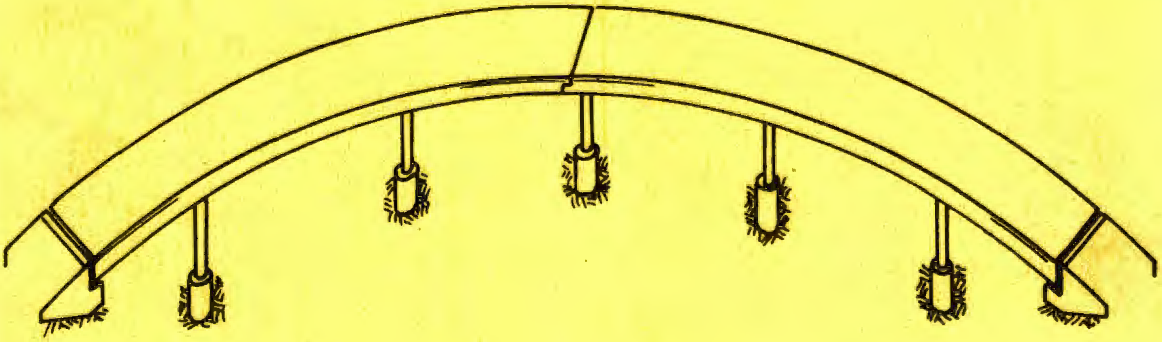
SEISMIC DESIGN OF HIGHWAY BRIDGES

WORKSHOP MANUAL



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16. Abstract

Highway Bridges serve as important links in our surface transportation network in that they provide a means for overcoming both manmade and natural obstacles. Following a severe earthquake, it is crucial that bridges not only remain intact, but continue to be capable of functioning in their intended lifeline role to provide for the efficient movement of traffic that will be required following a major disaster. A better understanding of earthquake resistant design on the part of practicing bridge designers is absolutely necessary if our bridges are to meet this standard.

A recent FHWA sponsored project consisting of the development of a curriculum for a one-week workshop in the seismic design of highway bridges and the "shakedown" of this curriculum in two pilot workshops was the first step that laid the groundwork for future nationwide training efforts to close the gap between theory and practice as it relates to seismic design of bridges.

This manual presents an introduction and overview of a broad spectrum of material as well as a "hands on" experience with the step-by step procedures of seismic analysis. Many resourses were used at the Califirnia Department of Transportation, California State University, McDonnell Douglas Automation Company, Federal Highway Administration, University of California, and countless other sources.

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TABLE OF CONTENTS

INTRODUCTION TO SEISMIC DESIGN OF BRIDGES

FUNDAMENTAL CONCEPTS IN STRUCTURAL DYNAMICS

BASIC SEISMOLOGY

APPLICATIONS - SEISMIC DESIGN METHODOLOGY

DESIGN AND RETROFITTING CONCEPTS

ADVANCED TOPICS FOR SEISMIC DESIGN OF BRIDGES

APPENDICES

LIST OF FIGURES

LIST OF TABLES

METRIC CONVERSION FACTORS

To Convert	To	Multiply By
inches (in.)	millimeters (mm)	25.40
inches (in.)	centimeters (cm)	2.540
inches (in.)	meters (m)	0.0254
feet (ft.)	meters (m)	0.305
miles (miles)	kilometers (km)	1.61
yards (yd)	meters (m)	0.91
square inches (sq in)	square centimeters (cm^2)	6.45
square feet (sq ft)	square meters (m^2)	0.093
square yards (sq yd)	square meters (m^2)	0.836
acres (acre)	square meters (m^2)	4,047
square miles (sq miles)	square kilometers (km^2)	2.59
cubic inches (cu in)	cubic centimeters (cm^3)	16.4
cubic feet (cu ft)	cubic meters (m^3)	0.028
cubic yards (cu yd)	cubic meters (m^3)	0.765
pounds (lb)	kilograms (kg)	0.453
tons (ton)	kilograms (kg)	907.2
one pound force (lbf)	newtons (N)	4.45
one kilogram force (kgf)	newtons (N)	9.81
pounds per square foot (psf)	newtons per square meter (N/m^2)	47.9
pounds per square inch (psi)	kilonewtons per square meter (kN/m^2)	6.9
gallons (gal)	cubic meters (m^3)	0.0038
acre-feet (acre-ft)	cubic meters (m^3)	1,233
gallons per minute (gal/min)	cubic meters per minute (m^3/min)	0.004
newtons per square meter (N/m^2)	pascals (Pa)	1.00

CHAPTER 1

INTRODUCTION TO SEISMIC DESIGN OF BRIDGES

1.1 INTRODUCTION

Bridges are important links in our surface transportation network because they provide the means for overcrossing both manmade and natural obstacles. It is crucial that they continue to function in this vital 'lifeline' role following an earthquake when protection of lives and property depends on the efficient movement of emergency traffic. This requires that bridges subjected to earthquakes maintain both structural integrity and accessibility. Critical areas in the seismic design process should be identified and improvements implemented into the design process as quickly as possible. The overall objective of this training effort is to provide the bridge engineer with some of the basic principles and tools needed to take the first step toward achieving this goal.

Prior to the San Fernando earthquake of February 9, 1971, very little damage to bridges resulted directly from seismically induced vibrational effects. Most of the damage on a worldwide basis had been caused by permanent ground displacement which resulted in [1.1]*:

1) Tilting, settlement and overturning of substructures
2) Displacement of supports and anchor bolt breakage
3) Settlement of approach fills and wingwall damage

Bridge damages sustained during the great Alaskan earthquake of March 27, 1964, are examples of this type of failure. Nearly all damages were caused by substructure failures resulting from large ground displacements, settlements, and loss of bearing capacity. Superstructure damages were primarily caused by substructure failures. These failure patterns are typical of the bridge failures suffered in nine major earthquakes which occurred in Japan prior to the San Fernando earthquake [1.1].

In California, earthquake damage of any type had been minimal, totaling approximately $100,000 [1.2] for the eleven most significant earthquakes (magnitudes 5.4 to 7.7) which occurred from

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* Numerals in brackets refer to reference numbers at the end of each chapter.

1933 to 1971. This general observation changed drastically, however, with the San Fernando earthquake (magnitude 6.6) which caused approximately $6.5 million damage to bridges, most of it due to vibration effects.

As a result of the San Fernando earthquake, there has been an increased public awareness of the potential of earthquake-induced damage to bridges. A reflection of this interest is a recognition of the need to design highway bridges that are more resistant to the damaging effects of seismic forces induced by ground vibration.

1.2 THE STATE OF SEISMIC DESIGN OF BRIDGES

The engineering community, sponsored at both the Federal and State levels, has been responding to a more intensified public mandate for improved seismic design of bridges in the years following the San Fernando earthquake. The state of knowledge with regard to the seismic design of bridges has been moving ahead on several fronts. A summary of some of the more pertinent events that have brought us to our present point are discussed in the paragraphs that follow.

1.2.1 THE PRACTICING PROFESSION (CALTRANS & ASSHTO)

Immediately following the earthquake, the Office of Structures, California Department of Transportation (CALTRANS) recognized the need to develop a more rational earthquake design criteria for bridges [1.3]. Efforts were initiated to develop new earthquake design guidelines that would consider seismicity and the vibrational properties of both the bridge and the underlying soil. This pioneering effort provided the basis for a new national seismic bridge design code that was subsequently accepted by the American Association of State Highway and Transportation Officials (AASHTO) (Appendix H.1). This code is, to a large degree, a designer's response to upgrade the seismic design methodology to cope with the catastrophic types of failures experienced in the San Fernando earthquake. Some of the primary features of the new code include: 1. Seismicity; 2. Dynamic Soil and Structural Response; 3. Risk; and 4. Ductile Design.

The new code provides for seismic analysis by the equivalent static force method for simple structures. Response spectrum or transient analysis is required for more complex structures. CALTRANS, now a leader in the seismic design of bridges, is currently using 3 dimensional response spectrum modal analysis on all of their structures, since they have found that the equivalent static force method is cumbersome to apply and generally yields unreliable results in most cases, particularly for the more complicated structures.

In addition to the development of a new seismic design criteria, the California Department of Transportation, also embarked on programs to:

1) Improve the deficient design details which precipitated some of the collapse-type failures.

2) Establish procedures for upgrading the earthquake resistance of existing structures.

3) Form a Structural Mechanics Group to research and implement state-of-the-art analytical techniques.

4) Purchase, develop, and implement computer programs utilizing the latest analytical techniques.

5) Intensify the training efforts for both designers and researchers.

6) Instrument existing structures.

The implementation of such design improvements has resulted in highway bridges with improved resistance to the hazards of earthquakes as became evident by the performance of the Incienso Bridge during the Guatemala earthquake of February 4th and 6th, 1976 [1.4].

Bridge designers throughout the world are continuing to develop innovative means to make bridges more earthquake resistant. The need of the designer to find immediate solutions to complex problems such as earthquake resistant design with the aid of available technological developments has resulted in a better understanding of the problem.

1.2.2 UNIVERSITY RESEARCH

Following the San Fernando earthquake, the U.S. Department of Transportation, Federal Highway Administration, recognized the need for increased understanding of the behavior of bridge structures during earthquakes. A research project entitled <u>An Investigation of the Effectiveness of Existing Bridge Design Methodology in Providing Adequate Structural Resistance to Seismic Disturbances</u> was initiated at the Earthquake Engineering Research Center, University of California, Berkeley. This investigation consisted of the following six phases:

1) A thorough review of the world's literature on seismic effects on highway bridge structure,

including damage to bridges during the San Fernando earthquake of February 9, 1971 [1.1].

2) An analytical investigation of the dynamic response of long multiple-span highway overcrossings of the type which suffered heavy damage during the 1971 San Fernando earthquake [1.5].

3) An analytical investigation of the dynamic response of short, single, and multiple span highway overcrossings of the type which suffered heavy damage during the 1971 San Fernando earthquake [1.6, 1.7].

4) Detailed model experiments on a shaking table to provide dynamic response data similar to prototype behavior which was used to verify the validity of theoretical response predictions [1.8].

5) Correlation of dynamic response data obtained from shaking table experiments and theoretical response. Modification of analytical procedures as found necessary to achieve correlation [1.9].

6) Case studies on three long-multiple span bridges subjected to strong seismic excitation. The dynamic responses were determined using the response spectral, linear time-history and nonlinear time-history approaches. Maximum response values are interpreted in terms of current design procedures and code provisions [1.10].

7) Preparation of recommendations for changes in seismic design specifications and methodology as necessary to provide adequate protection of reinforced concrete highway bridges against severe damage in future earthquakes.

The Federal Highway Administration also sponsored a study at the Illinois Institute of Technology to determine cost effective means for modifying existing intermediate size bridges to minimize the damaging effects of intense earthquake ground motions [1.11]. Research studies were performed to identify and define, through structural analysis, practical techniques and criteria for retrofitting the bridges selected during the program.

A follow-up project entitled "Seismic Retrofit Measure for Highway Bridges," [1.12, 1.13] was performed which included the additional two objectives:

1) Prepare design details and installation speci-

fications for retrofit measures that are to be employed on existing highway bridges to minimize earthquake damage.

2) Demonstrate, through a training curriculum, an approach in the application of the seismic analysis technique which can be used by the practicing bridge engineers to decide whether a bridge needs retrofitting and if it does, what type of retrofit measure(s) to employ.

To meet these objectives, the study produced design details and installation specifications for the following bridge retrofit measures:

1) Hinge restrainer for suspended concrete girders.

2) Displacement restrainer for stringers and girders bearing on bridge abutment.

3) Vertical displacement restrainers for steel girders.

4) Steel girder expansion joint restrainer.

5) Steel girder hinge restrainer.

6) Reinforced concrete column strengthening.

7) Girder bearing area widening technique.

8) Strengthening of abutment/pier footings.

In New Zealand, University of Canterbury researchers, working closely with the New Zealand Ministry of Works bridge design engineers, have been studying ductility of reinforced concrete bridge piers. A better understanding of ductility under the current design philosophy is crucial to understanding the seismic resistance of structures.

In addition to projects specifically related to bridges, several research efforts in fields relating to other structural types and to earthquakes in general have greatly improved the state of knowledge since the San Fernando event.

1.2.3 STRUCTURAL AND SOIL DYNAMICS COMPUTER CODES

The use of the digital computer for the solution of engineering problems has become very popular in the last few years. Large general purpose structural analysis programs have been developed which are capable of solving large linear structural problems. Fortunately for the design engineer new programs have emerged concurrent to the increased interest in the seismic design problem. Also the development of computer hardware, which now provides easy access of high speed computers through time-sharing networks, service bureaus and in/house installations, has put dynamic analysis computer codes at the fingertips of the engineer.

Most of the more popular structural analysis programs are capable of performing the types of dynamic analyses required for the seismic design of bridges. Some of the more popular computer codes are described in the following paragraphs.

1.2.3.1 STRUDL

The STRUDL (STRUctural Design Language) computer system was developed at the Massachusetts Institute of Technology, Department of Civil Engineering, in the late 1960's as part of the Integrated Civil Engineering System (ICES) Project. The project was jointly sponsored by several public and private organizations. The original version of the program, as released from MIT, was somewhat lacking in its dynamic analysis capabilities. Most of the problems with the dynamic analysis capabilities have been corrected, and in addition user enhancements have been added to the several versions of the program, which are currently available on a commercial basis. STRUDL uses a Problem Oriented Language which makes it easy for non-programmers to use. The program was originally coded for IBM hardware, but versions are currently available on UNIVAC and Control Data Corporation equipment.

1.2.3.2 SAP

The first version of program SAP (Structural Analysis Program), developed at the University of California by Professor E. L. Wilson, was published in 1970. Since that time several enhancements have been made. SAP IV, the current version, is designed to operate on CDC equipment. SAP VI, also a current version, was modified at the University of Southern California and is designed for IBM equipment. The SAP code is more straightforward from a programs point of view than other large systems and can be more easily modified for a particular organization's needs.

1.2.3.3 EASE2

EASE2, developed by Engineering Analysis Corporation, is an outgrowth of the SAP program. It is designed with the user in mind. It operates on CDC hardware and is very efficient in terms of computer resources required.

1.2.3.4 NASTRAN

The National Aeronautics and Space Administration (NASA) contracted with Computer Science Corporation and the MacNeal Schwendler Company to develop NASTRAN (NASA STRuctural ANalysis). This code is used by NASA and its contractors as well as many private engineering firms. NASTRAN operates on IBM, CDC, and UNIVAC hardware.

1.2.3.5 STARDYNE

STARDYNE is a general purpose structural analysis code for both static and dynamic analysis that was developed by Mechanics Research Incorporated, Los Angeles, California. First released in 1968 and available through the Control Data Corporation Cybernet System, STARDYNE could handle up to four thousand degrees of freedom. The present version of STARDYNE can handle up to twenty thousand degrees of freedom and includes a wide library of Finite Elements including three-dimensional solid elements.

1.2.3.6 Other General Purpose Structural Analysis Codes

There are other general purpose codes available, but they generally do not have as wide a popularity as the systems mentioned above. Some of these codes have very sophisticated capabilities not likely to be used by the bridge designer. Among the other codes available for dynamic analysis are systems such as MARC and ANSYS.

1.2.3.7 Special Purpose Dynamic Analysis Programs for Bridges

Programs specifically designed for the dynamic analysis of bridges have been written as research tools at some universities. These programs are currently in a form that is difficult and cumbersome to use. It is possible, with a substantial effort, to combine and enhance some of these programs to produce a computer code that would become a valuable production and learning tool for the bridge engineering community. The resulting code would also contain the optional ability to consider directly certain nonlinear aspects of bridge dynamic behavior.

1.2.3.8 SHAKE - A Soil Dynamics Computer Program

A valuable tool for determining the earthquake response of various soil configurations is the program SHAKE. This program was developed at the University of California and will be discussed in greater detail later in this text.

1.2.4 APPLIED TECHNOLOGY COUNCIL (ATC-6)

It is evident that a considerable amount of knowledge has been gained since the San Fernando earthquake. There is still a long way to go, however, before bridge seismic design ceases to be an art and becomes an exact science. Nevertheless, it was evident that the time had come to bring the current research and design experience together in one code of practice for bridge seismic design. This is currently being done in a project being conducted by the Applied Technology Council of Palo Alto, California under the sponsorship of the Federal Highway Administration. Preliminary recommendations for bridge seismic design are currently being drafted. Components being considered for components of these recommendations include:

1) A map defining coefficients for the expected peak ground motion in the United States.

2) The consideration of a bridge IMPORTANCE FACTOR in determination of design loadings.

3) All bridges shall be assigned to a SEISMIC PERFORMANCE CATEGORY based on the map area and the importance factor.

4) A limitation on acceptable bridge sites based on the potential for ground surface rupture.

5) A SITE COEFFICIENT based on the soil profile type at the bridge site.

6) A RESPONSE MODIFICATION FACTOR which will modify design forces for individual members based on their ability to behave as ductile components.

7) Clarification and improvement of required analysis procedures.

8) Special design and construction requirements for

reinforced concrete framing members and their connections.

1.3 PURPOSE OF THE WORKSHOP

The design of earthquake resistant bridges encompasses such inter-disciplinary fields as:

1) Seismology

2) Geology

3) Engineering Mechanics

4) Structural Dynamics

5) Soil Mechanics

The primary purpose of the workshop is to provide the bridge engineer with the skills required to:

1) Understand and use the new analytical approaches that have evolved to determine the motion and forces acting on highway bridges.

2) Design highway bridges to accommodate these seismically induced forces.

3) Analyze and design retrofitting features for existing bridges in order to decrease damage by seismic action.

The skills will include some of the more recently developed techniques associated with the expanding fields of seismic engineering. The materials presented here are in general currently used by the California Department of Transportation and as such believed to be workable in a bridge design shop.

The current AASHTO specification, included in Appendix H.1, involves a complete change in methodology from the previous specifications and is difficult to apply as it is presented. The AASHTO specifications will undoubtedly undergo modification in the near future as reflected in the both newly revised CALTRANS code and the ATC-6 provisions when they are completed. The materials presented will provide the designer with the skills necessary to apply the current codes and recognize the potential of new developments and recommendations as they become available. The format of the workshop is organized to present the following:

1) An overview of the current seismic design philosophy.

2) An introduction to basic theory.

3) An introduction to basic concepts and the results associated with the underlying theories.

4) A summary of some recent seismic design and retrofitting concepts used for bridges.

It is our intention to present an introduction and overview of the broad spectrum of material as well as a "hands on" experience with the step-by-step procedures of seismic analysis. The authors have played an active role in the development and implementation of the California seismic criteria for bridges since the 1971 San Fernando earthquake. We sense a real need to provide today's bridge engineer with usable information in applying current technology to the field of seismic analysis and design.

It would be impossible to apply such a rapidly developing technology without the support of many others. We have utilized resources at the University of California, Illinois Institute of Technology, Federal Highway Administration, California Department of Transportation, and countless other sources. The availability of information from these institutions and the willingness of their people to communicate their ideas has been a tremendous help.

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CHAPTER 2

FUNDAMENTAL CONCEPTS IN STRUCTURAL DYNAMICS

2.1 DYNAMIC LOADINGS AND THE BASIC EQUATIONS OF MOTION

Structural dynamics deals with the analysis and design of structures to resist the effects of time-dependent forces. Dynamic loads on bridges include:

1) Earthquakes
2) Wind loads
3) Moving live loads

Bridge structures that are subjected to time-varying forces experience vibrations. By employing current technology in the field of structure dynamics and recent computer program capabilities, and investigator can study the vibration response characteristics of bridges. The equations and mathematical derivations presented are discussed, but the emphasis will be on the physical concepts represented by the terms in the mathematical equations. Example computer problems included in Appendix A.2-A.4 illustrate both the physical concepts and computer capabilities associated with the basic theory. The basic concepts of structural vibrations that are needed to perform a dynamic analysis are presented in this chapter. A more rigorous treatment of the subject is given in text books listed in the references (see Section 2.7).

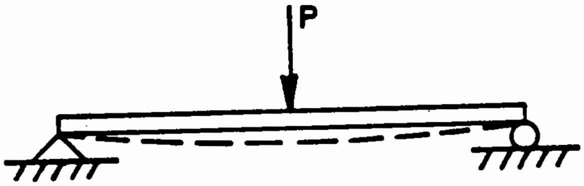
2.1.1 STATIC AND DYNAMIC LOADING

The basic differences in static and dynamic analysis are as follows:

1) Static Analysis (See Figure 2.1.1)

   a) Loads, displacements, and stresses are time independent.

   b) "Magnitude" of the load is independent of the response mechanism, i.e., the time rate of build-up of peak magnitude is large relative to the natural period of vibration of the structure.



STATIC PROBLEM

Figure 2.1.1

2) Dynamic Analysis (See Figure 2.1.2)

a) Loads, displacements, and stresses are time dependent.

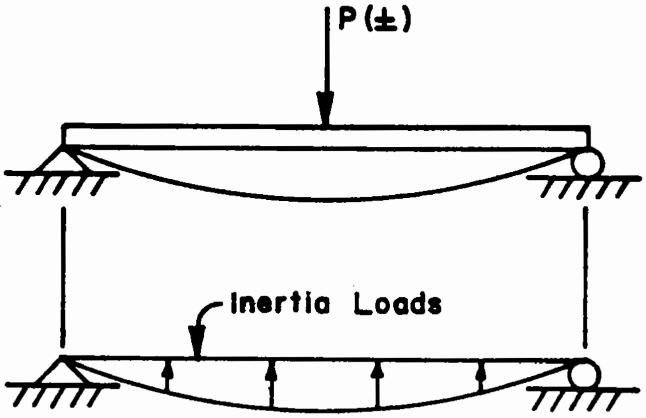
b) Inertia forces (and/or moments) are part of loading system forces (and/or moments), both depend upon, as well as contribute to, the deflection of the structure.

c) Damping forces are present which dissipate the induced motions. These forces are caused by:

i) Bodies moving through fluids at low velocities.

ii) Internal friction of the material.

iii) Relative motion between dry contact surfaces.



DYNAMICS PROBLEM

Figure 2.1.2

2.1.2 EQUILIBRIUM OF FORCES

A particle of mass acted upon by a system of forces as shown in Figure 2.1.3 experiences a linear acceleration proportional to the magnitude of the resultant and in the direction of this resultant (Newton's Second Law). The inertia force is given by:

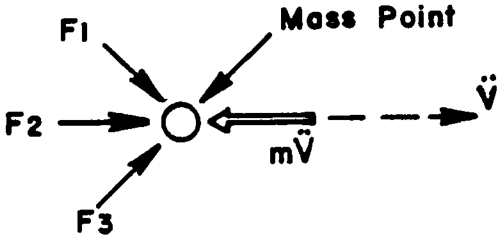
$$f_I = m \ddot{V}_t \tag{2.1.1}$$

where

V_t = total displacement

$\ddot{V}_t$ = total acceleration, the second time derivative of displacement.

This inertia force is directed through the mass point opposing acceleration as shown in the figure. The resultant sum of all the forces acting on the particle is zero.



PURE TRANSLATIONAL INERTIA FORCES

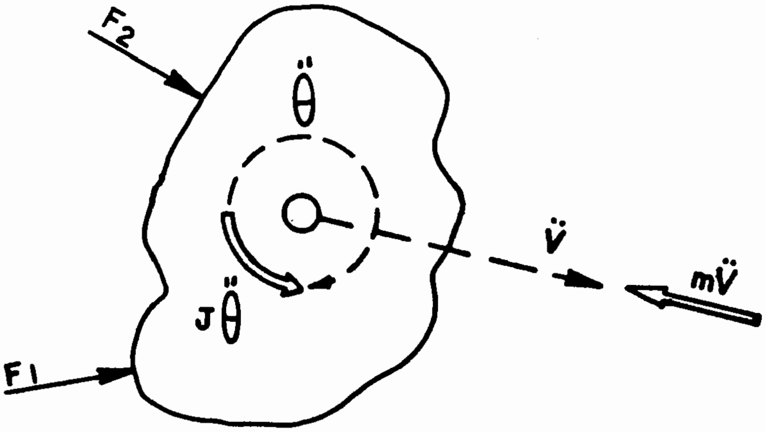
Figure 2.1.3

This principle may be extended to a rigid body by including the rotational inertia forces as shown in Figure 2.1.4.

Inertia forces are fictitious forces that are assumed to act on the body for the purpose of creating a condition of equilibrium. Considering the inertia effects in this manner, known as d'Alembert's Principle, provides a method of changing a problem in dynamics to an equivalent problem in statics.

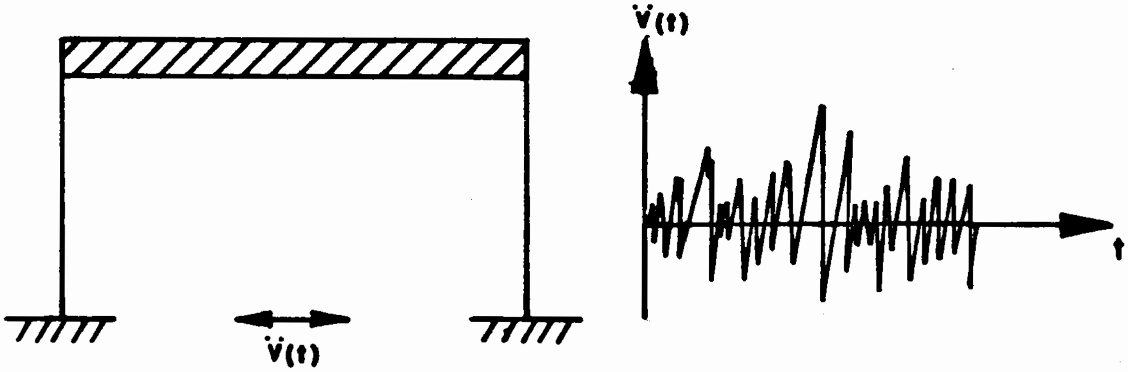
2.1.3 EQUILIBRIUM OF FORCES, SDOF SYSTEMS

Consider the single story frame subjected to a time history base motion as shown in Figure 2.1.5. The horizontal girder is assumed to be rigid and to include all the mass of the structure. The mass of this structure thus has a single-degree-of-freedom, the displacement v.



