model. Obviously, differences between zones

TABLE 2

ERROR SUM OF SQUARES FOR ALL TRIALS[a]

Method	Levels of Aggregation					
	Zone	Districts		Rings		Sector
		By Distance Ring	By Time Ring	By Distance	By Time	
DSGM						
Trial I	2.33	6.97	—	8.36	—	9.69
Trial II	2.41	—	4.43	—	4.07	3.02
Accessibility model						
Forecast	1.80	4.16	2.84	3.25	2.33	4.58
Fitted	1.79	3.98	2.76	2.18	1.99	4.46
Regression (fitted)	1.85	4.71	3.14	5.16	2.84	3.71
Stouffer model						
Forecast	2.21	6.45	4.22	5.57	3.48	11.25
Fitted	1.91	4.72	3.07	2.42	1.46	8.84
Schneider model						
Forecast	2.07	6.16	4.13	4.10	3.38	13.92
Fitted	1.95	4.65	3.08	1.91	1.65	10.18
Naive model	2.20	7.66	5.22	20.64	10.54	16.18

[a]All values have been multiplied by 10^{-6}.

TABLE 3

RATIO OF ALL ERRORS TO NAIVE MODEL ERROR

Method	Zone	Districts: By Distance Ring	Districts: By Time Ring	Rings: By Distance	Rings: By Time	Sector
DSGM						
Trial I	1.06	0.91	—	0.41	—	0.60
Trial II	1.10	—	0.85	—	0.39	0.19
Accessibility model						
Forecast	0.82	0.54	0.54	0.16	0.22	0.28
Fitted	0.81	0.52	0.53	0.11	0.19	0.28
Regression (fitted)	0.84	0.62	0.60	0.25	0.27	0.23
Stouffer model						
Forecast	1.01	0.84	0.81	0.27	0.33	0.70
Fitted	0.87	0.62	0.59	0.12	0.14	0.54
Schneider model						
Forecast	0.94	0.80	0.79	0.20	0.32	0.86
Fitted	0.89	0.61	0.59	0.09	0.15	0.63
Naive model	1.0	1.0	1.0	1.0	1.0	1.0

Levels of Aggregation

at this level are largely due to random variations not explainable by models. The districts represent a more reasonable level of detail at which to examine and compare accuracies. For the sake of comparison with transportation study practices, the average district (defined by distance rings) used in this study could be expected to have about 8,000 person trip-ends in 1948 (about 660 dwelling units with 3.2 persons per dwelling and 4 trip-ends produced per person).

Table 4 shows the relative accuracy of the accessibility model forecast at various levels in comparison to the size of the values being forecast. In this table the root-mean-square-error (RMSE) is used as the measure of error, since it can be compared with the magnitude of the forecast values: about two-thirds of the errors fall within RMSE values.

The RMSE is roughly half of the average 1960 dwelling units per zone, and about a third of the average 1960 dwelling units per district. Of course, these accuracies must be viewed in relation to the overall growth rate of 52 percent. Intuitively one would expect that the ratios of the RMSE's to the 1960 values might be nearly cut in half if the overall growth rate was half as large.

The accessibility model performed substantially better than other unfitted models at most levels of aggregation (Table 3); but the fitted Stouffer and Schneider models were

TABLE 4

COMPARISON OF ERRORS TO SIZE OF FORECAST VALUES ACCESSIBILITY MODEL FORECAST

Levels of Aggregation	RMSE $\sqrt{\frac{s \cdot s}{n}}$	Average 1960 DU (per areal unit)	Average Growth 1948-1960	Number of Areas
Zone	85	165	56	249
District[a]	381	1,006	342	41
Rings[a]	600	4,580	1,560	9

[a]By distance.

quite comparable to the fitted accessibility model. Somewhat surprisingly, the addition of several other explanatory variables in linear regression form did not improve the accuracy.

Results at the sector level are of interest because of the implications for forecasting radial corridor movements. Here the intervening opportunity models yield comparatively poor results, perhaps because they were not made sensitive to the distribution of employment, as were the accessibility model and regression equation.

Trial one of the DSGM assumed relative growth by sectors in proportion to available capacity—a weak assumption judging by comparison with the error of trial two. The importance of residential character in attracting additional growth apparently holds at all levels—between sectors as demonstrated by comparison of the two DSGM trials, and as a factor at the zone level as demonstrated by the statistical significance of that factor in the regression analysis.

Examination of Actual Patterns of Growth

All forecasts of 1960 density were based on the assumption that development in any zone would occur at the density indicated by a smooth line drawn through the 1948 density vs distance (or time) from the HVC. Figure 7 compares the actual 1960 density-distance gradient with that for 1948. There appears to have been a rather uniform amount of decrease in density at all distances, except for the core area where the decrease was substantial. This obviously accounts for some error in the forecasts which required estimates of 1960 density (DSGM and the opportunity models), especially in the core area.

The actual 1960 and 1948 percent saturation gradients are compared in Figure 8, along with the forecast curve used for trial one of the density-saturation gradient method. Not surprisingly, the actual 1960 curve does not follow as smooth a curve as for 1948, since the plot represents percentage of 1948 capacity rather than 1960 capacity. The most significant errors in the forecast appear to be due to the unexpectedly large decline in the core and the amount of growth that occurred in relatively remote portions of the area, ring 5 and 6. However, the general shape of the forecast curve is appropriate.

Figure 9 shows the same comparisons for the results of the accessibility and regression models. The agreement with the actual 1960 gradient is quite good, except for the obvious inability of these techniques, as used in this study, to predict decreases in the core.

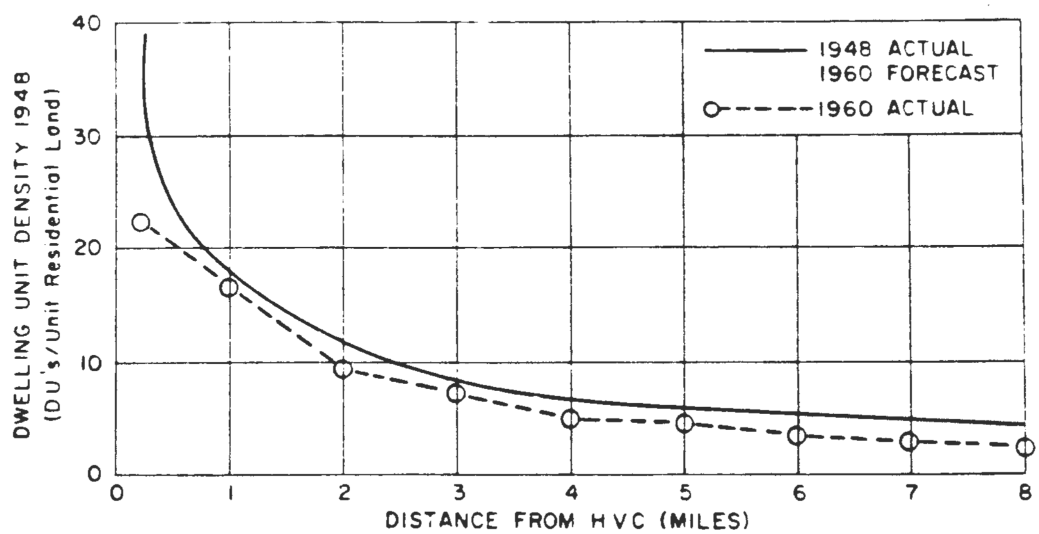


Figure 7. Dwelling unit density by distance bands.

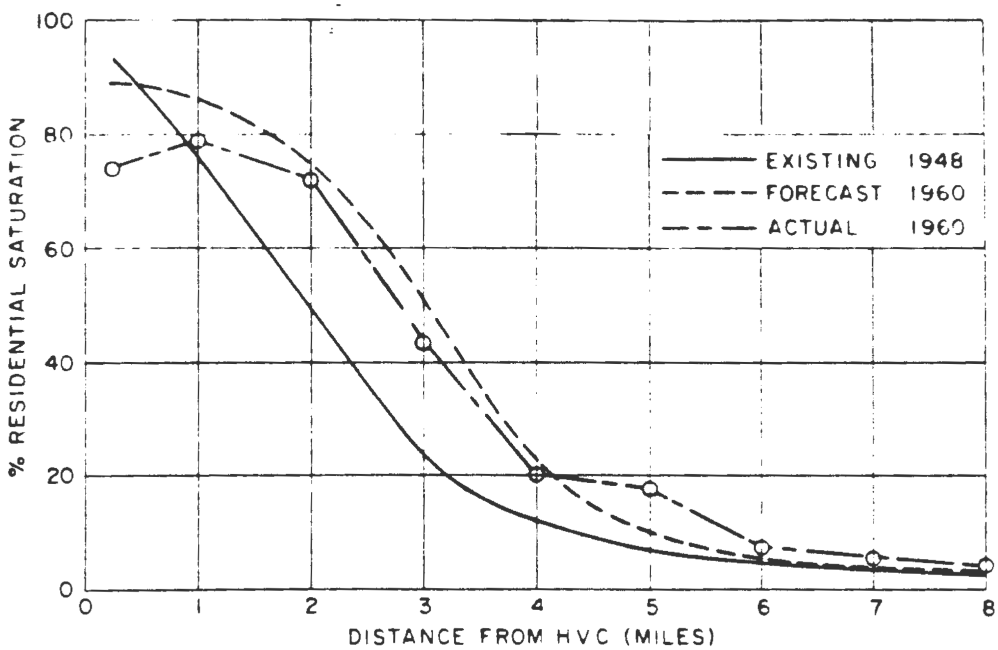


Figure 8. Percent residential saturation by distance bands.

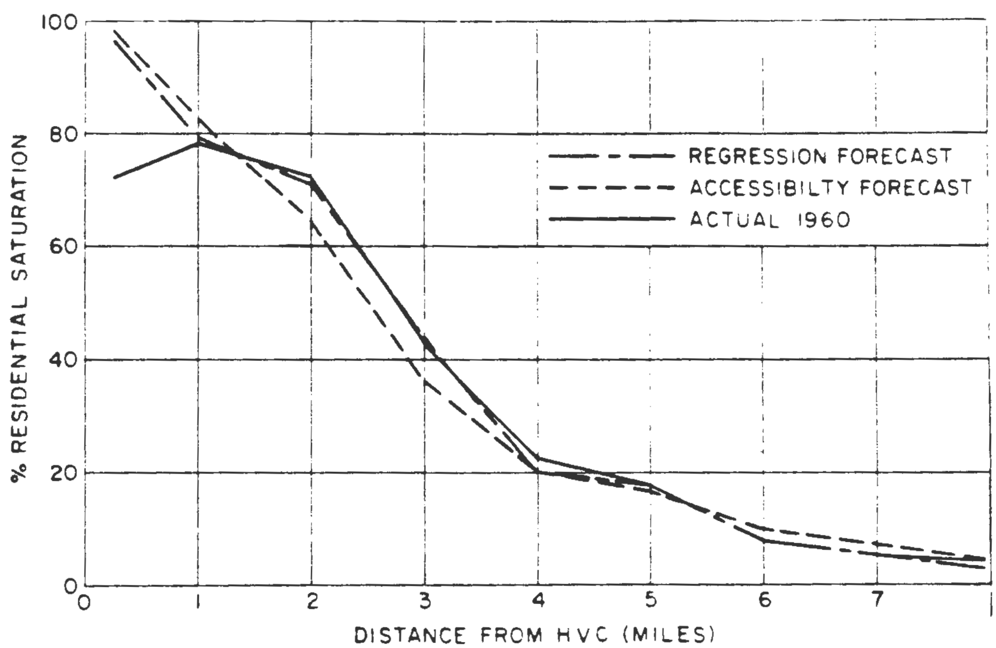


Figure 9. Percent residential saturation by distance bands.

In an attempt to picture how the residential density structure of the study region changed, Figure 10 was drawn. Using the data for total dwelling units and residential land area from the distance to HVC ring analysis, cumulative percent of total regional dwellings was plotted against cumulative percent total residential land area on a ring aggregate basis, proceeding outwards from the core ring. The plots for the actual conditions in 1948 and 1960 are shown. If smooth curves were drawn the slope at any point would represent the inverse of density for the marginal dwelling unit. A diagonal line drawn on Figure 10 would represent uniform residential density for the entire study area. The bowing of each of the curves below the diagonal indicates the decline in density as one proceeds outwards from the HVC. If densities in the inner area were to decline along with an increase in the dwelling unit densities in the outer rings, the region as a whole would be approaching a state of uniform density, and the curve would shift toward the diagonal. On the other hand, if the difference between inner and outer area densities were to increase substantially, then there would be a shifting of the plot down and to the right. Understanding that the plots in Figure 10 represent an overall increase from 1948 to 1960 of 52 percent, the rather minute change in the density structure of the study area as described by these plots is outstanding.

Although the two plots (Fig. 10) appear to coincide almost exactly, they should not be misread as indicating no change in the geographic distribution of dwelling units from 1948 to 1960. Each of the data points representing a distance ring has shifted downward and to the left from its 1948 position to 1960. That is, inasmuch as the majority of residential growth occurred in the suburban rings, the dwelling stock of the inner rings in 1960 represents a smaller proportion of the total region stock than in 1948 and also utilizes a smaller proportion of total residential land; hence, the shifting of the data points downward and to the left.

An interesting question is whether similar plots for other urban areas exhibit this same constancy as found in Greensboro. If this is found to be so, such plots could be quite helpful in residential forecasts.

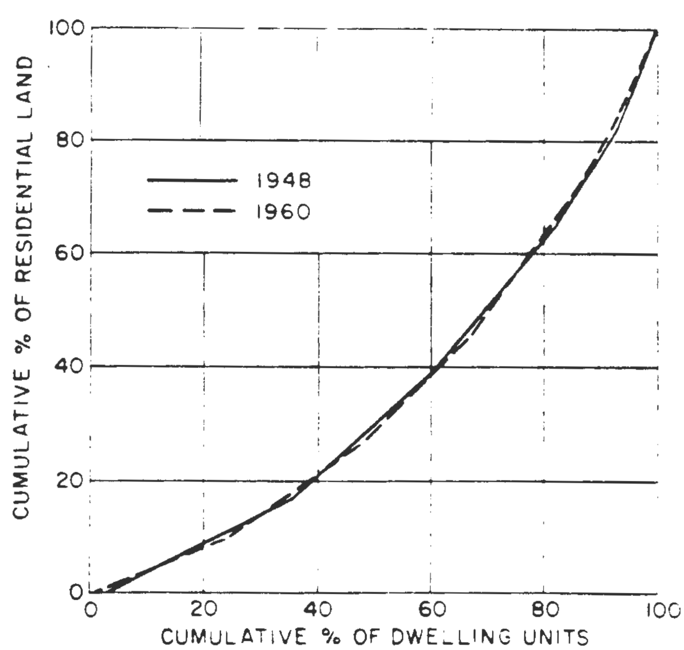


Figure 10. Cumulative dwelling vs cumulative residential land.

CONCLUSIONS

1. Simple, nonbehavioral residential land use forecasting models, which do not discriminate between the locational patterns of different types of households, are sufficiently accurate to be recommended for use in relatively small metropolitan areas of 100,000 population or larger. The Greensboro area's spatial structure and pattern of growth clearly demonstrates a degree of organization warranting analytical treatment in the planning process.

2. Land use forecasting with simple first generation models produced reasonably accurate results for levels of geographic aggregation where the average areal unit contained a population of about 2,000 persons. Efforts to forecast growth for much smaller areas may prove unjustified. At zone levels of about 300 population, these models appeared to offer little or no assistance in forecasting.

3. Differences in accuracy among the five forecasting methods are not large enough to warrant a strong recommendation for any single one in preference to others. Any of the methods would appear to be preferable to forecasting without the benefit of analytical techniques.

4. The simple accessibility model yielded the most accurate forecast of all methods used without benefit of calibration to time series data, for this one test. Errors in fitting were relatively insensitive to small changes in the exponent of accessibility.

5. None of the multiple linear regression models tested offered improvement over two-variable fitted models despite the fact that five or more factors were included in the regression equations.

6. Multiple regression models possess certain drawbacks. If the dependent variable is expressed as an extensive quantity (e.g., increase in dwelling units) then measured relationships with independent variables are influenced by pecularities of area definition and size, and may not conform satisfactorily with logical hypotheses regarding the land development process. Nonlinear transformations on the dependent variable such as logarithms or fractional power functions are unsatisfactory because the usual least squares criterion tends to bias the parameter estimates to produce good fits to small values and poor fits to large values. Expression of the dependent variable as an intensive quantity (e.g., dwelling unit increase per unit area) may be the most satisfactory operational solution except that relationships which are actually nonlinear may not be properly represented. Perhaps this might be handled by treating certain independent variables as sets of dummy variables.

7. Although the two intervening opportunity models performed satisfactorily as used in this study, some evidence pointed to the possibility of improvement by allocating growth from all major centers of employment rather than from just a single point, the CBD. In addition, each of the two models implies a different straight line plot on different semilogarithmic coordinates which did not hold true for Greensboro over the entire study area. Apparently the hypotheses are valid, but separate functions may be necessary for the built up, inner-city area, and the developing suburban area.

8. The forecasting approach used by CATS differed from the other models in important respects. It forces the analyst to become intimately familar with the study area before attempting to forecast. This is probably the strongest feature to recommend it. The graphical analyses that the method is based on represent excellent descriptions of the key spatial relationships of a metropolitan area—even for relatively small areas. The methods of analysis are useful tools regardless of the forecasting technique used. They can serve as checks on the reasonableness of forecasts made by less subjective models.

However, as applied in this study, the method is time-consuming, requiring considerable hand work and far more data manipulation. The method is less adaptable to the computer, and hence would be cumbersome for testing of alternative land use policies, or for recursive use in combination with other submodels.

9. The five techniques examined admittedly are far from representative of the extent of current land use forecasting research. They do represent the initial attempts and as such lack the sophistication and elegance of later thinking. These are descriptive models in that they do not involve themselves with the behavior of decision-makers;

nor do they possess any real theoretical content. It is highly probable that the key to increased forecasting accuracy for small subareas lies in the ability of the analyst to simulate the decision process of subpopulations of the region.

ACKNOWLEDGMENTS

The authors wish to express their gratitude to both Professor F. S. Chapin, Jr., of the University of North Carolina and to A. M. Voorhees of Alan M. Voorhees and Associates for making available the data utilized in this study.

Appendix

CALIBRATION OF ACCESSIBILITY MODEL

Two procedures were used in the attempt to estimate the optimal exponent of accessibility: linear regression on transformed variables and an iterative, nonlinear least squares fit of the untransformed dependent variable.

Linear Regression on Transformed Variables

Three transformed versions of the standard accessibility model were tested:

$$\log G_i = \log a + b \log V_i + c \log A_i \tag{1}$$

which, in nonlogarithmic form is

$$G_i = a\, V_i^{\,b}\, A_i^{\,c} \tag{2}$$

$$\log \left(\frac{G_i}{V_i}\right) = \log a + b \log A_i$$

or in nonlogarithmic form

$$G_i = V_i\, A_i^{\,b}$$

$$\log G_i - \log V_i = \log a + b \log A_i \tag{3}$$

which is the same as Eq. 2 in nonlog form.

The nonlogarithmic forms of Eqs. 2 and 3 are essentially equivalent to the standard form of the model as stated in the body of this report. They would be identical if the normal equations contained the condition that

$$a = \frac{G_T}{\sum_i V_i\, A_i^{\,b}}$$

Since a standard regression program was used, this condition may be violated, and equation estimates must be factored to sum to actual total growth. This holds for all three of the transformed versions of the model.

TABLE 5

RESULTS OF THREE VERSIONS OF LINEAR REGRESSION ON TRANSFORMED ACCESSIBILITY MODEL

Item	Eq. 1	Eq. 2	Eq. 3
Accessibility exponent (b)	3.52	1.63	2.29
Log a	-8.0	-3.2	-4.9
Vacant land exponent (c)	1.51	1	1
Sums of squares of error ($\times 10^{8}$)	2.21	1.89	1.78

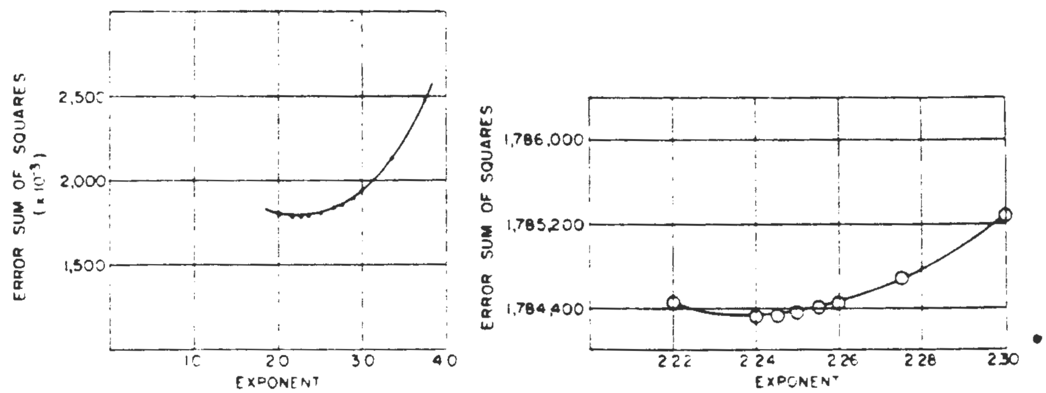


Figure 11. Accessibility model error vs accessibility exponent.

Eq. 1 also expresses vacant land as a power function in contrast to its linear form in the standard formula.

The basic problem, however, is that the least squares criterion is different for each version (the minimization of unexplained variance in the dependent variable) since the dependent variable is different for each. None is the correct criterion. The log transform tends to produce a bias toward better fits for small values of the untransformed dependent variable. Table 5 summarizes the results of the three versions.

The fairly wide variation in the accessibility exponent, as well as in the error term leads one to be suspicious of regression on transformed dependent variables.

Nonlinear Least Squares Fit of Exponent

A routine was programed to iterate toward the true least squares solution for the standard accessibility model

$$G_i = G_T \frac{A_i V_i^b}{\sum_i A_i V_i^b}$$

Figure 11 shows the results in the form of a plot of the sums of squares of error vs a range of exponents. A smooth curve with a minimum at $b = 2.24$ is apparent.

It is interesting to compare these results with the b value of 2.7 reported by Hansen for Washington, D.C. One might expect this value to increase with the size of the city.