TRANSLATIONAL AND ROTATIONAL INERTIA FORCES

Figure 2.1.4



STRUCTURE SUBJECTED TO TIME HISTORY GROUND MOTION

Figure 2.1.5

The displacement of this system can be expressed as:

$$V_t = V_g + V \tag{2.1.2}$$

where

V_t = total displacement of the mass

V_g = ground surface displacement

V = relative displacement

A free body diagram of the first floor is shown in Figure 2.1.6. Applying d'Alembert's Principle yields the following equation:

$$f_I + f_D + f_S = 0 \tag{2.1.3}$$

where

f_I = inertial force of the mass

f_D = damping force acting on the mass

f_S = elastic force exerted by the two columns

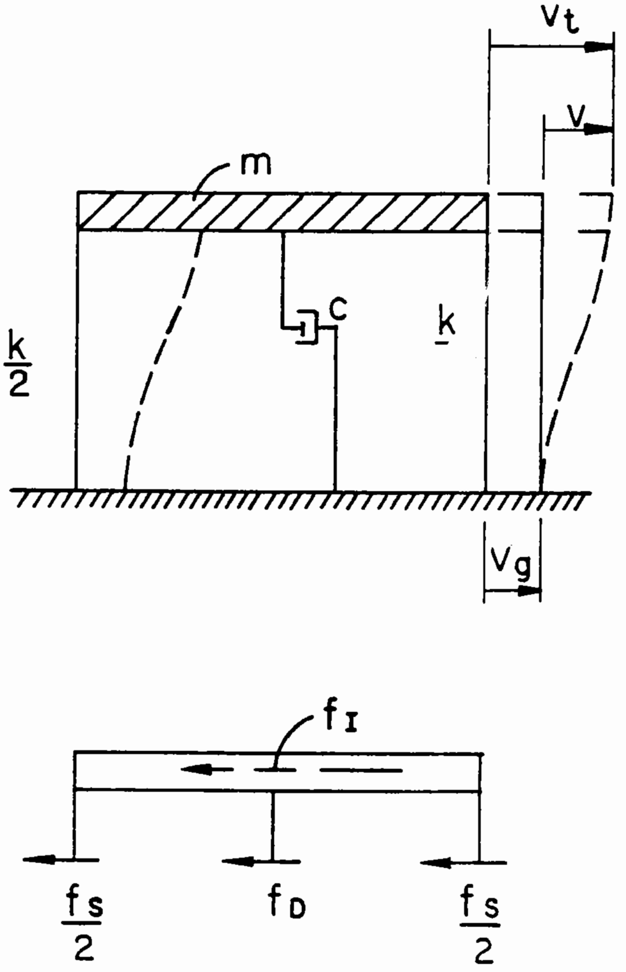
Each of the three forces acting on the first floor can be defined in terms of the motion of the structure as shown in Figure 2.1.7.

Substituting these values into Equation 2.1.3 yields:

$$m\ddot{V} + c\dot{V} + kV = 0 \tag{2.1.4}$$

The total acceleration of the structure, $\ddot{V}_t$, can be found by differentiating Equation 2.1.2 twice with respect to time:

$$\ddot{V}_t = \ddot{V}_g + V \tag{2.1.5}$$



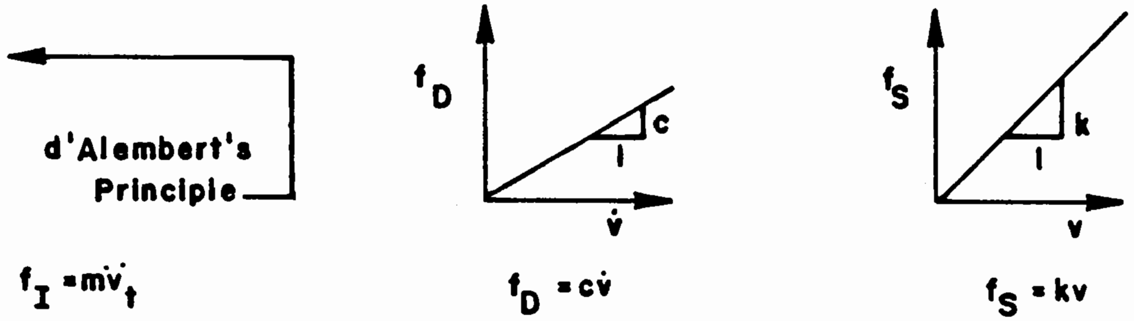
m = mass of the rigid deck

k = total stiffness of the two massless columns

c = coefficient of viscous damping

INFLUENCE OF SUPPORT EXCITATION
ON SDOF EQUILIBRIUM

Figure 2.1.6



INERTIA DAMPING STIFFNESS

GRAPHICAL REPRESENTATION OF
DYNAMIC EQUILIBRIUM FORCE TERMS

Figure 2.1.7

Notice that in this equation, $\ddot{V}_g$ is the ground acceleration which can be taken from the earthquake acceleration record. Substituting into Equation 2.1.4 yields:

$$m(\ddot{V}_g + \ddot{V}) + C\dot{V} + kV = 0$$

or

$$m\ddot{V} + C\dot{V} + kV = -m\ddot{V}_g = P_{eff}(t) \qquad (2.1.6)$$

where

P_{eff} = effective support loading

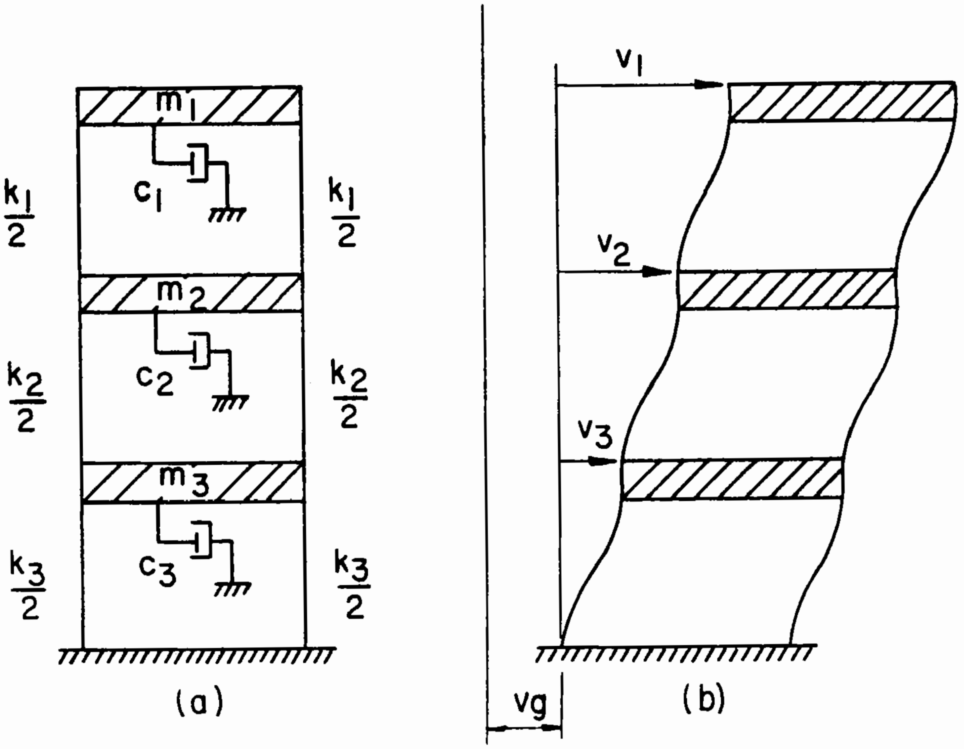
$-m\ddot{V}_g$ = effective support loading given in terms of the ground acceleration

Equation 2.1.6 is the differential equation of motion for the structure shown in Figure 2.1.5 responding to a ground acceleration. In this equation the effective support loading expressed as the product of the mass and the ground acceleration is analogous to an external load acting on the structure. The negative sign in the equation indicates that the effective force opposes the direction of ground acceleration. This has little significance in practice since the earthquake is generally assumed to act in both directions.

2.2 EXTENSION OF THE EQUATIONS TO MULTI-DEGREE OF FREEDOM SYSTEMS

The SDOF system idealization can be used to approximate the response of a bridge structure if the response shape can be assumed with some degree of accuracy and the loading is such that it excites only the assumed shape. Bridge systems having complex horizontal and/or vertical alignments, intermediate expansion joint hinges, varying column stiffness and skewed bents generally cannot be adequately idealized as SDOF systems. In these cases, the idealization of the bridge must take into consideration more than one degree of freedom.

In the development of the general equation of motion of multi-degree-of-freedom systems, the three-story frame structure shown in Figure 2.2.1 will be considered.



PROPERTIES OF A 3-STORY FRAME:
(a) Mass, Damping, and Stiffness
(b) Support Motion

Figure 2.2.1

The three horizontal girders are assumed to be rigid and to include the mass of the entire structure. Thus a degree of freedom is associated with each concentrated mass point at each story level. The equations of motion of the system are formulated by expressing the equilibrium of the effective forces associated with each degree of freedom. These equations may be written as follows:

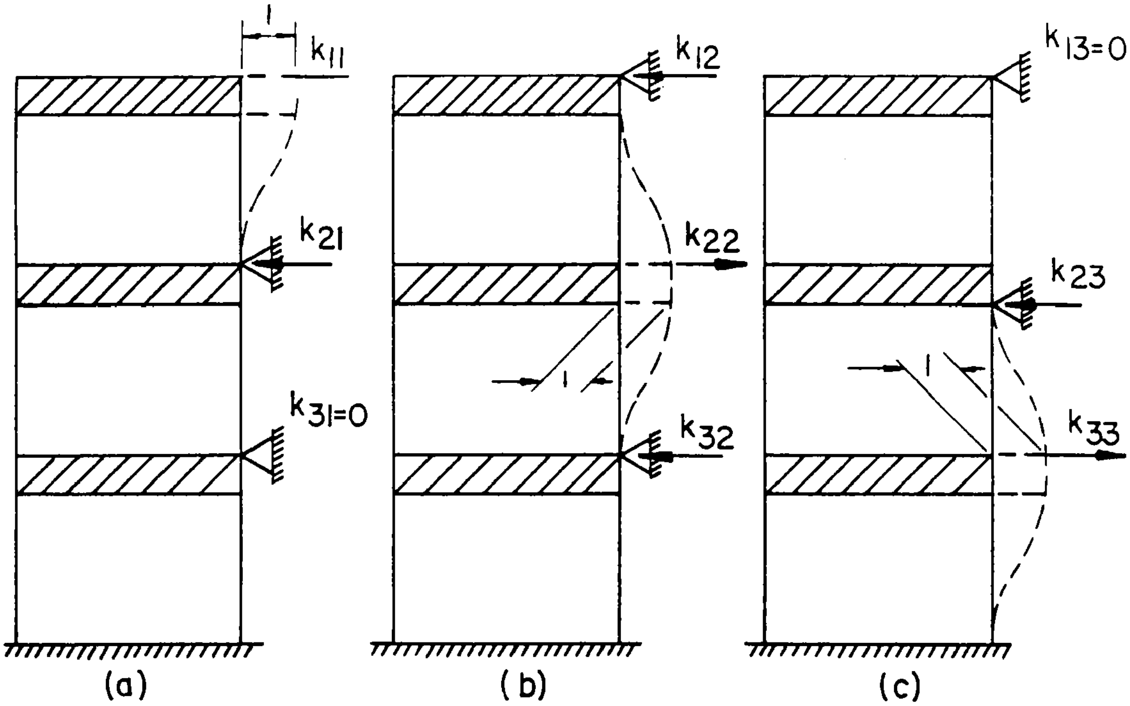
$$
\begin{aligned}
f_{I1} + f_{D1} + f_{S1} &= 0 \\
f_{I2} + f_{D2} + f_{S2} &= 0 \\
f_{I3} + f_{D3} + f_{S3} &= 0
\end{aligned}
\qquad (2.2.1)
$$

These forces may be expressed in matrix form as

$$\{f_I\} + \{f_D\} + \{f_S\} = 0 \qquad (2.2.2)$$

This is the MDOF equivalent of the SDOF Equation 2.1.2. Some basic definitions and theorems on matrix algebra are included in Appendix F.

The existing forces stated in each term of Equation 2.2.1 may be expressed by means of influence coeffcients. Consider, for example, the stiffness associated with each floor of the three-story structure. The force required to impose a unit displacement at coordinate point 1 (the top story) while restraining all other displacements (the remaining stories) as shown in Figure 2.2.2(a) is the influence coefficient k_{11} for the first story.



INFLUENCE COEFFICIENTS FOR THREE STORY STRUCTURE

Figure 2.2.2

The force developed at point 2 (the second story) due to a unit displacement at point 1 is the stiffness influence coefficient k_{21}. In general, the stiffness influence coefficient is defined as follows:

k_{ij} = force at coordinate i due to a unit displacement of coordinate j, all other coordinate displacements equal to zero.

Imposing displacements in a similar manner for the remaining two joints as shown in Figure 2.2.2(b) and 2.2.2(c), yields the stiffness influence coefficients for the remaining joints.

The complete elastic force relationships may be expressed in matrix form as:

$$\begin{Bmatrix} f_{S1} \\ f_{S2} \\ f_{S3} \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \end{Bmatrix} \tag{2.2.3}$$

or symbolically,

$$\{f_s\} = [k]\{V\} \tag{2.2.4}$$

The inertia forces may also be expressed by a set of influence coefficients called mass coefficients. The lumped mass matrix approach which assumes that the mass is concentrated at the joints generally yields accurate enough results for bridges if enough joints are considered. This simple procedure lumps the mass at each joint by considering the contribution from the structural elements connected to the joint.

The relationship between the accelerations at each point and the resulting inertia forces is expressed in matrix form as:

$$\begin{Bmatrix} f_{I1} \\ f_{I2} \\ f_{I3} \end{Bmatrix} \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix} \begin{Bmatrix} \ddot{V}_{t1} \\ \ddot{V}_{t2} \\ \ddot{V}_{t3} \end{Bmatrix} \tag{2.2.5}$$

or symbolically

$$\{f_I\} = [m]\{\ddot{V}_t\} \tag{2.2.6}$$

In general, the mass incluence coefficient is defined as:

m_{ij} = force at coordinate i due to unit acceleration of coordinate j, all other coordinate accelerations equal to zero.

Rotational inertia forces associated with the rotational degrees of freedom may or may not be included in the mass matrix formulation. For the dynamic analysis of bridge structures, these rotational inertia forces can generally be neglected.

Damping, which is assumed to be of the viscous type, depends on the velocity. These damping terms may be expressed by damping influence coefficients defined as:

c_{ij} = force at coordinate i due to a unit velocity of coordinate j, all other coordinate velocities equal to zero.

In general, the damping coefficient may be written symbolically as

$$\{f_c\} = [c]\{\dot{v}\} \qquad (2.2.7)$$

The C matrix is generally a diagonal matrix since it is usually assumed that there is no coupling in the damping terms. Substituting Equations 2.2.4, 2.2.6 and 2.2.7 into Equation 2.2.1 yields:

$$[m]\{\ddot{v}\}+[c]\{\dot{v}\}+[k]\{v\} = -[m]\{\ddot{v}_g\} \qquad (2.2.8)$$

For the three story frame of Figure 2.2.1 these equations of motion can be written in matrix form as follows:

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{V}_1 \\ \ddot{V}_2 \\ \ddot{V}_3 \end{Bmatrix} + \begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & C_3 \end{bmatrix} \begin{Bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \dot{V}_3 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1+k_2 & -k_2 \\ 0 & k_2 & k_2+k_3 \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \end{Bmatrix}$$

$$= - \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{V}_g \\ \ddot{V}_g \\ \ddot{V}_g \end{Bmatrix}$$

or

$$m_1 \ddot{V}_1 + C_1 \dot{V}_1 + k_1 (V_1 - V_2) = -m_1 \ddot{V}_g$$

$$m_2 \ddot{V}_2 + C_2 \dot{V}_2 + k_2 (V_2 - V_3) - k_1 (V_1 - V_2) = -m_2 \ddot{V}_g$$

$$m_3 \ddot{V}_3 + C_3 \dot{V}_3 + k_3 V_3 - k_2 (V_2 - V_3) = -m_3 \ddot{V}_g$$

Note that this results in a set of coupled differential equations. It is this coupling that makes the solution of the free vibration problem for multi-degree-of-freedom systems much more complex than the solution of the free vibration problem for a single-degree-of-freedom system.

The number of degrees of freedom required to model a bridge will depend on the geometric, stiffness and inertia characteristics of the overall system.

2.3 UNDAMPED FREE VIBRATION

In order to understand the behavior of a bridge structure during an earthquake, it is first necessary to understand how the structure will vibrate if given some initial excitation and allowed to vibrate freely unaffected by damping or externally applied forces. This type of motion will first be described for a single-degree-of-freedom system and then for multi-degree-of-freedom systems.

2.3.1 SINGLE DEGREE OF FREEDOM SYSTEMS

Motion of this type may be described mathematically for a single degree of freedom system by setting c and $p(t)$ equal to zero in Equation 2.1.6. Restating the equation

$$m\ddot{V} + c\dot{V} + kV = -m\ddot{V}_g = P_{eff}(t)$$

and substituting yields

$$m\ddot{V} + kV = 0$$

or

$$\ddot{V} + \frac{k}{m}V = 0 \qquad (2.3.1)$$

For a given structure k and m are constants. For convenience and for reasons that will become clear later, let:

$$\omega^2 = \frac{k}{m} \qquad (2.3.2)$$

Therefore Equation 2.3.1 becomes

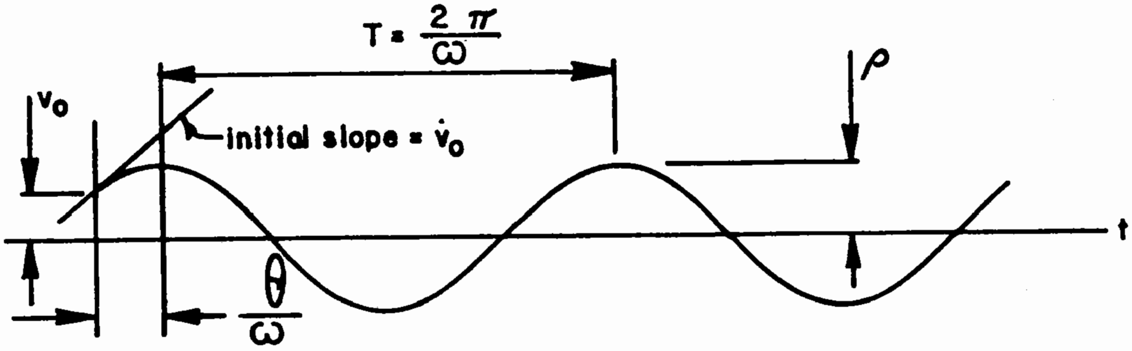
$$\ddot{V} + \omega^2 V = 0 \qquad (2.3.3)$$

Notice that ω^2 will be a constant that is associated with the structure and is dependent only on the mass and stiffness. Equation 2.3.3 is a second order homogeneous differential equation which has the following general solution:

$$V(t) = A\sin\omega t + B\cos\omega t \qquad (2.3.4)$$

The two constants A and B are evaluated from the initial conditions. Assuming the system starts from an initial position V_0 with an initial velocity $\dot{V}_0$; the solution becomes

$$V(t) = \frac{\dot{V}_0}{\omega}\sin\omega t + V_0\cos\omega t \qquad (2.3.5)$$



UNDAMPED FREE VIBRATION RESPONSE

Figure 2.3.1

Plotting the value for displacement vs. time yields the graph shown in Figure 2.3.1. This equation may also be expressed as:

$$v(t) = \rho\cos(\omega t - \theta)$$

where

$$\theta = \tan^{-1}\frac{\dot{v}_0}{\omega v_0} \quad \text{and} \quad \rho = \sqrt{v_0^2 + \left(\frac{\dot{v}_0}{\omega}\right)^2}$$

The motion portrayed is periodic (i.e., it repeats itself after a certain period of time). The previously defined constant associated with this motion is expressed as:

$$\omega = \sqrt{\frac{k}{m}} \qquad \text{circular frequency (radians per second)} \qquad (2.3.6)$$

Therefore, a single degree of freedom structure which is vibrating free of any external forces will vibrate at a constant frequency independent of the initial conditions and dependent only on its mass and stiffness. Most typically, the rate of vibration of structures such as bridges is defined in terms of:

$$f = \frac{\omega}{2\pi} \qquad \text{cyclic frequency (cycles per second or Hz)} \qquad (2.3.7)$$

or

$$T = \frac{2\pi}{\omega} \quad \text{period of vibration (seconds)} \qquad (2.3.8)$$

Since the frequency or period of free vibration is a function of the structure's properties, these values are often referred to as the natural frequency or natural period of the structure.

To further illustrate simple harmonic motion of undamped free vibration for a SDOF system, consider the frame shown in Figure 2.3.2 subjected to the following initial conditions:

$$V_0 = 1$$

$$\dot{V}_0 = 0$$

Substituting into Equation 2.3.5 yields

$$V(t) = \frac{\dot{V}_0}{\omega} \sin\omega t + V_0 \cos\omega t$$

thus

$$V(t) = \cos\omega t$$

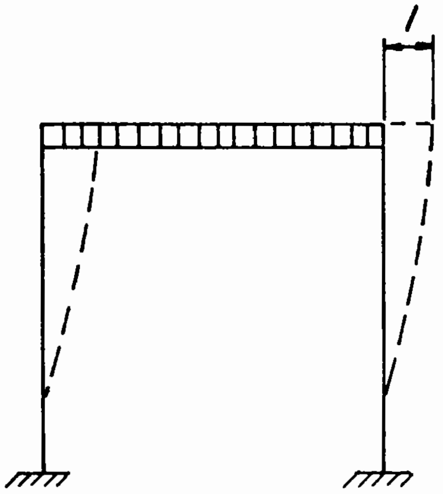
Differentiating to obtain the velocity yields

$$\dot{V}(t) = -\omega \sin\omega t$$

Differentiating again to obtain the acceleration yields

$$\ddot{V}(t) = -\omega^2 \cos\omega t$$

Harmonic motion of the system can be described by the motion of the projection on the horizontal axis of a particle moving at a constant angular velocity around the circumference of a circle of radius = 1, as shown in Figure 2.3.3a and 2.3.3b. At time t = 0, the particle is at its initial position $v_o = 1$ at point A. The diagram shows that the velocity vector is 90 degrees ahead of the displacement



UNDAMPED SINGLE DEGREE OF FREEDOM SYSTEM

FIGURE 2.3.2

vector. At time $t = t_1$, the displacement is the projection of OA' measured on the horizontal axis. The velocity is the projection of OB' and the acceleration is the projection OC'. The projection as a funtion of time is plotted for the displacement, velocity and acceleration, respectively in Figures 2.3.3c, 2.3.3d, and 2.3.3e.

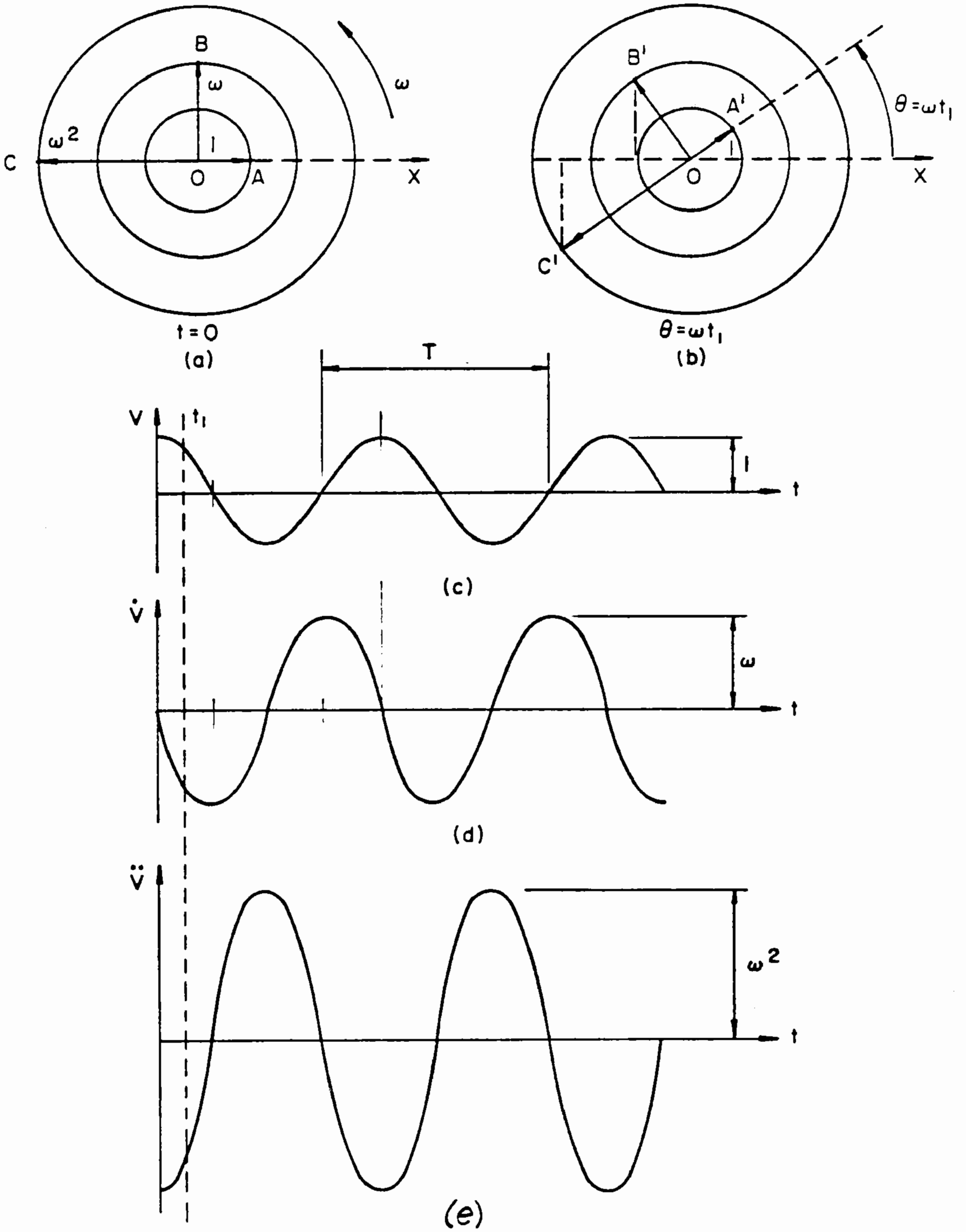
2.3.2 MULTI-DEGREE-OF-FREEDOM SYSTEMS

When a structure has several degrees of freedom which is typical of most bridge structures, the solution of the free vibration problem is much more difficult. In this case Equation 2.2.8, the matrix equation of motion,

$$[m]\{\ddot{v}\}+[c]\{\dot{v}\}+[k]\{v\}=-[m]\{\ddot{v}_g\}$$

reduces to

$$[m]\{\ddot{v}\}+[k]\{v\}=0 \qquad (2.3.9)$$



HARMONIC MOTION

Figure 2.3.3

If we assume the structure can vibrate in a periodic manner similar to the single degree of freedom structure, then the displacements of the various degrees of freedom for a structure vibrating in this manner are related as follows:

$$\{V(t)\}_m = \{\hat{V}\}_m \left(A \sin\omega_m t + B \cos\omega_m t\right) \tag{2.3.10}$$

where

$\{\hat{V}\}_m$ = the characteristic displacement shape assumed by the structure vibrating in this manner.

ω_m = the natural frequency of vibration of the structure vibrating in this manner.

For this type of vibration, the characteristic displacement shape would not change, only the amplitude would vary with time. Taking the second time derivative of Equation 2.3.10, the accelerations for this type of vibration are given by

$$\{\ddot{V}(t)\}_m = -\omega_m^2 \{\hat{V}\}_m \left(A \sin\omega_m t + B \cos\omega_m t\right) \tag{2.3.11}$$

Substituting Equations 2.3.10 and 2.3.11 into Equation 2.3.9 and rearranging yields:

$$\omega_m^2 [m] \{\hat{V}\}_m = [k] \{\hat{V}\}_m \tag{2.3.12}$$

Equation 2.3.12 is an Eigenvalue problem which has as many solutions as there are degrees of freedom. Each of these solutions represents a different <u>mode</u> of vibration that the structure can assume and still maintain the type of periodic motion described above. Associated with each mode is a characteristic displacement vector, $\{\hat{V}\}$, and a natural frequency of vibration, ω. The vector $\{\hat{V}\}$ is often referred to as the <u>eigenvector</u> or <u>mode shape</u>. This vector defines the characteristic shape that the structure will assume as it vibrates in the appropriate mode. It can be shown mathematically that each of these modes are <u>orthoginal</u> which means they are uncoupled and can be treated independent of one another. Physically this means that the displacements from the various modes at any instant in time can be superimposed to obtain the total displacements of the structure. The uncoupling of multi-degree-of-freedom systems in this manner greatly simplifies the solution of the free vibration problem. Actual free vibration of a complex structure is dependent on the extent to which each of the individual modes participate which is in turn dependent on the initial conditions.

When developing the equations of motion for a real structure, it is typically desirable to neglect the inertia effects for certain degrees of freedom. It is also desirable to consider the inertia effects to be uncoupled (i.e. the inertia effect at one degree of freedom does not have an effect on the other degrees of freedom). Physically, this results in the idealization of the structure as a weightless structural frame with concentrated mass at selected points. Experience has shown that this type of idealization can yield reliable results for bridge structures.

This practice results in a mass matrix with a significant number of zero coefficients. In many cases, economy can be realized in the eigenvalue solution if the matrices are condensed to include only the degrees of freedom with inertia effects. This is an easy problem for a lumped mass matrix, but the stiffness matrix requires considerable mathematical manipulation. Sample Problem 1 (Appendix A. 2) illustrates the computational steps necessary to solve a simple free vibration problem. It also illustrates the corresponding STRUDL commands needed to solve the same probem. By reviewing this problem, the student can best understand the nature of free vibration and therefore relate to the meaning of the commands used in STRUDL.

The solution of the free vibration problem which yields the mode shapes and frequencies is the most difficult computational task associated with the dynamic analysis. Reviewing free vibration results can tell an experienced designer familiar with dynamics many things about the accuracy of the idealization and the probable behavior of the structure during an earthquake. Because of the difficulty and the amount of computation required, the digital computer is indispensible to the designer performing a dynamic analysis on a complex structure.

2.4 STRUCTURAL DAMPING

The amplitude of vibration of a damped structure, not subject to external exciting forces, will be attenuated by resisting forces developed during motion. These forces are associated with:

1) Bodies moving through fluids, such as air, at low velocities

2) Internal friction of the material

3) Relative motion between dry contact surfaces

The resisting forces dissipate energy in time as the vibration dies out. The causes for these actions are seldom fully understood and are generally evaluated directly by experimental methods for most structural systems. Since the true damping characteristics are very complex and difficult to define, it is common practice to express the damping in terms of equivalent viscous damping which is proportional to the relative velocity of the system.

If for a single degree of freedom system the effective support loading equals zero, Equation 2.1.6 can be written as:

$$m\ddot{V}+C\dot{V}+kV=0 \qquad (2.4.1)$$

This is a homogeneous, second order, linear ordinary differential equation. The soluton is of the form:

$$V(t)=Ge^{st} \qquad (2.4.2)$$

This yields values for velocity and acceleration of:

$$\dot{V}(t)=Gse^{st} \qquad (2.4.3)$$

$$\ddot{V}(t)=Gs^2e^{st} \qquad (2.4.4)$$

Substituting these values into Equation 2.4.1 and dividing out the constants yields:

$$ms^2 + cs + k = 0 \qquad (2.4.5)$$

The values of s can therefore be calculated by solving for the roots of this quadratic equation:

$$s = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 + \frac{k}{m}} \qquad (2.4.6)$$

Three types of damped motion can occur depending on the value of the quantity under the square root sign.

$$\left(\frac{c}{2m}\right)^2 - \frac{k}{m} = 0$$ CRITICAL DAMPING

$$\left(\frac{c}{2m}\right)^2 - \frac{k}{m} < 0$$ UNDERDAMPING

$$\left(\frac{c}{2m}\right)^2 - \frac{k}{m} > 0$$ OVERDAMPING

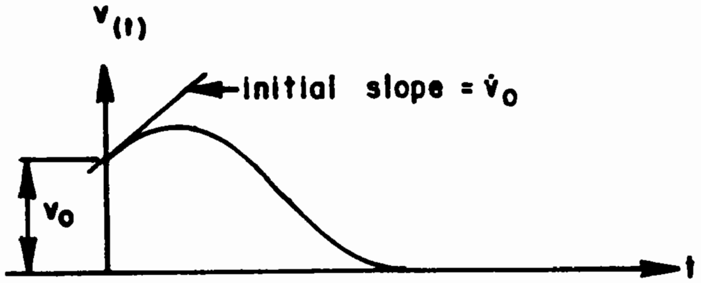
Only the first two types of damping are of practical importance. Critical damping is the minimum amount of damping required to prevent a structure from oscillating. The value of damping required to do this is:

$$C_c = 2m\omega \text{ = critical damping coefficient} \qquad (2.4.7)$$

The damping ratio, ξ, is the ratio of actual damping to critical damping:

$$\xi = \frac{c}{C_c} \text{ = damping ratio} \qquad (2.4.8)$$

The motion of a critically damped system is shown in Figure 2.4.1. Structures with damping values less than critical are known as underdamped structures. Most real structures fall in this category. These structures will continue to vibrate at an ever decreasing amplitude until all the energy is dissipated.



MOTION OF CRITICALLY DAMPED SYSTEM

Figure 2.4.1

It can be shown mathematically that the frequency of vibration of a damped structure will vary as a function of the damping ratio.

$$\omega_D = \omega \sqrt{1-\xi^2} \qquad 2.4.9$$

However, for real structures the value of ξ seldom exceeds 0.20. By substituting this value of into Equation 2.4.9, it is obvious that the difference between ω_D and ω is very small. Therefore, in practice, the frequency of damped structures is assumed to be equal to the frequency of undamped structures.

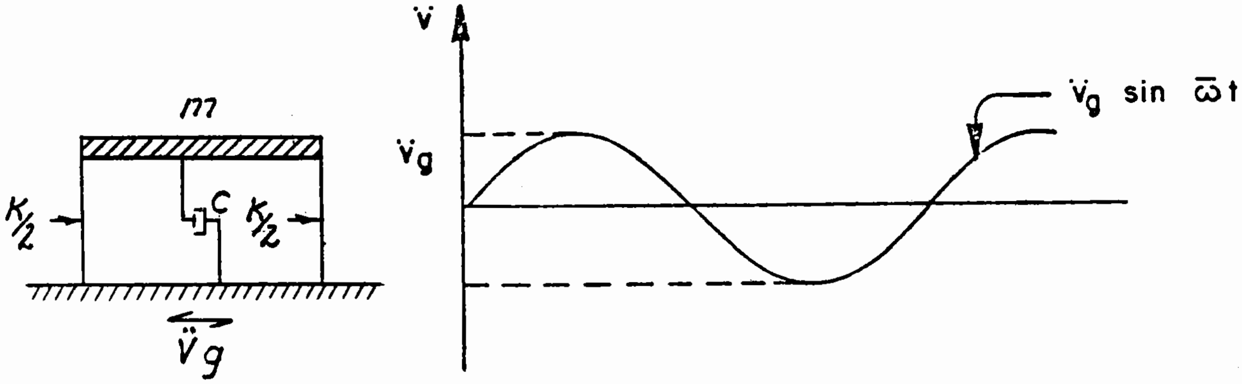
$$\omega_D \approx \omega \text{ or } T_D \approx T \qquad (2.4.10)$$

2.5 FORCED VIBRATION AND SUPPORT MOTION

So far, we have not considered the effect of a time varying forcing function on the dynamic response of a structural system. Dynamic forces in structures can be applied in several different ways. Earthquake forces are induced in a structure by motion of the supporting soil or rock. The equations of motion for a single degree of freedom structure subject to a support motion were developed in Section 2.1. The following discussion will explore this type of motion in greater detail.

2.5.1 RESPONSE TO HARMONIC SUPPORT MOTION

Assume the structure previously discussed in Section (2.1) and shown below in Figure 2.5.1 is subjected to a harmonically varying support acceleration.



HARMONIC SUPPORT MOTION

Figure 2.5.1

Equation 2.1.6 becomes:

$$m\ddot{V}+C\dot{V}+kV=-m\ddot{V}_g\sin\bar{\omega}t$$

where

$\bar{\omega}$ = frequency of input motion

$\ddot{V}_g$ = amplitude of input motion

If for the present we assume that no damping exists, the equation of motion is reduced to:

$$m\ddot{V}+kV=-m\ddot{V}_g\sin\bar{\omega}t$$

The general solution to this differential equation can be shown to be:

$$V(t)=A\sin\omega t+B\cos\omega t-\frac{\ddot{V}_g}{\omega^2}\left(\frac{1}{1-\beta^2}\right)\sin\omega t$$

and

$$\beta = \frac{\bar{\omega}}{\omega}$$

where

β = the ratio of the applied frequency to the natural free vibration frequency.

For a system starting at rest, the constants A and B take the following values:

$$A = \frac{\beta \ddot{v}_g}{\omega}\left(\frac{1}{1-\beta^2}\right) \; ; \; B = 0$$

Thus, the response to the given harmonically varying support motion for undamped vibration becomes:

$$v(t) = \frac{\ddot{v}_g}{\omega^2}\left(\frac{1}{1-\beta^2}\right)\left(\beta \sin \omega t - \sin \bar{\omega} t\right) \qquad (2.5.1)$$

The terms in this equation have the following physical significance.

$\left(\frac{1}{1-\beta^2}\right)$: This term is known as the magnification factor. As the value of β approaches unity, this factor becomes very large.

$\beta \sin \omega t$: This is a free vibration response term. It represents a portion of the response caused by the structure vibrating at its natural frequency.

$\sin \bar{\omega} t$: This term represents a portion of the response due to the structure vibrating n at the frequency of the support motion. It is known as the steady state response term.

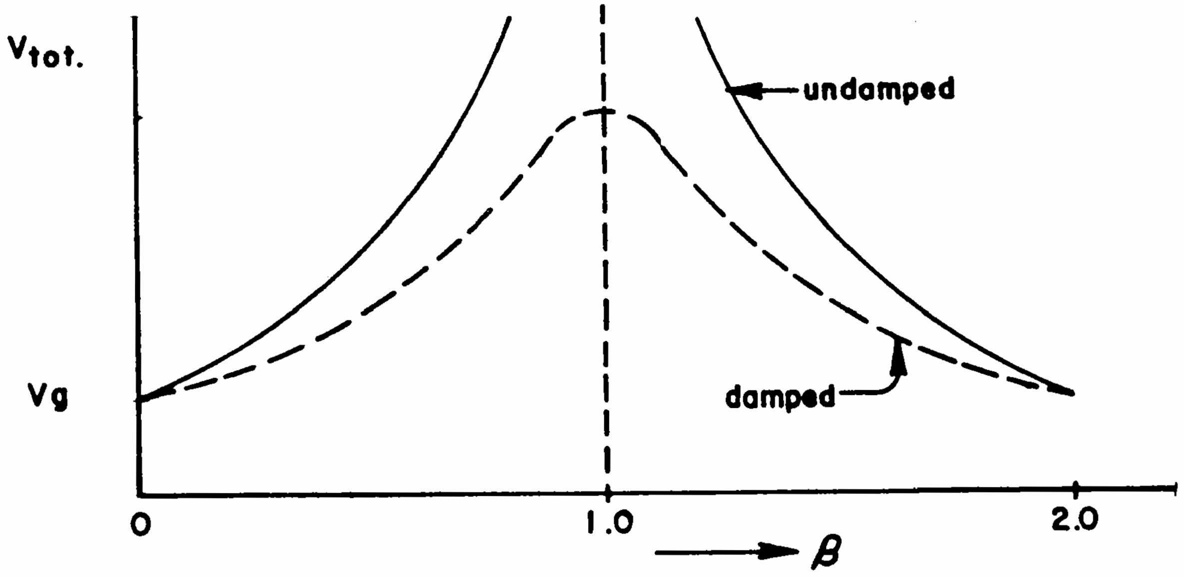
If $\bar{\omega}$ approaches zero, so does the value of β, and Equation 2.5.1 becomes:

$$v(t) = \frac{\ddot{v}_g}{\omega^2}$$

This represents a constant support acceleration which will result in a constant relative displacement.

As $\bar{\omega}$ aproach ω , β approaches unity and the displacement becomes infinitely large. This phenomenon is known as resonance.

A plot of displacement vs. the frequency ratio, β , is shown in Figure 2.5.2.



DYNAMIC MAGNIFICATION

Figure 2.5.2

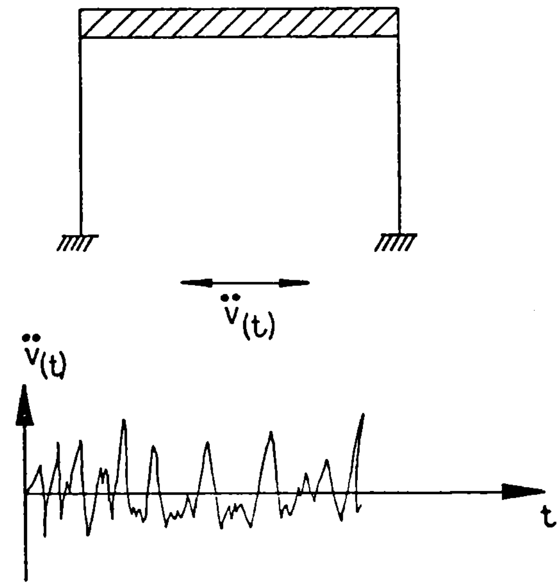
When damping is included, the mathematics becomes much more difficult and will not be covered here. Many of the principles of undamped response still apply, however, and will be used to explain damped response to harmonic motion. Magnification, free vibration and steady state vibration terms still are present in the equation of motion. When damping is present, however, free vibration will tend to be dissipated after a time and only a damped structure will assume a steady state response at a frequency equal to the frequency of the support motion. The presence of damping also has the effect of reducing the magnification factor. The response of a damped structure is shown in Figure 2.5.2 along with the undamped response for comparison.

The magnification factor concept also extends to multi-degree of freedom systems. In this case, there will be several modes of vibration each of which will have a corresponding natural frequency of vibration. When the frequency of the support motion approaches one of these natural frequencies, and when the motion is in a direction which can excite the corresponding vibration mode, the structural response in that mode will be magnified. This fact is important in seismic design since earthquake support motion for a given location tends to contain predominant frequencies within a given range.

Sample Problem 3 shown in Appendix A.4 illustrates the effect of varying frequencies of harmonic excitation for both a force applied at the first floor and for support acceleration.

2.6 DUHAMEL INTEGRAL

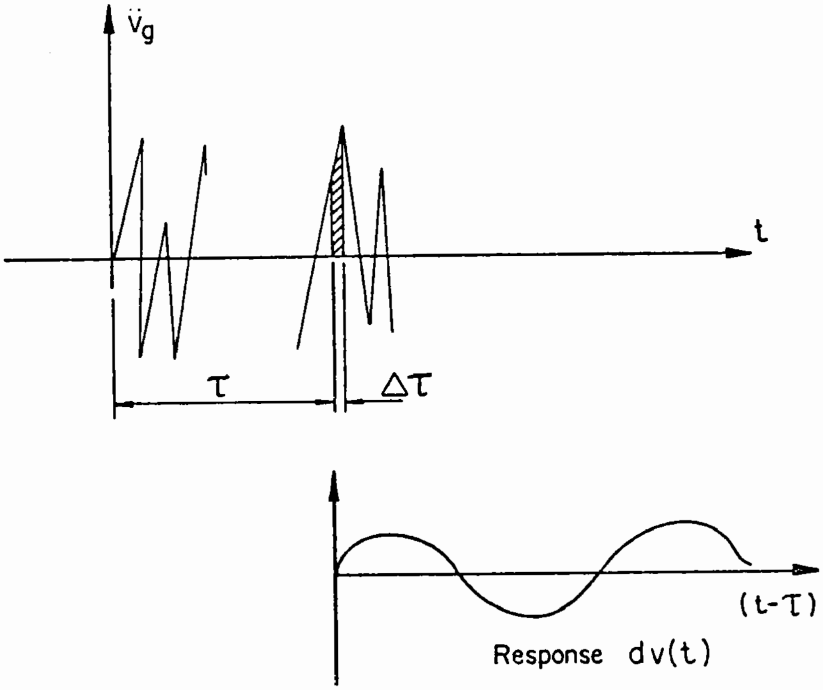
In a real earthquake the ground motion is not harmonic but varies randomly as shown in Figure 2.6.1. To evaluate the response of a structure to this ground motion, first consider the response of the system to a portion of the ground motion over a very short interval of time as shown in Figure 2.6.2.



SDOF STRUCTURE SUBJECTED TO TIME-HISTORY MOTION

Figure 2.6.1

The acceleration acting during the time interval, $d\tau$, produces a short duration impulse load, $m\ddot{v}_g(\tau)d\tau$, on the structure. This procedure is approximate for impulses of finite duration but becomes exact as the duration of the loading approaches zero.



DERIVATION OF DUHAMEL INTEGRAL

Figure 2.6.2

The differential response for a single impulse of duration, $d\tau$, at $t = \tau$ is also shown in Figure 2.6.2. This response, beginning at the termination of the impulse load, is simply a free vibration response and may or may not include the effects of damping. It can be easily obtained by considering impulse momentum relationships.

The loading history may be considered as a succession of such short impulses, each producing its own differential response. For a linearly elastic system, then, the total response can be obtained by summing all the differential responses developed during the loading history. Therefore, the following equation is obtained for the response of an undamped structure:

$$v(t) = \frac{1}{\omega}\int_0^t \ddot{v}_g(\tau)\sin\omega(t-\tau)d\tau \qquad 2.6.1$$

This equation is known as the Duhamel Integral for an undamped system. It is used to evaluate the response of an undamped SDOF system for any support motion.

For the damped SDOF system, the equation is similar to the undamped analysis except that the free-vibration response initiated by the impulse, $m\ddot{v}_g(t)d\tau$, is subjected to an exponential decay. Therefore, the damped response may be expressed as:

$$v(t) = \frac{1}{\omega}\int_0^t \ddot{v}_g(\tau)e^{-\xi\omega(t-\tau)}\sin(t-\tau)\,d\tau \qquad 2.6.2$$

In the case of arbitrary loading such as an earthquake, direct integration of the Duhamel Integral is impossible and the integration has to be performed numerically.

2.6.1 RESPONSE SPECTRUM (DEFINITION)

For a given earthquake motion, the response of single degree of freedom systems with various natural frequencies and specified damping can be determined using the Duhamel Integral. The maximum response of each system (i.e., displacement, velocity, and acceleration) can be found by examining the time history of the responses. A response spectrum is a plot of the maximum responses of various single degree of freedom systems (See Figure 2.6.3) versus the undamped natural period of the systems.

The maximum value of response relative to the ground is taken as the measure of earthquake intensity. This maximum may be expessed as

$$S_d = \left|v(t)\right|_{max} \qquad (2.6.3)$$

Substituting into Equation 2.6.2

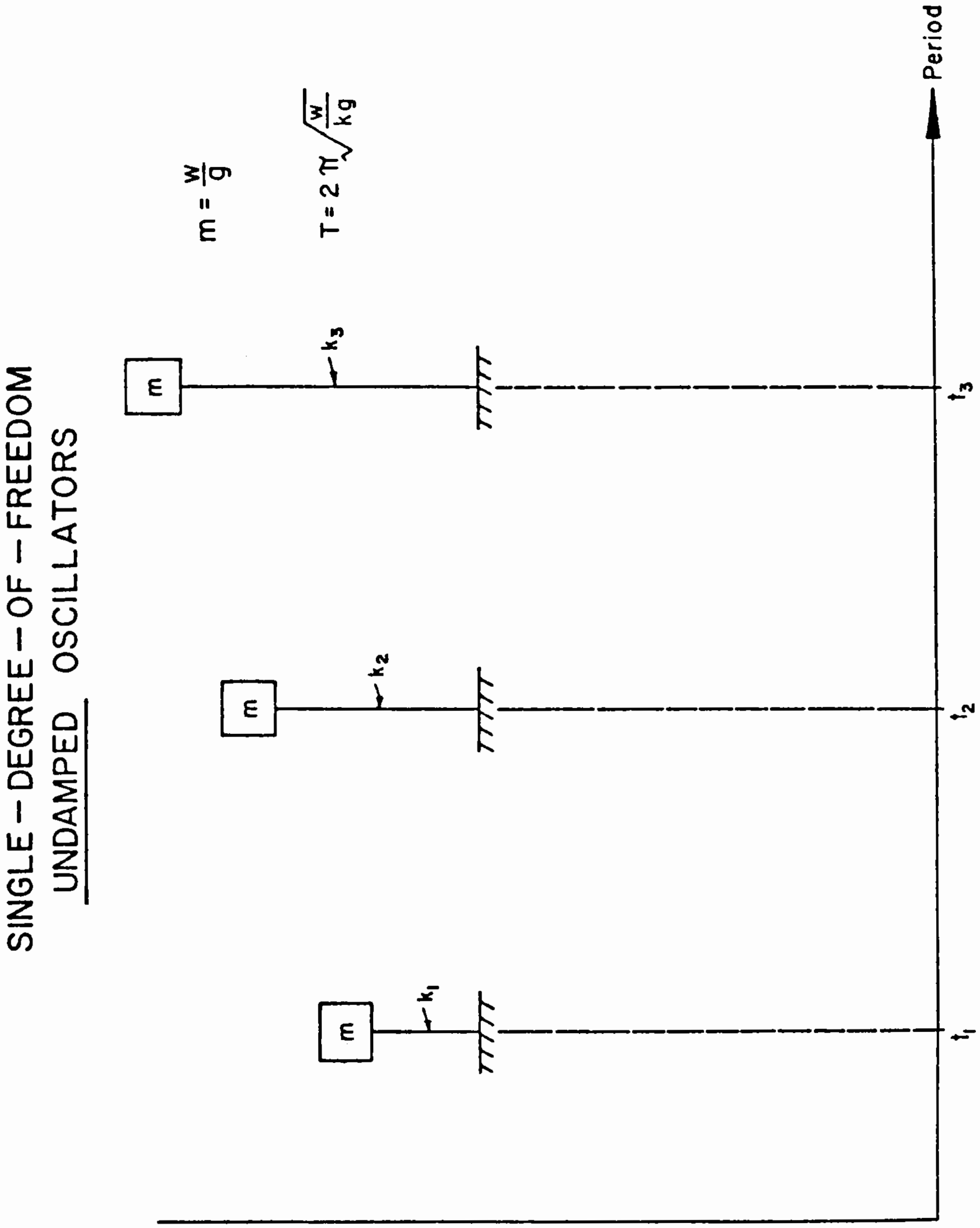
$$S_d(\xi,\omega) = \left[\frac{1}{\omega}\int_0^t \ddot{v}_g(\tau)e^{-\xi\omega(t-\tau)}\sin(t-\tau)\,d\tau\right]_{max} \qquad (2.6.4)$$

The spectral velocity S_v and the spectral acceleration defined by

$$S_v = \omega S_d \qquad (2.6.5a)$$

and

$$S_a = \omega S_v = \omega^2 S_d \qquad (2.6.5b)$$



SINGLE-DEGREE-OF-FREEDOM
UNDAMPED OSCILLATORS

Figure 2.6.3

may also be obtained by evaluating Equation 2.6.4 using these simple relationships. As indicated by Equation 2.6.4, S_d depends not only on the ground motion history, but also on the frequency of vibration and the damping of the system.

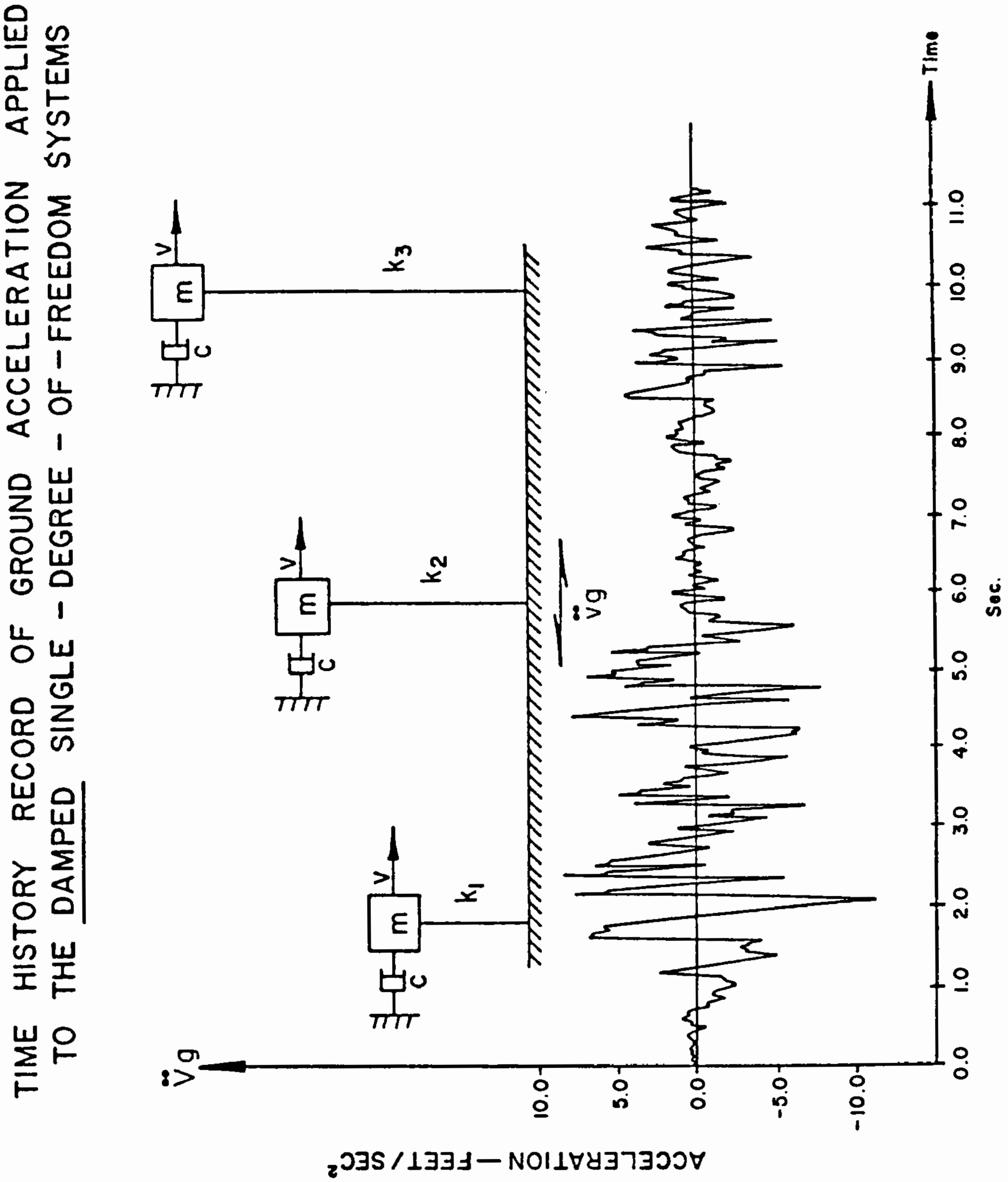
As an illustration, consider the ground motion record and SDOF systems shown in Figure 2.6.4. The acceleration response time history of any one system can be determined and the maximum response selected as shown in Figure 2.6.5. As shown in Figure 2.6.6, a response spectrum of acceleration is obtained by plotting the maximum response of a whole series of systems. A response spectrum can be used to predict the response of a multi-degree of freedom structure thus eliminating the need to determine the time history of response of a large structure to a specified support motion.