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APPENDIX C

Methodology For Developing Activity Distribution Models (Extracted from Transportation Research Board Record Number 126)

Methodology for Developing Activity Distribution Models by Linear Regression Analysis

DONALD M. HILL, Senior Research Analyst, and
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•A PROPOSED mathematical framework for developing urban activities distribution models is described. The models distribute forecast regional totals of socio-economic variables to small zones; for example, resident population by various income levels would be distributed to traffic zones. The distribution is carried out as a function of future public policies relating to highway and rapid transit improvements, public open space, etc.

To calibrate activities distribution models, information over a historic time interval on growths and declines of the activities to be distributed is needed. Thus changes in zonal values of activities, and similar changes in the policy variables to be tested are the information with which the models are calibrated.

This paper describes a methodology for developing an activity distribution model by linear regression analysis. A simple example of the regression model is the linear equation constructed with three variables

$$\Delta R = a + b_1 \Delta Z_1 + b_2 \Delta Z_2$$

where R is the measurement of growth or decline of a land use activity; ΔZ_1 and Z_2 reflect changes in measurable and causal factors; and a, b_1 and b_2 are parameters derived by application of the least squares principle. The best values of a, b_1 and b_2 are established to minimize the expected error of estimate of ΔR by solution of the equation with known values of ΔZ_1 and ΔZ_2.

However, by the use of linear regression analysis, it is frequently argued that the model builder is seriously limited in the flexibility of the model's construction. Critics of regression analysis are quick to point out the following troublesome restrictions of regression analysis.

1. Linear relationships must exist between the dependent variable ΔR and the independent variables ΔZ_1 and Z_2.
2. The effects of the independent variables are additive and the ΔZ_1 and ΔZ_2 variables must not be interrelated with one another. Furthermore, the errors of estimate of ΔR from values of ΔZ_1 and ΔZ_2, must be normally distributed with mean zero and constant variance.

In view of these restrictions, it is argued that the advantages of regression analysis are soon canceled by the violation of one or more of the above restrictions in using a particular data set.

Evidence is presented that the above restrictions are not insurmountable obstacles in the development of a linear regression model. If any of the restrictions are violated due to the nature of the data, which appear to invalidate the construction of a linear model, then the model can be reformulated to avoid such violations. For example, the following precautionary procedures are possible:

1. Nonlinear relationships between ΔR and ΔZ variables can be linearized by breaking up the single ΔZ variable into several ΔZ variables, i.e., ΔZ_1, ΔZ_2, ΔZ_3, etc. By

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doing so, a linear relationship will exist between ΔR and each ΔZ. Transformation of the ΔZ variable by logarithms, cosines, etc., can achieve the same results.

2. The application of factor analysis techniques can create from highly interrelated ΔZ_1 (adj) and ΔZ_2 (adj) variables which are independent of one another. In so doing, the assumption of additive effects of independent variables is confirmed. If such techniques are not available for use or not preferred, then the expected errors of estimate of ΔR which have unequal variances can be dealt with satisfactorily by suitable transformations of the ΔR and/or ΔZ variables to insure constant variance for expected errors of estimate.

Explicit analysis of locational behavior can be incorporated in the model's design. Regression models do not have to depend primarily on a blanket interpretation of past events. The model's development can be shaped in accordance with a theory of allocation of growth of activities or urban development. The researcher in the development of the model will be back and forth between the theory of the model and tests of its behavior with data. Adjustments of the theory will result to improve the model's application with empirical data. However, the theory of the model should not be warped or distorted solely to achieve a best fit to the data.

Development of the model can be achieved by applying several types of regression analysis techniques; for example: (a) ordinary least squares, (b) indirect least squares, (c) limited information-single equation method, (d) 2-stage least squares, (e) simultaneous least squares, and (f) full information maximum likelihood method.

While method (a) deals with single equation models, methods (b) to (f) deal with models formulated as systems of simultaneous equations. If single equation models are formulated, method (a) is adequate and the one to use. However, most activity distributions require models formulated as systems of equations—methods (b) to (f). The relative efficiency of each of the methods for parametric estimation is discussed in the case of simultaneous equation models.

There are distinct computational and economic advantages associated with the use of linear regression analysis. Readily available analysis methods and economical computer programs can be used by the researcher for the model's development. Also, through the economies and flexibility of regression analysis techniques, several test models can be easily evaluated. In general a great deal of knowledge and modeling experience can be gained from constructing and testing regression models.

MODEL DESIGN BY LINEAR EQUATIONS

In the typical model design, one must choose a mathematical framework to describe a hypothesized set of structural relationships. This framework will comprise the variables chosen and specify the ways in which these variables are interrelated. A model framework convenient for use is a linear structural equation as follows:

$$\Delta R = b_1 \Delta Z_1 + b_2 \Delta Z_2 + \ldots + b_k \Delta Z_k + u \tag{1}$$

Here ΔR is an urban activity variable dependent on the measurements of a number of independent variables, $(\Delta Z_1, \Delta Z_2, \ldots, \Delta Z_k)$. The parameter set $(b_1, \ldots, b_k)$, describes the relationship between the dependent variable and the independent variable set. The error term, u, occurs due to the imperfect fit of a mathematical equation to observed phenomena of urban development. It is the principle of model calibration to estimate the parameter set $(b_1, \ldots, b_k)$, so as to minimize overall the error terms, u, as well as to eliminate systematic bias in the error terms.

Eq. 1 accommodates adequately the situation where the dependent variables, ΔR, to be predicted, are not interrelated with one another. However, many model designs are premised on the occurrence of interrelationships between the dependent variables to be predicted. In accordance with this design requirement, it is desirable to formulate a framework of simultaneous linear equations; for example:

$$
\begin{aligned}
\Delta R_1 + a_{12}\,\Delta R_2 + \ldots + a_{1m}\Delta R_m &= b_{11}\,\Delta Z_1 + \ldots + b_{1k}\,\Delta Z_k + u_1 \\
a_{21}\,\Delta R_1 + \Delta R_2 + \ldots + a_{2m}\Delta R_m &= b_{21}\,\Delta Z_1 + \ldots + b_{2k}\,\Delta Z_k + u_2 \\
a_{m1}\Delta R_1 + a_{m2}\Delta R_2 + \ldots + \Delta R_m &= b_{m1}\Delta Z_1 + \ldots + b_{mk}\Delta Z_k + u_m \qquad (2)
\end{aligned}
$$

Within this framework, it is possible to account for the interrelationship between the dependent variables, $(\Delta R_1, \ldots, \Delta R_m)$, as well as accommodate the dependency of each ΔR variable on the independent variable set, $(\Delta Z_1, \ldots, \Delta Z_k)$. As in the case of Eq. 1 (which of course is a special case of equation system Eq. 2 where $a_{ij} = 0$ for $i \neq j$) the error terms, u, must account for the imperfect fit by the mathematical equation. The parameter sets $(a_1, \ldots, a_k)$ and $(b_1, \ldots, b_k)$ are estimated so that the overall errors (u_i) are minimized by the regression process of least squares.

The selection and formulation of variables in the model is critical in the model's design. The dependent variables should measure adequately the distribution which we propose to predict. The independent variables should provide adequate explanation of the distribution to be predicted, as well as retaining their separate identity with respect to one another. In particular, the following two criteria are suggested for the formulation of variables:

1. The variables formulated for incorporation into the model should be the same type. That is, variables which are changed in basically different ways by changes in definition of subregional areas and size should not be mixed in a single model. Variables will in general be of two types, i.e., point variables and aggregate variables. Point variables do not tell anything about area aggregates unless multiplied by some base quantity such as total land or total activity. Examples of point variables are densities, accessibilities, and area rate of growth. Area aggregate variables, on the other hand, refer to measurable magnitudes or quantities. Examples of aggregate variables are total population, and total employment or total land area.
2. The construction of the variables should be such that their interpretation is clear. The variables must be capable of being measured and named. Data categories assimilated to form a variable should furnish it with a logical name or explanatory description.

The formulation of variables should simplify the design of the model wherever possible. If two or more variables demonstrate similar locational characteristics and otherwise appear to cluster together due to a similarity in name and procedure of measurement, it is desirable to aggregate the variables into a single variable. Clustering or aggregating dependent variables will simplify the model design by reducing the number of estimating equations of the system. There must be one equation in the model for every dependent variable to be predicted. By aggregating dependent variables, it is possible (all else being equal) to increase substantially the predictive accuracy of the model over what might be achieved with a more complex model. Aggregation of independent variables which are highly interrelated is preferred for other reasons.

CRITERIA FOR APPLYING LINEAR REGRESSION ANALYSIS IN MODEL DESIGN

Linear regression analysis is simply defined as the estimation of the value of one variable (Δ R) from the values of other given variables (other ΔR and/or ΔZ) via a framework of some chosen linear equation. Descriptions of various regression techniques suggested for use in distributing urban activities are described hereinafter. Such regression analysis may be used provided the following criteria are met:

1. It is hypothesized in the construction of the activity distribution model that linear relationships exist between the dependent and independent variables.
2. It is hypothesized that the influences of the variables are additive. While the dependent variables are assumed to be interrelated with each other as well as being

related to the independent variables, it is desirable for the independent variables not to be interrelated with each other.

Linear Influences of Variables

In the application of regression analysis to estimate the parameters of a model it is essential that there is a linear relationship between the expected value of the dependent variables and the independent variables. Fortunately, even when this condition does not apply, it is often possible to modify the original variables in some way so that the new variables meet the requirement. The modifications or transformations of data most commonly applied are the logarithmic, the square root, or the reciprocal.

One of the assumptions of the linear model is the serial independence of the error terms, u, that is, covariance (u_i, u_{i+j}) = 0 for all observations i and j, where j = 0. However, there are circumstances in which the assumption of a serially independent error term may not apply. It is possible that one may make an incorrect specification of the form of the relationship between dependent and independent variables. For example, one may specify a linear relationship between the ΔR and ΔZ variables when the true relation is quadratic. While the error term in the true relationship may be non-autocorrelated, the new quasi-error term associated with the linear relationship must contain a term in ΔZ^2. If serial correlation exists in the ΔZ-values (i.e., characteristic of time series variables), then serial correlation will occur in the quasi-error terms.

In cases of autocorrelated errors, there are three main consequences of applying straight-forward regression processes without transforming the variables affected:

1. While the estimates of the parameters will be unbiased, their error variances could be larger than those achievable by applying suitable transformations in the estimation process.
2. The estimates of the error variances associated with parameters will be understated.
3. Inefficient predictions with large errors of estimation will be obtained.

The satisfactory manner of testing for linear relationships between the dependent and independent variables is by plotting the relationships between pairs of variables on graph paper. Based on the results, a decision can be made on the value of transforming variables, so as to linearize their influences.

Additive Influences of Independent Variables

Two variables exhibiting a high degree of interrelationship are said to introduce non-additive influences on the dependent variables. Unless interactance terms descriptive of the interrelationships are introduced in the model, there occurs serious ambiguity in the calibration process in separating the influences of the two variables. This ambiguity can be reflected in large fluctuations in the parameters associated with each model derived from calibrations with different aggregations of the subregions and variable sets, etc. Also, the signs associated with parameters of the affected variables may disagree from that expected from a priori reasoning.

Nonadditivity of a particular variable, unless previously eliminated, will frequently cause heterogeneity of error variance which is associated with the estimating equation for a particular dependent variable. This should not occur as it can have a serious effect on the parametric estimation achieved by regression analysis. Regression analysis may only be validly performed provided the error variance of the estimates of the expected value of a dependent variable is constant for all values of the independent variable (i.e., homogeneity of error variance is important).

The degree of interrelationship between variables can be measured in two ways: graphical analysis by plotting pairwise relationships on graph paper, and calculation of bivariate correlation coefficients. The value of the correlation coefficient will vary between minus unity and plus unity, and in either case as it approaches its limits, a high degree of interrelationship or correlation is indicated.

If two independent variables are correlated, one of three courses may be followed (a) eliminate the one variable considered least important to the model design, or which one believes a priori to be less important; (b) combine the two variables, provided the new aggregate variable can be named and measured; (c) substitute a scale of a variable which is natural (i.e., which experience or theory suggests is additive) to reduce and even eliminate interactance between variables. Examples of transformation by logarithms or reciprocals have been shown to reduce interrelationships.

If it is considered important to include all variables in the model, then course (b) or (c) is preferred.

If course (b) is followed, factor analysis can be useful in aggregating variables into independent, and therefore, additive influences. The basis for conducting factor analysis is a matrix of correlation coefficients describing the pairwise relationships between all variables affected. Factor analysis processes will construct factors comprising a linear function or equation of the variables whose pairwise correlations are being analyzed. The principle for constructing these factors is such that the factors are statistically independent of one another. The factors should be able to be named and associated with an aggregate influence on urban development.

Heterogeneity of error variance, caused by nonadditivity, will usually be reflected by a relationship of the error variance to the mean (m) or expected value of the dependent variable for a particular independent variable. The choice of a suitable variable transformation will frequently depend on the relationship between the error variance

TABLE 1

SUMMARY OF TRANSFORMATIONS[a]

Variance in Terms of Mean m	Transformation	Approximate Variance on New Scale in Absence of Heterogeneity
m	$\sqrt{x}$ or $\sqrt{x + 1/2}$	0.25
$\lambda^2 m$	for small integers	$0.25\lambda^2$
$\lambda^2 m^2$	$\log_e x$, $\log_e (x + 1)$	λ^2
	$\log_{10} x$, $\log_{10} (x + 1)$	$0.189\lambda^2$
$2m^2/(n - 1)$	$\log_e x$	$2/(n - 1)$
$m(1 - m)/n$	$\sin^{-1} \sqrt{x}$ (degrees)	$821/n$
	$\sin^{-1} \sqrt{x}$ (radians)	$0.25/n$
$km(1 - m)$	$\sin^{-1} \sqrt{x}$ (radians)	$0.25k$
$\lambda^2 m^2 (1 - m)^2$	$\log_e [x/(1 - x)]$	λ^2
$(1 - m^2)^2/(n - 1)$	$\frac{1}{2} \log_e [(1 + x)/(1 - x)]$	$1/(n - 3)$
$m + \lambda^2 m^2$	$\lambda^{-1} \sinh^{-1} (\lambda\sqrt{x})$, or $\lambda^{-1} \sinh^{-1} (\lambda\sqrt{x - 1/2})$ for small integers	0.25
$\mu^2(m + \lambda^2 m^2)$	$\lambda^{-1} \sinh^{-1} (\lambda\sqrt{x})$, or $\lambda^{-1} \sinh^{-1} (\lambda\sqrt{x + 1/2})$ for small integers	$0.25/\mu^2$

[a]Bartlett, M. S. "The Use of Transformation," Biometrics 3, 39-52, 1947.

and the mean of observations. This relationship is usually determined by empirical analysis with subregional data.

Table 1 gives transformations that have been found to have practical value.

SCOPE OF MODEL DESIGN

The design of the activity distribution model (1, 2) is based on a combination of deductive and inductive reasoning based on observations of urban development patterns. It represents an itera· ·e procedure in which the analyst begins with general observations of subject matter, develops a hypothesis or theory of the causal system which explains the behavior of his subject matter; tests this hypothetical structure for its power to explain the observed data of his field, in this case urban development; studies carefully the discrepancies between the explanation provided by his hypothetical structure and the observed data; revises his hypothetical structure on the basis of these discrepancies; tests the structure again; etc. The analyst is thus back and forth between his theoretical explanation of the causal system and his observation of all possible aspects of the subject matter on urban development. His goal in this iterative process is to reduce the discrepancies between theory and observation to a minimum.

Identification of Equation Systems

The problem of identification in a system of causally interrelated variables is connected with making an empirical estimation of the system from observed data. The problem only exists for systems of simultaneous equations, and does not occur when the area of study can be fully explained by a single equation. Each equation in the system will be designed to explain one dependent variable of the system in terms of those causes which exert a direct or approximate influence on it. These causal variables include both other dependent variables, and independent variables.

The essential meaning of identification can now be stated. Any particular equation in our system is identified if it is sufficiently different from all of the other equations, i.e., in its form, the variables included in it, and any restrictions on the values which its parameters can take. By "sufficiently different" we mean that it must be impossible to arrive at an equation which "looks like" the particular equation we are testing by any linear combination of other equations in the system, or of all of the equations including the one being tested.

Sample Identification Problem. Suppose that our system consists of two dependent variables, ΔR_1, ΔR_2, and three independent variables, ΔZ_1, ΔZ_2, ΔZ_3. Suppose that we are assuming linear relations, and that we have as yet no clear ideas about structure specification. We might then simply put all variables in the system into each equation.

$$\begin{aligned} &(a)\quad a_{11}\Delta R_1 + a_{12}\Delta R_2 + b_{11}\Delta Z_1 + b_{12}\Delta Z_2 + b_{13}\Delta Z_3 = u_1 \\ &(b)\quad a_{21}\Delta R_1 + a_{22}\Delta R_2 + b_{21}\Delta Z_1 + b_{22}\Delta Z_2 + b_{23}\Delta Z_3 = u_2 \end{aligned} \qquad (1)$$

The a's and b's are constant coefficients or parameters, and the u's can be treated here as either constant terms or as random disturbances. We can assume that Eq. (a) is supposed to explain ΔR_1 and that Eq. (b) is intended to explain ΔR_2. Let us further assume that the system we are analyzing is represented by a sample of observed data.

$$\left[\Delta R_{it}\right], \left[\Delta Z_{jt}\right] \; (i = 1, 2;\ j = 1, 2, 3;\ t = 1, 2, \ldots, T) \qquad (2)$$

We now attempt to use these data to estimate the parameters of our system (1) above. But since the two equations look exactly alike, when we apply our observed data to the estimation of parameters we get exactly the same result for each equation. There is no way of distinguishing the behavior of one part of the system from that of the other using empirical methods.

Suppose, next that we do more work on the theory of our system, and arrive at a specification which excludes ΔZ_2 and ΔZ_3 as variables from (a) and ΔZ_3 from (b). Let us call the new equations (c) and (d). Now the two equations "look different" from each other. We have restricted b_{12}, b_{13} and b_{23} to zero. This is the most common kind of restriction which aids identification. But is there still any danger of getting those two equations mixed up in empirical estimation? Suppose we test by making a linear combination of (c) and (d). Thus suppose we form $\ell(c) + m(d)$ where ℓ and m are arbitrary multipliens. The resulting equation has the form

$$a_1\Delta R_1 + a_2\Delta R_2 + b_1\Delta Z_1 + b_2\Delta Z_2 = v \tag{3}$$

This is different from the new specification we have made for (c), for it excluded both ΔZ_1 and ΔZ_2, but it is no different from our new specification for (d). In our new system (c) is completely distinguishable empirically from the rest of the system, but (d) is not. Therefore (c) is identified, and (d) is not identified.

Now suppose that our theoretical specification had removed ΔZ_2 and Z_3 from (a) and ΔZ_1 from (b), giving equations (e) and (f). Suppose we make a linear combination $\ell(e) + m(f)$,

$$a_1\Delta R_1 + a_2\Delta R_2 + b_1\Delta Z_1 + b_2\Delta Z_2 + b_3\Delta Z_3 = w \tag{4}$$

This form does not look like either (e) or (f), and both equations in our system are fully distinguishable and hence identified.

In conclusion, the main basis for identification is the inclusion of only the main causal variables in each equation, and the exclusion of irrelevant variables, both dependent and independent. But there are other bases for obtaining distinguishability of one equation from all others, and these include cases like the following. It might be that there is a natural restriction that two parameters in the equation have a preordained ratio to each other, or that one or more parameters have preordained values, indicated by theory, or arrived at by separate studies. Sometimes a nonlinearity in an equation may insure identifiability, or even a specification of differences in the variances of the random components in particular equations may achieve this.

A necessary, but not sufficient, condition of an identified system of m equations is that in each equation, at least m - 1 of the variables are restricted, usually by setting them to zero. This is known as the "order" condition of identifiability. If fewer than m - 1 variables are restricted in any equation, the system is said to be under identified, and cannot be solved by the parameter estimation programs. If more than m - 1 variables in some equations and at least m - 1 variables in all equations are restricted, the system is said to be over identified. This will usually be the case with activity distribution models.

Methods of Identifying a Model. By and large, the identification of the system of simultaneous equations which comprise the model will be determined by a priori reasoning in support of a particular theory of urban development. These are, however, empirical tests which can be applied as a guide in choosing an appropriate identification for the model.

Tests of Model Design

The testing of the model is usually carried out by regression processes, such as least squares (LS) or maximum likelihood (ML). Their purpose is to make the best possible tests and estimates of the structural parameters associated with variables of the model. In doing so, a complete separation is sought between the systematic part of the relationships and the random part. Generally, testing can profitably begin with an examination of our estimates of the random component.

An examination is conducted of nonsystematic residuals of the equation which the estimation process may have produced. If these reveal any trend, cycle or sawtoothed behavior then the model design (i.e., its identification) is on this basis rejected. It is concluded that the model does not contain all of the systematic forces which affect the dependent variable being explained, or it may contain some forces which should not be there.

Next, one examines the standard errors of the parameters attributable to variances associated with the observed data and conducts accompanying t-tests of significance. Here one tests again the model design, this time to see which variables test out as significant and as causes affecting dependent variables. But these tests can only be suggestive rather than rigorous, if our residual has already tested to be nonrandom and containing systematic elements.

In making the tests of significance of parameters (and hence of the associated causes), the model design can be open to two types of error. First, the test may reject a design which is really appropriate. This is the well-known Type I error. It can arise because the source of data is not complete or adequately representative of subregional development patterns. Application of more representative data, with an appropriate level of significance can reduce this danger.

A second kind of error which one may make is to accept a design which is false. This is the Type II error. Some other identification of the model is correct, but the one chosen has produced estimates which happen to fall into the range of acceptance for the model. Here we have an identification error which could slip by the tests.

Finally, one tests the results at this stage through reapplying to them one's knowledge of the subject matter. On the basis of general observation of the pattern of development, and of the tests of the primary model design on this basis, one achieves concepts about the sizes and signs of the parameters associated with the variables of the model. If the regression tests produce results which are markedly different from expected, one must take this as a rejection of the model design, or otherwise as some combination of data error and error in the model's identification. Consequently, it is such rejections which lead the analyst forward in the iterative process of model testing.

During this process of iterative revisions of the identification, there is always the danger of warping the theory, and hence design, to make the model fit the particular data source. This is a real trap, and no doubt one could fall into it. But there is a defense against it. The defense lies in carefully preserving the strength, logic and realism of the model's design. It is only when the observed data, and the discrepancies or residuals between observed data and the systematic explanation, reveal some clearly relevant but hitherto unsuspected force or omitted force that the identification should be revised. Design should never be altered merely to get a good statistical fit when the theoretical underpinning of such alterations is weak, illogical, and unrealistic.