Note that the spectrum shown by the jagged line in Figure 2.6.6 represents the spectrum for a single earthquake motion. Since each earthquake motion will have slightly different characteristics, the use of a single earthquake motion for design purposes leaves some uncertainty as to the significance of the response it will produce. For design purposes, it is more meaningful to take a smoothed average of several records as shown by the smooth line. The design response spectrum usually provides an adequate basis for the seismic design of most bridge structures. The spectral displacement S_d represents the maximum relative displacement that a SDOF system will experience for a given input motion and damping. This maximum displacement corresponds to a condition of zero kinetic energy and the maximum potential energy or strain energy given by

$$V_{max} = \frac{1}{2} k S_d^2 \qquad (2.6.6)$$

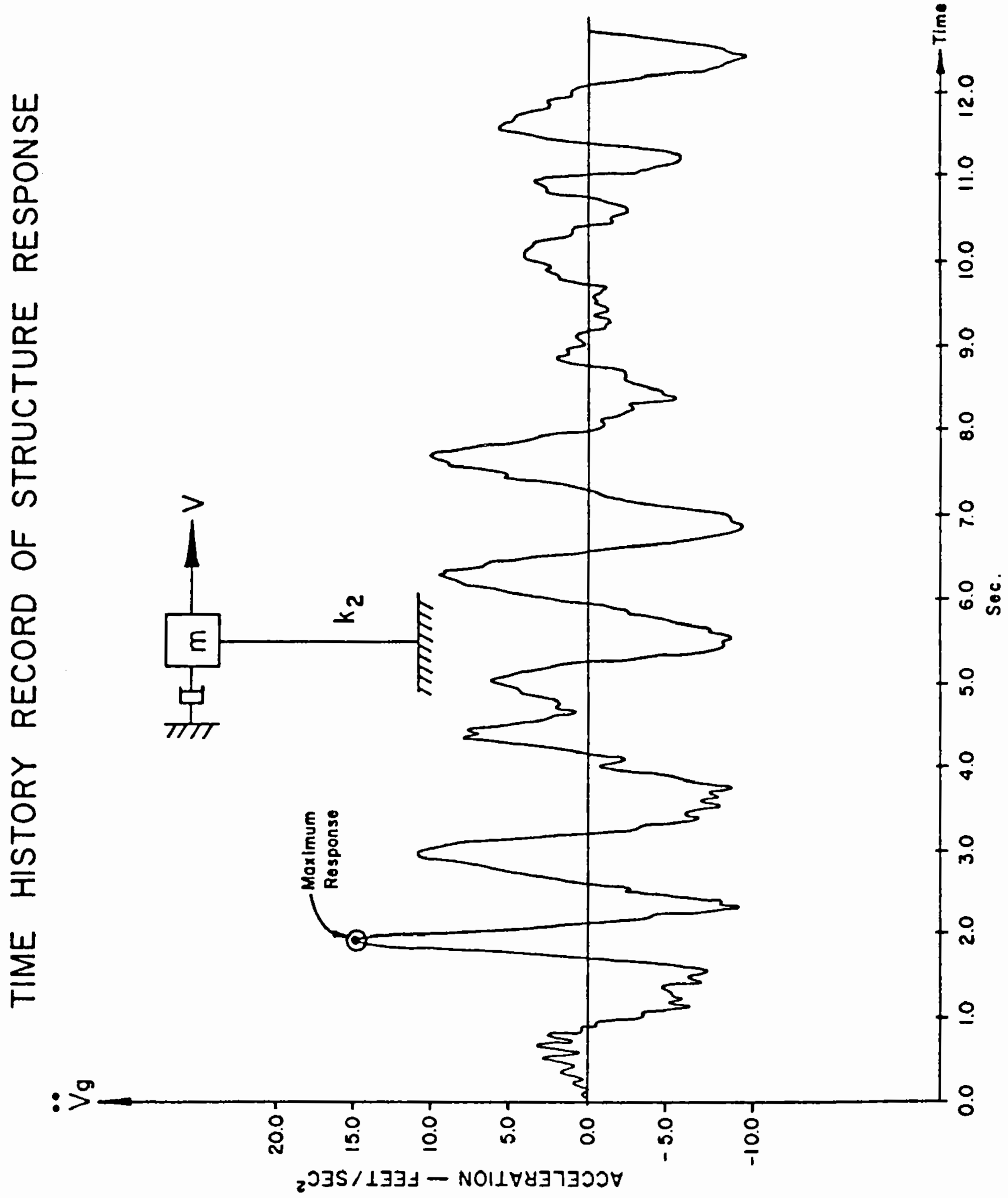
In a conservative system, the maximum strain energy equals the maximum kinetic energy. The maximum relative velocity that the system will experience is defined as:

$$S_v = \left| \dot{v}(t) \right|_{max} \qquad (2.6.7)$$



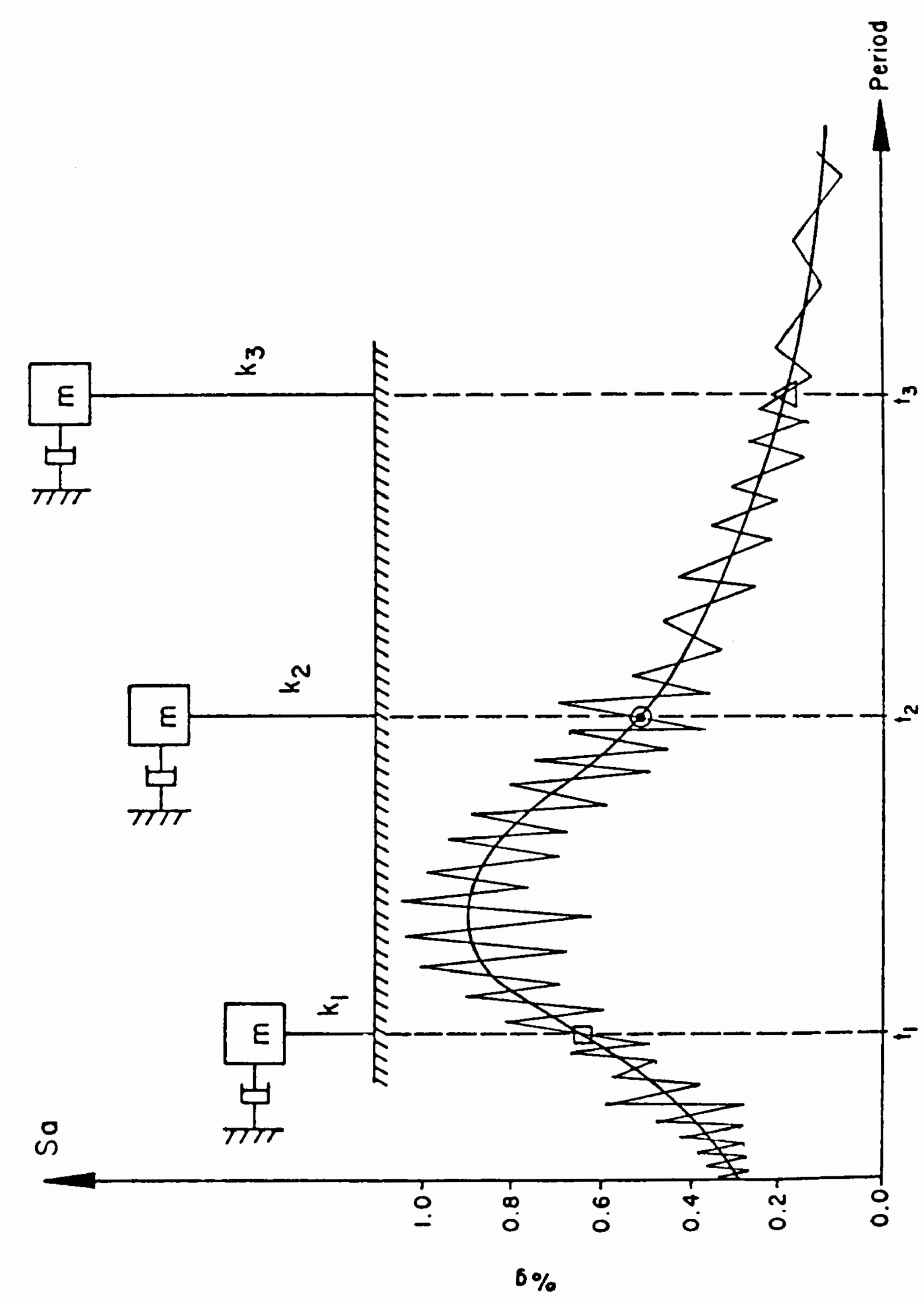
TIME HISTORY RECORD OF GROUND ACCELERATION APPLIED
TO THE DAMPED SINGLE-DEGREE-OF-FREEDOM SYSTEMS

Figure 2.6.4



TIME HISTORY RECORD OF STRUCTURE RESPONSE

Figure 2.6.5



MAXIMUM RESPONSES OF A FAMILY OF SINGLE-DEGREE-OF-FREEDOM SYSTEMS FOR A CONSTANT VALUE OF DAMPING VS PERIOD YIELDS A RESPONSE SPECTRUM FOR A GIVEN GROUND ACCELERATION

MAXIMUM RESPONSES OF SINGLE-DEGREE-OF-FREEDOM SYSTEMS

Figure 2.6.6

The maximum kinetic energy of the system is given by

$$T_{max} = \frac{1}{2} m S_v^2 \tag{2.6.8}$$

Recalling that for a conservative system

$$T_{max} = V_{max} \tag{2.6.9}$$

thus

$$\frac{1}{2} m S_v^2 = \frac{1}{2} k S_d^2$$

$$S_v = \frac{k}{m} S_d$$

or

$$S_v = \omega S_d \tag{2.6.10}$$

The response parameter S_v is called the spectral psuedo velocity because it does not actually represent the maximum velocity of the SDOF system, but is only an approximation of the relative velocity. The psuedo velocity S_v is approximately equal to the maximum relative velocity for systems with moderate or high frequencies, but it may differ considerably from the maximum relative velocity for low frequency systems. An expression for the true relative velocity would be obtained by differentiating Equation 2.6.2. The psuedo velocity is, however, generally used.

An expression for the spectral acceleration S_a can be obtained in a similar way by neglecting the effects of damping. The maximum total acceleration that the system will experience is given by

$$S_a = |\ddot{V}_t(t)|_{max} \tag{2.6.11}$$

We can write Equation 2.1.6 as:

$$m\ddot{V} + kV = -m\ddot{V}_g$$

$$m(\ddot{V} + \ddot{V}_g) + kV = 0$$

Recalling that the total acceleration is expressed as

$$\ddot{V}_t = \ddot{V} + \ddot{V}_g$$

thus

$$m\ddot{V}_t + kV = 0$$

or

$$\ddot{V}_t = -\frac{kV}{m}$$

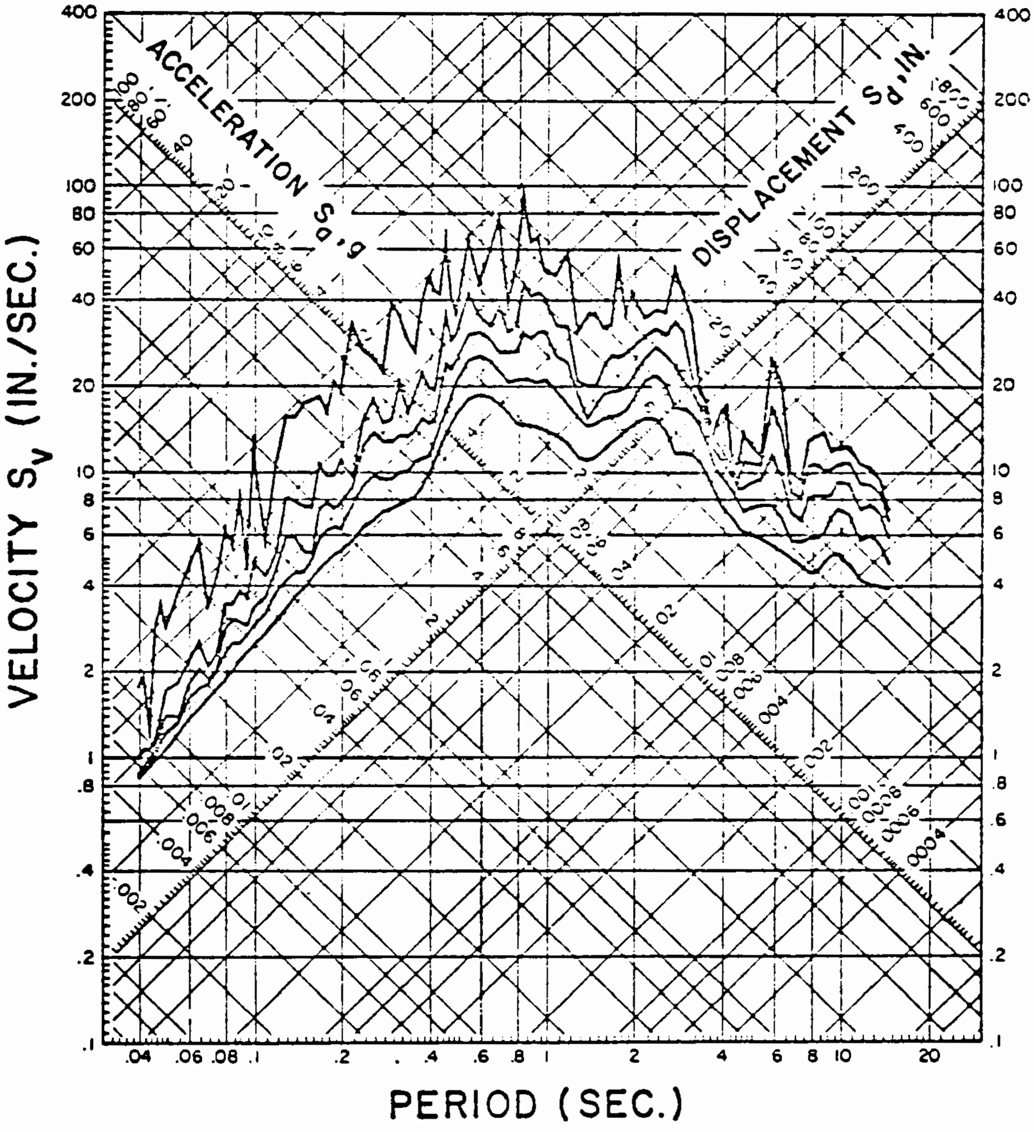
$$\ddot{V}_t = -\omega^2 V$$

since

$$S_a = \left|\ddot{V}_t(t)\right|_{max} \tag{2.6.12}$$

$$S_a = \omega^2 S_d$$

The simple relationships given in Equation 2.6.10 and 2.6.12 permit the representation of displacement, psuedo-velocity, and acceleration on a single plot of four-way log paper as shown in Figure 2.6.7.



IMPERIAL VALLEY EARTHQUAKE MAY 18, 1940
EL CENTRO COMPONENT SOOE

DAMPING: 0, 2, 5, 10 AND 20% OF CRITICAL

TRILOG PLOT - RESPONSE SPECTRUM

Figure 2.6.7

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CHAPTER 3

BASIC SEISMOLOGY

3.1 PLATE TECTONICS

Only in the past 25 years has it been possible to detect earthquakes on a uniform, worldwide basis. During this time, a very distinct structural pattern has emerged at both the mid-ocean ridges and the continental tectonic zones.

The concentration of earthquakes around the boundaries of some of the major continental blocks and along the mid-ocean ridges can readily be seen in Figure 3.1.

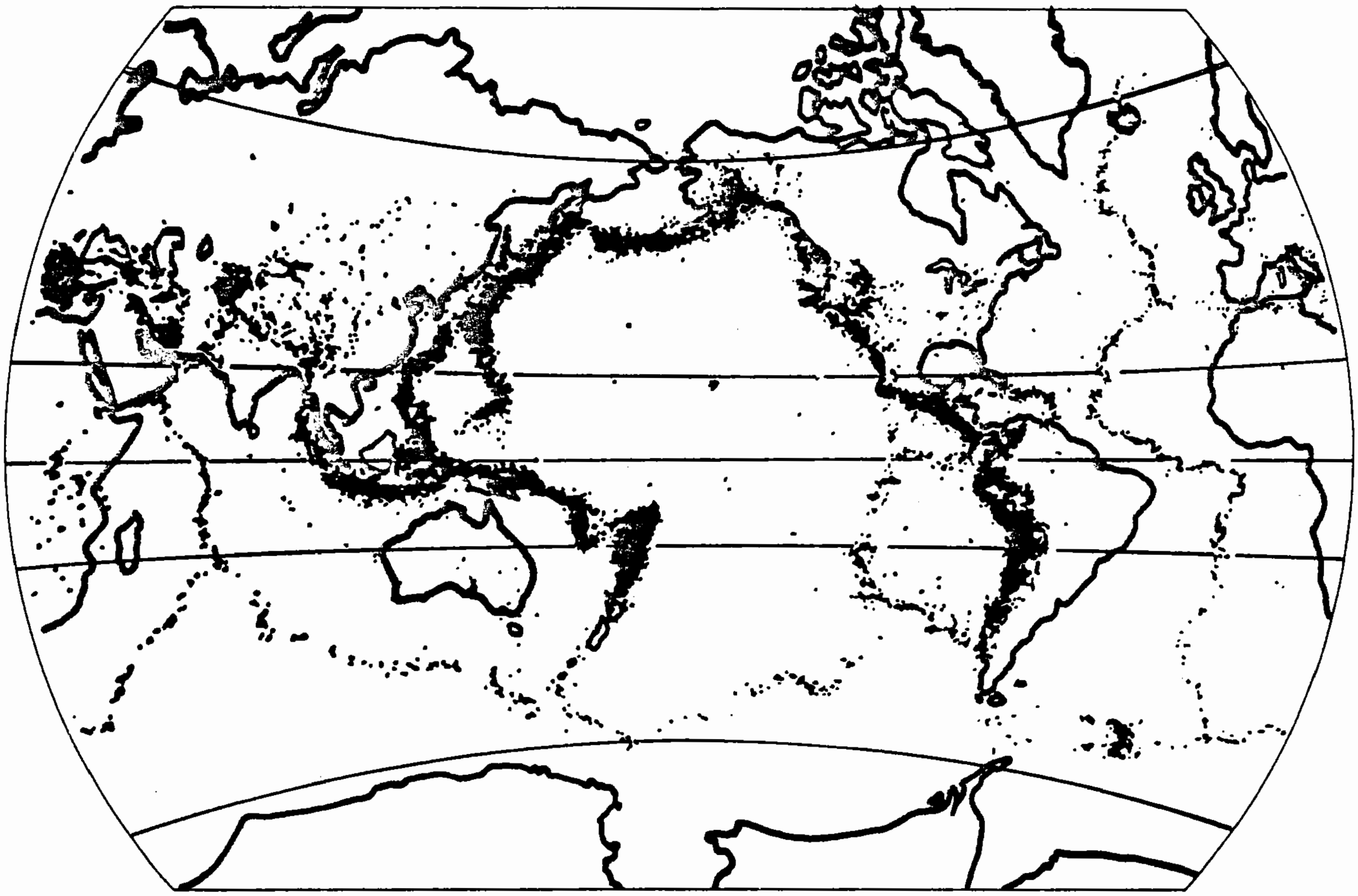
The emerging field of plate tectonics has provided considerable insight to aid in the understanding of the major global sources of earthquakes.

The theory of plate tectonics was first presented by German meteorologist, Alfred Wegener, in 1912 [3.1]. Wegener's theory was that if the earth could flow vertically in response to vertical forces, it could also flow laterally. He showed an amazing number of fossil, rock and structure correlations on both sides of the Atlantic.

Between 1920 and 1930, Wegener's theory generated much discussion. This debate set the stage for the discovery of sea floor spreading in the early 1960's. Harry H. Hess of Princeton University [3.2] and Robert S. Dietz of the USGS independently [3.3] proposed that the ocean floor might be in motion. At the same time, Mason, Raff, and Vacquier of the Scripps Institute of Oceanography, discovered that the ocean floor off the West coast of North America had a very striking pattern of variations in magnetic intensity [3.4]. These long strips of magnetic variation have been traced parallel to the mid-ocean ridge and are symmetrical about the ridge. This magnetic "tape-recorder" shows that the sea floors have spread outwards from the ridges over millions of years. They ultimately plunge beneath the ocean trenches, such as those near Chile and the Aleutians.

The Earth's crust (or lithosphere) is divided into a number of Geological plates such as the North American plate which is between the mid-Atlantic ridge and the San Andreas fault.

These plates are 30 to 60 miles (50-100 km) thick and are thinner in ocean regions. Evidence indicates that about 350 million years ago, North America, Europe, and Africa were joined, and



WORLDWIDE SEISMICITY

Figure 3.1

that about 200 million years ago a major continental breakup began which formed the Atlantic Ocean [3.5]. It is believed that these plates are propelled by sea-floor spreading caused by either (1) radial contraction or expansion of the Earth and/or (2) slow convective motion of the material within the earth's mantle. It is not fully understood how these movements led to the formation of seas and mountains [3.6]. It is not known how motion of the lithosphere as a whole relates to motion of individual plates. The boundaries of these plates are marked by major geologic features such as:

1) rifts and ridges
2) fold mountains
3) island arcs
4) transform faults

Rifts and ridges are locations of deep cracks in the earth and represent zones where plates are drifting apart. Molten rock rises to the surface along these zones and solidifies into new rock. This new rock drifts apart since some is attached to each moving plate.

Fold mountains may indicate the edge of a plate since they are formed by great forces acting in compression.

Island arcs like the Aleutian Islands are bounded by deep trenches and are the sites of much volcanic activity, faulting and earthquakes. These are areas in which the edge of one plate is being pushed beneath another plate. The subducted plate enters the mantle and dissolves when it reaches an area of sufficient heat.

Transform faults cut across the rifts and ridges and offset them. These faults mark the boundaries of adjacent plate segments which slide by each other parallel to the faults. The San Andreas fault is an example of a large transform fault.

An ocean basin is formed where plates move apart. Molten rock wells up from the rift between the plates and this material is constantly moved to either side. The Gulf of California is considered to be a young ocean in the process of being formed by the rifting apart of adjacent plates.

Mountains are formed by the collision of two plates or by the subduction of an ocean plate under a continental plate. In either case, the excess material is folded or deposited to form mountain ranges.

These movements of plates cause an almost continuous relocation of the continents and ocean basins. Major earthquakes usually occur at the junction of these plates. Although earthquakes occur mainly along the edges of plates and at the mid-ocean ridges, there are known occurrences of catastrophic earthquakes in other regions. In the United States, the major 1811-1812 New Madrid earthquakes in Missouri and the 1886 Charleston, South Carolina, earthquake are examples of exceptions to the heavy concentration of earthquake epicenters at plate boundaries.

3.2 THE ELASTIC REBOUND THEORY

Seismologists are not in complete agreement on the detailed mechanism of tectonic energy release. It seems certain that the primary cause is a sudden movement of rock masses along weak zones called faults. These movements can adequately explain the resulting shaking and displacements that have been observed during a large number of earthquakes.

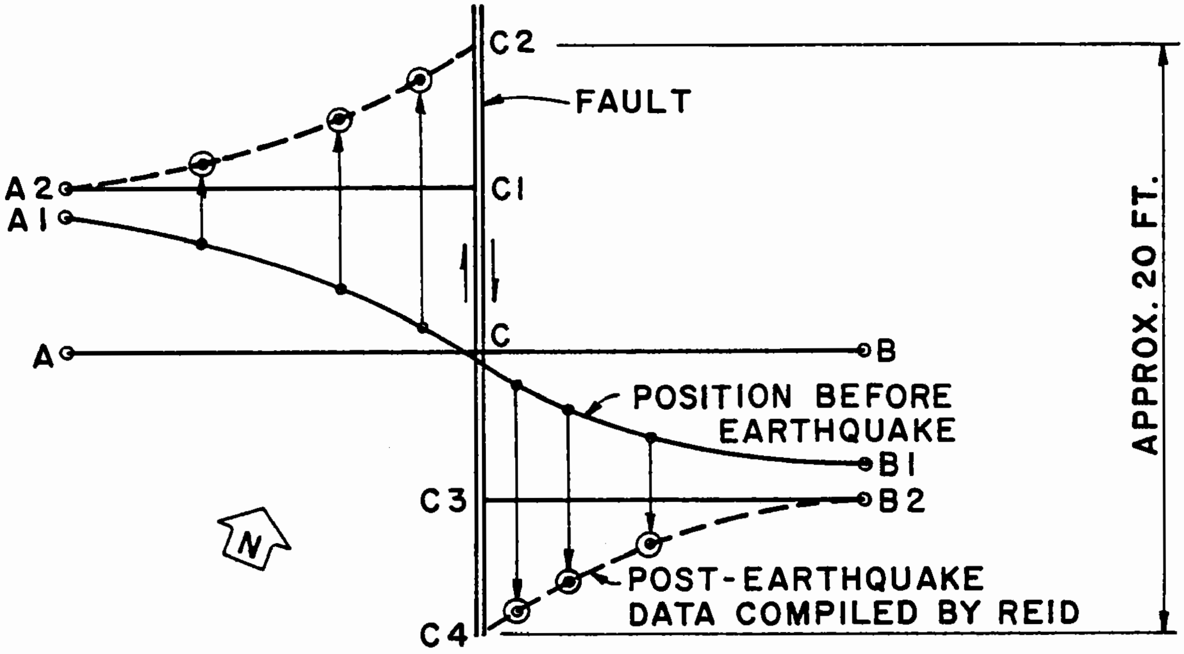
The majority of earthquakes, however, are not accompanied by visible fault displacement and are usually assumed to have faulting at depth. All seismologists now agree that major earthquakes arise from movements on faults, even when movement is undiscernible at the ground surface [3.7, 3.8].

Studies by Reid and Law [3.9] after the 1906 San Francisco earthquake suggested that although the fault displacement was sudden, the energy it released had probably been slowly accumulating in the rocks for a long period of time. Reid said, "The fracture of the rock which causes a tectonic earthquake is the result of elastic strains greater than the strength of the rock can withstand, produced by relative displacement of neighboring portions of the earth's crust." The only mass movements at the time of the earthquake are the sudden elastic rebounds of the sides of the fracture towards portions of no elastic strains, and these movements gradually diminishing away from the fault surface, extend, even in large earthquakes, to distances of only a few miles from the fracture.

Reid showed that careful analysis of precise survey data of the San Francisco area fully supported this theory. The USCGS had performed triangulation surveys in 1851-1865, 1874-1892 and in 1906-1907. Reid grouped the data according to the average distance of the several stations from the fault.