When the scientific process has reached a terminal stage, one should have minimal identification errors, and hence the estimates of standard errors of estimates should be realistic. During the process one has resisted rejecting a good theory on the basis of statistical tests, while at the same time one has been even more resistant to warping a design solely to get good statistical fits. The systematic model should be in agreement without general observations and knowledge about the subregional development. And finally, the residuals should be in a purely random sequence, with mean zero and constant variance.

The test of successful estimation of the true model comes partly in its explanatory power, and partly in its predictive power. If one has found satisfactory causal explanation of development, and if the model is performing in a known way, one should be able to make satisfactory predictions.

REGRESSION PROCESSES

Development of the model can be achieved by applying several types of regression analysis methods.

Ordinary Least Squares

One applies ordinary least squares to a single equation in a model (3, Chap. 4, pp. 106-138), i.e.,

$$\Delta R = B\Delta Z + u \tag{1.1}$$

where

ΔR = vector of dependent variables;
ΔZ = vector of independent variables;
B = parameter associated with independent variables; and
u = residual error.

If, however, there are two or more dependent variables in each equation one does not know which dependent variable to select as the primary dependent variable of an equation, i.e.,

$$A\Delta R = B\Delta Z + u \tag{1.2}$$

where A = parameters associated with the dependent variables. The remaining dependent variables are always correlated with the error term in the equation because of the simultaneous nature of the equations in the model. Therefore, ordinary least square estimators are always biased (estimate does not equal true value) and they will also be inconsistent—for increasing numbers of sample observations, the estimates continue to be biased (3, Chap. 6, pp. 148-150).

For these reasons ordinary least squares is considered to be an unsuitable estimation method for dealing with systems of simultaneous equations. On the other hand, when dealing with a single equation containing one dependent variable, it is the method to use.

Indirect Least Squares

In the situation where a system of simultaneous equations is exactly identified, this is the proper estimation method to use. The other simultaneous estimation methods to be mentioned below always provide identical estimators to the indirect least squares method for the case of exact identification (exactly m - 1 of the parameters are set equal to zero where m is the number of dependent variables in total). The indirect least squares method is less complicated than the other methods, hence it provides definite computation economies.

The procedure (4, Chap. 4.4, pp. 135-137) is to estimate the parameters of the reduced form equations by application of the ordinary least squares method. A reduced form equation has only one dependent variable which is defined as the primary dependent variable, i.e.

$$\Delta R = D\Delta Z + u \tag{2.1}$$

By deciding that a certain number of the parameters in each equation of the simultaneous equation system are zero, the reduced form equations are converted into a simultaneous system where each equation contains one or more dependent variables, i.e., multiply (2.1) by A to obtain

$$A\Delta R = AD\Delta Z + Au \tag{2.2}$$

Write B = AD; therefore (2.1) is converted into a simultaneous system

$$A\Delta R = B\Delta Z + u$$

To recap, the exact number of parameters per equation which are set equal to zero is m - 1.

Limited Information Estimation Methods

Limited Information Single Equation Method (LISE) or Least Variance Ratio Method (LVR). This is a limited information maximum likelihood approach. It is a maximum likelihood approach (4, Chap. 6.2, pp. 166-167) in that the logarithmic likelihood function for the dependent variable is defined, i.e.

$$L(\alpha) = \tfrac{1}{2}\log AWA' - \tfrac{1}{2}\log \alpha M\alpha' + k - \tfrac{1}{2}\log \text{determinant } W \qquad (3.1)$$

where

$$\alpha = [A, B]$$

$$M = \begin{bmatrix} M_{\Delta R \Delta R} & M_{\Delta R \Delta Z} \\ M_{\Delta Z \Delta R} & M_{\Delta Z \Delta Z} \end{bmatrix}$$

$$M_{\Delta R \Delta R} = M = \frac{1}{T}\sum_{t=1}^{T} \Delta R_t^2 \text{ etc., } (T = \text{number of observations})$$

$$W = M_{\Delta R \Delta R} - M_{\Delta R \Delta Z}\, M^{-1}_{\Delta Z \Delta Z}\, M_{\Delta Z \Delta R}$$

Next, the function is maximized to yield uniquely the ratios of the parameters associated with the dependent variables of each equation. By setting one of the parameters equal to unity the remaining parameters are defined. The parameters of the independent variables are determined by solving a mathematical identity of dependent variable parameters and values of both dependent and independent variables.

The application of the method requires the user to know the specification of the single equation being estimated (i.e., which parameters are zero), and the independent variables appearing in the remaining equations which are assumed to have non-zero parameters. The detailed specification concerning the parameters of dependent variables in remaining equations is assumed unknown. Hence, only limited information needs to be known to obtain the estimators.

Two-Stage Least Squares

The basic idea (3, Chap. 9.5, pp. 258-260) of the 2-stage least squares (TSLS) is to select a dependent variable in each equation of the system and set its parameter equal to unity; i.e., rewrite $A\Delta R = B\Delta Z + u$

$$\Delta R_1 = A_2\Delta R_2 + B_2\Delta Z + u \qquad (3.2)$$

Next replace the remaining dependent variables by their estimates based on ordinary least squares regression between each dependent variable and all independent variables in the model.

$$\Delta R_2 = \Delta Z(\Delta Z'\Delta Z)^{-1}\Delta Z'\Delta R_2 \qquad (3.3)$$

Finally ordinary least squares is applied to the selected dependent variable, the regression estimates of the remaining dependent variables, and the independent variables in each single equation.

There is a basic similarity between LISE and 2-stage least squares as they both make use of all the independent variables in the model in order to estimate the parameters of a single equation, but do not require a detailed specification of the dependent variables in the remaining equations of the model. Both methods are consistent estimating methods. For large numbers of sample observations, both methods provide unbiased estimates of the parameters. It is reported that for special cases with a small number of observations, the 2-stage least squares method may provide more efficient estimators than LISE—estimators with smaller limiting variance (5).

Full Information Method (FI)

This method implies the use of full information concerning the specification of the simultaneous equation system. The FI methods are anticipated to provide the most efficient estimators of all the methods. There are two different techniques which comprise the FI method, simultaneous least squares and maximum likelihood.

Simultaneous Least Squares (SLS)

SLS (6) is a distribution free method of estimation (no assumption is made about the distribution of residual error). The method is the simultaneous equation counterpart of ordinary least squares. It takes completely into account the simultaneous interactions of all dependent variables in the system:

$$A\Delta R = B\Delta Z + u \tag{4.1}$$

It is a least squares method in that the sum of the squared deviations between observed and estimated dependent variables are minimized; i.e. minimize

$$E^2 = \sum_{t=1}^{T} \sum_{i=1}^{N} u^2_{sit}$$

where

$$u_S = A^{-1} u$$

Maximum Likelihood Technique (ML)

Complete information on the simultaneous system is taken into account (4, Chap. 5, pp. 143-162). The likelihood function for the dependent variables, conditional upon the values of the independent variables, is determined for the complete model. By assuming that the residuals of the estimating equations are multivariate normally distributed, the logarithmic likelihood function is defined,

$$L(\alpha, \sigma) = \log \det B - \tfrac{1}{2} \operatorname{trace} (\alpha' \sigma^{-1} \alpha M) + k - \tfrac{1}{2} \log \det \sigma$$

where

$\det$ = determinant;
α = $[A, B]$;
σ = non-singular covariance matrix of residual error u; and
trace matrix R = $\sum_i r_{ii}$ (sum of diagonal elements).

Maximizing the logarithm of the function with respect to the parameters of the model and its residuals lead to difficult estimating equations.

There are two assumptions involved in the use of ML, which may restrict its application. The first is the assumption that the residual errors are multivariate normally distributed. While the distributions of the residuals are probably bell-shaped and may be asymptotically normal (a property of large samples), the assumption of normality is not closely met with small samples of data. The second assumption (7) concerns the optimal properties of structure estimation. If the residual errors are normally distributed, both maximum likelihood and least square techniques lead to identical results which are linear unbiased estimates. However, where the residuals are non-normal then the ML and LS estimators are quite different. Nevertheless, the LS estimates are still the best linear unbiased estimates.

In conclusion, SLS is preferred to ML because of its distribution free properties and secondly because of anticipated computer economies. The computation economies are achieved by using a truncated procedure of SLS. This gain will, of course, be at the small expense of loss of accuracy in estimation. In truncated SLS the results are accepted after two or three stages of the recursive procedure of SLS estimation.

FACTOR ANALYSIS PROCESS

Variables which possess high statistical association are grouped together in clusters called factors. In particular, the intercorrelations among all the variables under study constitute the basic data for factor analysis (8).

All variables are assumed to be in standardized form, i.e., each has a mean value of zero and a variance of unity. It is the object of factor analysis to represent a variable in terms of several underlying factors, by a simple mathematical model of the linear form,

$$\Delta R_j \text{ or } \Delta Z_j = a_{j1} F_1 + a_{j2} F_2 + \ldots + a_{jf} F_f$$

Naming the Factors

The factors are not named by the process, and this anonymity must be removed before the statistical association indicated by factor analysis can be evaluated against the planner's a priori knowledge of cause and effect relationships. The variables which are most closely associated with (those which supposedly make the most significant contribution to) each cluster should help in naming the factor.

Significance of Factors

The relative importance of each factor is indicated by its eigenvalue, which represents a measure of the total contribution of the factor to the variances of all the variables being analyzed. Eigenvalues for all factors are produced by the technique. An eigenvalue of unity or greater is considered to indicate a significant factor. Experience with our prototype activities distribution models (1, 2) has shown, however, that there are a small number of factors with very high eigenvalues, a few more with eigenvalues of unity or more, and a large number of factors which have eigenvalues less than unity. The latter, strictly speaking, are considered little more than statistical "noise," whose contribution to the variances of the variables will generally be insignificant.

Selection of Factors

The factor analysis process provides for specifying the number of factors to be used. Normally the process discards all factors with an eigenvalue less than unity. In some instances, the arguments for using unity as a cutoff are marginal, and in some cases a factor with an eigenvalue less than one may be significant. With a small number of input variables, a factor with an eigenvalue of less than one could make a significant contribution to the variance of the variables. In such cases, one may specify the number of factors required.

Regardless of which cutoff option is employed, the eigenvectors associated with eigenvalues of the factors are computed and are normalized so that the squared eigenvector coefficients associated with each factor add to unity or less. The normalized eigenvector coefficients associated with each factor (known as factor loadings) are produced in array form.

Structure of Each Factor

The construction of factors is established by a regression procedure, based on the array of factor loadings. Each factor is presented as a linear function of the variables. An array is produced which indicates for each factor the statistical importance of each variable in its construction.

Factor Rotation

There is a possibility that several factors will look very much alike, and possess similar eigenvalues. In order to sharpen the picture of the system as much as possible, a varimax method of rotation is utilized in factor analysis processes. The rotation should maximize large factor loadings and minimize small ones, and the distinction between factors should be much sharper in the rotated than in the unrotated case. Both the unrotated and rotated arrays are true shadows of the same shape taken in different lights. Traditionally, the multiplicity of true shadows offered by factor analysis has deterred investigators from using the method as a "proof." The cautious investigator has assured himself that he uses it (in moderation) only to stimulate his insight into a mass of data; that is, to prompt a review of his logic.

It is emphasized that use of factor analysis is always subject to demand for a logical explanation of clustering. However, it seems intuitively attractive with the large amount of data available in computer-size models to suppose that the surface of the factor sl. pe is sufficiently regular that the maxima found by rotation, if not the best view, is at least one of the good views.

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APPENDIX D

UTOWN Data

UTOWN Data

by

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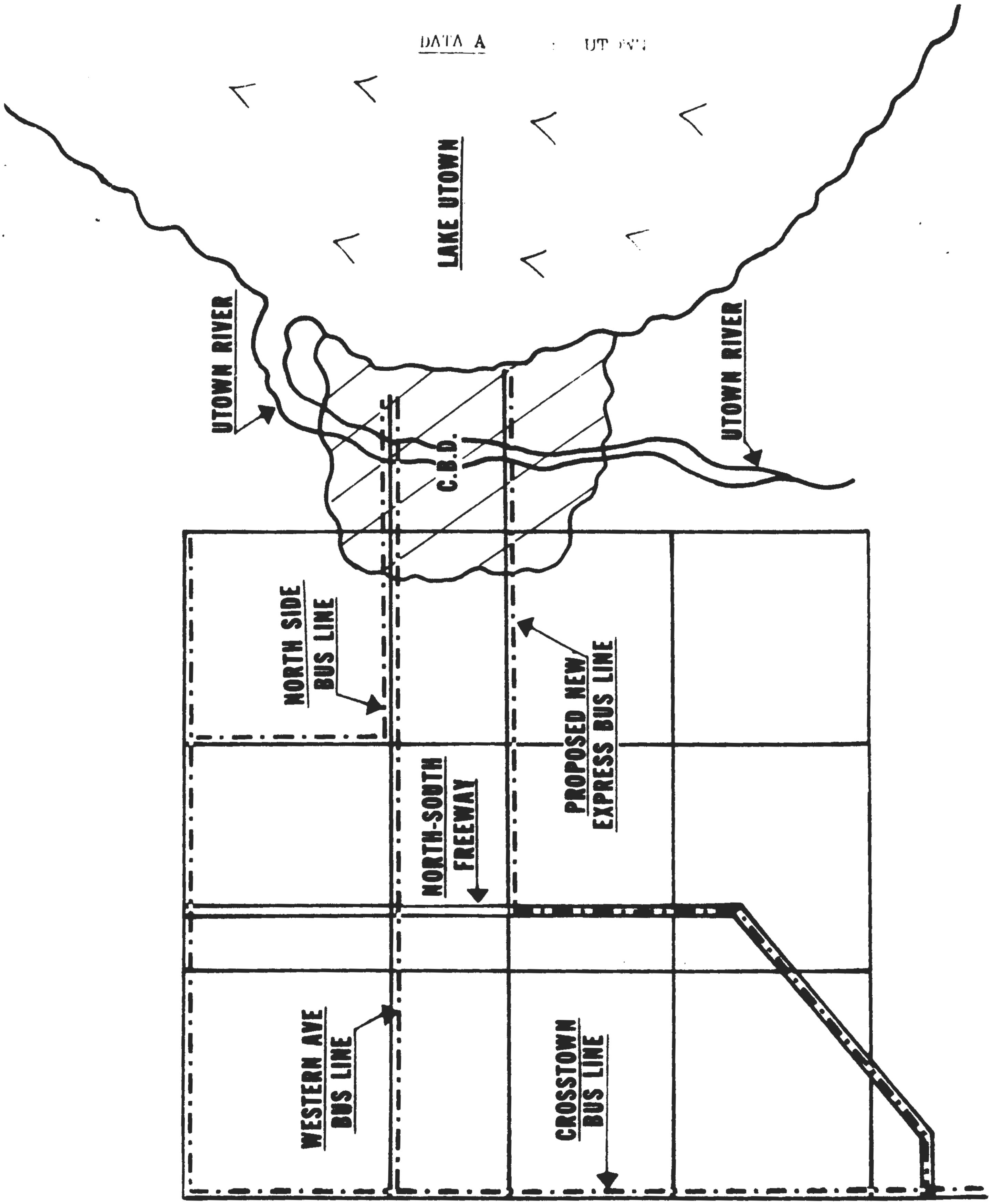
OUTLINE

PAGE

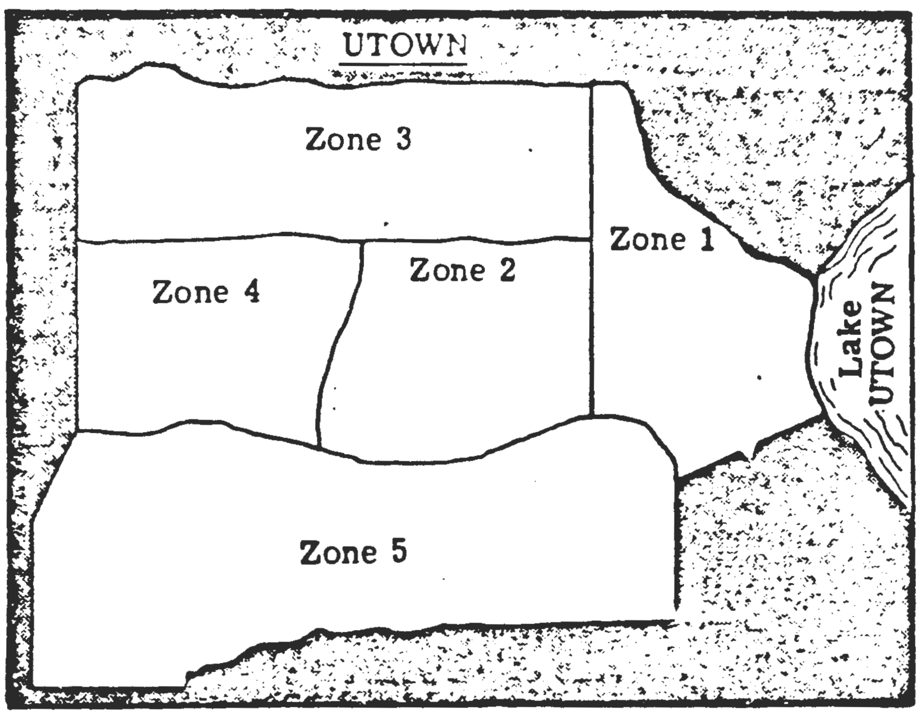
DATA A : UT



LAKE UTOWN
UTOWN RIVER
UTOWN RIVER
C.B.D.
NORTH SIDE BUS LINE
PROPOSED NEW EXPRESS BUS LINE
NORTH-SOUTH FREEWAY
WESTERN AVE BUS LINE
CROSSTOWN BUS LINE

DATA B

UTOWN
CASE STUDY
ZONE MAP



DATA C

UTOWN Socio-Economic and Land Use Data

Activity	Zone 1	2	3	4	5	Total
1. Households						
1970	1,000	10,000	6,000	4,000	4,000	25,000
1977	1,000	8,000	8,000	5,000	5,000	27,000
2000						33,572
2. Average Household Size						
1970	3.14	3.14	3.14	3.14	3.14	
1977	2.92	2.92	2.92	2.92	2.92	
2000	2.50	2.50	2.50	2.50	2.50	
3. Population						
1970	3140	31,400	18,840	12,560	12,560	78,500
1977	2920	23,360	23,360	14,600	14,600	78,840
2000						83,930
4. Total Employment						
1970	18,000	5,000	1,200	1,200	600	26,000
1977	25,000	5,800	1,400	1,800	1,000	35,000
2000						64,572

Activity	Zone 1	2	3	4	5	Total
5. Retail Employment						
1970	4,000	1,000	200	600	200	6,000
1977	6,000	800	200	800	200	8,000
2000						14,572
6. Non-Retail Employment						
1970	14,000	4,000	1,000	600	400	20,000
1977	19,000	5,000	1,200	1,000	800	27,000
2000						50,000
7. Average Income						
1970	10,000	4,000	12,000	6,000	15,000	
1977	11,000	5,000	15,000	8,000	18,000	
2000	(14,300)	(8,286)	(24,857)	(14,571)	(27,857)	
8. Total Land (Acres)	67.4	318.0	253.2	367.4	893.4	1899.4

DATA C (Continued)

Activity	Zone 1	2	3	4	5	Total
9. Residential Land (Acres)						
1970	13.4	190.3	168.2	228.4	502.4	1102.7
1977	14.4	193.3	171.2	231.4	510.4	1120.7
2000						
10. Non-Residential Land (Acres)						
1970	36.0	41.7	16.0	40.0	40.0	173.7
1977	36.0	43.7	17.0	42.0	43.0	181.7
2000						
11. Vacant Land (Acres)						
1970	18.0	86.0	69.0	99.0	351.0	623.0
1977	17.0	81.0	65.0	94.0	340.0	597.0
2000						
12. Population / Residential Acre						
1970	235	165	112	55	25	
1977	203	121	137	63	29	
2000						

DATA C (Continued)

Activity	Zone 1	2	3	4	5	Total
13. Total Employment Non-Residential Acre						
1970	500	120	75	30	15	
1977	695	133	82	43	23	
2000						
14. Access to Population						
1970	98.0	266.4	139.6	124.9	89.5	
1977	91.0	183.7	191.3	138.8	99.5	
2000						
15. Access to Employment						
1970	56.1	69.8	35.7	31.6	20.2	
1977	186.1	75.1	36.8	33.3	20.8	
2000	(316.1)	(90.1)	(46.8)	(41.3)	(25.8)	
16. Airline Distance from CBD (miles)	0	3.4	6.2	9.2	14.2	

DATA D

Income Submodel:

The income submodel relates average zone income to the number of households in a particular income group. UTOWN has the following characteristics which might have been obtained from home interviews or census data.