Figure 3.2 shows a simplified map of Reid's results and how they are explained by the elastic rebound theory. Line ACB represents the time when no elastic strain existed in the rocks of the region. As the regional displacement began, the point A moved slowly north and point B moved south. Just before the earthquake, the line ACB was located at A1, C, B1 and was curved as shown. When the fault finally gave way and the stress was released with explosive violence the line A1, C straightened out to A2, C1. Similarly, the line B1, C on the other side straightened. Because of the large masses of rock involved this movement continued past the point C1 to point C2.

The elastic rebound theory provided the first realistic model to aid in determination of duration, amplitudes, and frequencies of seismic shaking at various locations from the fault zone.



SIMPLIFIED MAP OF 1906 SAN FRANCISCO OFFSET

Figure 3.2

3.3 FAULTS

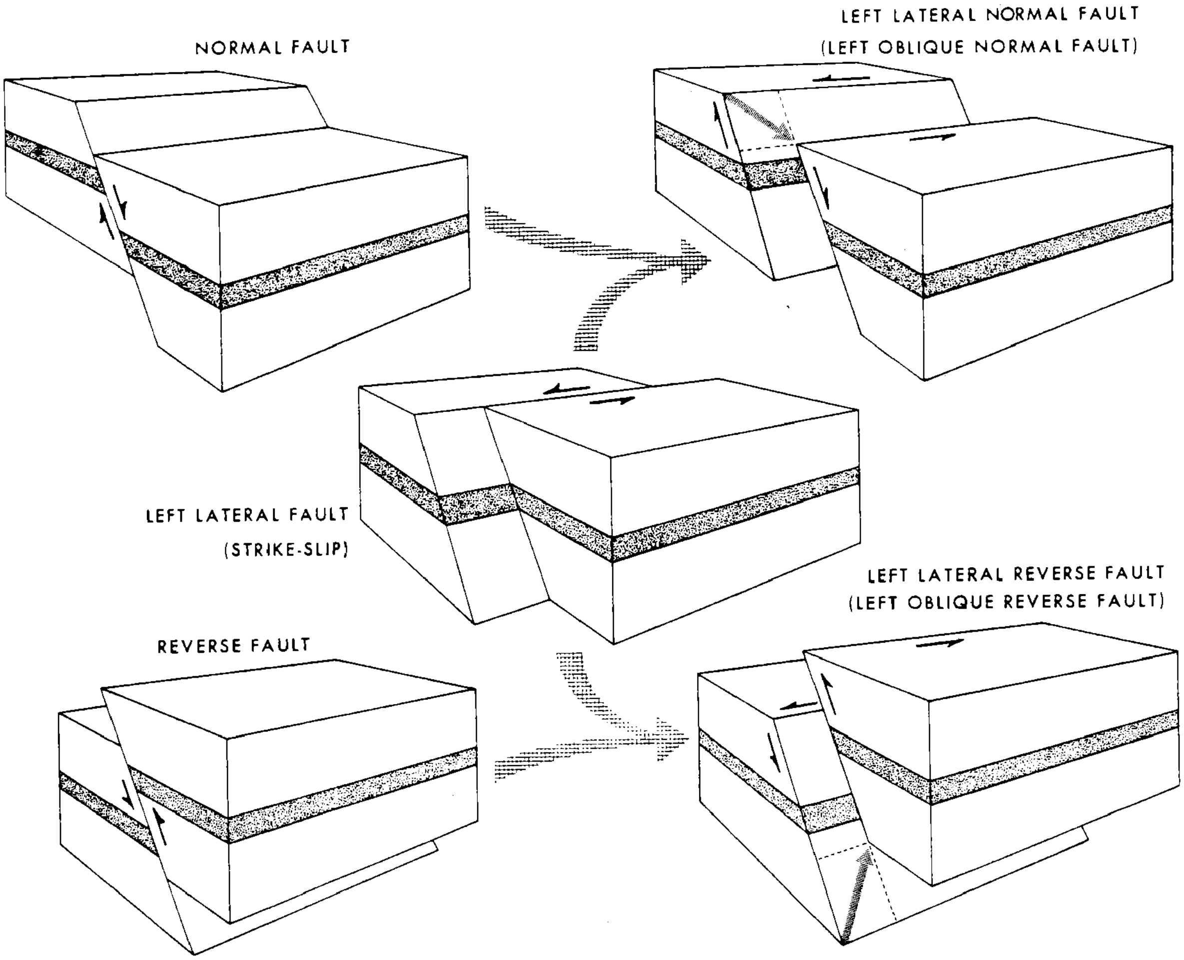
Three basic types of faults, based on relative movement can be identified [3.8]:

1) Normal Fault, in which the block above the fault has moved down relative to the block below.

2) Reverse (or Thrust) Fault, in which the block above the fault has moved up relative to the block below.

3) Lateral (or Strike-Slip) Fault, in which the rocks on either side of the fault have moved laterally past each other.

In reality, most faults exhibit a combination of vertical and lateral movement, and are called oblique faults. (Fig. 3.3)



FAULT TYPES

Figure 3.3

3.4 SEISMIC WAVES

The sudden release of strain energy initiates vibrations in the earth's crust. These vibrations propagate in all directions from the fracture.

The earth can transmit vibrational waves in a variety of ways, but in general, they can be classified as BODY WAVES and SURFACE WAVES.

3.4.1 BODY WAVES

Body waves are waves which travel through the body of the material. Two well-known body waves are (1) the longitudinal or P wave and (2) the Transverse or S wave.

1) Longitudinal (Compressional or P) Waves. The symbol P (for primary) is used to denote these fast waves, because they are the first recorded on the seismograph. These stress waves are created by the motion of particles moving back and forth (tension and compression) in the direction in which the waves are progressing. They travel at a speed of between 4.3 and 8.6 miles a second. Their period of vibration varies from 0.1 to 0.2 sec. close to the energy source to as high as 7 to 15 sec. at a distance from a major event.

2) Transverse (shear, rotational or S) waves. The S (for secondary) waves are slower than P waves. They travel at a little less than 3 miles per second. A particle in the path of a Transverse wave may oscillate in any direction in the plane normal to the direction of the advance of the wave. Although slower than longitudinal waves, transverse waves transmit more energy than P waves. The S waves have a longer period of vibration than P waves, varying from 0.2 to 0.5 sec. close to the focus and from 10 to 20 sec. at large distances.

3.4.2 SURFACE WAVES

Surface waves are generated from body waves reflecting off the ground surface. These waves then propagate along the surface of the ground. The physics of these waves is very complex. There are two common types: (1) Rayleigh waves and (2) Love waves.

1) Rayleigh (or R) waves. In these waves, the ground particles move vertically and radially in the plane of propagation. This wave is a combination of the P and S waves.

2) Love (or Q) waves. These waves vibrate transversely to the direction of wave travel and have no vertical component.

Both Rayleigh and Love waves are dispersed, which means their velocities are dependent on their wave lengths. Their periods range from 2 to 3 seconds close to the focus to as much as one hour at some distance from a major event.

Because of the reflection, refraction, and dispersion of the seismic waves, the wave train that arrives at a distant seismograph station is understandably complex. Detailed study of these records permits accurate determinatin of the location of the earthquake, the origin time, and the energy release. Figure 3.4 illustrates the four types of earthquake waves.

The Earthquake Engineer is interested in the vibrations which can cause damage to structures. These large vibrations knock the sensitive seismograph out of commission. Strong-motion accelerographs are used to record these strong motions.

3.5 EARTHQUAKE SIZE

When a seismologist records an earthquake on his seismograph, he is able to measure the exact amplitude and period of the waves which were received at his station. This record is a measure of the absolute size of the earthquake. By taking into account the distance of the station from the earthquake, it is possible to express this size in terms of the total energy released at the focus.

3.5.1 MAGNITUDE

One scale which has been developed to represent the instrumental measure of an earthquake's size is the Richter magnitude. The Richter magnitude (M) is defined as:

$$M = \log_{10} \frac{A}{A_0} \tag{3.1}$$

where

A = maximum amplitude recorded by a Wood Anderson torsion seismometer at a distance of 100 km. from the earthquake

A_0 = amplitude of .001 mm.

In practice, the recording is made at distances that are larger than 100 km. Therefore, the recorded amplitude is extrapolated to a distance of 100 km. from the earthquake epicenter.

Seismologists use a number of magnitudes, depending on the type of seismic wave being measured. For example, an underground nuclear explosion which has the same body wave magnitude as an earthquake, will have a smaller surface wave

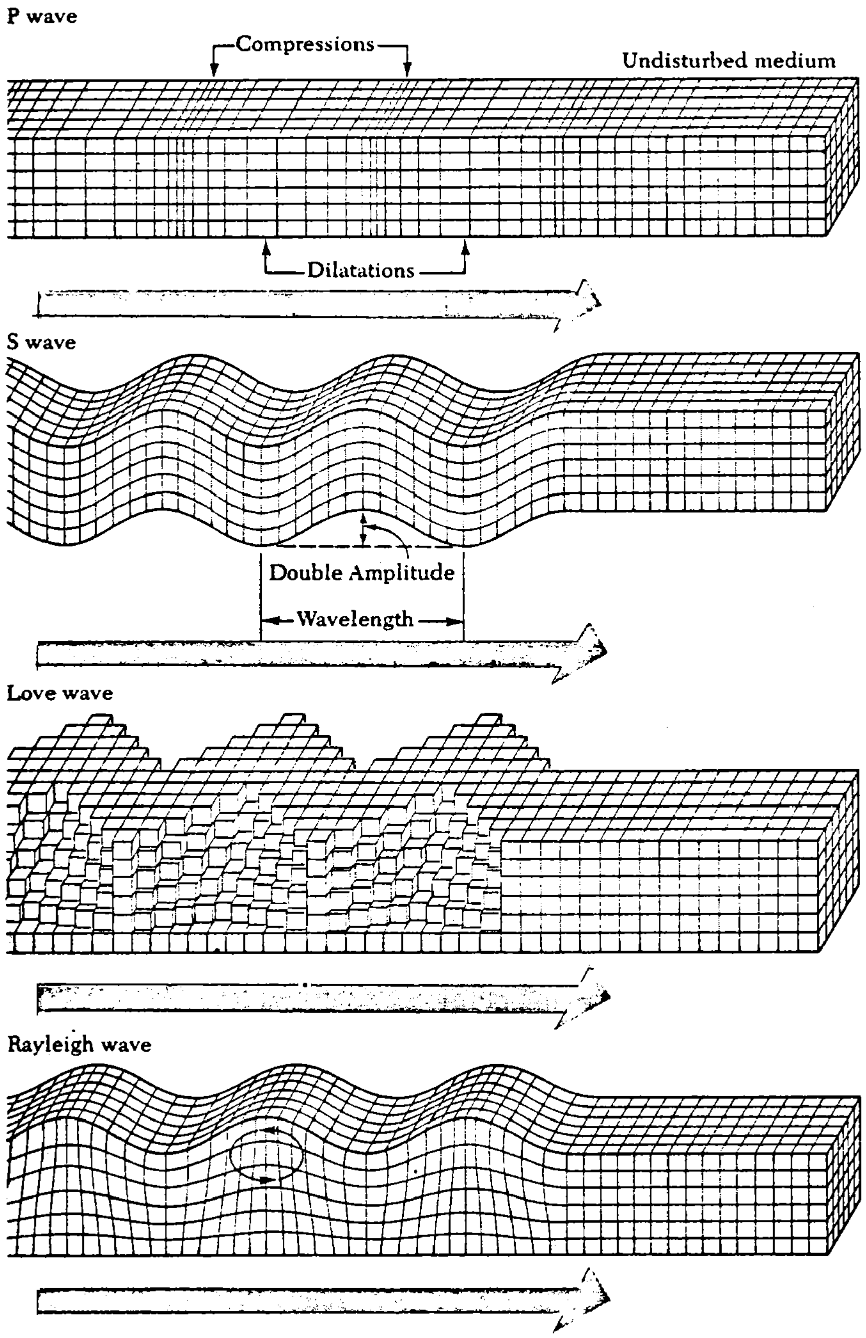


Diagram illustrating the forms of ground motion near the ground surface in four types of earthquake waves. [From Bruce A. Bolt, *Nuclear Explosions and Earthquakes*. W. H. Freeman and Company. Copyright © 1976.]

Figure 3.4

magnitude.

The Richter magnitude, however, uses the maximum wave amplitude regardless of wave type which is one of its seismological limitations.

3.5.2 ENERGY

The currently accepted relationship between energy and magnitude is:

$$\log_{10} \text{ENERGY (ergs)} = 11.4 + 1.5\,M \qquad (3.2)$$

where

M = The Richter magnitude

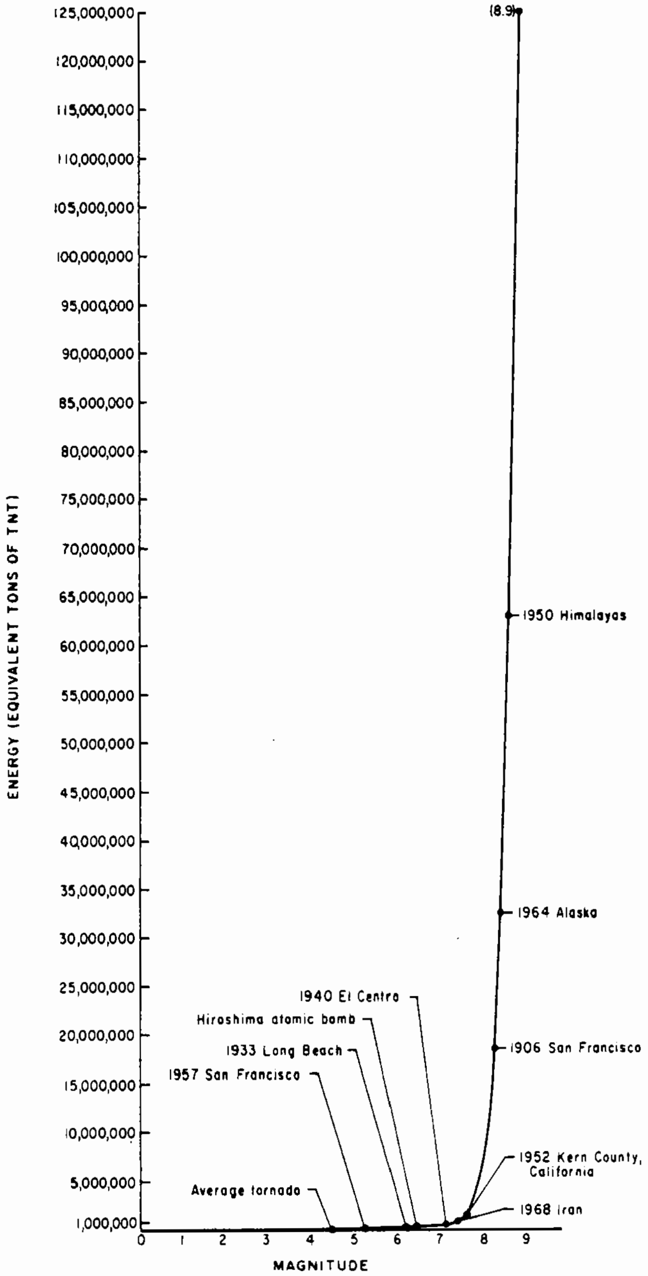
Because of the logarithmic factor, the energy released increases very rapidly with the magnitude (approx. 32 times per unit); thus a magnitude 8 earthquake releases approximately one million times the energy of a magnitude 4 event. Although the Richter magnitude is open ended, the largest recorded value is 8.9. Figure 3.5 illustrates the relation between magnitude and energy in equivalent tons of TNT.

The energy released from a large earthquake is proportional to the length of faulting involved and to the square of the fault slip. Fault length is not directly related to the strong ground motion in the region near the fault, however. The energy involved in the close-in locations is best considered in terms of the useful portion of the frequency spectrum of the wave motion.

During faulting, the stress on the fault drops by amounts ranging up to 3000-5000 psi in shallow focus earthquakes. The stress drop is proportional to the rock rigidity and the fault offset.

3.5.3 INTENSITY

Intensity is a measure of the effects of an earthquake on man and his environment. This scale of measurement is not dependent on instrumental measurement, but depends on the observer's interpretation of events he sees or feels. Consequently, no intensity rating is given for an earthquake,



ENERGY VS. MAGNITUDE

Figure 3.5

even one of high magnitude, that occurs far from civilization and is not felt by man.

One of the first generally-used intensity scales was developed in Europe in the 1880's by DeRossi of Italy and Forel of Switzerland. The Rossi-Forel scale, with values from I to X, was widely used for about 20 years. This scale's main defect was that it lumped a great deal of damage into the upper level (or X) classification.

In 1902, the Italian seismologist, Mercalli established the scale which in modified form is widely used today. This scale (the Modified Mercalli), is shown in Figure 3.6. It is based on a I to XII range and provides for a more refined analysis of major damage.

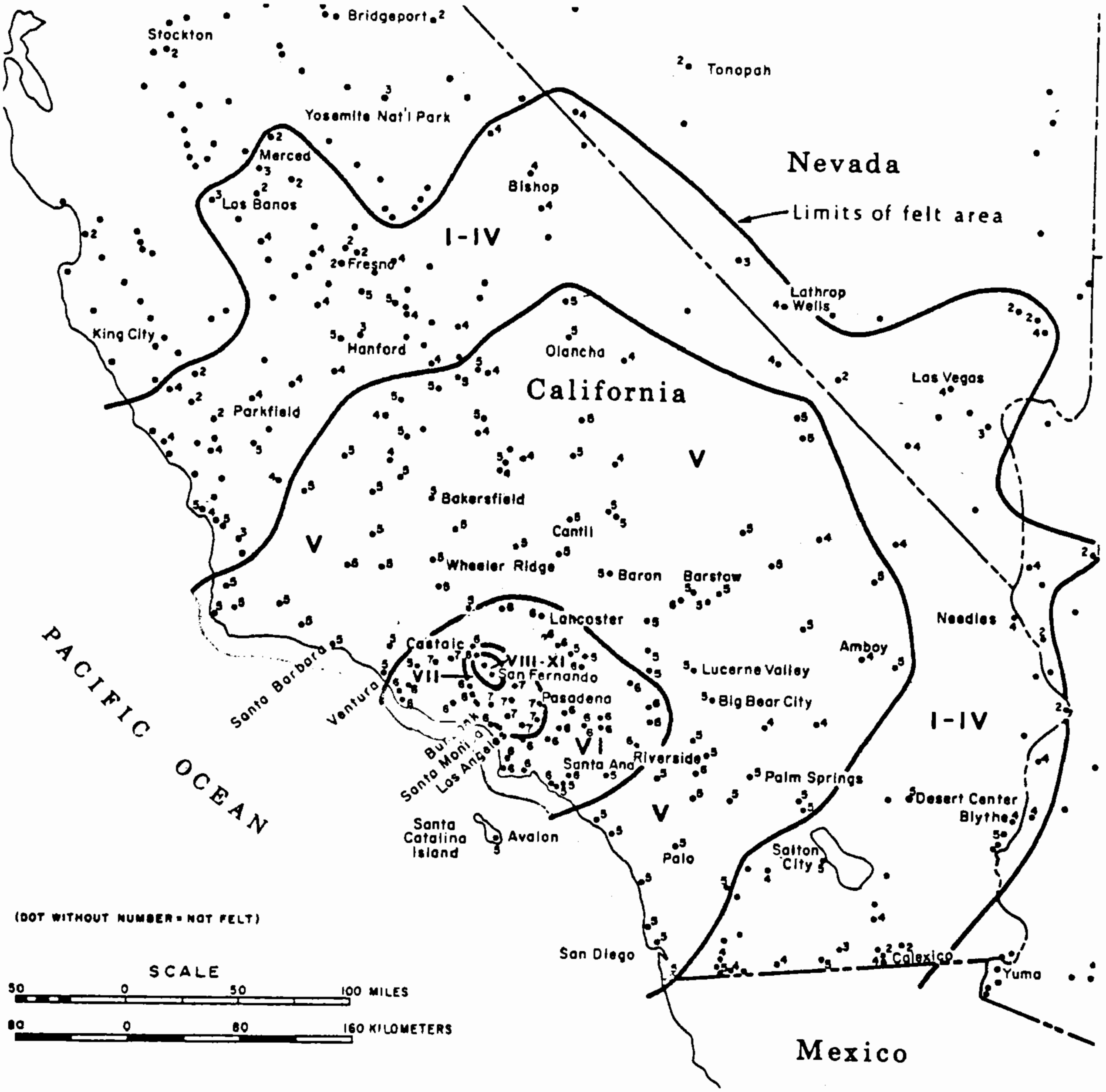
The varying intensity grades are frequently expressed in an isoseismal map with roughly circular lines drawn to enclose areas of equal intensity. Figure 3.7 shows the preliminary intensity map developed for the February 9, 1971, San Fernando earthquake. This map was developed from results obtained from over 2000 questionnaires by the USGS Seismological Field Survey and indicates a felt area of approx. 80,000 sq. miles over California, Arizona, and Nevada.

If most of these effects are observed	*then the intensity is:*
Earthquake shaking not felt. But people may observe marginal effects of large distance earthquakes without identifying these effects as earthquake-caused. Among them: trees, structures, liquids, bodies of water sway slowly, or doors swing slowly.	*I*
Effect on people: Shaking felt by those at rest, especially if they are indoors, and by those on upper floors.	*II*
Effect on people: Felt by most people indoors. Some can estimate duration of shaking. But many may not recognize shaking of building as caused by an earthquake; the shaking is like that caused by the passing of light trucks.	*III*
Other effects: Hanging objects swing. *Structural effects:* Windows or doors rattle. Wooden walls and frames creak.	*IV*
Effect on people: Felt by everyone indoors. Many estimate duration of shaking. But they still may not recognize it as caused by an earthquake. The shaking is like that caused by the passing of heavy trucks, though sometimes, instead, people may feel the sensation of a jolt, as if a heavy ball had struck the walls. *Other effects:* Hanging objects swing. Standing autos rock. Crockery clashes, dishes rattle or glasses clink. *Structural effects:* Doors close, open or swing. Windows rattle.	*V*
Effect on people: Felt by everyone indoors and by most people outdoors. Many now estimate not only the duration of shaking but also its direction and have no doubt as to its cause. Sleepers wakened. *Other effects:* Hanging objects swing. Shutters or pictures move. Pendulum clocks stop, start or change rate. Standing autos rock. Crockery clashes, dishes rattle or glasses clink. Liquids disturbed, some spilled. Small unstable objects displaced or upset. *Structural effects:* Weak plaster and Masonry D* crack. Windows break. Doors close, open or swing.	*VI*
Effect on people: Felt by everyone. Many are frightened and run outdoors. People walk unsteadily. *Other effects:* Small church or school bells ring. Pictures thrown off walls, knicknacks and books off shelves. Dishes or glasses broken. Furniture moved or overturned. Trees, bushes shaken visibly, or heard to rustle. *Structural effects:* Masonry D* damaged; some cracks in Masonry C*. Weak chimneys break at roof line. Plaster, loose bricks, stones, tiles, cornices, unbraced parapets and architectural ornaments fall. Concrete irrigation ditches damaged.	*VII*
Effect on people: Difficult to stand. Shaking noticed by auto drivers. *Other effects:* Waves on ponds; water turbid with mud. Small slides and caving in along sand or gravel banks. Large bells ring. Furniture broken. Hanging objects quiver. *Structural effects:* Masonry D* heavily damaged; Masonry C* damaged, partially collapses in some cases; some damage to Masonry B*; none to Masonry A*. Stucco and some masonry walls fall. Chimneys, factory stacks, monuments, towers, elevated tanks twist or fall. Frame houses moved on foundations if not bolted down; loose panel walls thrown out. Decayed piling broken off.	*VIII*
Effect on people: General fright. People thrown to ground. *Other effects:* Changes in flow or temperature of springs and wells. Cracks in wet ground and on steep slopes. Steering of autos affected. Branches broken from trees. *Structural effects:* Masonry D* destroyed; Masonry C* heavily damaged, sometimes with complete collapse; Masonry B* is seriously damaged. General damage to foundations. Frame structures, if not bolted, shifted off foundations. Frames racked. Reservoirs seriously damaged. Underground pipes broken.	*IX*
Effect on people: General Panic. *Other effects:* Conspicuous cracks in ground. In areas of soft ground, sand is ejected through holes and piles up into a small crater, and, in muddy areas, water fountains are formed. *Structural effects:* Most masonry and frame structures destroyed along with their foundations. Some well-built wooden structures and bridges destroyed. Serious damage to dams, dikes and embankments. Railroads bent slightly.	X
Effect on people: General panic. *Other effects:* Large landslides. Water thrown on banks of canals, rivers, lakes, etc. Sand and mud shifted horizontally on beaches and flat land. *Structural effects:* General destruction of buildings. Underground pipelines completely out of service. Railroads bent greatly.	*XI*
Effect on people: General panic. *Other effects:* Same as for Intensity X. *Structural effects:* Damage nearly total, the ultimate catastrophe. *Other effects:* Large rock masses displaced. Lines of sight and level distorted. Objects thrown into air.	*XII*

- *Masonry A: Good workmanship and mortar, reinforced designed to resist lateral forces.
- Masonry B: Good workmanship and mortar, reinforced.
- Masonry C: Good workmanship and mortar, unreinforced.
- Masonry D: Poor workmanship and mortar and weak materials.