Zone	Average Income	% in Low ($0-7999)	% in Medium ($8-11,999)	% in High ($12,000 and Greater)
1	10,000	10	50	40
2	4,000	75	23	2
3	12,000	6	31	63
4	6,000	38	52	10
5	15,000	2	22	76

DATA E

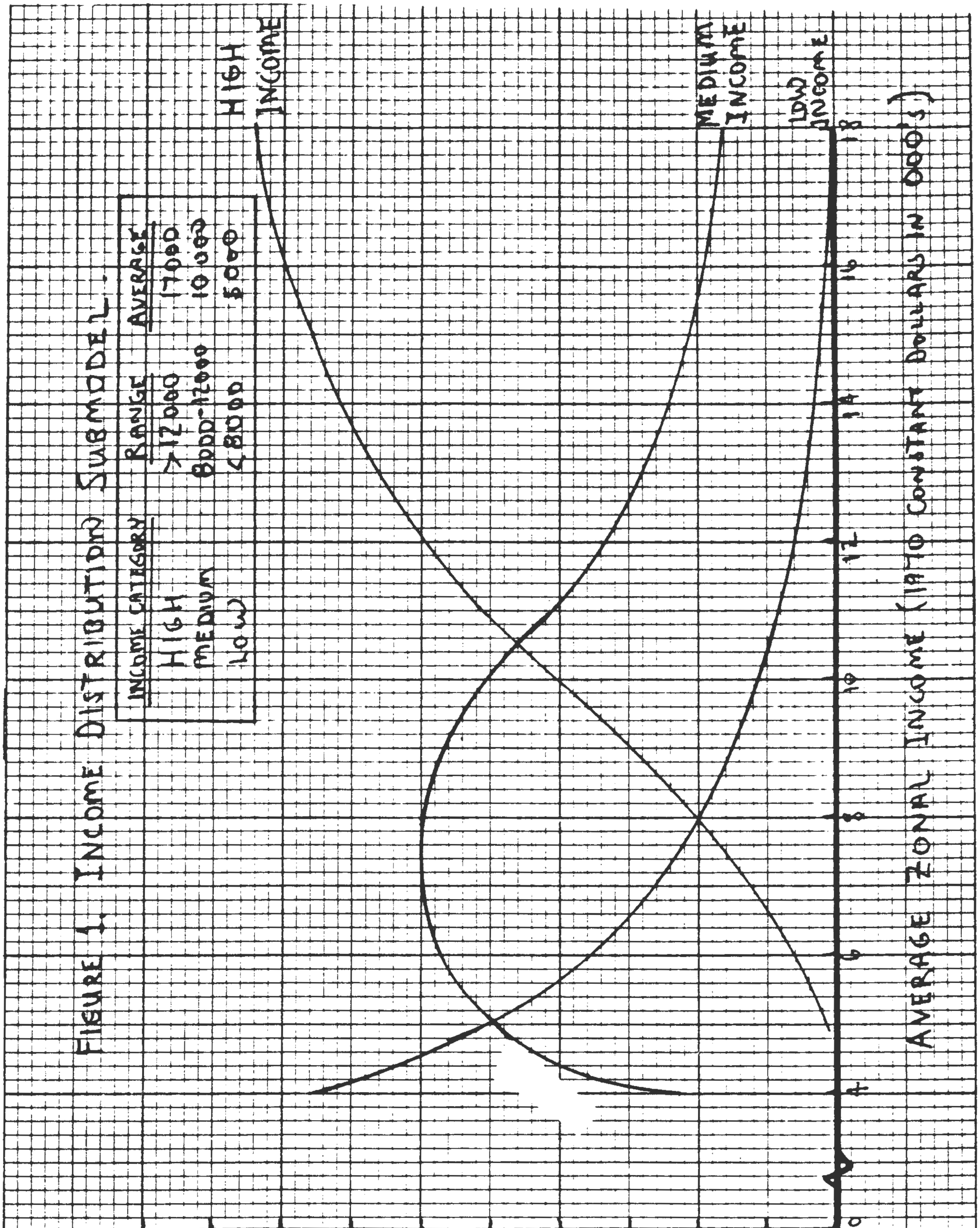
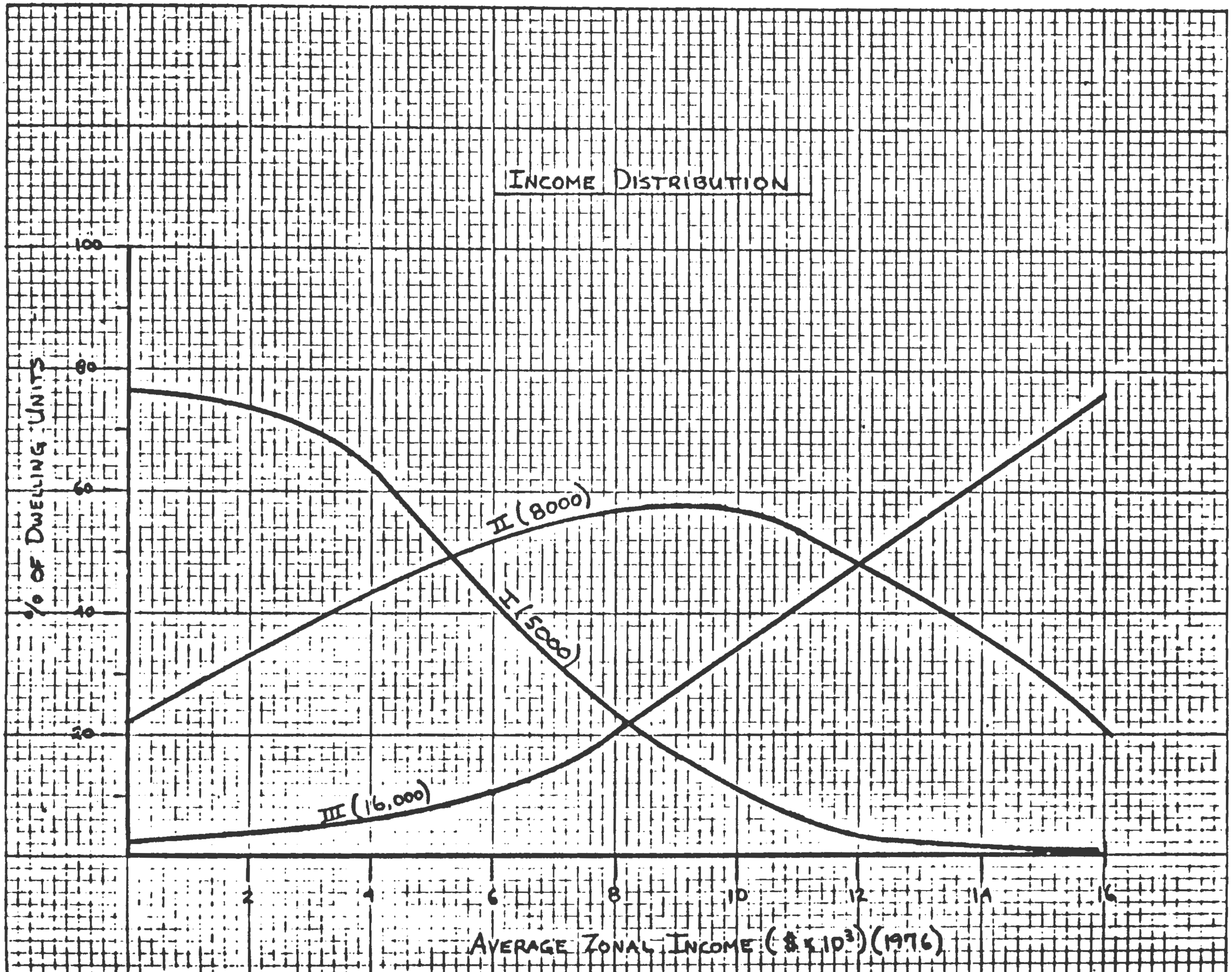


FIGURE 1. INCOME DISTRIBUTION SUBMODEL
INCOME CATEGORY RANGE AVERAGE
HIGH >12000 17000
MEDIUM 8000-12000 10000
LOW <8000 5000
HIGH INCOME
MEDIUM INCOME
LOW INCOME
PERCENT OF HOUSEHOLDS
0 10 20 30 40 50 60 70 80 90 100
0 4 6 8 10 12 14 16 18
AVERAGE ZONAL INCOME (1970 CONSTANT DOLLARS IN 000'S)

DATA F

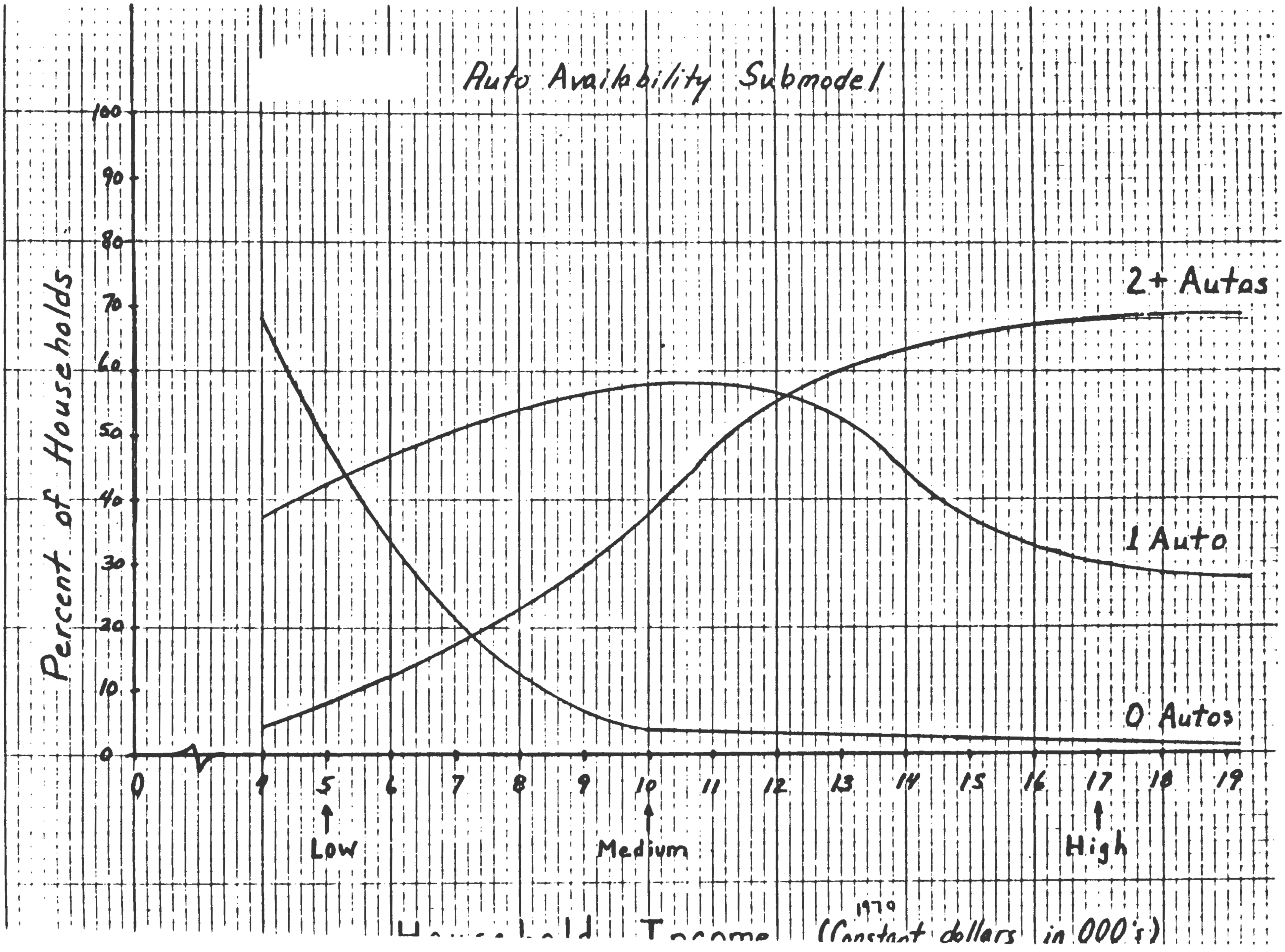

INCOME DISTRIBUTION
% OF DWELLING UNITS
100
80
60
40
20
II (8000)
I (5000)
III (16,000)
2
4
6
8
10
12
14
16
AVERAGE ZONAL INCOME ($ x 10³) (1976)

Auto Availability Submodel:

The auto availability submodel relates household income to auto availability and predicts the percent of households in each auto availability category. This submodel is calibrated from the same data base as the income submodel, that is, home interviews or census data.

For the purpose of this example, the following data about UTOWN households is assumed.

Income Range	Percent Households without auto	Percent Households with 1 auto	Percent Households with 2+ autos
Low	50%	42%	8%
Medium	4%	58%	38%
High	2%	30%	68%

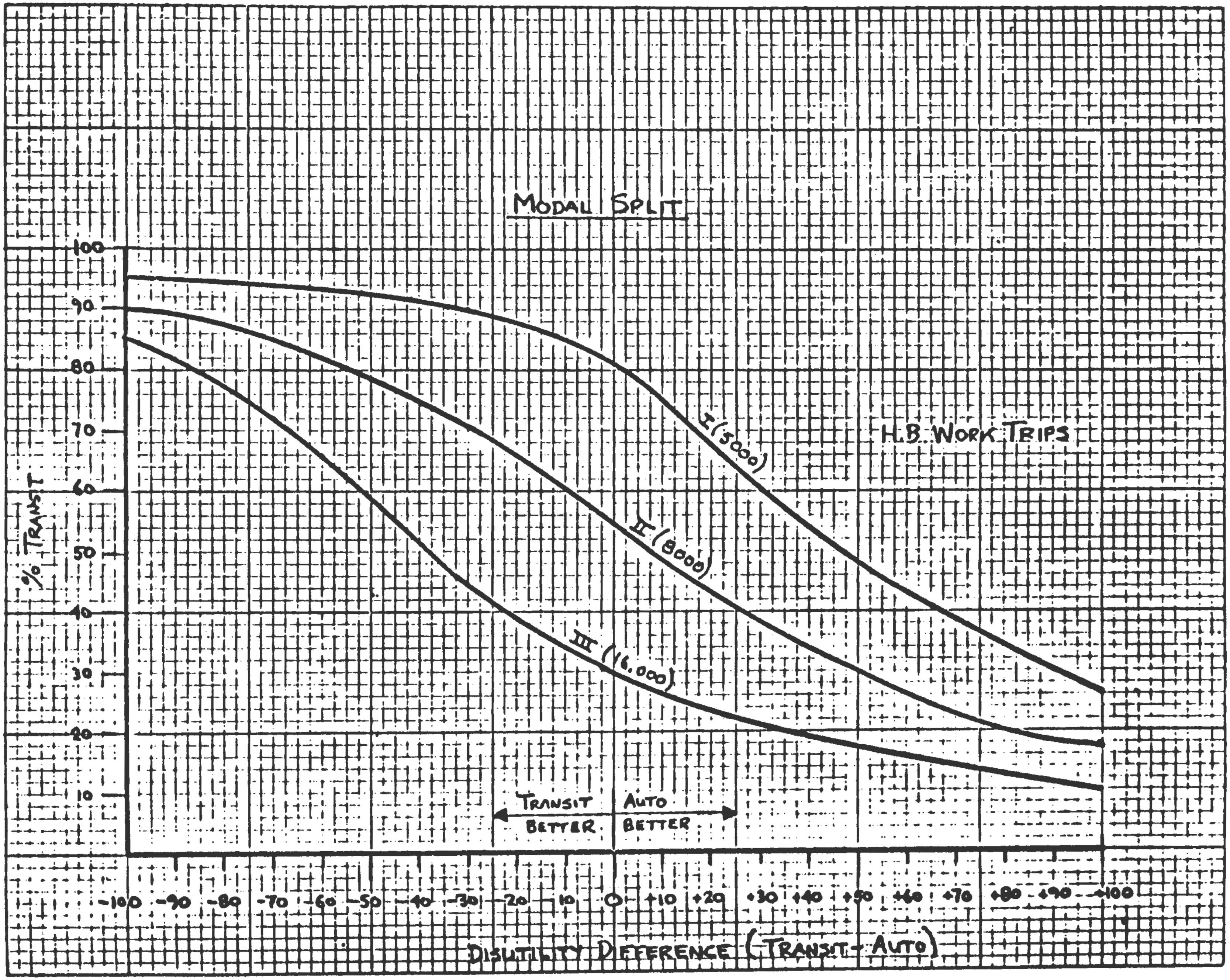

Auto Availability Submodel
2+ Autos
1 Auto
0 Autos
Percent of Households
100
90
80
70
60
50
40
30
20
10
0
0
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
Low
Medium
High
1970
(Constant dollars in 000's)

DENSITY FUNCTION

(Residential Acres * Non-Residential Acres) 10^{-3}

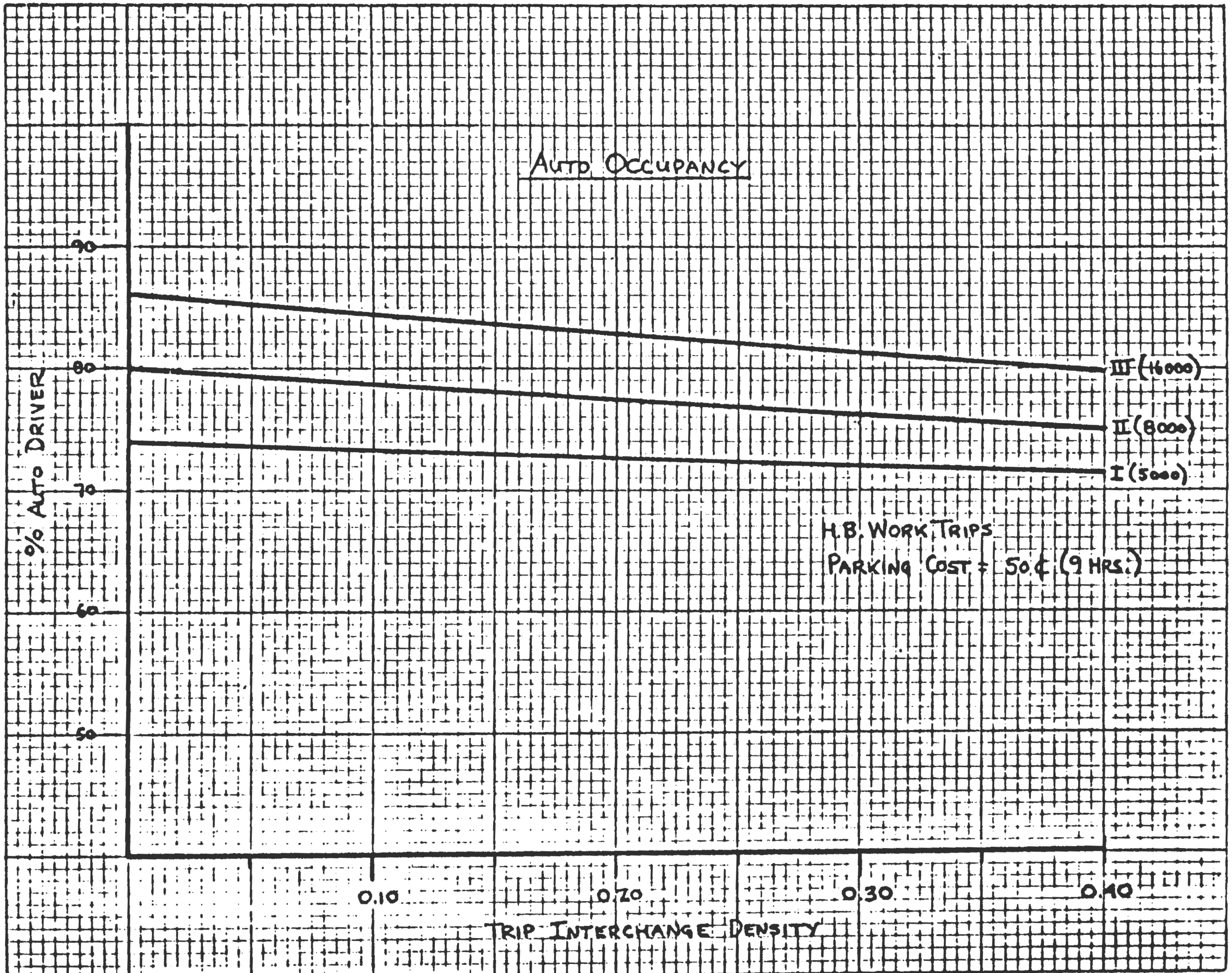
	1	2	3	4	5
1	0.2	0.2	0.4	0.7	0.3
2	1.4	1.2	2.8	4.8	2.2
3	3.4	2.9	7.0	12.0	5.5
4	1.2	1.0	2.6	4.4	2.0
5	3.9	3.3	8.1	13.8	6.3

DATA J



MODAL SPLIT
H.B. WORK TRIPS
I (5000)
II (8000)
III (16,000)
% TRANSIT
100
90
80
70
60
50
40
30
20
10
TRANSIT BETTER
AUTO BETTER
-100 -90 -80 -70 -60 -50 -40 -30 -20 -10 0 +10 +20 +30 +40 +50 +60 +70 +80 +90 +100
DISUTILITY DIFFERENCE (TRANSIT - AUTO)

DATA K


AUTO OCCUPANCY
% AUTO DRIVER
90
80
70
60
50
III (16000)
II (8000)
I (5000)
H.B. WORK TRIPS
PARKING COST = 50¢ (9 HRS.)
0.10
0.20
0.30
0.40
TRIP INTERCHANGE DENSITY

DATA L

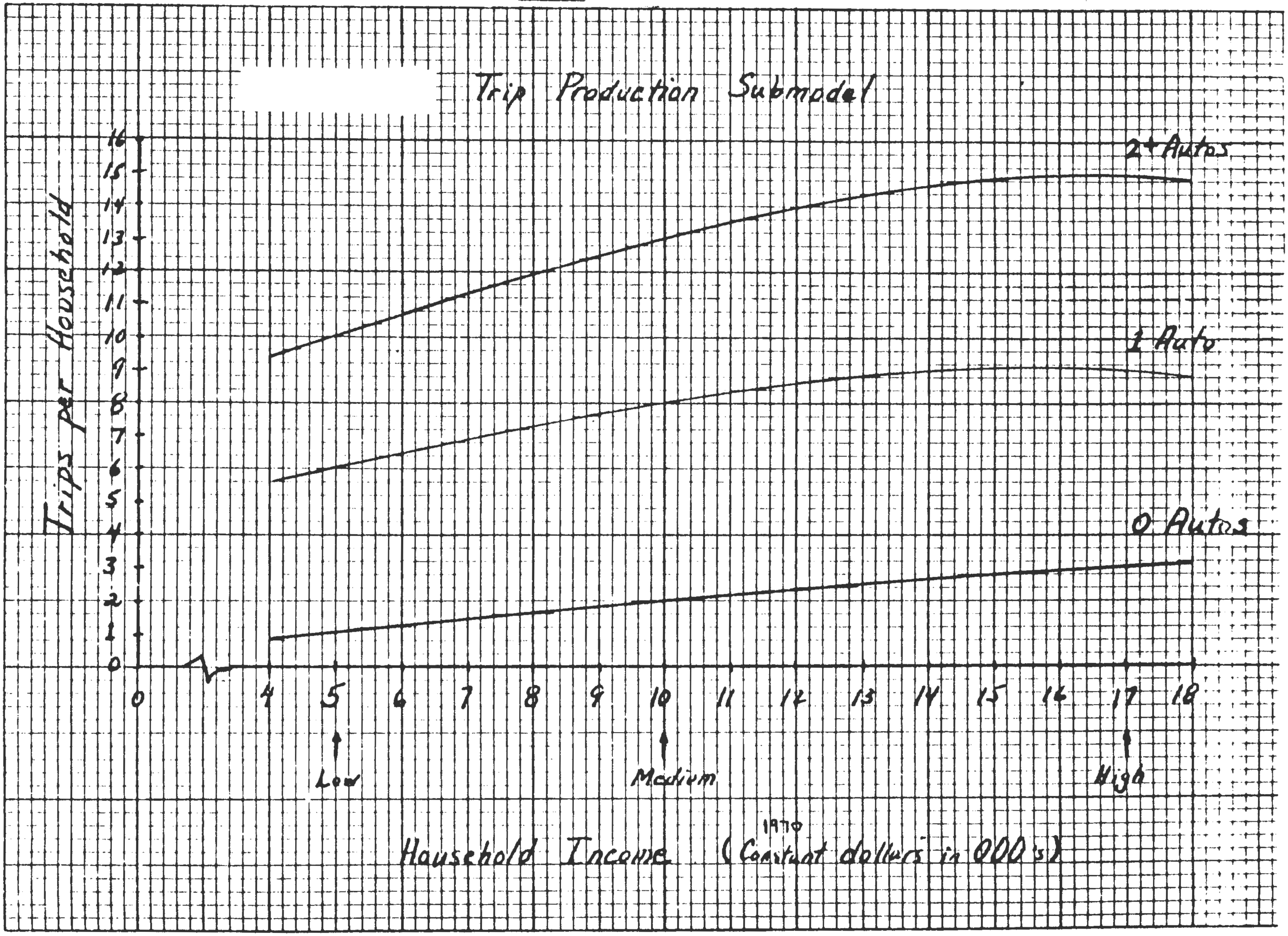
Trip Production Submodel:

Using data collected about household trip making, this submodel relates trip productions to income and auto availability characteristics. This data has in the past generally been gathered in home interview surveys. In some cases the data is still valid for calibrating models; in others, the planner must use judgment and possibly conduct a small sample survey to validate any assumptions made.