MODIFIED MERCALLI INTENSITY SCALE

Figure 3.6

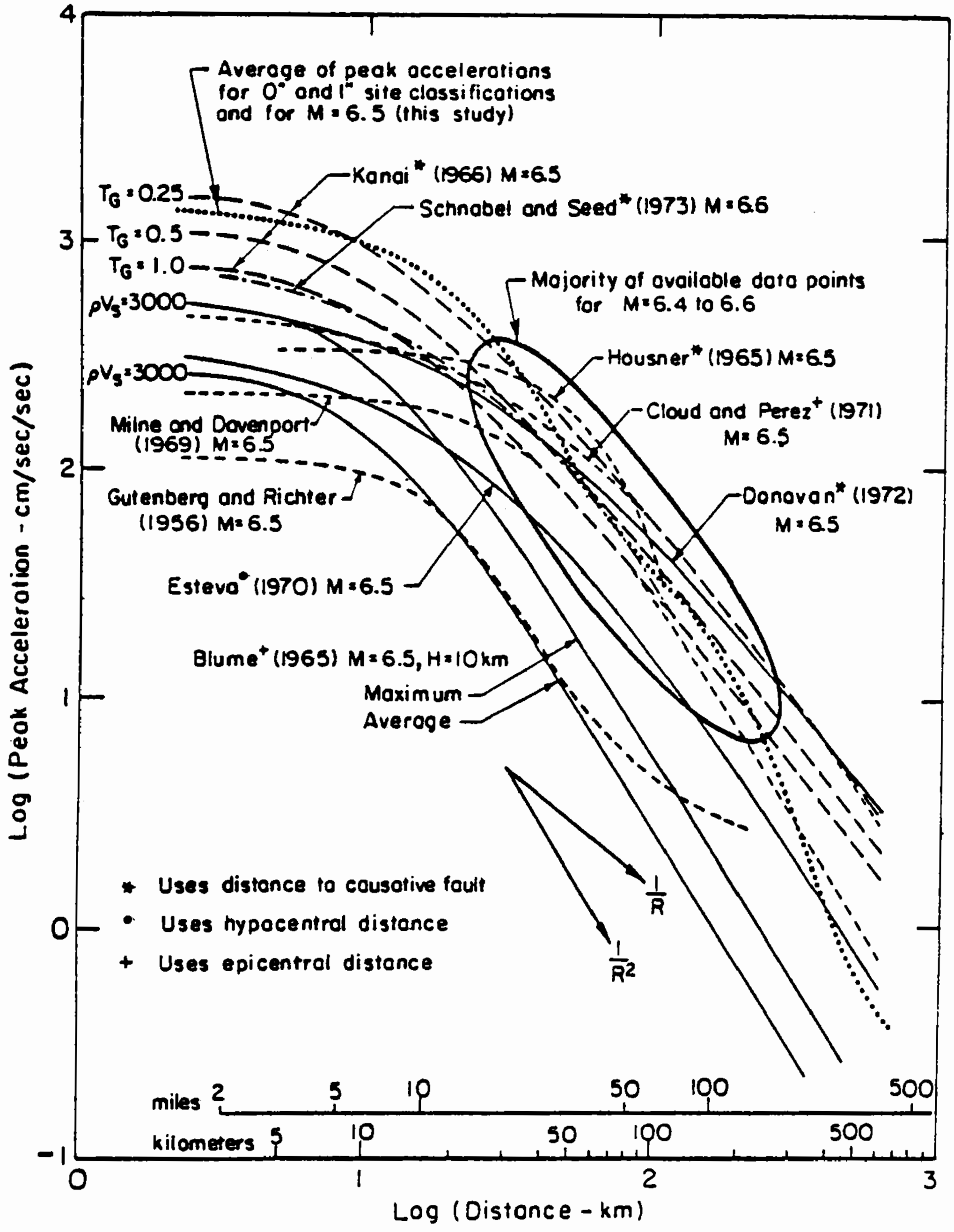


INTENSITY MAP OF SAN FERNANDO EARTHQUAKE
(FEBRUARY 9, 1971)

Figure 3.7

3.6 ATTENUATION OF ROCK MOTIONS

The variations in rock motions which occur at a given distance from the epicentral region can vary considerably. A recent study was completed by Trifunac and Brady (1975) [3.10] in which different attenuation relationships were evaluated. Figure 3.8 shows a compilation of these relationships by various investigators. Such variations make the prediction of maximum rock accelerations for any given site extremely difficult, and similar variations can be expected to occur in frequency characteristics. For this reason, it appears that the probabilistic approaches will continue to provide the most rational method to determine the rock-motions at a site. Past records must be used to provide a guide to the general characteristics of the motions. These records can then be used to lead to envelopes of motions which anticipate the full range of shaking effects which might develop at a site. The Schnabel-Seed attenuation curves[3.11] fall somewhat midway in the evaluation plot and coupled with the conservatively enveloped "R" spectra in the criteria, the resulting AR spectra in rock are conservative.



RELATIONSHIPS BETWEEN PEAK ACCELERATION AND DISTANCE

Figure 3.8

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CHAPTER 4

APPLICATIONS - SEISMIC DESIGN METHODOLOGY

4.1 CURRENT SEISMIC DESIGN PROVISIONS FOR BRIDGES

4.1.1 INTRODUCTION

The current AASHTO seismic design criteria is at best difficult to apply. The use of new terminology and increased complexity have created an entirely new generation of criteria for the bridge engineer.

This chapter will discuss the development of the criteria, some of the factors considered, and the future directions of Bridge Seismic Criteria.

The Appendix presents recent versions of highway bridge seismic criteria and their commentaries:

1) The 1977 AASHTO criteria. (Appendix E.1)

2) The 1977 California criteria. (Appendix E.2)

4.1.2 RELATIONSHIPS BETWEEN AASHTO AND CALTRANS SEISMIC CRITERIA

The 1975 AASHTO criteria is generally based on a 1975 California criteria, the difference being the determination of peak rock acceleration. In the absence of a suitable map of peak rock-acceleration for a particular area, the user of the AASHTO criteria selects an "A" value generally based on the UBC zone of his site. There are two limitations encountered when applying this procedure:

1) The UBC zones and their equivalent accelerations were developed for surface motions and may tend to give overly conservative answers when applied to rock spectra.

2) Housner's maximum acceleration of 0.5g close to the causative fault was determined prior to the San Fernando earthquake and may be low for certain types of faults.

The 1977 California version requires the use of the elastic spectrum (ARS) for analysis. Subsequent reductions for ductility and risk are then made selectively for each member type.

The "C" curves developed for the criteria were originally "pre-reduced" by "Z" for the convenience of the engineer to obtain equivalent column forces similar to those used in building codes. The engineer then had to be cautioned that the "C" coefficient values did not represent the actual loads that the structure receives but had been pre-reduced to facilitate an elastic column design. In practice, especially when applying the "C" curves to the response spectrum method, the engineer obtains deceptively low deflections and forces in members and abutments.

When performing a response spectrum analysis, the engineer is forced to make modeling assumptions at the abutment supports and hinges, which lead directly to forces and deformations in these areas. The use of unreduced (ARS) spectra gives the engineer a more realistic picture of the actual deformations in the system.

Reductions to the design level can then be made, depending on the component under consideration. For example, an abutment key is more brittle than a ductile column. This component then would require a much lower reduction factor (i.e. higher design force). However, if it were determined that failure of the key would not contribute to a collapse condition, it could be designed to fail before excessive forces reached the abutment.

This refinement in the arrangement of the criteria puts examination of collapse mechanisms, relative component importance, system deformations and energy absorbing characteristics of each structural element in the hands of the engineer. Different reductions for columns, keys and restrainers are included in the criteria.

4.1.3 FUTURE DIRECTIONS OF BRIDGE SEISMIC CRITERIA

An improved national seismic regionalization map was developed by the Applied Technology Council (ATC-3) [4.1]. The resulting maps present in some ways a radical departure from previous risk maps in the United States, both in format and in being based on probabilistic arguments. These maps are used to determine surface accelerations which have a 10 percent probability of being exceeded in 50 years.

This information gives these maps the ability to consider not only earthquake size, but the frequency of occurence as well. The California rock acceleration map [4.2] considers only the maximum credible (possible) earthquake regardless of frequency of occurence.

In the authors' opinion future criteria developments should not lose sight of this maximum credible event especially when evaluating collapse mechanisms and failure potentials; however, the probabilistic approach has merit in evaluating the short-term seismic response of the structure. For example; the importance of many lifelines may require that certain structures remain operational immediately following a "probable" event. By considering the importance of the structure, a corresponding exposure period can be chosen.

Knowing the exposure period, the engineer can then perform an analysis based on the probable event in that period.

4.2 DETERMINATION OF SITE SPECTRA

4.2.1 INTRODUCTION

It has long been recognized that the intensity of ground shaking during earthquakes and the associated damage to structures are greatly influenced by local geologic and soil conditions.

During the last 15 years, investigations have taken two general paths toward characterizing the influence of soil conditions on ground motions:

1) Statistical Procedures

A number of accelerometer records have now been obtained at a number of locations in the same general area to show the major effects of variations in local soil conditions on the characteristics of strong ground surface motions. Studies of records from the 1957 San Francisco earthquake, the Caracas earthquake of 1967 and others clearly demonstrate that there is a definite influence of regional and local geology on the characteristics of the motions developed in rock formations under a site and that the overlying soils modify these rock motions. Recent studies by Seed and others have further refined the available data which appear to have agreement with other previous studies.

2) Analytical Procedures

Several methods for evaluating the effect of local soil conditions on ground response during earthquakes are presently available. Most of these methods are based on the assumption that the main responses in a soil deposit are caused by the upward propagation of shear waves from an underlying rock formation. Analytical procedures based on this concept which incorporate the effects of non-linear soil behavior have been shown to give results in agreement with field observations in a number of cases.

4.2.2 STATISTICAL PROCEDURES TO DETERMINE SOIL RESPONSE

Housner in 1959 proposed the first set of average spectral shapes based on a statistical evaluation of eight accelerometer records from four earthquakes [4.6]. Twelve years later Hayashi, Tsuchida, and Kurata from Japan presented some preliminary results from low-level accelerograms recorded from 38 Japanese earthquakes [4.7]. Recently Newmark, Blume, and Kapur made two independent studies to determine the AEC Regulatory Guide Spectrum [4.8]. With these records as background, a comprehensive study of 104 accelerometer records was recently performed in Berkeley in 1974 by Seed and others [4.3].

The records selected were divided as follows:

1) Rock Sites--28 records

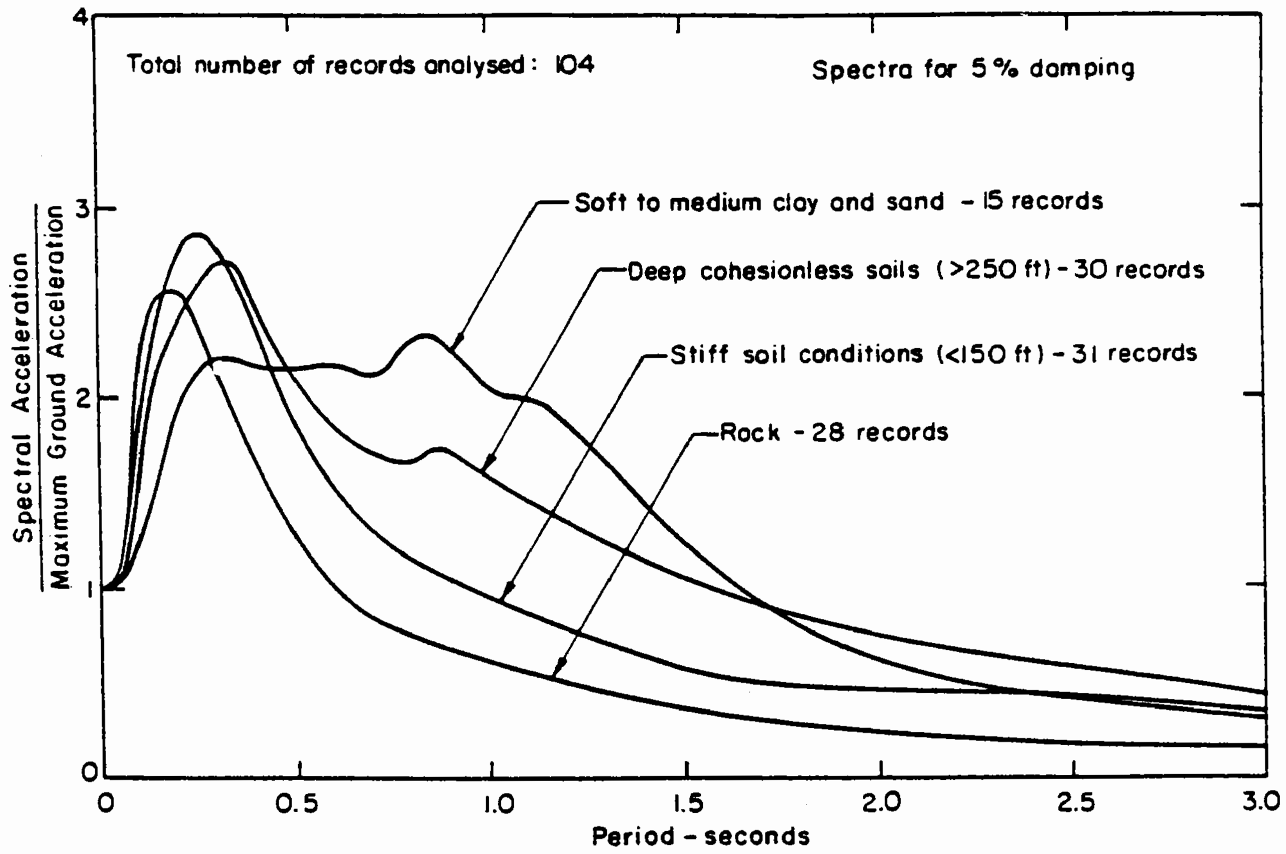
2) Stiff Soil Sites--31 records

3) Deep Cohesionless Soil Sites--30 records

4) Soft to Medium Clay and Sand Sites--15 records

Most of the records were obtained from western U.S. sites and from a small number of Japanese sites.

The average response spectra shown in Figure 4.2.1 represents a compilation of this data [4.3]. This study and others [4.5] permit continued refinement of elastic spectra for structures including bridges.



AVERAGE ACCELERATION SPECTRA
FOR DIFFERENT SITE CONDITIONS

Figure 4.2.1

4.2.3 ANALYTICAL PROCEDURES TO DETERMINE SOIL RESPONSE

The analytical procedure generally involves the following steps:

1) Determine the characteristics of the motions likely to develop in the rock which is under the site and select an acceleration record with these characteristics for the analysis.

2) Determine the dynamic properties of the soil deposit.

3) Compute the response of the soil deposit to the base-rock motions.

The wave propagation program, SHAKE, developed by Schnabel, Lysmer, and Seed in Berkeley in 1972 [4.4] which is based on a one-dimensional model appears to be the most efficient present procedure for computing the response of a soil profile to vertically traveling shear waves. Appendix D.1 contains both a description of this program and the user instructions. More irregular soil deposits, especially those with heavy imbedded structures such as nuclear power plants, may require a finite-element analysis using similar programs extended to 2 and 3 dimensions such as QUAD4 [4.9] and FLUSH [4.10].

4.2.4 DYNAMIC SOIL PROPERTIES

In analyzing a particular site for ground response during an earthquake using the SHAKE program, the following soil parameters are required:

1) The shear wave velocity at a low-strain level for each layer in the system.

2) Unit weight of each layer.

3) Layer thicknesses.

4) Location of ground water level.

5) Material type (rock, sand or clay).

6) Shear moduli and damping relationships for increasing strain levels.

The shear-wave velocity is the single most important parameter needed to assure the program of an accurate estimate of the elastic shear modulus in which:

$$E_s = DS^2 \tag{4.2.1}$$

where

E_s = Dynamic Elastic Shear Modulus

D = In-situ density

S = Shear wave velocity

The relationship of shear moduli and damping for various levels of strain is discussed in Appendix D.4.

4.2.5 DETERMINATION OR SHEAR WAVE VELOCITIES

The determination of the shear modulus is best performed in an undisturbed in-situ location and is accomplished through a variety of geophysical exploration methods and equipment. The basic idea is to generate, identify, isolate, and measure the time-rate of travel of a shear wave from a given source to various monitoring positions. At each step or phase of the test there are significant problems which tend to cloud or mask the result. Recent improvements in geophysical equipment coupled with developmental work at CALTRANS and the University of California, have worked together to produce consistent and reliable S-wave velocities to depths of 200 feet at sites throughout California. This technique [4.11] utilizes a downhole sensor to which a shear wave is propagated from the surface.

Recording is done using a seven-channel light beam oscillograph with 700 hz flat (within five percent) response and approx. 4 cm. full-scale galvonometer deflections. Signals are conditioned using a six-channel amplifier with gain levels from 5 to 2000 giving record sensitivities of 0.1 to 500 MM/MV. Newly developed signal enhancement instruments are also used in this work with good success.

The signal source for generation of the shear wave at the surface is an 8-foot-long wooden plank with a nominal 8 x 12 inch cross-section. The plank weighs 100 lbs. and is placed on smoothed, level ground approximately 10 feet from the plastic cased hole. The front wheels of a vehicle are placed on the plank, resulting in a pressure on the ground of one or two psi. S waves are generated by light to moderate hammer blows delivered to the ends of the plank using a 15 lb. wooden hammer. This technique generates a clean signal which is rich in S waves.

A total suite of records is obtained at 5 to 10 ft. intervals from the bottom of the hole. At each depth, a set of records is made for:

1) Horizontal blows at each end of plank for S waves.

2) Vertical blow for P waves.

Approximately 8 man hours are required to survey a 200 foot hole.

Data reduction must be performed carefully, utilizing all data plotted serially with depth. Opposite phased records are overlaid to verify S wave presence. The use of the actual boring log for the hole is also a very important aid to data reduction.

4.2.6 DEVELOPMENT OF SITE SPECTRA

At the site, a series of down-hole survey locations are selected to reasonably represent the area. It must be remembered that a series of single-dimension SHAKE analyses will be utilized to develop the site spectra.

Extensive geologic knowledge of the site is required. If the site extends for some distance a number of representative soil columns will need to be developed. Foundation borings can be useful in filling in gaps between down-hole survey locations. In any case, a representative soil column for a site or portion of a site can then be analyzed by the SHAKE program. The soil column should be excited by different levels of rock motion, because this parameter, next to depth, is the most significant parameter affecting surface motions. A rock motion which gives as smooth and representative spectrum as possible, (such as described by Romstad) [4.5] should be used (see Appendix D.3). Soil amplification curves ("S") as described in Appendix D.2 are then computed from the SHAKE results.

Elastic surface response spectra (ARS) can then be computed for each configuration of column and acceleration by multiplying "A" by the "R" curve by the "S" curve. (Refer to example problem, Appendix D.2).

Figure (4.2.2) illustrates the development of a complex spectra for a long river crossing:

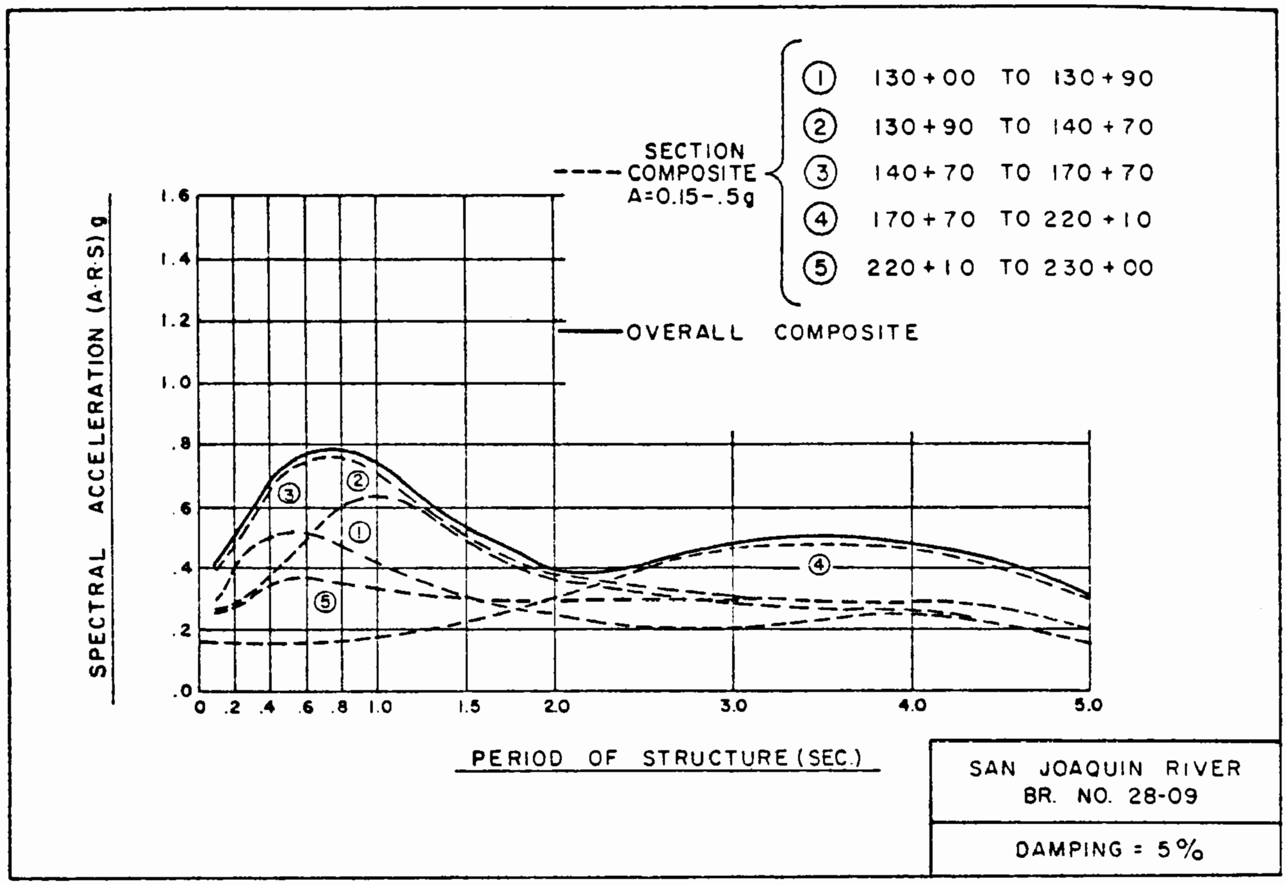
1) The geologists divided the soil profile into 5 zones for seismic response computation.

2) Analysis was performed for each zone subjected to rock motion with peak accelerations from 0.15g to 0.5g.

3) An envelope spectrum was then determined for each zone (shown dashed in Figure (4.2.2)).

4) Finally an overall composite spectrum was drawn over the total length of the project.

Note that Zone 5 is a relatively stiff abutment area while Zone 4 is an extremely soft river area which accounts for its long period.



SPECTRA FOR SAN JOAQUIN RIVER BRIDGE

Figure 4.2.2

4.3 EQUIVALENT STATIC FORCE METHODS FOR BRIDGES

4.3.1 INTRODUCTION

The development of a realistic simplified equivalent static load approach for the dynamic analysis of bridges that would suffice for the final design of simple bridges and could even be used for preliminary design on the more complex bridges, is desirable for the following reasons:

1) Simple extensions of what is currently used and would be easy to implement

2) Does not require a computer

3) Quick and easy to apply

The determination of seismic response by the equivalent static force method basically involves three steps:

1) Calculating the period of the first mode of vibration in the direction under consideration.

2) Obtaining the corresponding response coefficient "C".

3) Distributing the resulting equivalent static earthquake force to the substructure elements.

The formula for the natural undamped period of vibration for the simple single degree-of-freedom systems shown in Figure 4.3.1 is:

$$T = 2\pi\sqrt{\frac{M}{K}} \qquad (4.3.1)$$

where:

M = The mass of the system = W/g

K = The stiffness of the structure, i.e., the force P required to deflect the mass one unit.

Substituting g in units of inches into Equation 4.3.1:

where:

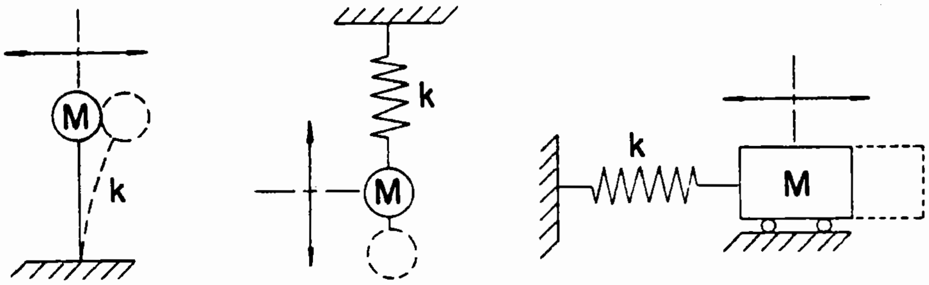
$$T = 2\pi\sqrt{\frac{W/g}{K}}$$

$g = 386.4$ in/sec

$k = P/\Delta$ for $\Delta = 1''$

yields

$$T = 0.32\sqrt{\frac{W}{P}} \qquad (4.3.2)$$



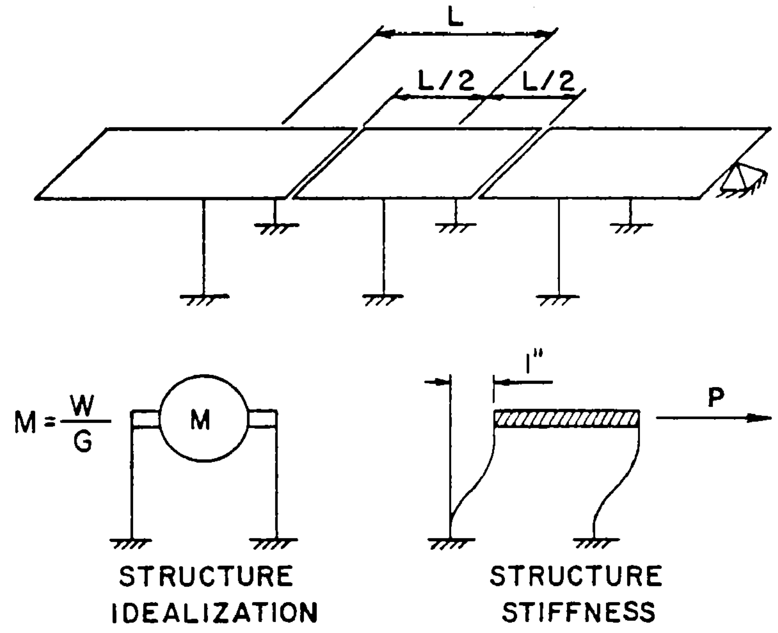
SINGLE-DEGREE-OF-FREEDOM LUMPED MASS SYSTEMS

Figure 4.3.1

4.3.1.1 Lollipop Method

Prior to the San Fernando Earthquake of 1971, bridges were generally designed for earthquake forces using an equivalent static force approach known as the Lollipop Method. In other words, the bridge bents were assumed to act independent of one anotner as single-degree-of-freedom oscillators with a lumped mass equivalent to the tributory deck mass as shown in Figure 4.3.2. Both structure period and load distribution were generally determined using this approach.

The idealization for the Lollipop Method implied the following simplifying assumptions about the dynamic behavior of a bridge:



"LOLLIPOP" IDEALIZATION

Figure 4.3.2

1) Each bent vibrates in its own natural period, independent of the other bents.

2) The transverse bending and torsional stiffness of the superstructure do not contribute to the stiffness of the system.

There are several obvious over-simplified assumptions in this approach. Even for bridges of simple geometry, the assumptions are somewhat questionable. Inaccuracies in the calculation of structural period may result in unrealistic values for the equivalent static earthquake force. In addition, the distribution of this force may also be in error. The main advantage in using this technique is that it was simple and easy to apply.

4.3.1.2 Uniform Load Method

To overcome the deficiencies in the Lollipop Method, an empirical approach, called the Uniform Load Method, was devised with the following objectives:

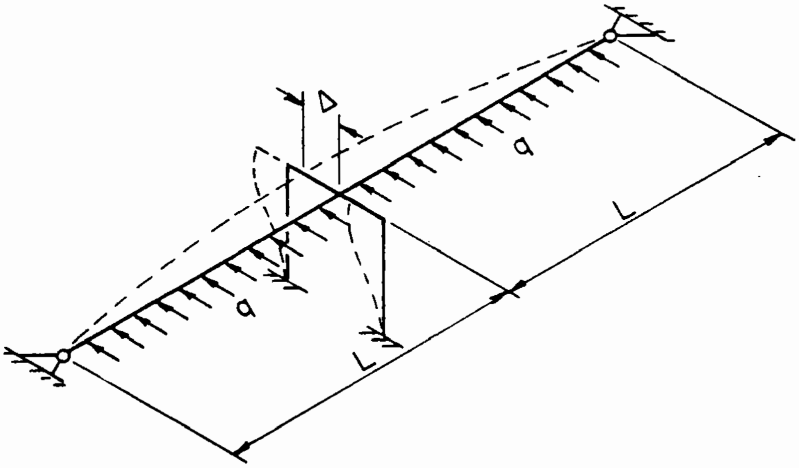
1) Maintain continuity of the superstructure in determining the natural period of the system.