Assuming UTOWN has a reliable survey available, the following information is available to calculate <u>trips per household</u> in the column at the far right.

Income	Auto Ownership	Number of Households	Number of Trips	Trips per Household
Low	0	239	239	1
	1	201	1206	6
	2+	38	380	10
Medium	0	15	30	2
	1	221	1768	8
	2+	145	1885	13
High	0	8	24	3
	1	117	1053	9
	2+	266	3990	15

DATA M

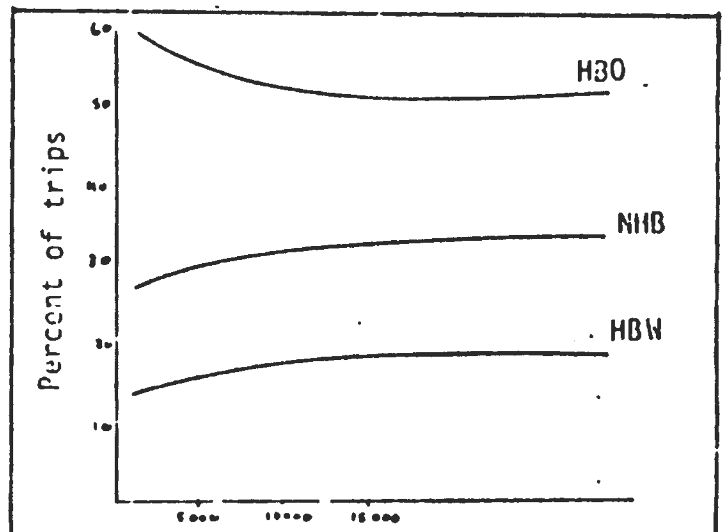

Trip Production Submodel
2+ Autos
1 Auto
0 Autos
Trips per Household
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
0
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
Low
Medium
High
Household Income (1970 Constant dollars in 000's)

Trip Purpose Submodel:

The final submodel in the trip production process relates trip purpose to income in such a way that the trip productions can be divided among these various purposes. In order to establish this relationship, data from existing home interview surveys and other sources, if necessary, are tabulated as shown for UTOWN below.

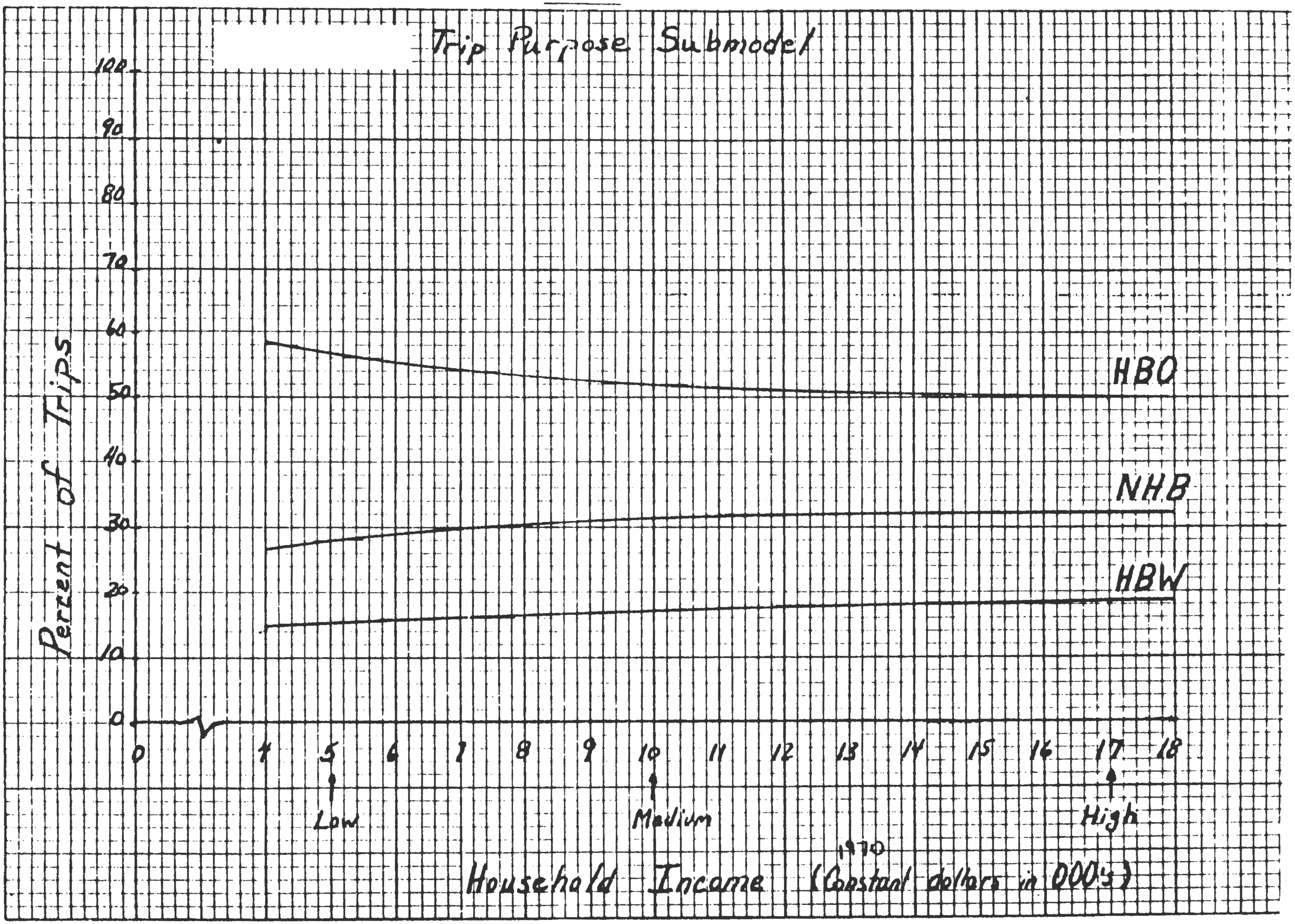
Income Level	Percent Home Based Work	Percent Home Based Other	Percent Non-Home Based
Low	15	57	28
Medium	17	52	31
High	18	50	32

These data can be plotted and curves smoothed as shown at the right to provide the planner with relationships to forecast trip purpose. Once again these relationships can be borrowed from similar areas if existing data are found inadequate.



Household Income
Data for curves from O-D Survey

DATA O


Trip Purpose Submodel
Percent of Trips
100
90
80
70
60
50
40
30
20
10
0
HBO
NHB
HBW
0
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
Low
Medium
High
Household Income (1970 Constant dollars in 000's)

TRIP ATTRACTION RATES

Trip Purpose	Attractions Per Household	Attractions Per Nonretail Employee	Attractions Per Downtown Retail Employee	Attractions Per Other Retail Employee
Homebased work	Negligible	1.7	1.7	1.7
Homebased other	1.0	2.0	5.0	10.0
Nonhome based	1.0	1.0	3.0	5.0

DATA Q

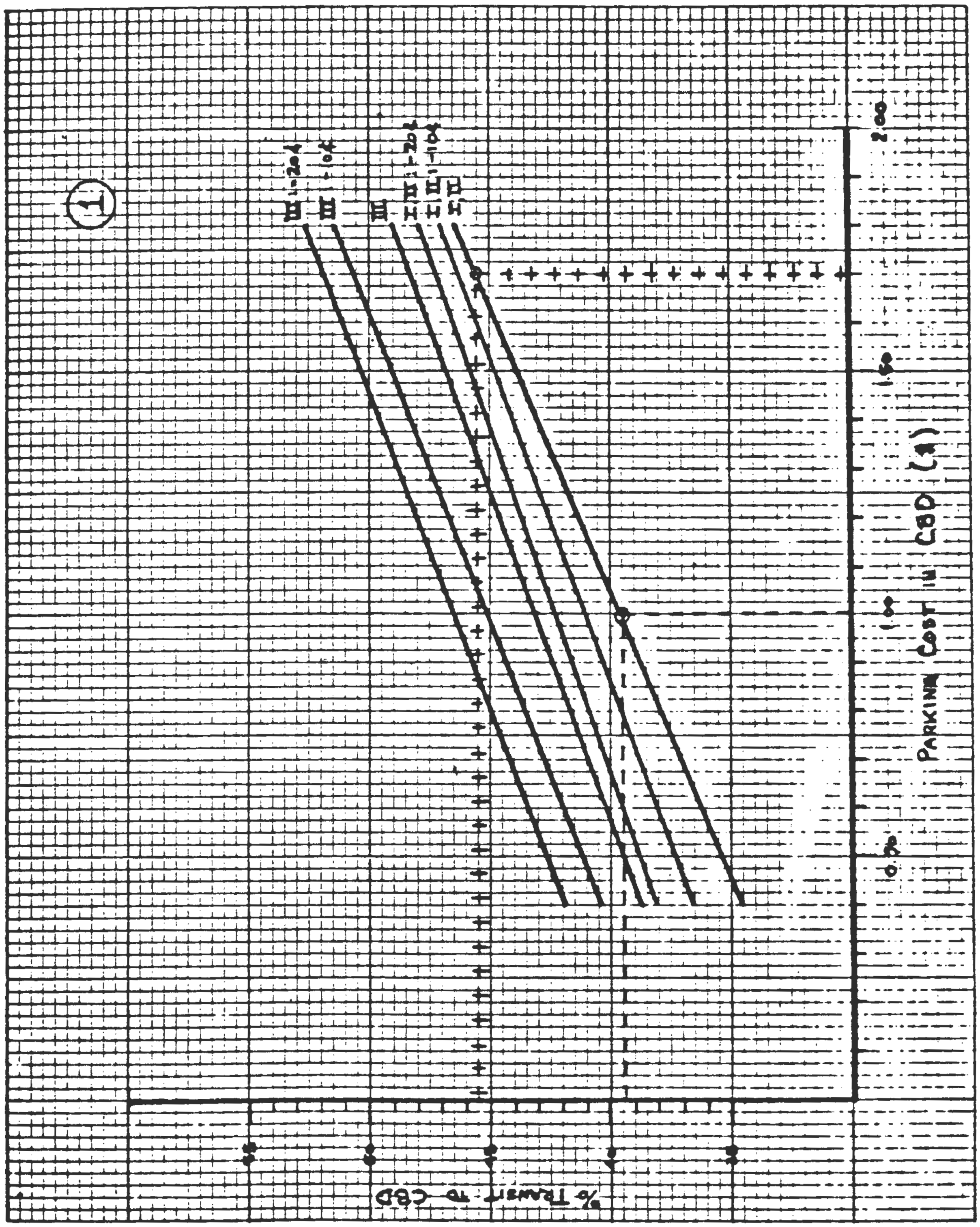
FINAL ADJUSTED P'S & A'S

	Productions			Attractions		
Zone	HB-Work	HB-Other	NHB	HB-Work	HB-Other	NHB
1	1,917	5,554	39,139	33,532	72,697	39,139
2	8,916	28,917	17,509	7,779	27,393	17,509
3	17,166	48,588	10,506	1,878	13,064	10,506
4	7,833	23,726	10,300	2,414	15,804	10,300
5	11,112	31,234	7,004	1,341	9,061	7,004
Total	46,944	138,019	84,458	46,944	138,019	84,458

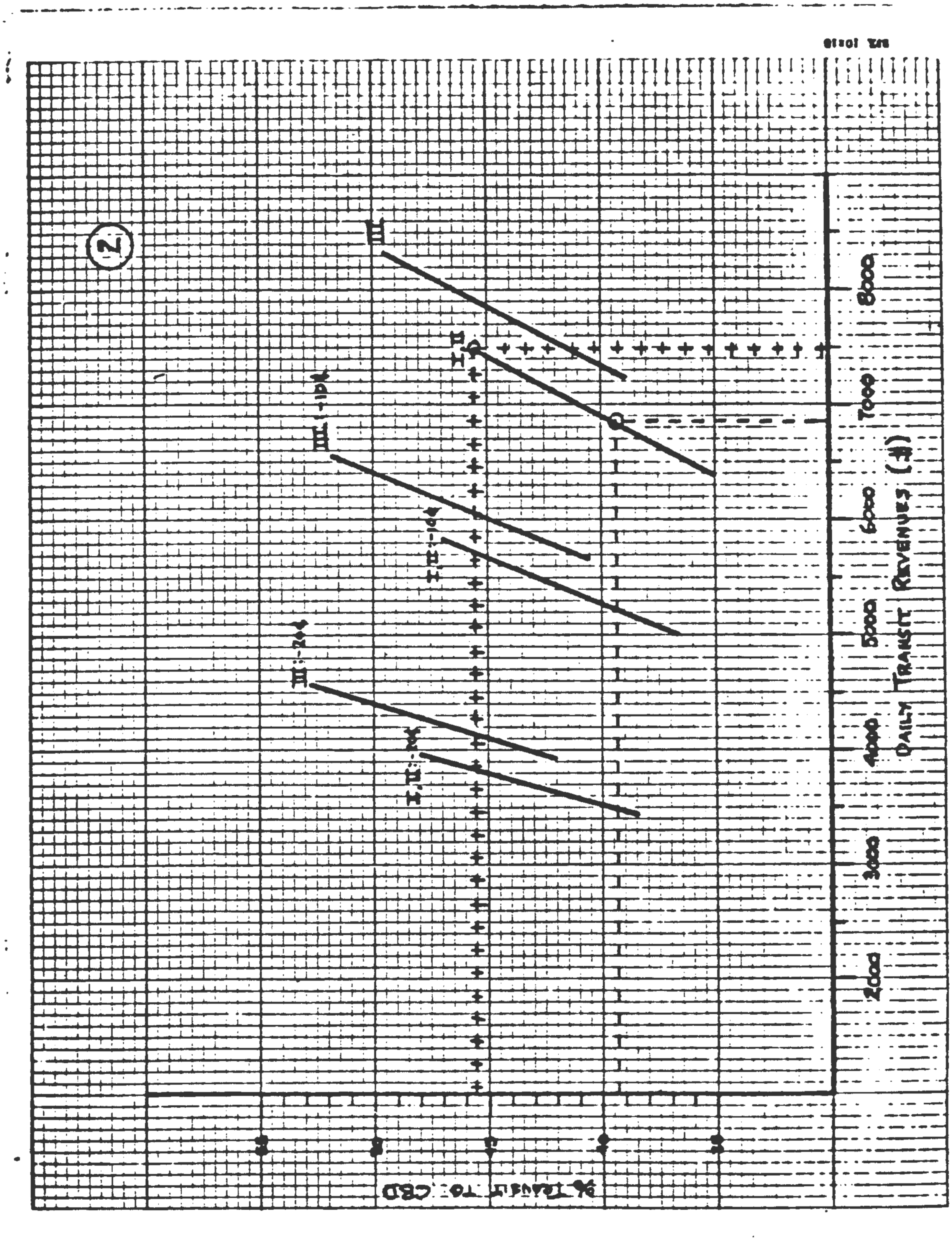
PARKING COST (CENTS)

ZONE	BASE	PARKING #	EXPRESS BUS
1	100	120	100
2	80	80	80
3	50	50	50
4	0	0	0
5	0	0	0

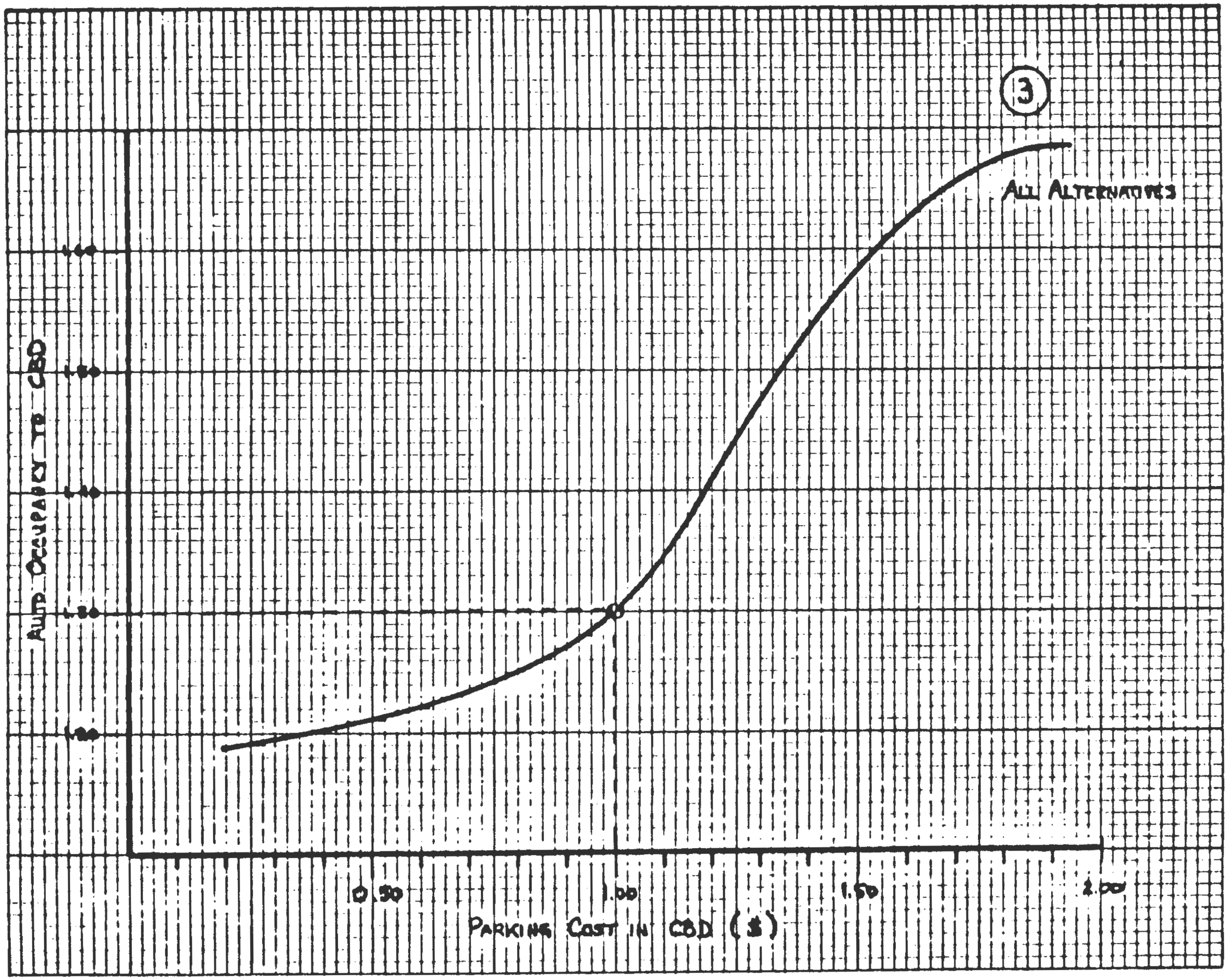
DATA S


1
III - 20¢
III - 10¢
III
II - 20¢
II - 10¢
II
2.00
1.50
1.00
0.50
PARKING COST IN CBD ($)
% TRANSIT TO CBD

DATA T

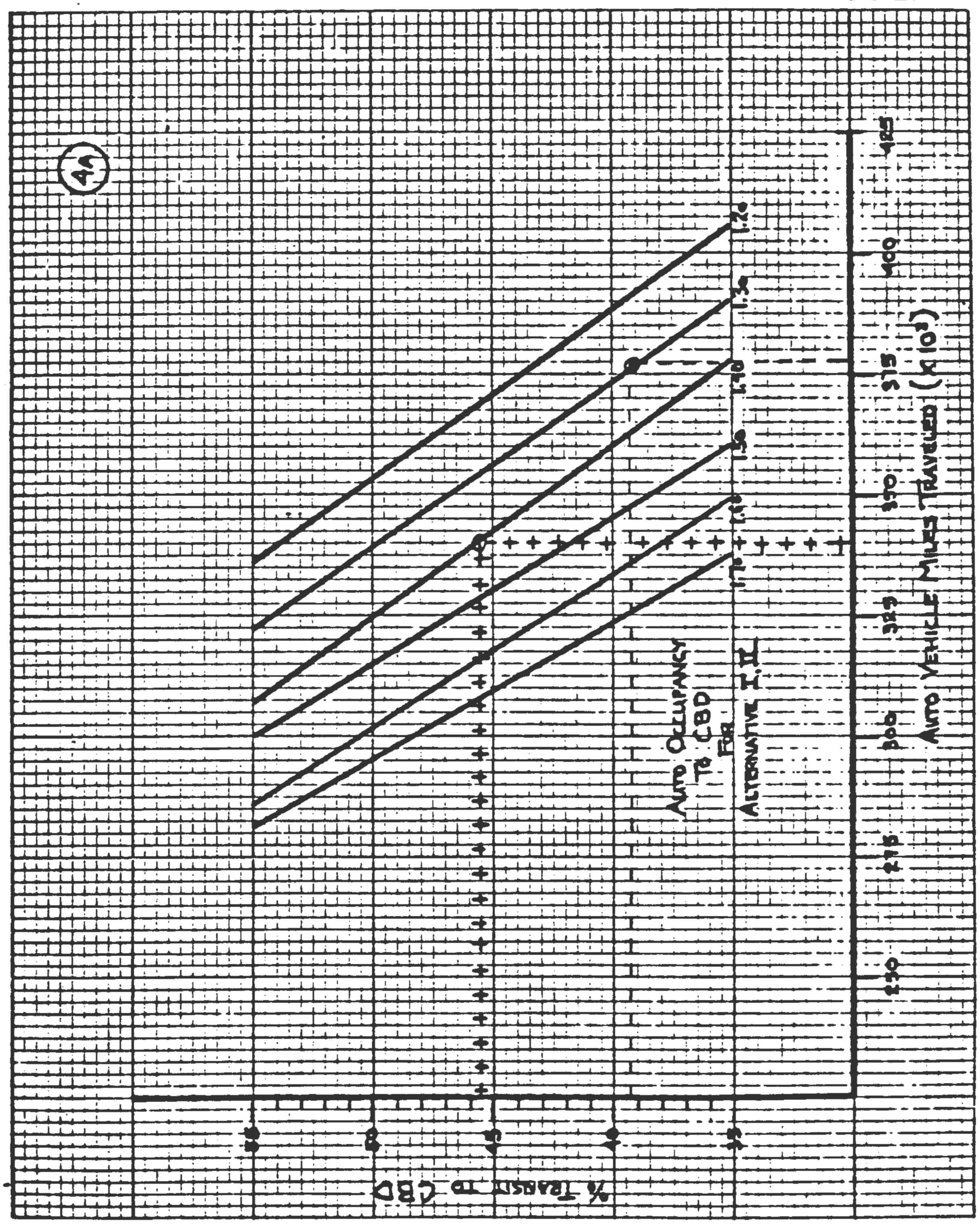

2
% Transit to CBD
2000
3000
4000
5000
6000
7000
8000
Daily Transit Revenues ($)

DATA U

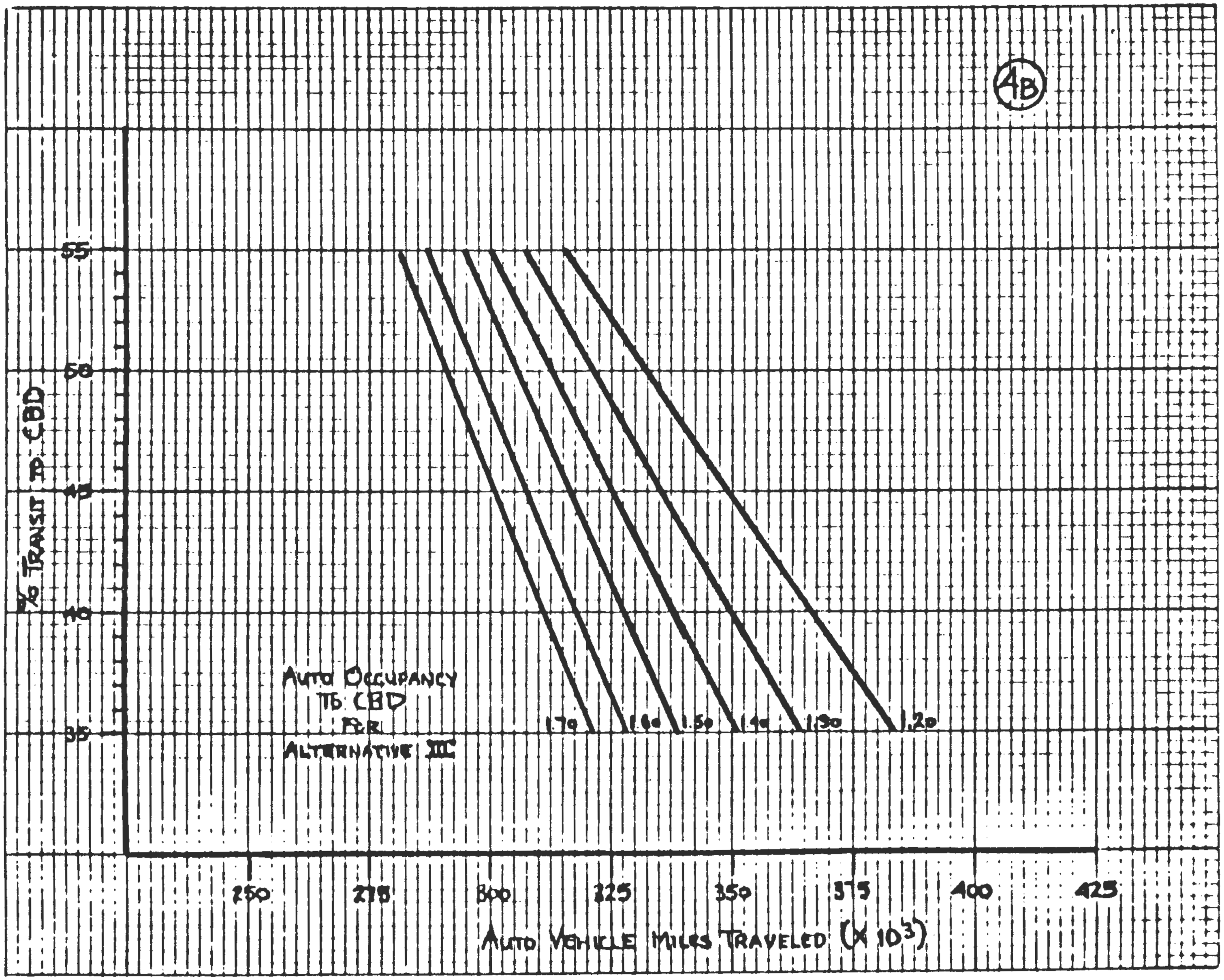


3
ALL ALTERNATIVES
AUTO OCCUPANCY TO CBD
0.50
1.00
1.50
2.00
PARKING COST IN CBD ($)

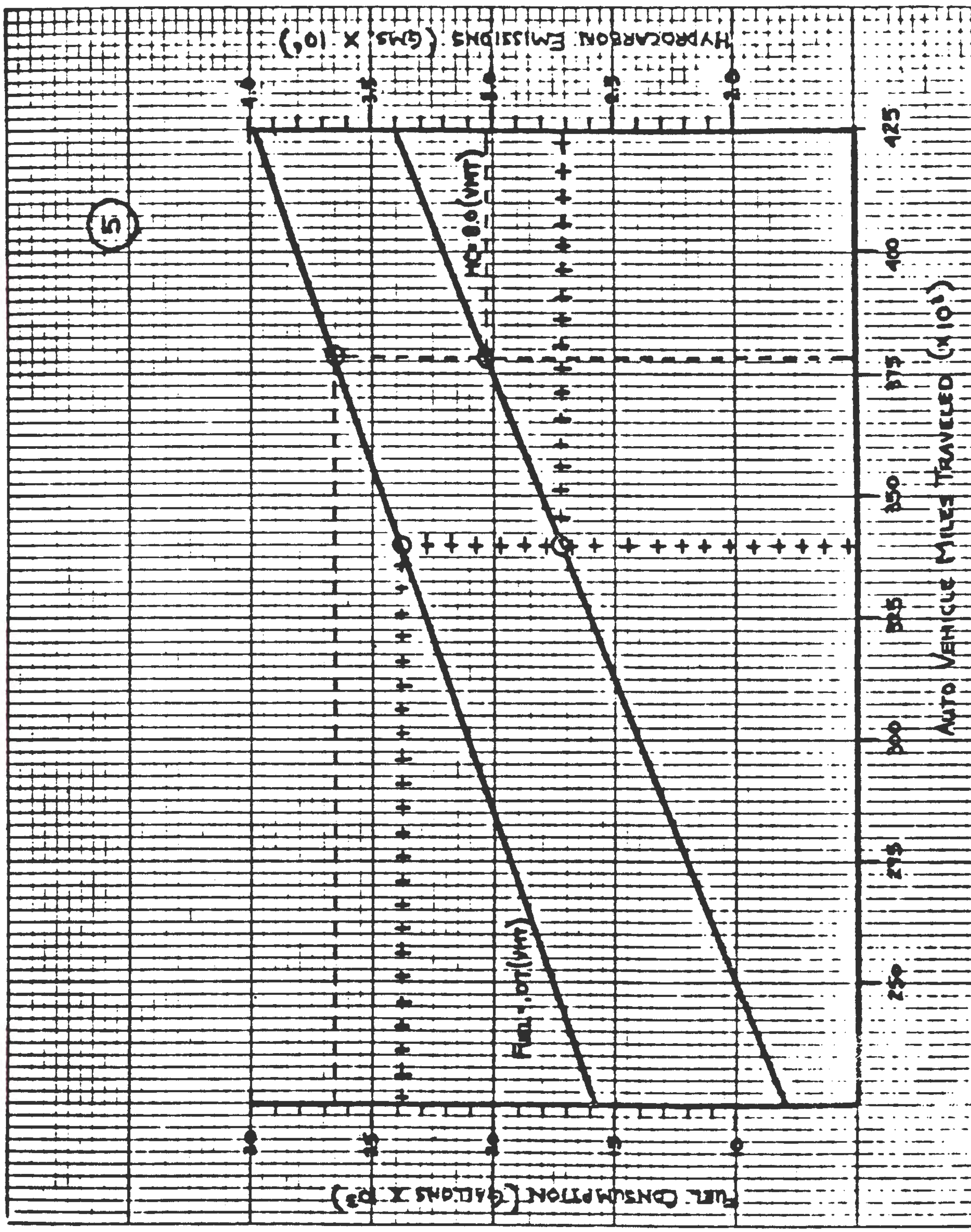
DATA V


4A
Auto Occupancy To CBD For Alternative I, II
1.20
1.30
1.40
1.50
1.60
1.70
250
275
300
325
350
375
400
425
Auto Vehicle Miles Traveled (x 10³)
% Transit to CBD

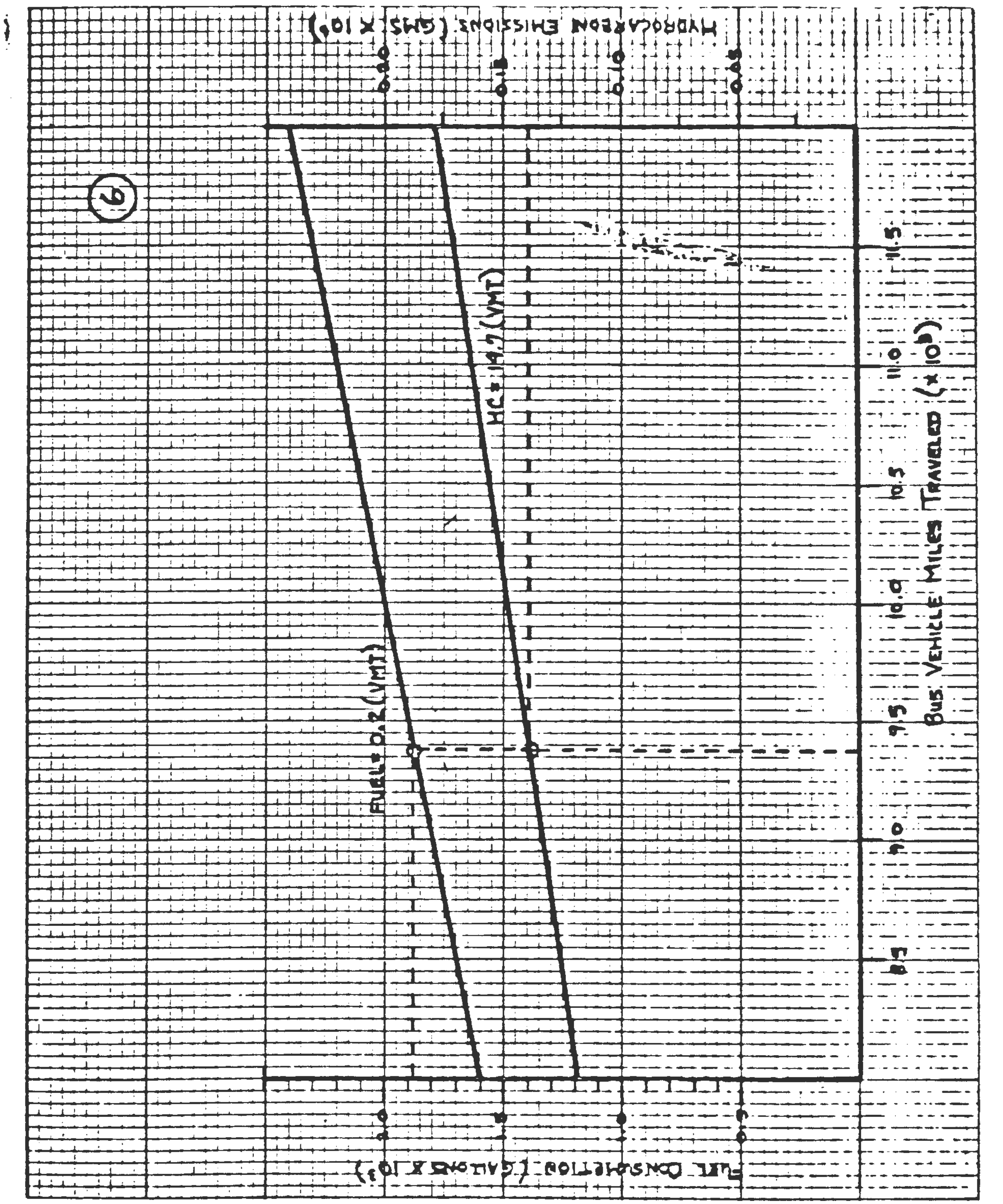
DATA W


4B
% TRANSIT TO CBD
55
50
45
40
35
AUTO OCCUPANCY
TO CBD
FOR
ALTERNATIVE III
1.70
1.60
1.50
1.40
1.30
1.20
250
275
300
325
350
375
400
425
AUTO VEHICLE MILES TRAVELED (X 10^3)

DATA X



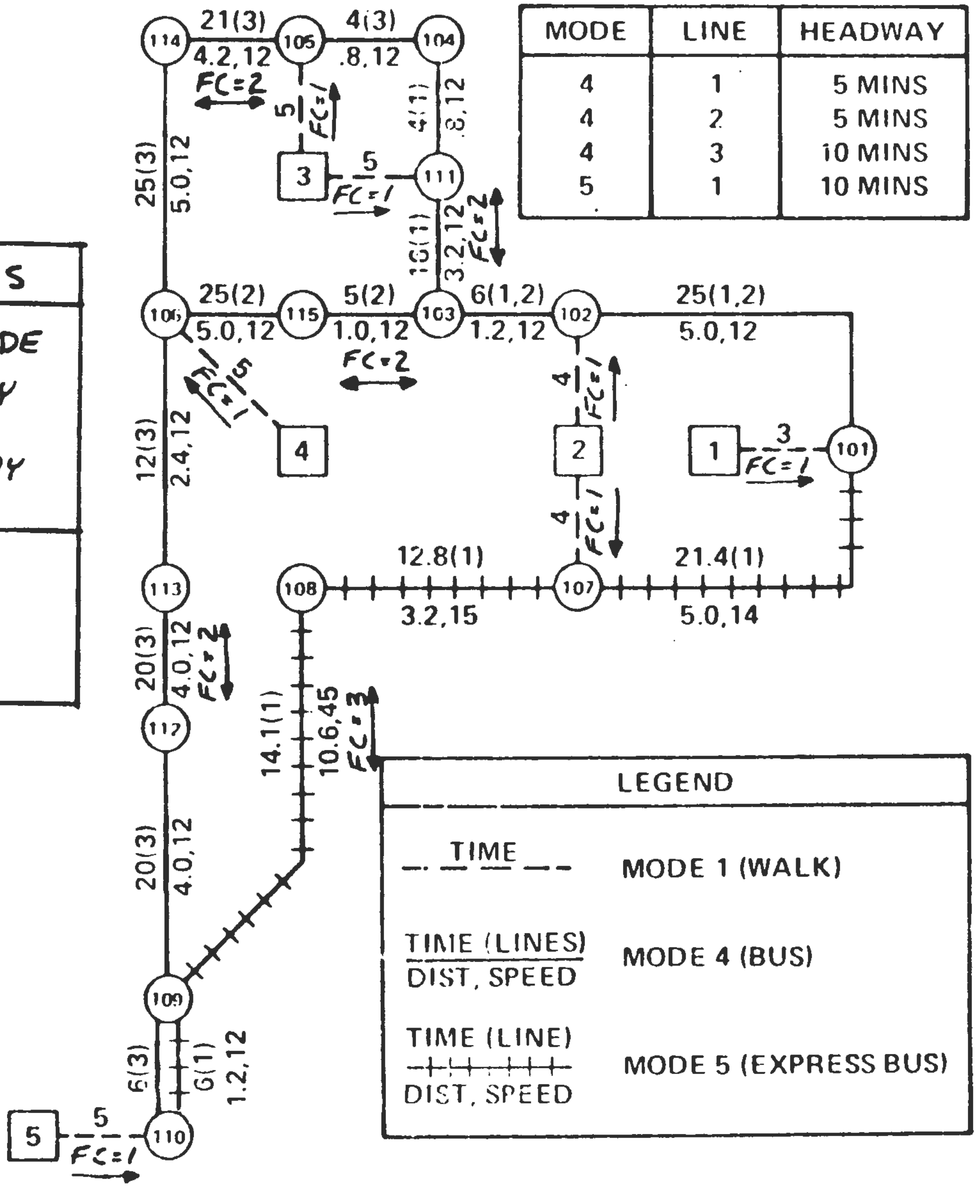
DATA Y



6
HYDROCARBON EMISSIONS (GMS X 10^6)
0.20
0.15
0.10
0.05
HC = 14.7 (VMT)
FUEL = 0.2 (VMT)
FUEL CONSUMPTION (GALLONS X 10^3)
2.0
1.5
1.0
0.5
8.5
9.0
9.5
10.0
10.5
11.0
11.5
BUS VEHICLE MILES TRAVELED (x 10^3)