2) Distribute the earthquake force to all of the participating elements of the bridge.

3) Allow for ease of application using seismic design coefficients and static analysis techniques.

The steps in the Uniform Load Method approach can be summarized as follows:

1) Apply a uniform horizontal load q (usually taken as unity) to the structure in the direction of vibration as shown in Figure 4.3.3.



IDEALIZATION FOR UNIFORM LOAD TECHNIQUE

Figure 4.3.3

2) Perform a static analysis on the structure to determine the resulting displacements and member forces due to the applied uniform load q.

3) Adjust the maximum displacement to 1 inch. Using this adjustment factor, adjust the uniform load q to correspond to a maximum displacement of 1 inch.

4) Multiply the adjusted uniform load q by the length of the structure. This is the value for stiffness

which, along with the total dead load W of the structure, can be used to compute the fundamental transverse period of the structure.

5) Having obtained the period, determine the response coefficient "C" from the response curves.

6) Determine the total earthquake force acting on the structure by combining the response coefficient with the framing factor and the total dead load.

7) Convert the total earthquake force into an equivalent uniform load.

8) To determine forces in the members due to this uniform earthquake loading, prorate the forces in the members from the original uniform loading applied to the structure.

The desirability of using a simple approach employing a seismic coefficient in a static analysis, rather than a complex dynamic analysis, has provided the impetus for implementing the Uniform Load Method. Recent experience has shown that this empirical approach gives accurate results for certain types of simple bridges, but it may in some cases require more effort than a response spectrum dynamic analysis. This is because the Uniform Load Method requires a space frame analysis for all but very simple structures to properly analyze the transverse stiffness of the columns interacting with the superstructure.

Several case studies [4.], included in Appendix G, were performed to evaluate the accuracy and limitations of the Uniform Load Method as compared to a response spectrum dynamic analysis. For comparison, the Lollipop Method was also included in these case studies. In selecting bridges for these case studies, different structural and geometric characteristics were considered in order to evaluate the effect of the following parameters:

1) Number of spans

2) Ratio of span lengths

3) Number of columns per bent

4) Curvature

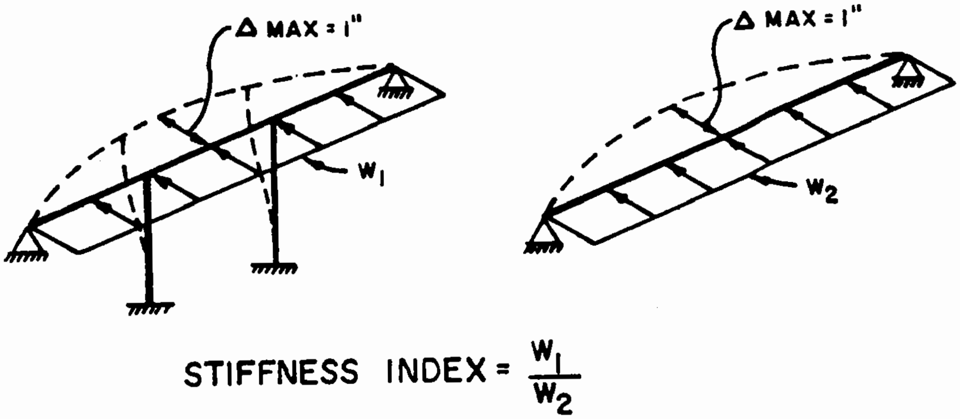
5) Skew

6) Structure width

7) Column length and fixity

An attempt was made to categorize the types of structures which could be accurately analyzed by the Uniform Load Method. It was found that the single most important criterion for categorizing the structure was the relative stiffness between the superstructure and substructure. In order to quantify this criterion, a stiffness index was established.

The Stiffness Index relates the relative contribution of the columns to the transverse stiffness of the entire structure. As illustrated in Figure 4.3.4, the Index is found by taking the ratio of the transverse stiffness of the entire structure, including the columns, to the stiffness of the superstructure alone, acting as a simple beam.



DEFINITION OF STIFFNESS INDEX

Figure 4.3.4

Based on the cases considered, it was observed that the Uniform Load Method can yield accurate results for structures with certain characteristics. Continuous structures on a straight, non-skewed alignment could generally be analyzed using this approach provided the stiffness index was 2 or less. However, for structures with a stiffness index greater than 2, only those with balanced span lengths and equal column stiffnesses could be accurately analyzed. This method was not satisfactory for structures with skewed supports, intermediate hinges, or curved alignments.

Since there are several limitations to the Uniform Load Method and since it generally requires a space frame analysis, there is a need to develop a simple but effective means for applying the equivalent static force approach to bridge structures.

In the development of an equivalent lateral force analysis procedure, it is necessary to determine the period of a structure and the distribution of the resulting lateral force. A reliable method for calculating the period must include the effective stiffness of the deck, restraining devices and soil springs, and the discontinuity of expansion joints, in addition to the individual column stiffnesses. In short, the true dynamic behavior of the bridge should be considered. The period should, if estimated, be an underestimated value to provide a conservative estimate of the equivalent lateral force. It is unlikely all bridge types will lend themselves to simplified techniques, but a large percentage of common types of bridges should be covered. Both longitudinal and transverse modes should be considered. Above all, the method should not require the use of a computer.

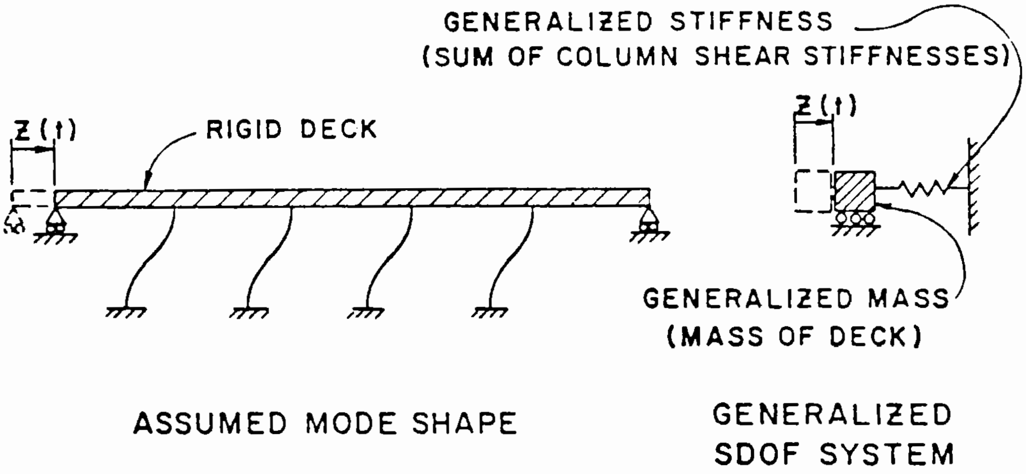
4.3.1.3 Generalized Coordinate Method

Another equivalent static force approach, that shows promise, can also be used to determine the period and earthquake response of certain types of bridges by applying energy principles to a generalized single-degree-of-freedom systems.

For the longitudinal mode of vibration, the structural displacement is characterized by the behavior of a rigid deck, limiting all the columns to equal longitudinal displacements as shown in Figure 4.3.5. This is the classical approach which has been used in the past to determine the longitudinal earthquake force for design.

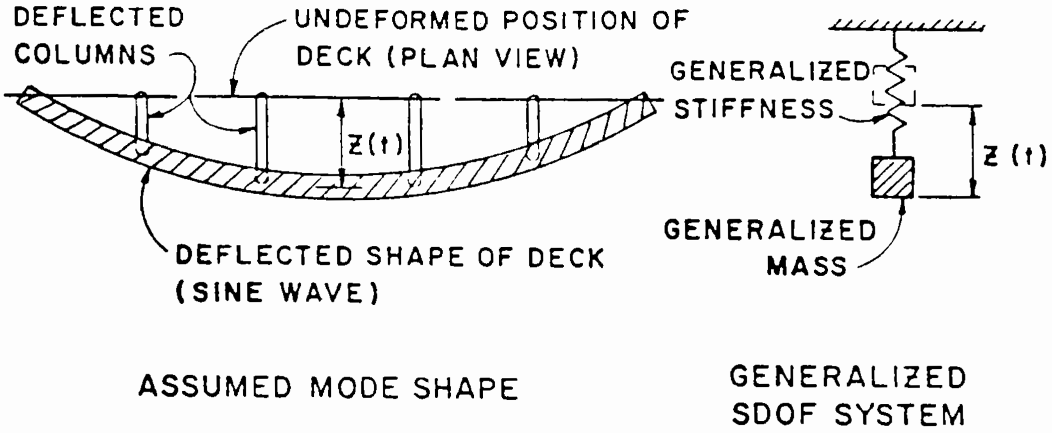
The transverse mode of vibration is more complex in that the transverse displacement of the columns are not all equal, but rather, are functions of their position along the superstructure as shown in Figures 4.3.6 and 4.3.7. In addition to this, the continuous superstructure will undergo bending and will, thus, make a contribution to the potential energy of the system.

The reliability of this method depends on the ability to predict and define the structure's mode shape. The effective application of this technique also requires that one mode dominate in each direction. Fortunately, many of the simpler bridges being designed today satisfy both of the requirements.



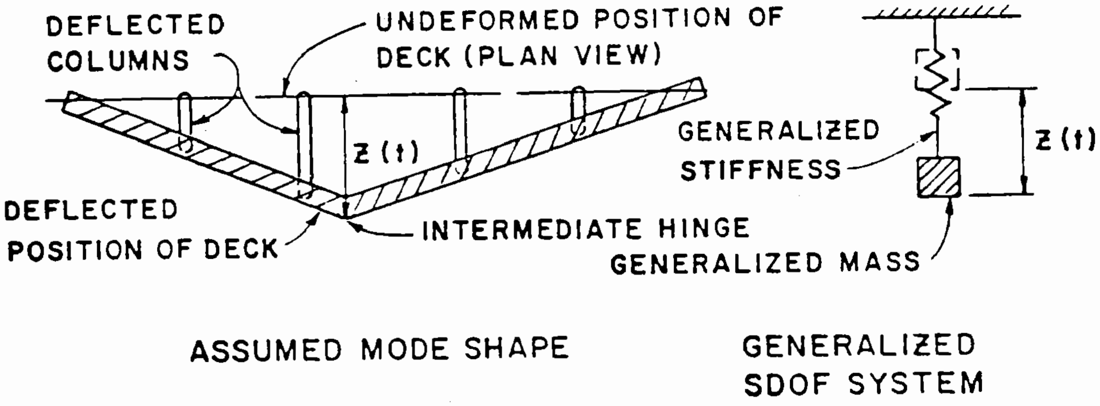
GENERALIZED COORDINATE APPROACH
LONGITUDINAL MODE

Figure 4.3.5



GENERALIZED COORDINATE APPROACH
TRANSVERSE MODE (CONTINUOUS DECK)

Figure 4.3.6



GENERALIZED COORDINATE APPROACH
TRANSERSE MODE (INTERMEDIATE HINGE)

Figure 4.3.7

The method may be applied to girder deck bridges with no more than one intermediate hinge and having the following characteristics:

1) Tangent or nearly tangent alignment.

2) Deck length to width ratio less than 15.

3) Skew angles of the abutments and supports less than twenty degrees.

4) Approximately uniform span lengths and column stiffness.

The basic approach of this method is outlined in the following steps:

1) Assume the predominate mode of vibration and define a generalized coordinate at the location of maximum displacement in the direction under consideration.

2) Calculate virtual work done by external forces and internal member forces as the structure vibrates through a unit virtual displacement at the assumed generalized coordinate.

3) Equate work to zero and solve for the structure period of the predominate mode in terms of the "Generalized Mass" and the "Generalized Stiffness".

4) Determine the seismic coefficient from the appropriate response spectrum chart.

5) Determine the earthquake excitation factor and scale the seismic coefficient.

6) Determine the maximum generalized displacement.

7) Determine the individual column forces using the generalized displacement calculated.

8) Calculate member forces, apply ductility factors and design the member.

It should be noted that the first three steps given above are used only in the development of the formulas. The designer need not repeat these steps for each design since they are implied in the use of the formulas.

This approach was tested on several bridges which has previously been analyzed by the response spectrum technique. In most cases where this approach could be applied, the results compared well with those from the response spectrum analysis. In almost all cases, the comparison was better than was obtained using either the Uniform Load Method or the Lollipop Method.

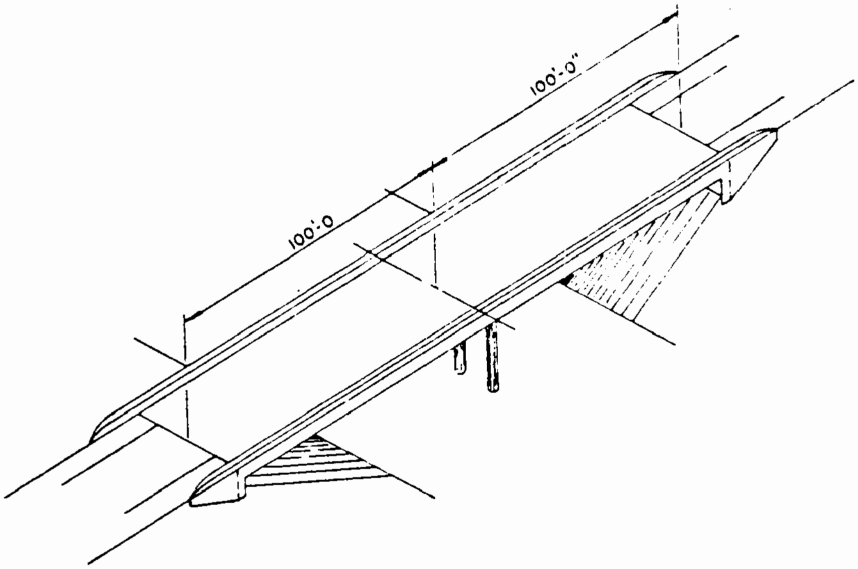
Although the generalized coordinate approach to the equivalent static force method is not widely used, it appears to be a definite improvement over the other two methods.

4.3.2 Applications

To demonstrate the use of the AASHTO Code and application of equivalent static load techniques, consider the structure shown in Figure 4.3.8. The structure and site data are given in Table 4.3.1.

4.3.2.1 Lollipop Method

The structure is idealized as shown in Figure 4.3.9 and the steps, as outlined in Section 4.3.1, are followed as shown below:



EXAMPLE - 2 SPAN BRIDGE

Figure 4.3.8

STRUCTURE DATA:

Span lengths = 100'

Column heights = 40'

E = 3,000 ksi = 432,000 ksf

Column I = 50 ft^4

Dead Load = 7.5 k/ft.

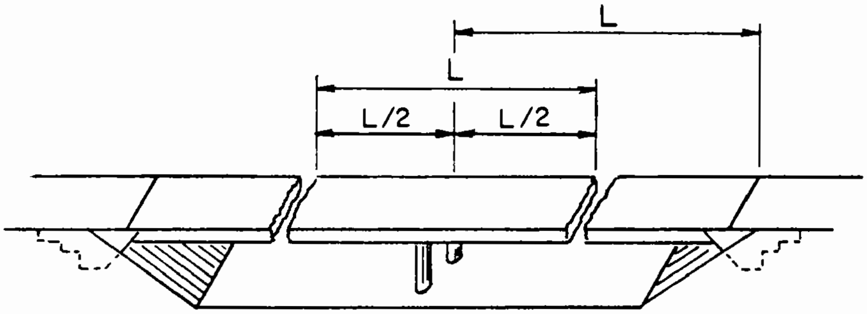
SITE DATA:

Peak Rock Acceleration = 0.5g

Depth of Alluvium to rock-like material=60'

STRUCTURE AND SITE DATA

Table 4.3.1



STRUCTURE IDEALIZATION-LOLLIPOP METHOD

Figure 4.3.9

1) Period calculation in the transverse direction

Determine force required to deflect the two columns 1" for a fixed-fixed end condition (see Appendix F.3):

$$P = (2)\frac{12EI\Delta}{h^3} = \frac{(2)(12)(432000)(50)(.0833)}{(40)^3}$$

$$P = 675 \text{ kips}$$

The contributing deadload W is computed using the tributary as depicted in the figure:

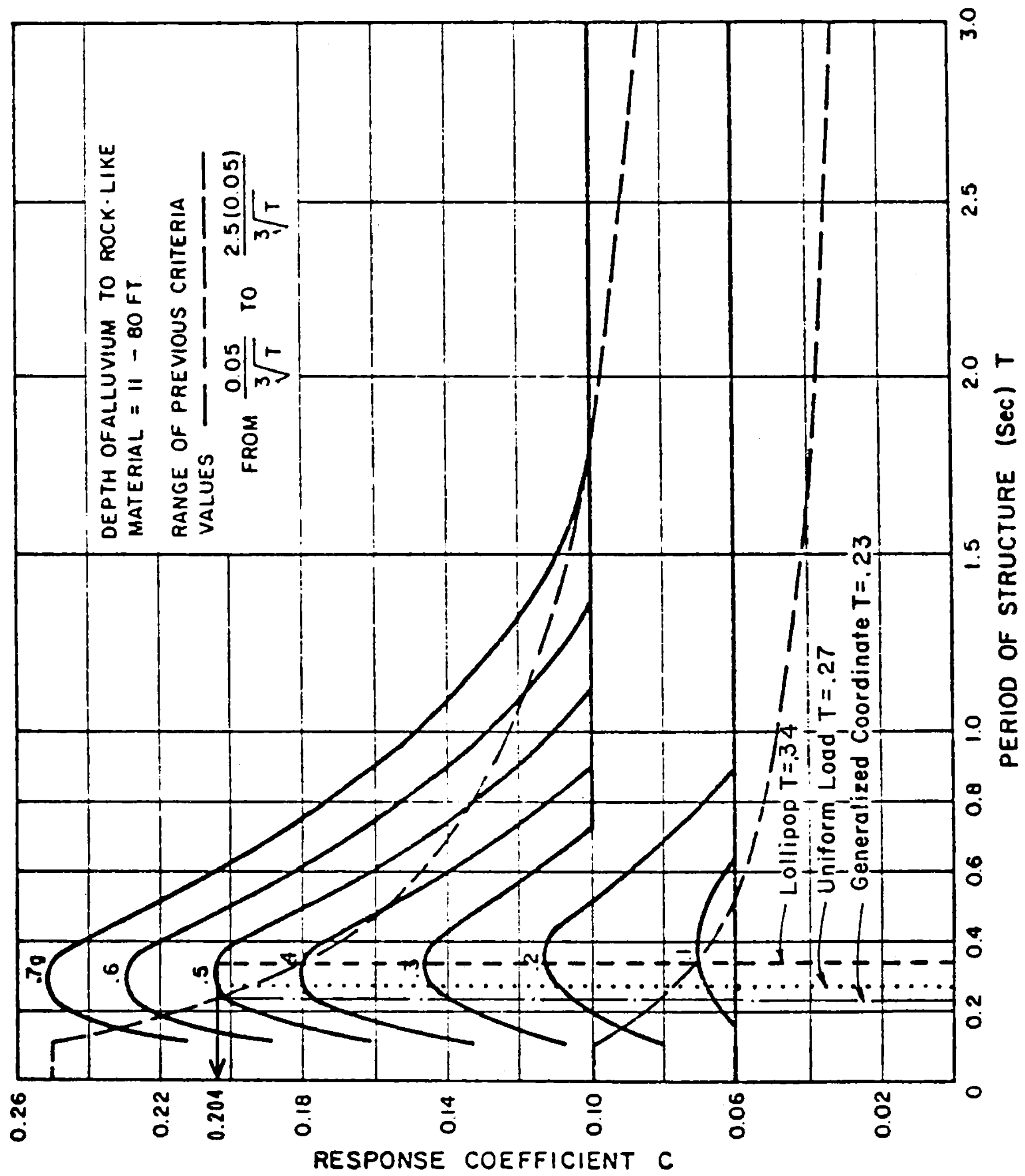
$$W = 7.5 \times 100 = 750 \text{ kips}$$

Substituting into Equation 4.3.2 yields:

$$T = 0.32\sqrt{\frac{W}{P}} = 0.32\sqrt{\frac{750}{675}}$$

$$T = 0.34 \text{ sec}$$

2) From the appropriate AASHTO Response Coefficient Curve, included in Figure 4.3.10, obtain a Response Coefficient C of 0.204.



RESPONSE COEFFICIENT "C" FOR VARIOUS
VALUES OF PEAK ROCK ACCELERATION "A"

(Depth of Alluvium to Rock-Like Material = 11-80 ft.)

Figure 4.3.10

The earthquake force acting on this portion of structure is found from Equation 1 Section 1.2.20 of the AASHTO Specifications (see Appendix H.1).

where:

EQ = C F W

C = Response Coeff. = 0.204

F = Framing Factor = 0.8
(multi-column bent)

W = Dead Load = 750 kips

EQ = 0.204(0.8)(750K) = 122 kips

3) Distributing the Design Earthquake Shears and Moments to the columns yields:

Design Shear = 122K/2 = 61 kips

Design Moment =(61)(20) = 1220 kip-ft

4.3.2.2 Uniform Load Method

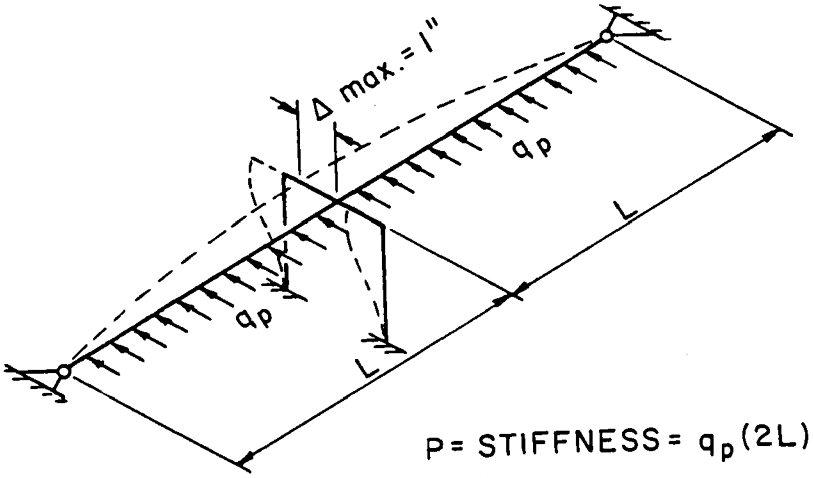
To demonstrate the application of the Uniform Load Method referred to in the AASHTO Code, consider the same bridge again shown in Figure 4.3.8. The steps outlined in Sections 4.3.1.2 are followed to determine the final seismic forces as shown below.

1) Apply a uniform load of intensity q directed as shown on the idealized structure in Figure 4.3.11.

2) Determine the displacements due to the assumed load.

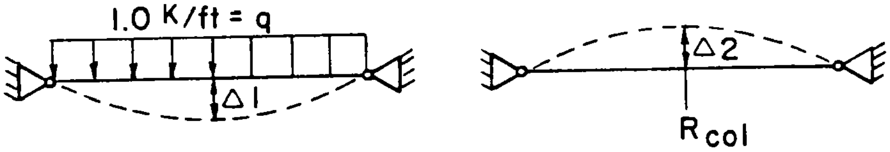
Because of symmetry, the maximum deflection due to a uniform transverse loading will occur at the bent.

By superposition, the deflection at the bent is composed of the deflection due to the uniform load, assuming the columns were not present, minus the deflection caused by a point load equal to the shearing force in the bent. The loads are applied to the superstructure as indicated in



STRUCTURE IDEALIZATION FOR UNIFORM LOAD METHOD

Figure 4.3.11



SUPERPOSITION OF LOADS

Figure 4.3.12

Figure 4.3.12. Notice in this case a uniform load of 10K/Ft was used. The total displacement is given by:

$$\Delta = \Delta_1 - \Delta_2$$

and

$$\Delta_1 = \frac{5qL^4}{384EI} \quad ; \quad \Delta_2 = \frac{RL^3}{48EI}$$

Assuming the columns to be fixed, the shearing force, R, in the bent would be:

$$R = \frac{12EI_c\Delta}{h^3}\ (2)$$

Substituting this value of R into the above equation for and solving for Δ we get:

$$\Delta = \frac{5qL^4}{384EI_s} - \frac{24EI_c\Delta L^3}{48EI_s}$$

or

$$\Delta = \frac{10qL^4h^3}{384E(12I_sh^3+I_cL^3)}$$

Substituting the appropriate values and solving we get the deflection due to the assumed uniform load q:

$$\Delta = 0.007874 \text{ ft}$$

3) Adjust the uniform load to produce a 1" deflection:

$$q_p = \frac{(0.0833)(q)}{\Delta} = \frac{(0.0833)(1.0)}{0.007874} = 10.585\ \frac{\text{kip}}{\text{ft}}$$

4) The force required for a 1" deflection is:

$$P = q_p(2L) = 10.585(200) = 2117 \text{ kips}$$

Contributing dead load W is computed for the entire structure:

$$W = 7.5(200) = 1500 \text{ kips}$$

The period of the structure is:

$$T = 0.32\sqrt{\frac{W}{P}} = 0.32\sqrt{\frac{1500}{2117}} = 0.27 \text{ sec}$$

5) From the AASHTO Response Coefficient Curve, included in Figure 4.3.8, we obtain a Response Coefficient "C" of 0.204.

6) The total earthquake force acting on the structure is computed from Equation 1 Section 1.2.20 of the AASHTO Specification.

$$EQ = C\ F\ W$$

$$C = 0.204$$

$$F = 0.80 \text{ (Multi-Col. Bent Framing Factor)}$$

$$W = 1500 \text{ (Contributing Dead Load)}$$

$$EQ = (0.204)(.8)(1500) = 245 \text{ kips}$$

7) The equivalent uniform Earthquake load is:

$$245/200 = 1.225 \text{ kips/ft}$$

The deflection at the bent due to this equivalent static uniform Earthquake loading is:

$$\Delta_E = \frac{1.225}{1.0}(0.007874) = 0.00965 \text{ ft}$$

8) Design Earthquake shears and moments per column are obtained by prorating the forces obtained from the original loading.