DATA Z

UTOWN TRANSIT NET. DISTANCES, SPEED, TIMES, HEADWAYS, FARES



MODE	LINE	HEADWAY
4	1	5 MINS
4	2	5 MINS
4	3	10 MINS
5	1	10 MINS

FARE CODES

FC = FARE CODE

→ = ONE WAY FARE

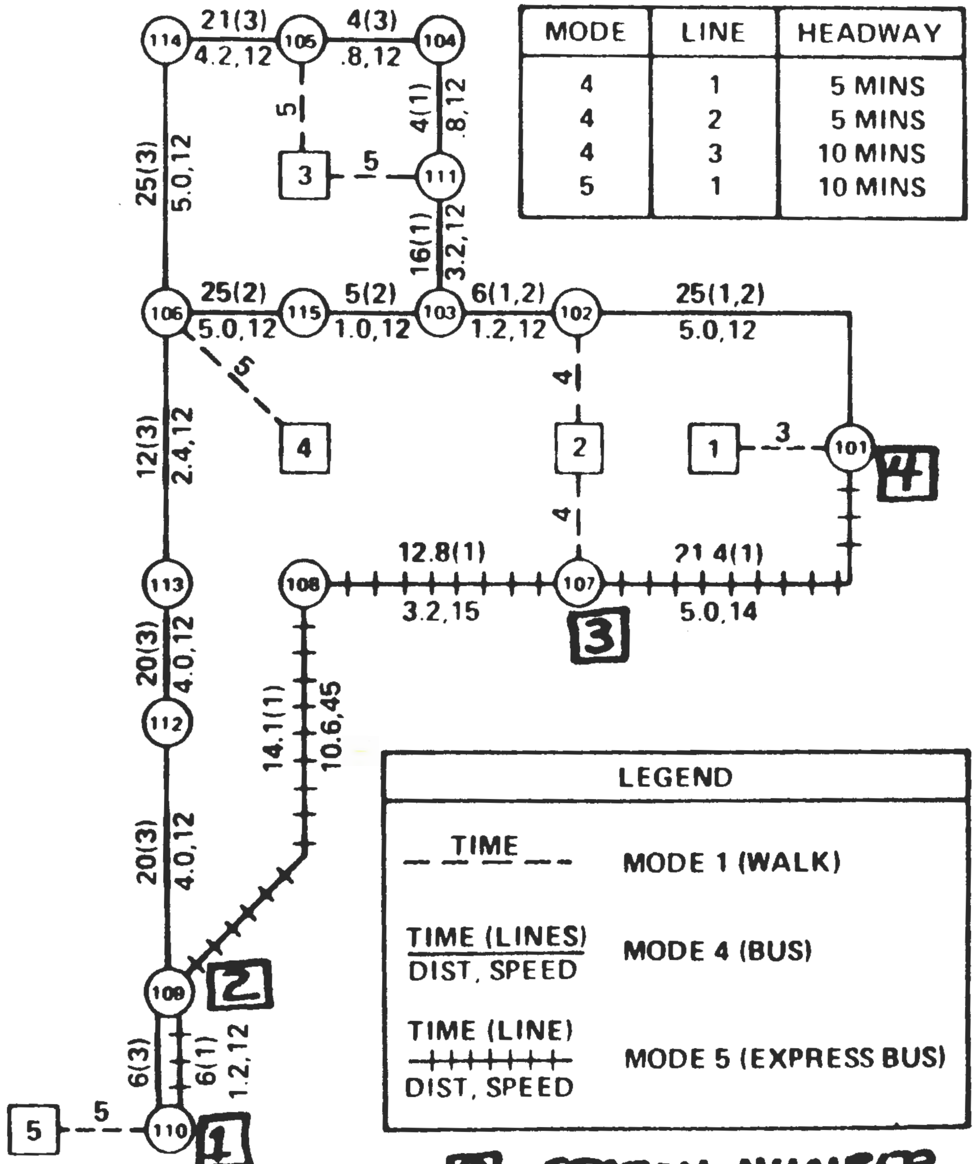
↔ = TWO WAY FARE

1 = 30¢

2 = 15¢

3 = 25¢

UTOWN TRANSIT NET. SELECTED STATIONS

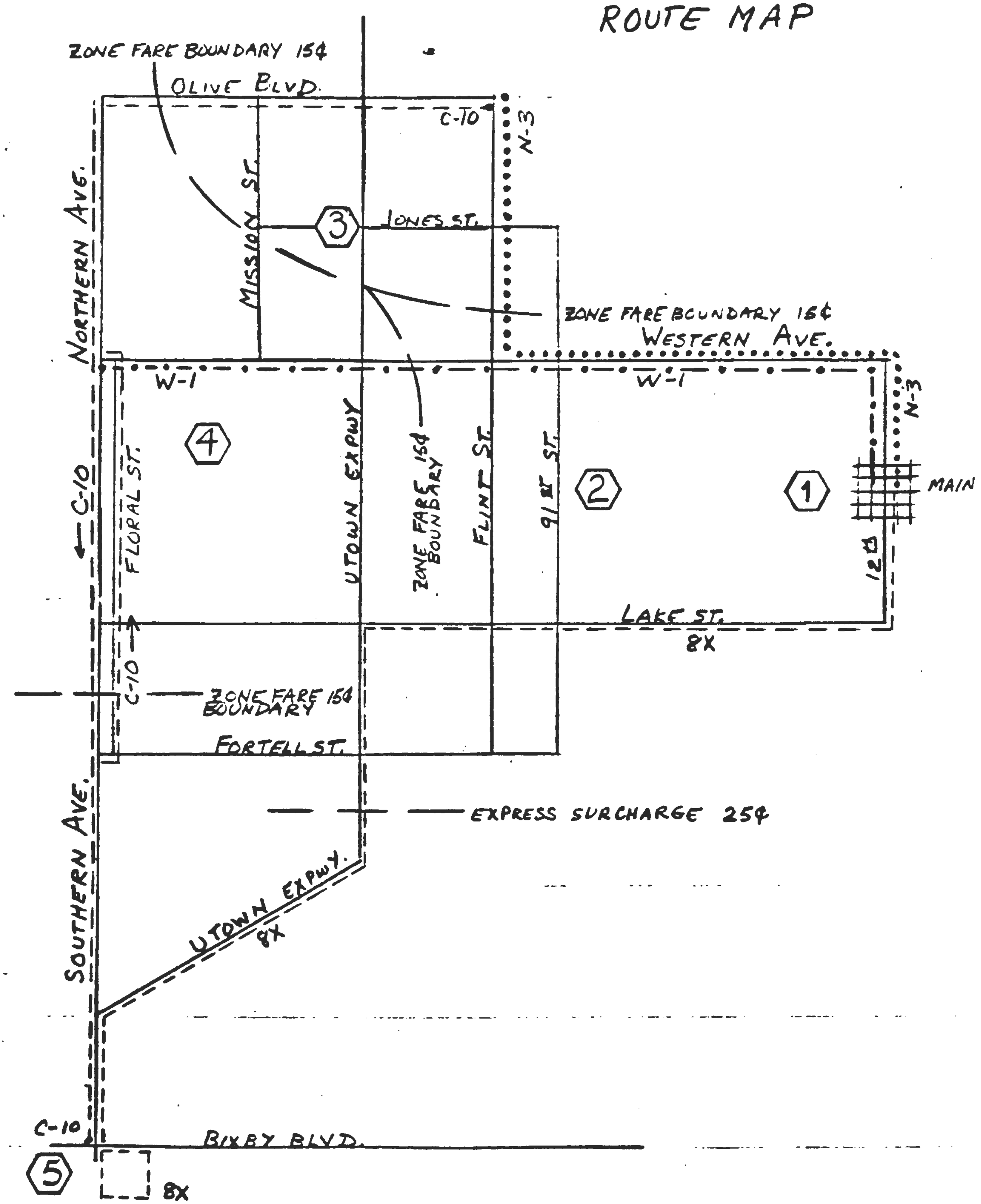


MODE	LINE	HEADWAY
4	1	5 MINS
4	2	5 MINS
4	3	10 MINS
5	1	10 MINS

[2] = STATION NUMBER

DATA BB

UTOWN TRANSIT AUTHORITY ROUTE MAP



BASE YEAR (DOWN TRANSIT (NO RADIO BUS)

A.M. PEAK IMPEDANCES

EXCESS TIME (MIN)
RUNNING TIME (MIN)
FARE (CENTS)

FROM \ TO ZONE	1	2	3	4	5
1	7 12 30	8 25 30	11 47 45	11 61 45	16 119 60
2	8 25 30	9 14 30	12 22 45	12 36 45	17 94 60
3	11 47 45	12 22 45	13 15 30	15 46 45	15 104 60
4	11 61 45	12 36 45	15 46 45	15 22 30	15 58 45
5	16 119 60	17 94 60	15 104 60	15 58 45	15 24 30

future to expo bus

DATA DD

UTOWN TRANSIT

A.M. PEAK IMPEDANCES

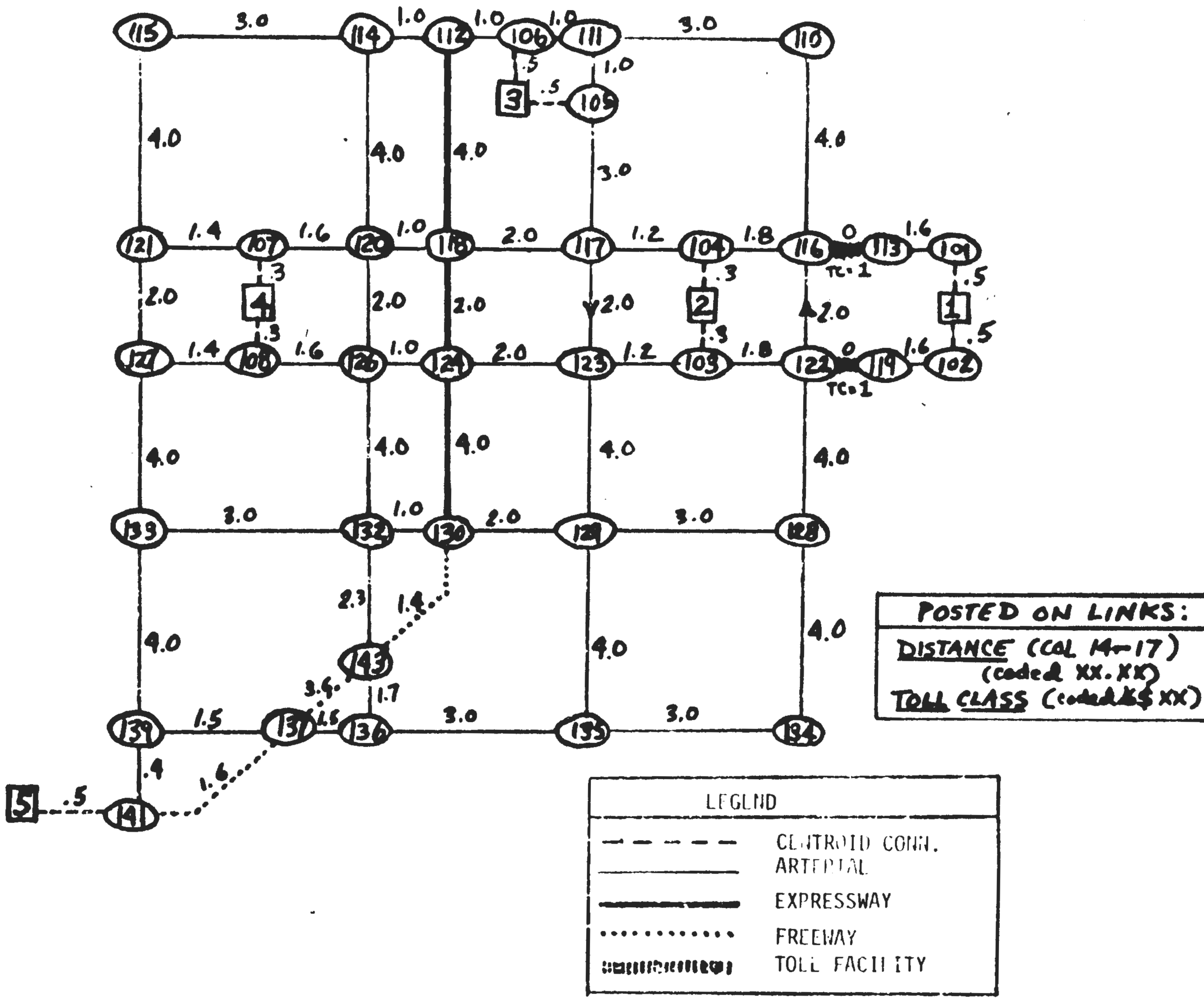
Excess Time (Min.)
Running Time (Min.)
Fare (Cents)

TO ZONE

FROM ZONE	1	2	3	4	5
1	7 12 30	8 25 30	11 47 45	11 61 45	13 * 54 55
2	8 25 30	9 14 30	12 22 45	12 36 45	14 * 33 55
3	11 47 45	12 22 45	13 15 30	15 46 45	18 * 101 70
4	11 61 45	12 36 45	15 46 45	15 22 30	15 * 58 45
5	13 * 54 55	14 * 33 55	18 * 101 70	15 * 58 45	15 * 24 30

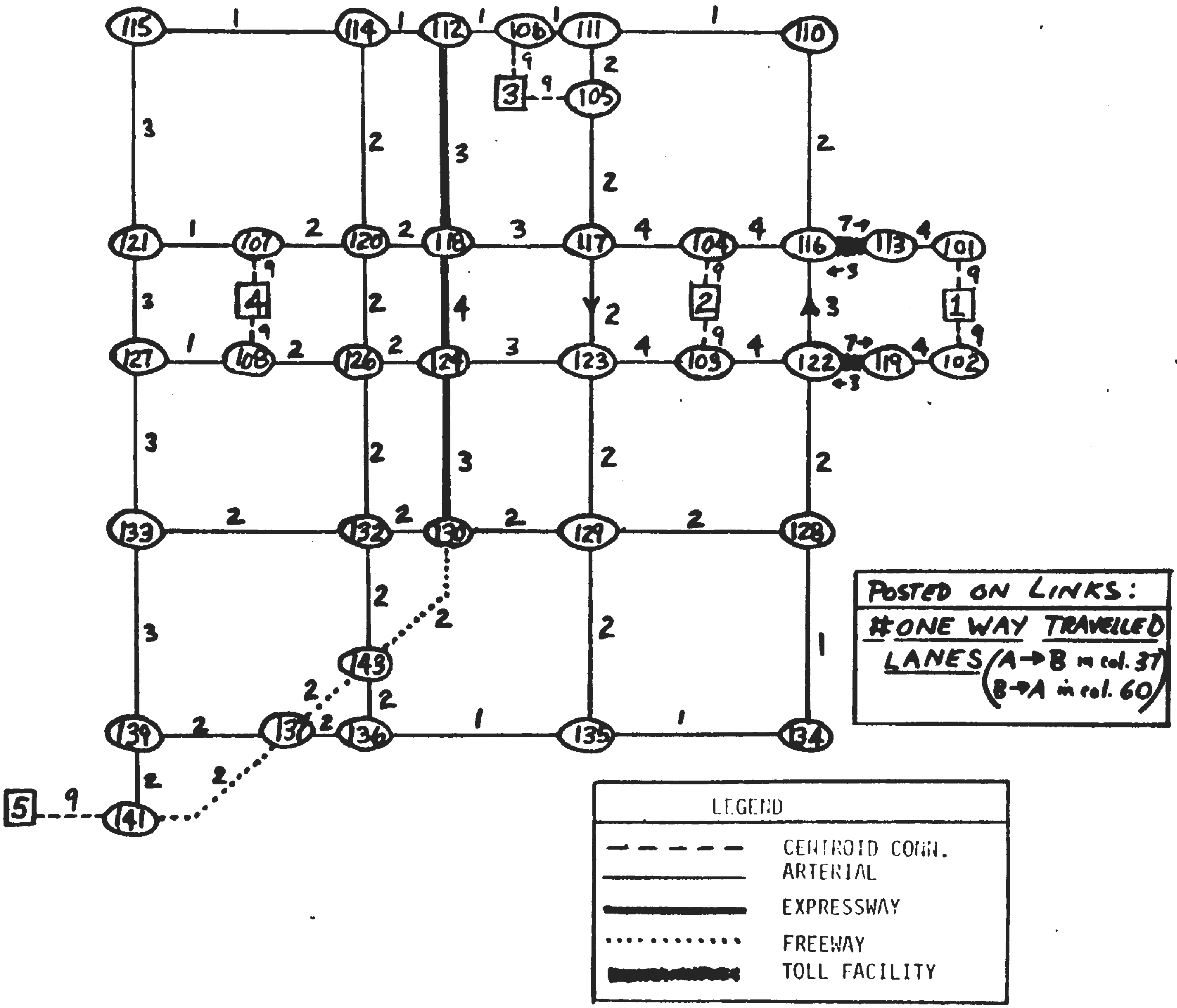
DATA EE

UTOWN HIGHWAY NET. DISTANCES



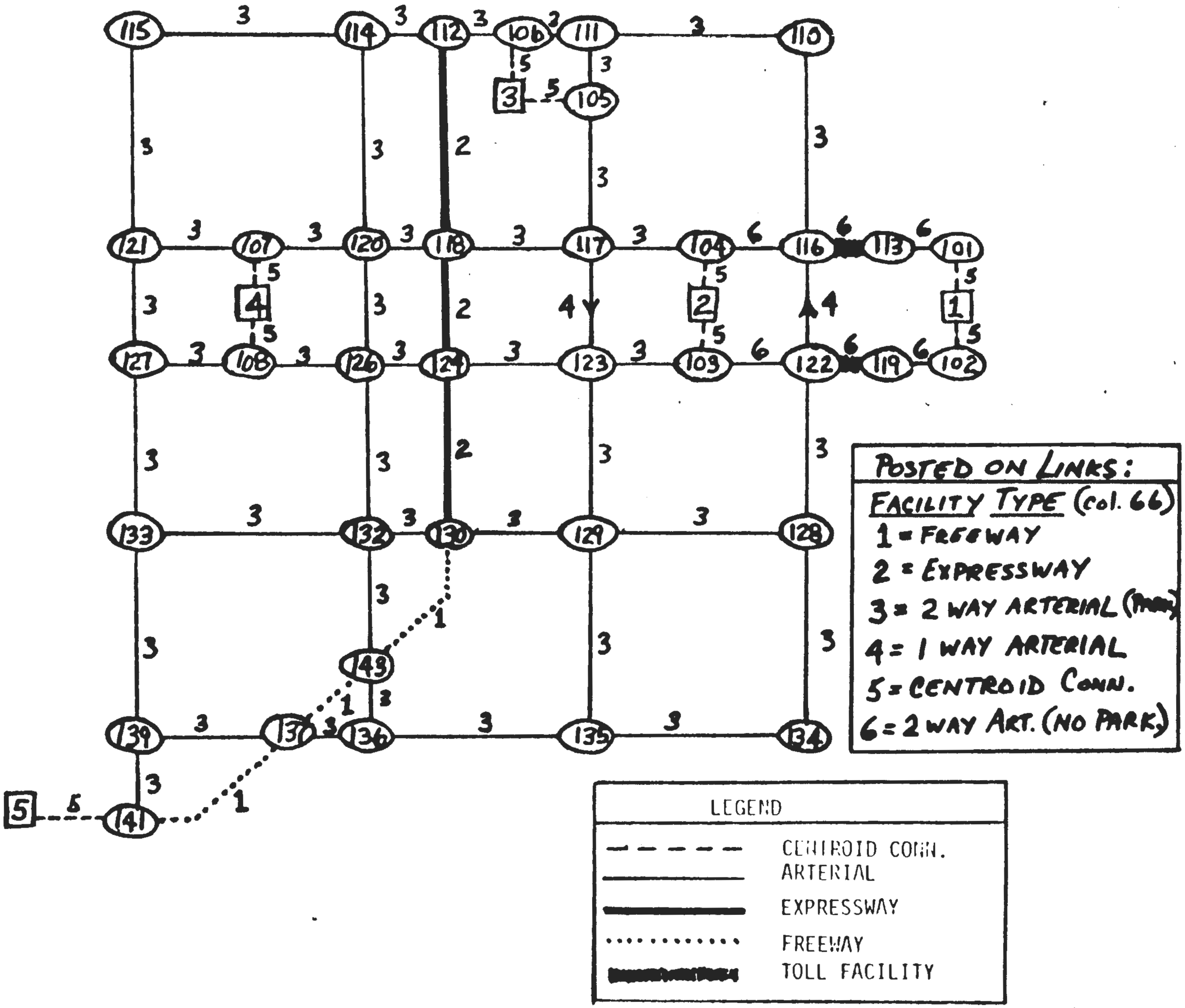
DATA FF

UTOWN HIGHWAY NET. LANES



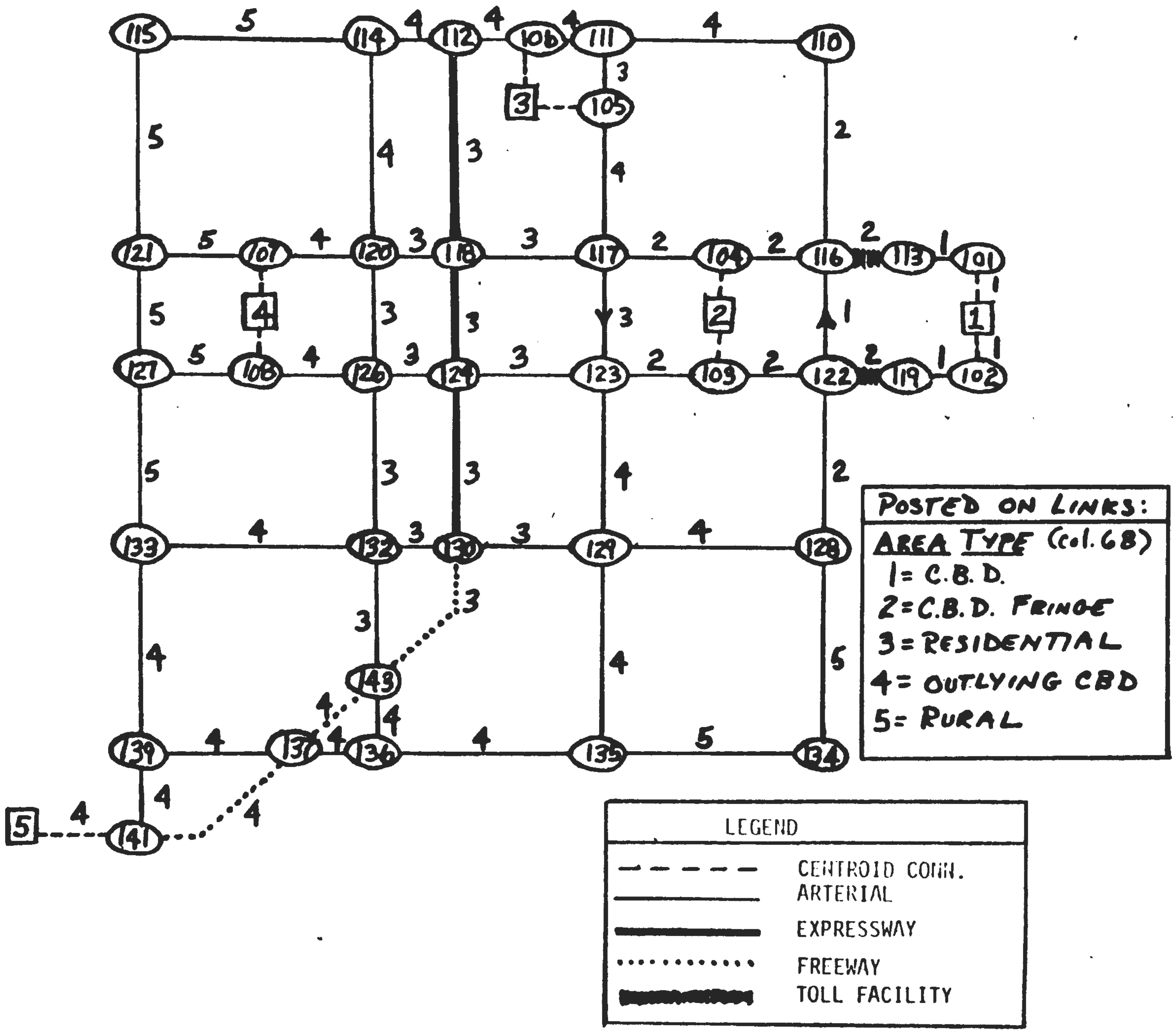
DATA GG

UTOWN HIGHWAY NET. FACILITY TAPES



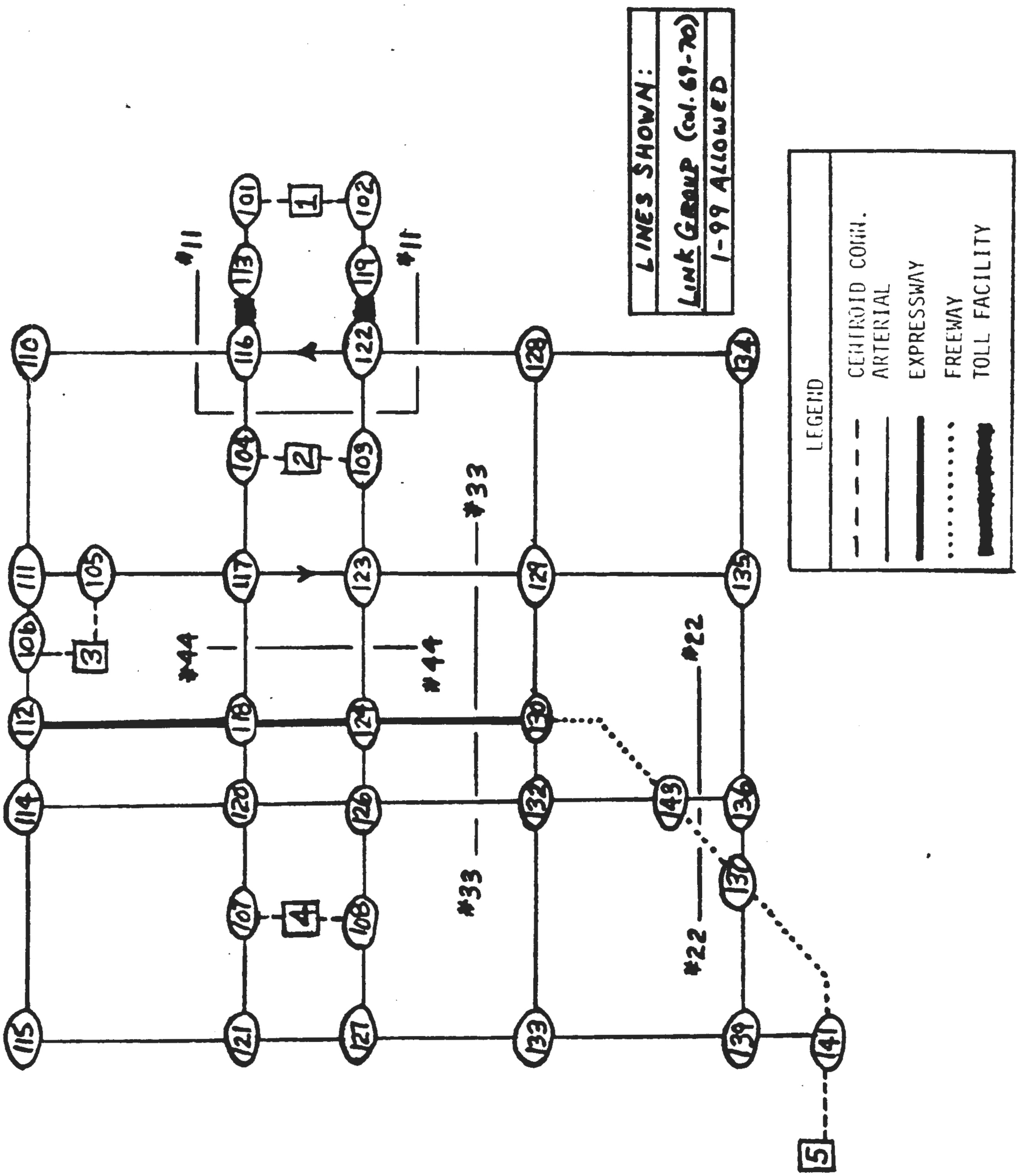
DATA IIB

UTOWN HIGHWAY NET. AREA TYPES



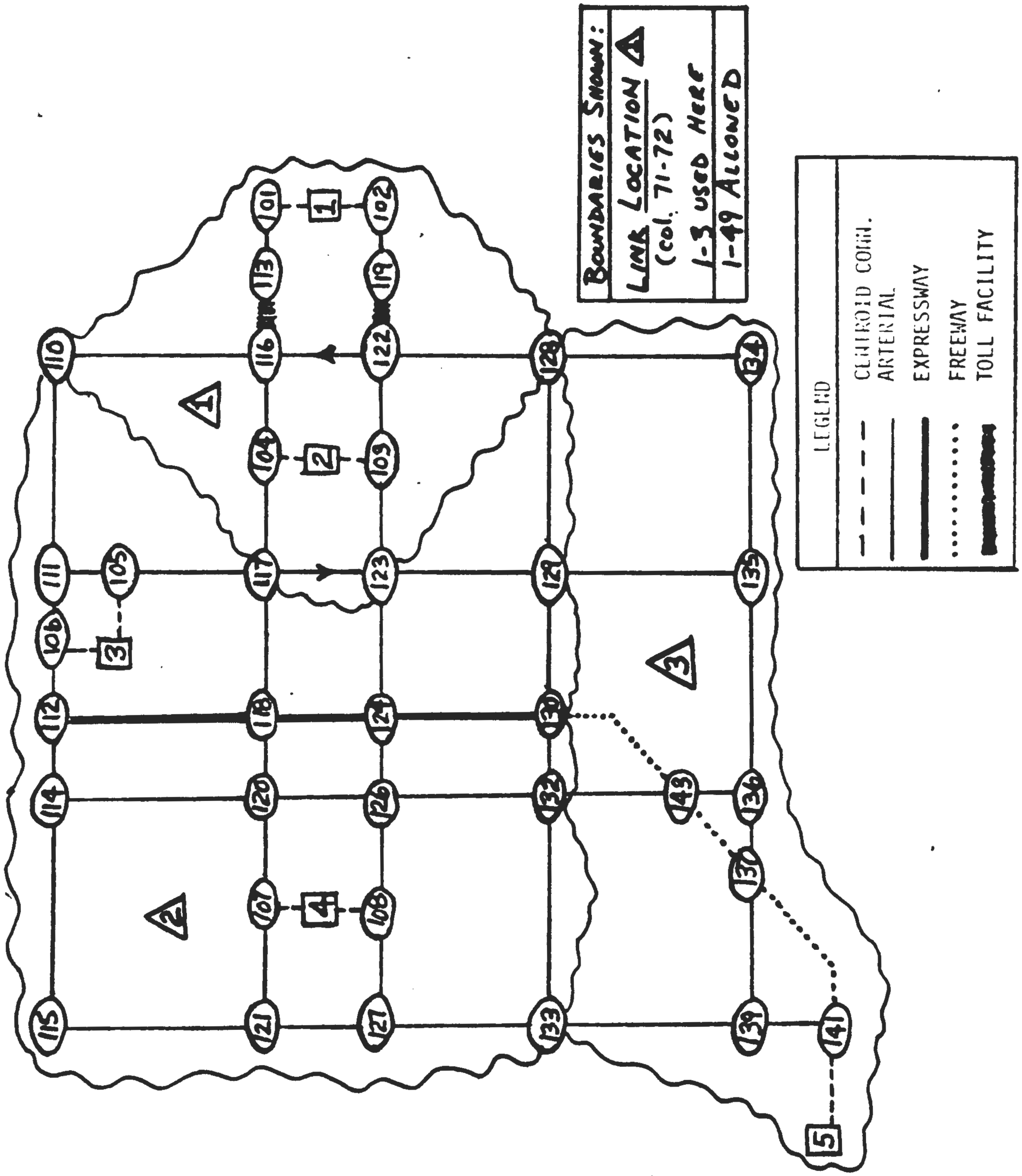
DATA II

UTOWN HIGHWAY NET. LINK GROUPS



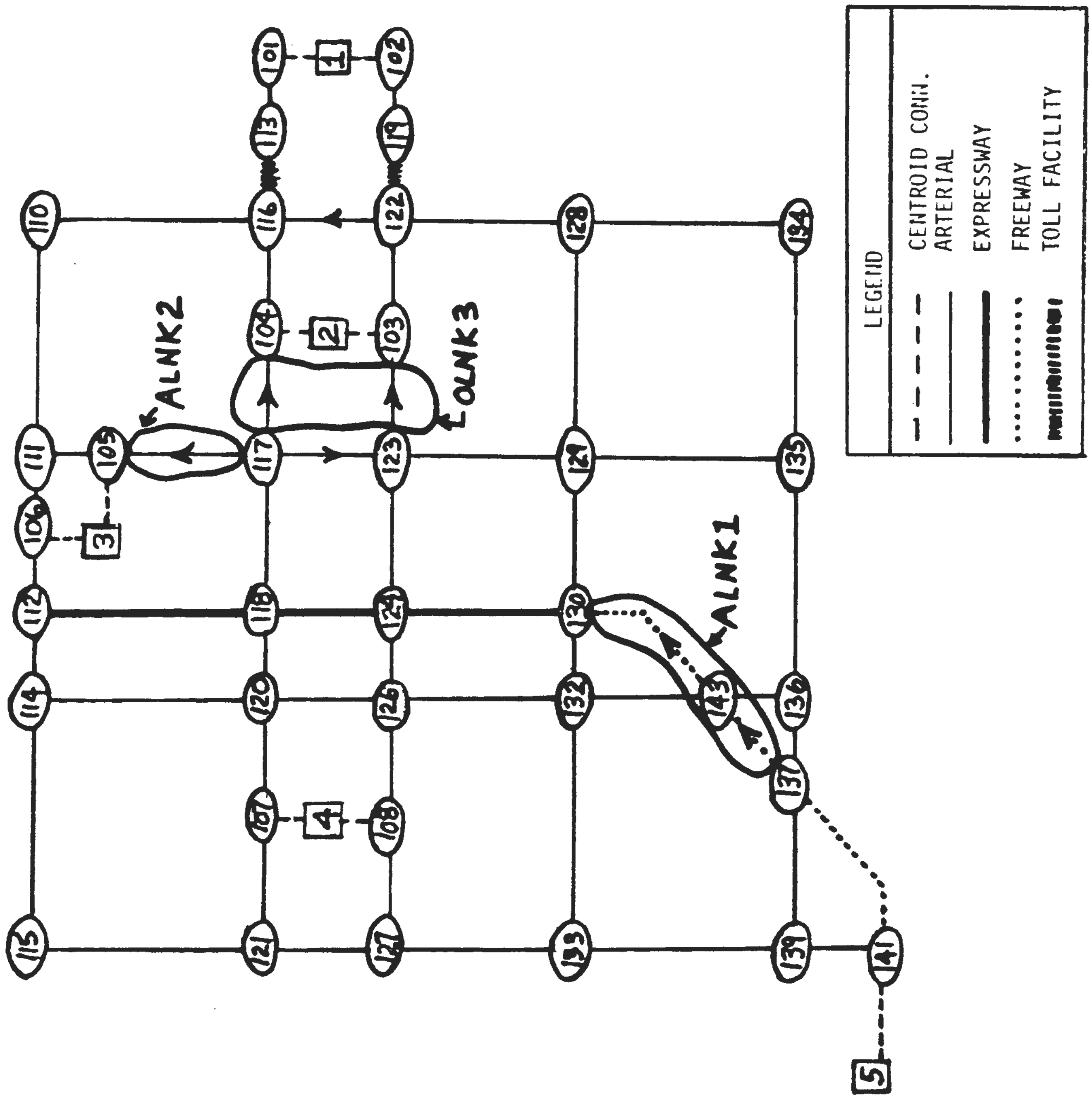
DATA JJ

UTOWN HIGHWAY NET. LINK LOCATIONS



DATA KK

UTOWN HIGHWAY NET. SELECTED LINKS



DATA LL

UTOWN AUTO A.M. PEAK DRIVE TIMES (MINUTES)*

TO

FROM

Zone	1	2	3	4	5
1	4	17	33	35	43
2	17	9	19	21	29
3	33	19	10	23	35
4	35	21	23	12	29
5	44	30	35	29	13

* Times estimated at .75 x free flow speed as taken from UROAD speed tables. These do not include terminal times.

DATA MM

UTOWN AUTO TERMINAL TIMES

Zone	Time (Min.)
1	4
2	3
3	2
4	2
5	2

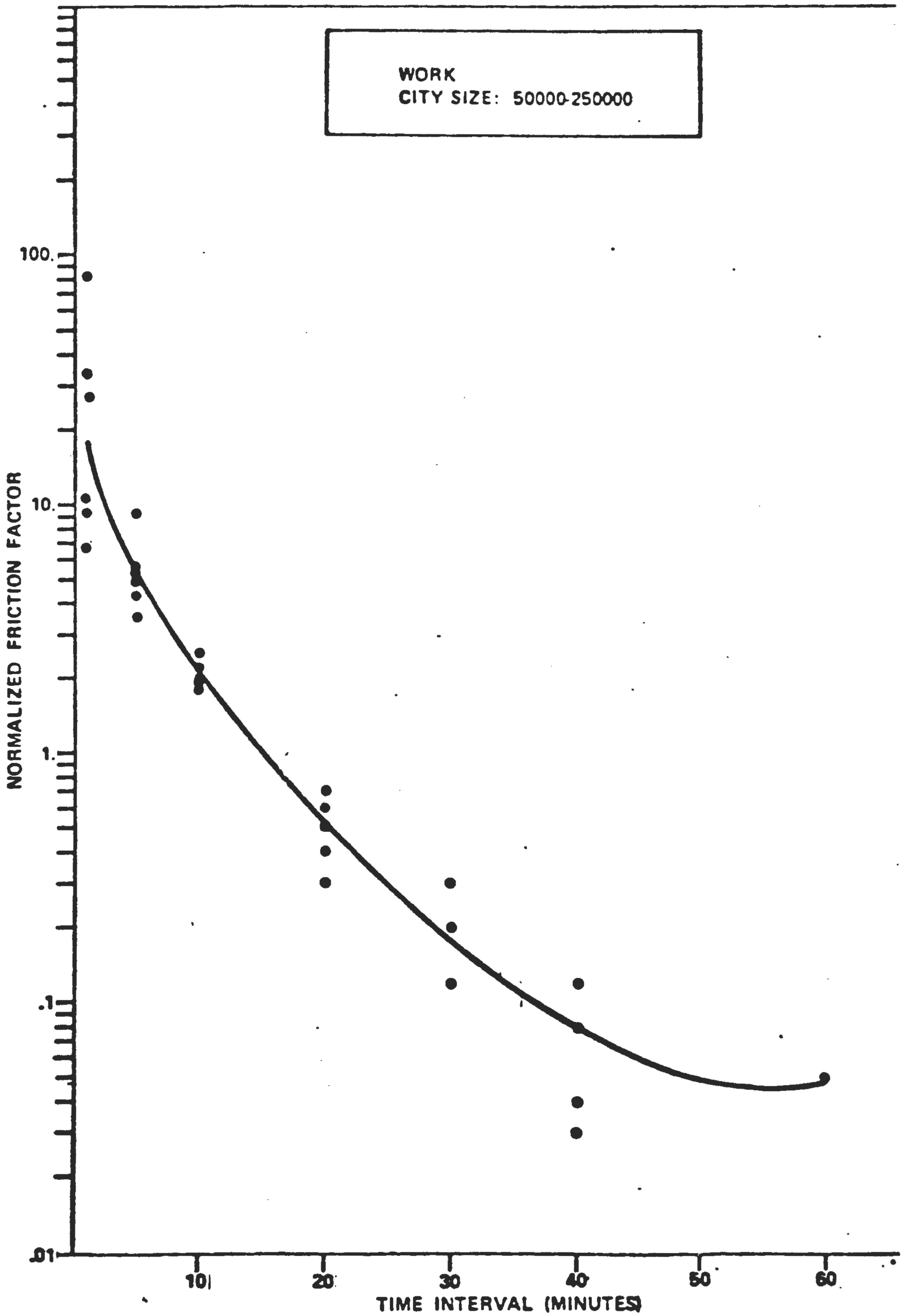
DATA NN

UTOWN AUTO A.M. PEAK TOTAL TRAVELTIMES (MIN.)

TO

	Zone	1	2	3	4	5
F	1	12	24	39	41	49
R	2	24	15	24	26	34
O	3	39	24	14	27	39
M	4	41	26	27	16	33
	5	50	35	39	33	17

DATA 00



WORK
CITY SIZE: 50000-250000
NORMALIZED FRICTION FACTOR
100.
10.
1.
.1
.01
10
20
30
40
50
60
TIME INTERVAL (MINUTES)

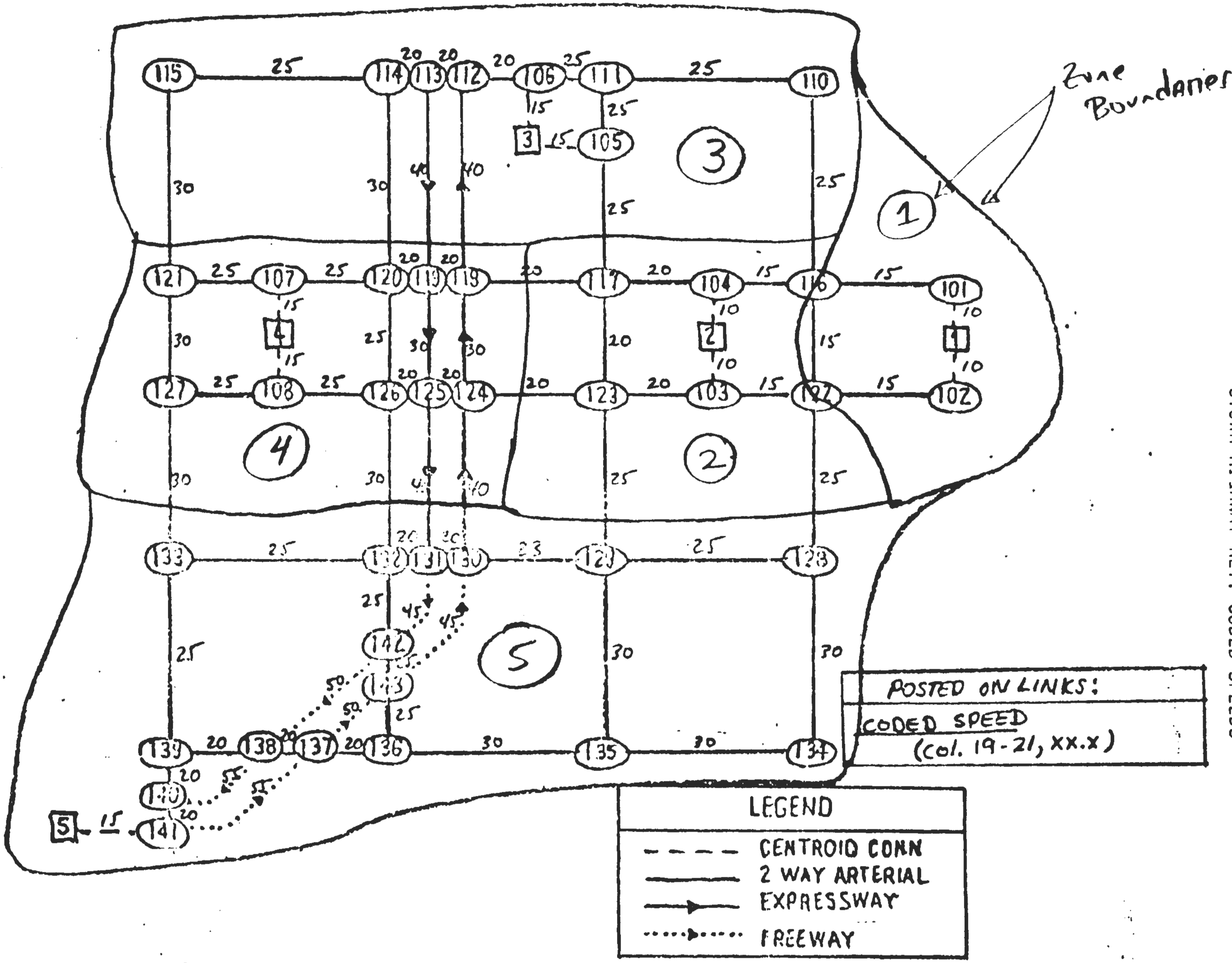
DATA PP

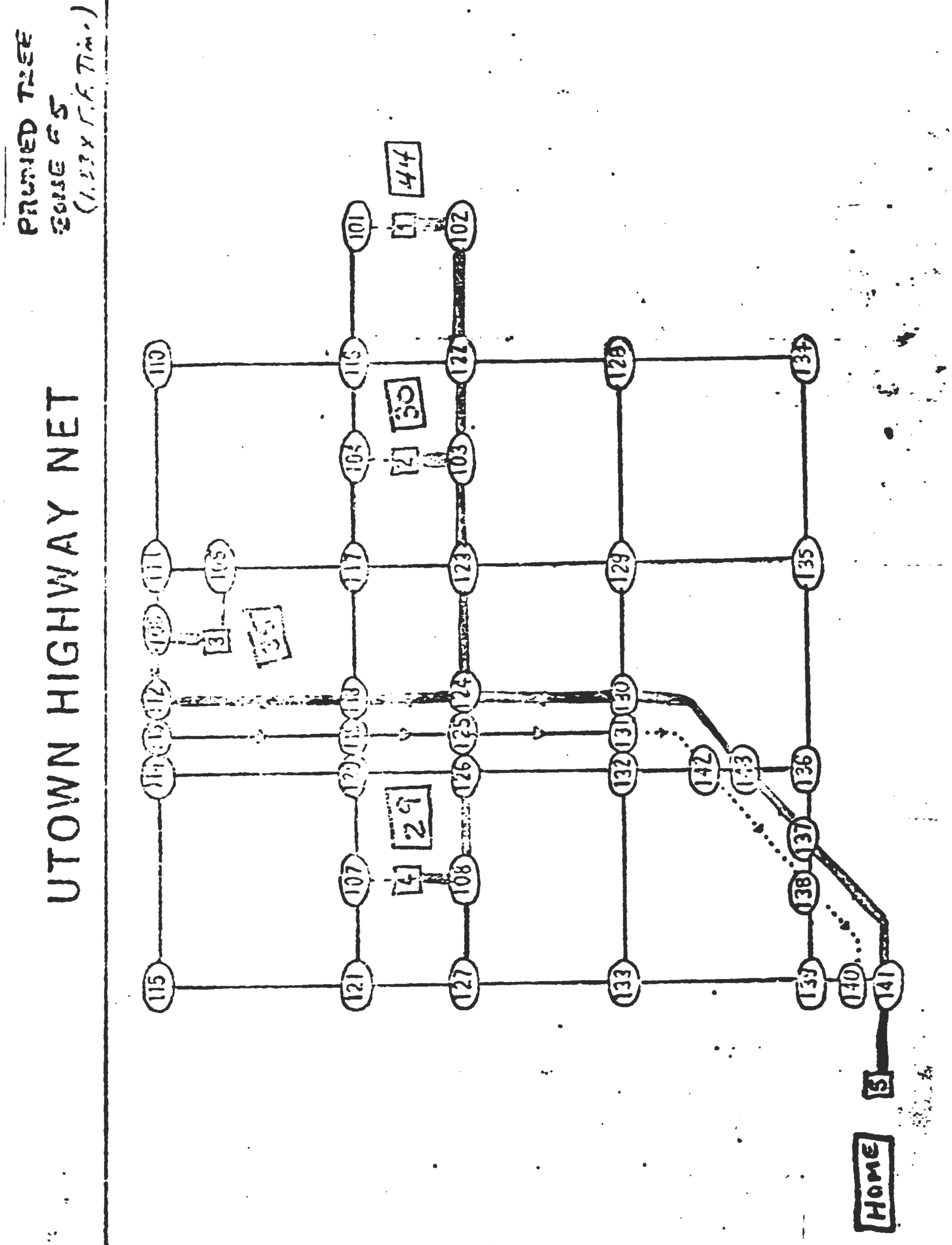
UTOWN TRAVELTIME FACTOR TABLE

TRAVELTIME (Min.)	TRAVELTIME FACTOR (F_{ij})
12	1.60
14	1.25
15	1.00
16	0.85
17	0.75
24	0.34
26	0.26
27	0.24
33	0.14
34	0.13
35	0.12
39	0.084
41	0.075
49	0.051
50	0.048

DATA QQ

UTOWN HIGHWAY NET. CODED SPEEDS




UTOWN HIGHWAY NET
PRUNED TREE
ROUTE #5
101
102
1
44
110
116
122
128
134
104
103
2
30
111
117
123
129
135
3
124
130
125
131
126
132
142
136
137
138
107
108
4
29
115
121
127
133
140
141
5
HOME

74