<u>Design Shear</u> $= \frac{12EI\Delta_E}{L^3} = \frac{(12)(432{,}000)(50)(0.00965)}{(40)(40)(40)} = 39$ kips

<u>Design Moment</u> $= (39)(20) = 780$ kip-ft

4.3.2.3 Generalized Coordinate Approach

The method is based on the premise that the mode shape of the vibrating structure can be assumed and expressed mathematically in terms of a single generalized coordinate taken at the point of maximum displacement. It is at least theoretically a better approximation of the dynamic character of a vibrating bridge than the previous two methods. The application of the method consists basically of the following steps:

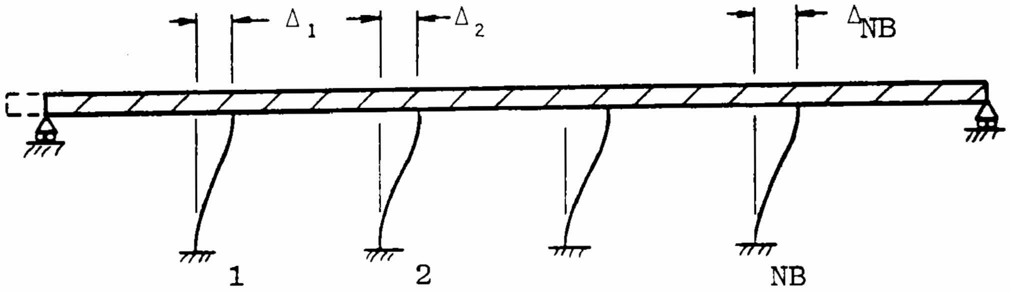
A) Determine the period of vibration for the assumed mode.

B) Determine the corresponding seismic response coefficient "C" from the AASHTO Specifications.

C) Determine the maximum displacement due to the seismic loading.

D) Determine the component forces corresponding to the maximum displacement.

Longitudinal Period - For the longitudinal period, the assumed mode of vibration is characterized by the behavior of a rigid deck, limiting all the columns to equal longitudinal displacements as shown in Figure 4.3.13.



ASSUMED LONGITUDINAL MODE OF VIBRATION

FIGURE 4.3.13

Thus, the displacements are:

$$\Delta_1 = \Delta_2 = \Delta_i = \Delta_{NB} = 1 \qquad (4.3.3)$$

where:

Δ_i = the displacement of Bent "i" set equal to 1.

NB = the total number of Bents supporting the section.

The longitudinal period of a multiple-span bridge can be determined as follows:

1) Define a section to be the portion of the superstructure between abutments and/or, hinges.

2) Calculate the effective generalized mass, M , of the section as the total weight, W, of the section divided by g, gravitational acceleration.

Thus:

$$M^* = \frac{W}{g} \qquad (4.3.4)$$

where:

M^* = the total effective mass of the section in $\frac{\text{kips-sec}}{\text{ft.}}$

W = the total weight of section in kips

$g = 32.2$ ft/sec ^{2}

3) Calculate the total contributing stiffness to the generalized K^*, as the sum of the longitudinal bent stiffness for the bents that support the given section. Column end conditions must be taken into account when calculating longitudinal column stiffness.

Thus the total effective generalized stiffness is given by:

$$K^* = K_1+K_2+\ldots K_{NB} = \sum_{i=1}^{NB} K_i \qquad (4.3.5)$$

where:

K_i = longitudinal stiffness contribution at Bent "i" (i.e., rigidity in shear) kips/ft.

NB = number of bents supporting the section.

4) The longitudinal period T is then given as:

$$T = 2\pi\sqrt{\frac{M^*}{K^*}} \qquad (4.3.6)$$

Transverse Period - The procedure for determining the period of vibration in the transverse direction is governed by the presence of an intermediate hinge. There are two basic cases included:

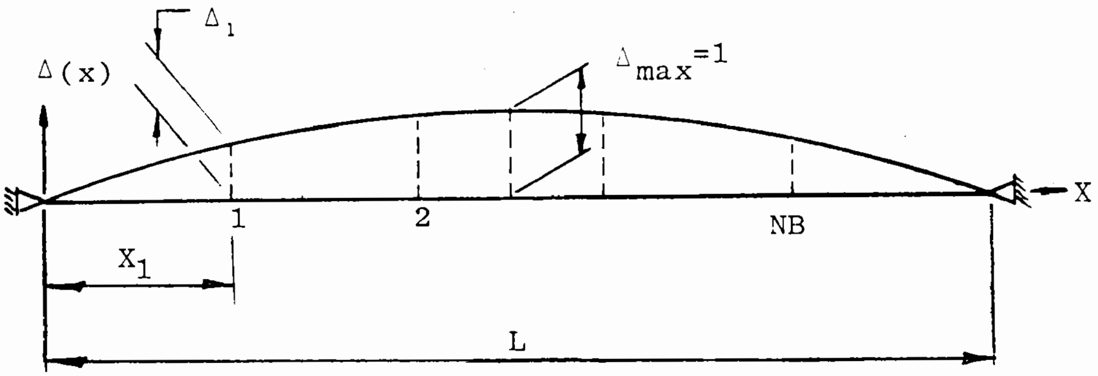
A) Superstructure without an intermediate hinge.

B) Superstructure with one intermediate hinge.

Procedures for transverse period determination for each of these two cases are described below:

Case A: Bridge without an intermediate hinge

The assumed mode of vibration is a half sine wave, which is used to approximate the transverse displacement pattern characterized by a relatively stiff bridge superstructure pinned at the abutments as shown in Figure 4.3.14.



ASSUMED TRANSVERSE MODE SHAPE-NO INTERMEDIATE HINGE

Figure 4.3.14

The displacement function for the assumed mode shape is expressed as:

$$\Delta(x) = \sin \pi \frac{x}{L} \tag{4.3.7}$$

where:

$\Delta(x)$ = transverse displacement at x assuming a unit maximum displacement at the center.

L = total length of superstructure in feet.

x = length along superstructure with the origin located at the left abutment.

The transverse period for Case A may be determined as follows:

1) Calculate the displacements at the bents due to a unit displacement at the generalized coordinate.

$$\Delta_i = \sin \frac{\Delta x_i}{L} \qquad (4.3.8)$$

where:

Δ_i = transverse displacement of bent "i" assuming a unit maximum displacement at the center in feet.

x_i = distance to bent "i" from the left abutment in feet.

2) Calculate the columns' contribution to the generalized stiffness (see Appendix F.3 for appropriate stiffness coefficients):

$$K_c^* = K_1\Delta_1^2 + K_2\Delta_2^2 + \ldots K_{NB}\,\Delta_{NB}^2 = \sum_{i=1}^{NB} K_i\,\Delta_i^2 \qquad (4.3.9)$$

where:

K_i = transverse bent stiffness in kips/ft at bent "i" (assuming for multiple column bents no deck rotation and for single column bents top of column is free to rotate).

3) Calculate the superstructure's contribution to the generalized stiffness due to bending:

$$K_s^* = \frac{48EI}{L^3} \qquad (4.3.10)$$

where:

E = Young's Modulus in ksf.

I = moment of inertia of the superstructure structure for bending in the transverse direction in ft .

4) Calculate the effective generalized stiffness:

$$K^* = K^*_c + K^*_s \qquad (4.3.11)$$

5) Calculate the generalized mass term:

$$M^* = \frac{W}{2g} \qquad (4.3.12)$$

where:

W = the total weight of the superstructure in kips

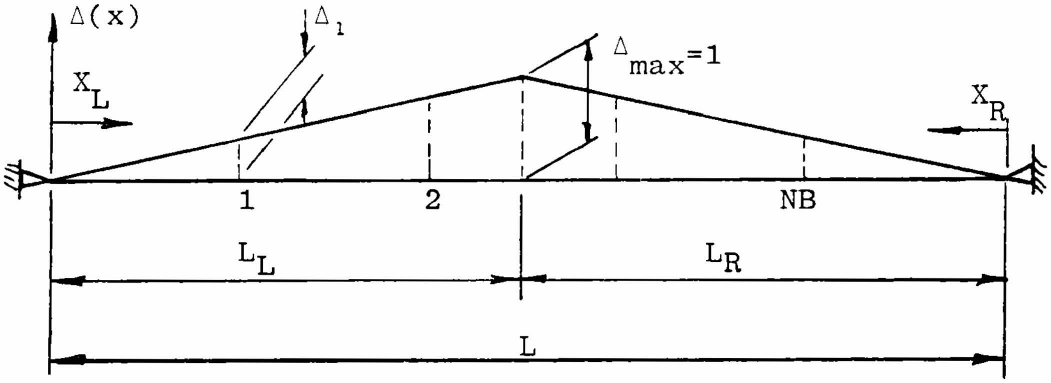
6) The transverse period is then given as:

$$T = 2\pi\sqrt{\frac{M^*}{K^*}} \qquad (4.3.13)$$

Note that even though torsional effects in the superstructure are assumed to have a negligible effect on the total strain energy stored in the system, torsion in the superstructure is assumed to have an effect on column boundary conditions. Thus, for bridges with multiple column bents, torsional rotation of the superstructure at the bents is constrained and the columns are assumed to displace without rotation of the column tops (i.e., a fixed condition is assumed for the column tops). For bridges with single column bents, however, torsional rotation of the superstructure is not constrained and thus, the columns are assumed free to rotate at the top (i.e., a pinned condition is assumed for the column tops).

Case B: Bridges with one intermediate hinge.

The mode of vibration is assumed to be triangular as shown in Figure 4.3.15. Due to the discontinuity created by the presence of an intermediate hinge, bending in the deck is assumed to make a negligible contribution to the generalized stiffness K*. Thus for the assumed mode shape, the displacement function is expressed as:



ASSUMED TRANSVERSE MODE SHAPE
ONE INTERMEDIATE HINGE

Figure 4.3.15

$$\Delta(x) = \frac{X_L}{L_L} \text{ or } \Delta(x) = \frac{X_R}{L_R} \qquad (4.3.14)$$

where:

$\Delta(x)$ = transverse displacement of x a unit maximum displacement at the hinge.

L_R, L_L = length of the right or left section of the superstructure between the hinge and the right or left abutment respectively.

x = distance from the right or left abutment.

The transverse period for Case B may be determined as follows:

1) Calculate the displacements at the bents

$$\Delta = \frac{x_i}{L_s} \qquad (4.3.15)$$

2) Calculate the columns' contribution to the generalized stiffness:

$$K_c^* = K_1\Delta_1^2+K_2\Delta_2^2+\ldots K_{NB}\Delta_{NB}^2=\sum_{i=1}^{NB} K_i\ \Delta_i^2 \tag{4.3.16}$$

where:

K_i = transverse bent stiffness at bent "i" in kips/ft (assuming) for multiple column bents no deck rotation and for single column bents top of column is free to rotate).

NB = total number of bents supporting the superstructure.

3) Calculate the generalized mass term M

$$M^* = \frac{W}{3g} \tag{4.3.17}$$

where:

W = the total weight of the superstructure in kips.

4) The transverse period is then given as:

$$T = 2\pi\sqrt{\frac{M^*}{K^*}} \tag{4.3.18}$$

Earthquake Response Displacement-The maximum displacement in feet at the point of maximum displacement (i.e., the generalized coordinate) is calculated for each of the assumed mode shapes using the following formulas:

Longitudinal

$$Z_{max} = 0.82CT^2 \text{ (ft)} \tag{4.3.19}$$

Transverse - No Hinge

$$Z_{max} = 1.04CT^2 \text{ (ft)} \tag{4.3.20}$$

Transverse - One Hinge

$$Z_{max} = 1.222CT^2 \text{ (ft)} \qquad (4.3.21)$$

Where "C" is taken directly from the AASHTO seismic response coefficient curves

<u>Member Forces-Longitudinal Mode</u> The column displacement in the longitudinal direction are all equal to the maximum displacement Z . The column shears and moments are calculated using these displacements calculated from Equation (4.3.19), and the elastic properties of the members.

$$V_i^L = Z_{max} \qquad (4.3.22)$$

where:

V_i^L = Longitudinal earthquake response displacement at Bent i.

<u>Member Forces-Transverse Mode-No Intermediate Hinge</u> The column shears and moments are computed using the maximum displacement calculated at the generalized coordinate or point of maximum displacement obtained from Equation (4.3.20). The displacements of the individual columns are calculated using the following expression from which the member forces may be calculated using the elastic properties of the members:

$$V_i^T = Z_{max} \sin\Delta\frac{X_i}{L} \qquad (4.3.23)$$

where:

V_i^T = the transverse response displacement at Bent "i".

The resisting forces at the abutments may be obtained from statics by applying the resisting shear forces at each bent location and an equivalent static uniform loading given by the following expression:

$$q_o = 0.81CW \qquad (4.3.24)$$

Note that q_o is an equivalent static uniform loading to be used only for calculation of the resisting forces at the abutments for the superstructure assumed to vibrate in a half sine mode shape.

Member Forces-Transverse Mode-Intermediate Hinge The column shears and moments can be computed using the maximum transverse displacement obtained from Equation (4.3.21). The transverse displacements at the individual bents are calculated using the elastic properties of the members and the displacements obtained by a simple linear ratio of the maximum displacement at the hinge as given by the following:

$$V^T_{Li} = \frac{x_{Li} Z_{max}}{L_L} \qquad (4.3.25)$$

$$V^T_{Ri} = \frac{x_{Ri} Z_{max}}{L_R}$$

where:

L_L = length of the left section of the superstructure.

L_R = length of the right section of the superstructure.

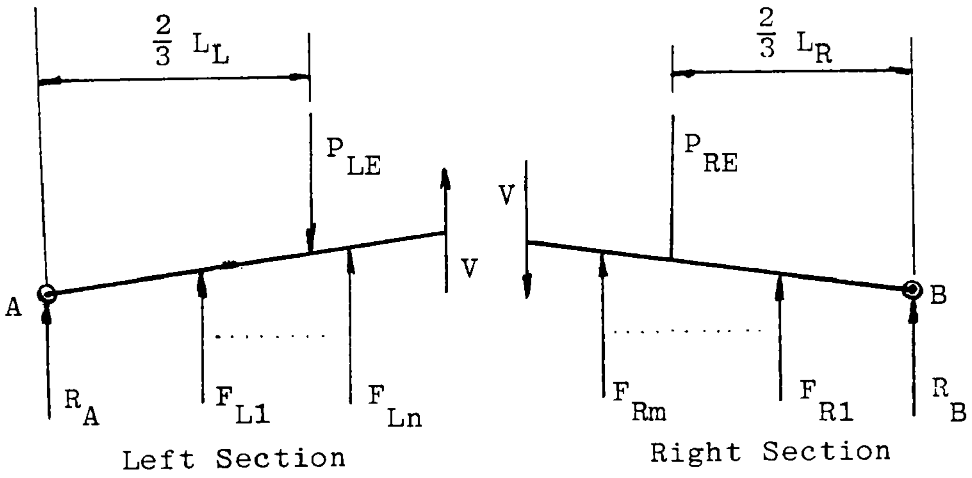
x_{Ri} = distance from the right abutment to the bent.

x_{Li} = distance from the left abutment to the bent.

The resisting forces at the abutments and the shear forces at the hinge may be obtained from statics by applying the resisting shear forces calculated at each bent and equivalent concentrated loads for each superstructure section as shown in Figure 4.3.16. The equivalent concentrated loads are obtained from the following expression:

$$P_{LE} = \frac{0.75 L_L CW}{L} \quad \text{and} \quad P_{RE} = \frac{0.75 L_R CW}{L} \qquad (4.3.26)$$

To demonstrate the application of the Generalized Coordinate Method, consider again the two span bridge shown in Figure 4.3.8. As in the previous two cases, the seismic response to transverse loading will be determined following the steps outlined above.



FREE BODY DIAGRAMS SHOWING LOCATIONS OF
EQUIVALENT STATIC LOADS

Figure 4.3.16

A) Transverse Period

The structure is continuous and assumed to have a transverse vibration mode shape as described for Case A.

1) For this bridge the generalized coordinate is located at the bent. Then the bent is assigned the unit maximum displacement.

$$\Delta_1 = 1$$

2) The columns contribution to the generalized stiffness is given by Equation 4.3.9.

$$K_c^* = K_1 \Delta_1^2$$

and

$$K_1 = (2)\frac{12EI}{L^3} = (2)\frac{(12)(432000)(50)}{(40)^3}$$

thus $K_1 = 8100(1) = 8100$ kips/ft

3) Using Equation 4.3.10, the superstructure stiffness contribution is calculated as:

$$K_s^* = \frac{48EI}{L^3} = \frac{(48)(432{,}000)(3000)}{(200)(200)(200)}$$

$$= 7890 \text{ kip/ft}$$

4) Adding the calculated contributions yields the total effective generalized stiffness:

$$K^* = 8100 + 7890 = 16{,}990 \text{ kips/ft}$$

5) The generalized mass is calculated from Equation 4.3.12 as:

$$M^* = \frac{W}{2g} = \frac{2(7.5)(100)}{2(32.2)} = 23.29 \ \frac{\text{kip-sec}^2}{\text{ft}}$$

6) Substituting K* and M* into Equation 4.3.13, yields:

$$T = 2\pi\sqrt{\frac{M^*}{K^*}} = 2\pi\sqrt{\frac{23.29}{16990}}$$

$$= 0.23 \text{ sec}$$

B) <u>Seismic Coefficient</u>

The seismic coefficient "C" determined from the AASHTO Response Coefficient is 0.203.

C) The maximum displacement at the generalized coodinate, or bent in this case, is given by Equation 4.3.20.

$$Z_{max} = 1.04CT^2 = (1.04)(.203)(.23)^2 = .0112 \text{ ft.}$$

D) The shear force at the bent corresponding to the calculated displacement is:

$$F = \frac{2(12EI)(.8)}{L^3} = \frac{(2)(12)(432{,}000)(50)(.0112)(.8)}{(40)(40)(40)}$$

$$= 72.37 \text{ kips}$$

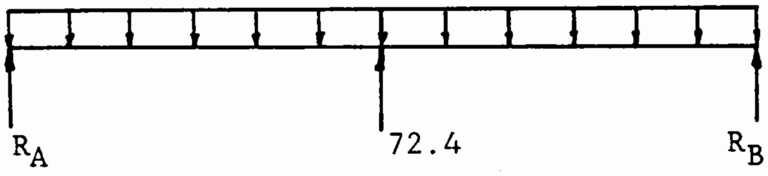
or 36.18 kips/column

Design Shear = 36.18 kips

Design Moment = 724 kip ft

The shear force at the abutment may be obtained from statics by applying the shear force calculated at the bent and an equivalent uniform load given by Equation 4.3.24.

$$q_o = 0.81CWF = (0.81)(.203)(7.5)(.8) = 0.986 \text{ kips/ft}$$



$$R_A = R_B = .5(0.986 \times 200 - 72.4) = 62.4 \text{ kips}$$

4.4 RESPONSE SPECTRUM ANALYSIS

4.4.1 INTRODUCTION

The basic concepts of the response spectrum and its use for dynamic analysis of bridges that can be considered single-degree-of-freedom systems were discussed in some detail in Chapter 2. A response spectrum was defined as the graphical representation of the maximum response of single-degree-of-freedom elastic systems to earthquake ground motions versus the periods or frequencies of the systems. The effect of damping is considered in determining response of the various systems. The most usual measures of response are maximum displacement, V , maximum pseudo relative velocity, $\dot{V}$, and maximum pseudo acceleration, $\ddot{V}$.

As was mentioned in Chapter 2 the response spectrum for a particular earthquake motion is generally irregular in shape with peaks and valleys at different periods. For design purposes, response spectrums are generally smoothed and include the effects of several earthquake motions.

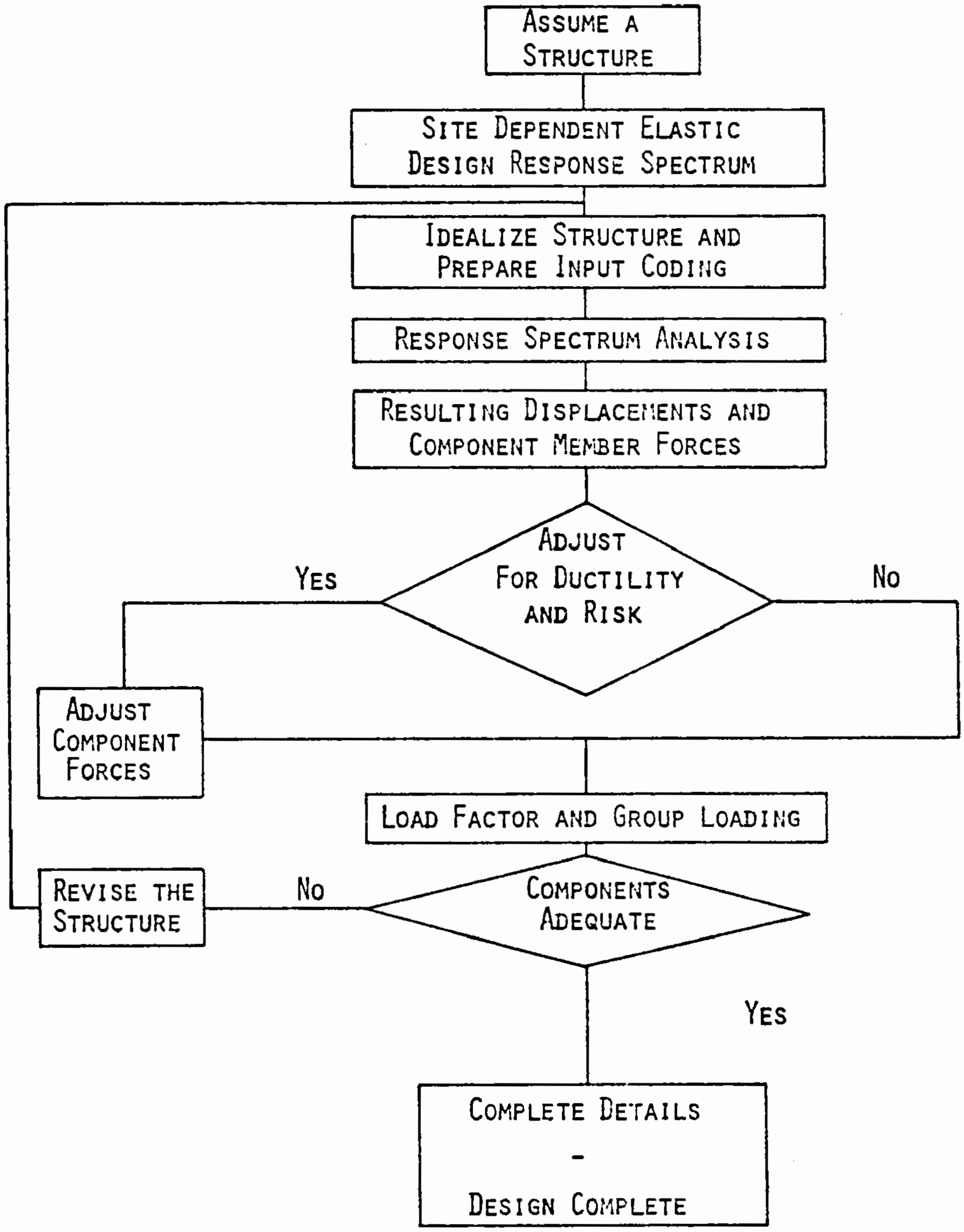
For structures that cannot be idealized as single degree of freedom systems, it is generally necessary to perform a computer analysis of the dynamic response. The response spectrum method appears to be a satisfactory approach for the seismic design of bridges.

When the modes and frequencies of the system have been obtained, the modal responses for a design earthquake loading are determined for each mode considering the participation for that mode. It must be remembered that modal responses as determined above are maximums and generally will not occur simultaneous to the maximum responses of other modes. It is therefore necessary to combine the various modal responses in some statistical manner in order to obtain a realistic value of the actual maximum response of the total structure at a given location.

Since bridges are important links in our surface transportation network, they should maintain both structural integrity and accessibility following an earthquake. It is generally not economical, however, to design a bridge structure to withstand earthquake forces elastically. Current design practice is to rely on the post-elastic behavior of the supporting members to resist structure collapse during a maximum credible event. To maintain the required structural integrity it is necessary to limit post-elastic behavior to an acceptable level. Actual determination of post-elastic response is a complicated problem that is approximated only by the elastic response spectrum technique. Currently this is done by applying a response modification factor that includes the effects of available ductility and risk.

The current seismic design process employing the response spectrum technique as practiced at California Department of Transportation is shown in Figure 4.4.1. The response spectrum analysis is performed with a general purpose linear structural analysis computer program. Unlike the response spectrums currently used by AASHTO, CALTRANS now uses response spectrums unaltered for ductility and risk. Elastic response is determined by taking the Root Mean Square of the individual modal responses. The resulting component member forces are then reduced for ductility and risk depending on the nature of the component. This is also the approach proposed by ATC-6.

The reduced forces are combined with dead-load and other forces for individual component design by the load factor method.



CALTRANS CURRENT SEISMIC DESIGN PROCESS

Figure 4.4.1

4.4.2 IDEALIZATION OF BRIDGE STRUCTURES

As was mentioned in Chapter 2, the analysis of a multi-degree of freedom system such as a complex bridge requires that the structure be modelled to include a finite number of inertia effects lumped on a weightless structural frame. Several general analysis computer programs are commercially available to perform the complicated mathematics associated with the analysis.

Proper modelling of a structural system for a response spectrum analysis generally does not resemble the analytical model used for a static load analysis. Since the dynamic response of a structure is dependent on the inertia forces of the system, the degrees of freedom which must be included in a dynamic analysis are those corresponding to significant inertia contributions. The stiffness effects are sometimes condensed to these degrees of freedom to simplify the solution. For most structures, the rotational inertia contributions are not significant and therefore the rotational inertias at each joint are usually neglected. In the latter cases, the dynamic degrees of freedom are the translational degrees of freedom at each joint. However, in some instances the rotational inertias of certain members are significant in their contribution to dynamic response. In such instances, the rotational inertias are included in the analyses. In general, complex structures should be modelled as a three dimensional space frame. If the computer program selected doesn't have a banding algorithm to optimize the joint numbering, care must be taken to number the joints to minimize the banding.

The number of mass lumps to be included is critical to the analysis. Too few will result in unsatisfactory answers, and too many will increase the computer costs unneccessarily. In general, for deck/column type structures, masses lumped at the quarter points of spans and the third points of columns will yield satisfactory results at the least cost.

Depending on the type of program used for analysis, it is important that the designer keep in mind the effect of various masses. For example, a problem can arise in the modelling of an intermediate expansion joint hinge. Most programs allow for the automatic calculation of mass effects by lumping a portion of the adjoining member masses (usually half) at the joint. To accurately model the expansion joint it is necessary to provide an axial member force release in the member used to model the deck at the hinge. Unless care is taken, a portion of the mass may inadvertently be lumped on the wrong section of the bridge. This can be overcome by including a short member attached to joints on either side of the hinge. Similar problems will occur at abutments and at piers with expansion bearings.

Expansion joint hinges are generally fitted with restrainers to prevent excessive movement and loss of support during an earthquake. The actual behavior of the hinge at an expansion joint is very complex since gaps are provided for normal deck movement. Restrainers are generally only effective in tension, and become active only after the gaps are taken up. Banging of adjacent superstructure sections and forces generated in the bearings present further complications. In an elastic analysis it has been found that modelling the restrainers as space truss members effective in both tension and compression yields acceptable results for overall structural response, and restainer forces which are conservative. In certain programs such as STRUDL it is very easy to consider restrainers eccentric to the centerline of the deck.

There are many considerations in modelling that will become more evident in the example problems and discussions included in Appendices A thru C. The designer must keep in mind that a dynamic analysis is sensitive to inertia as well as stiffness effects. The flexibility of foundations is important and will be considered further in Chapter 6. Rotational inertia effects, soil interaction, effects of water, etc. may also important in some cases. The designer must use good judgment in preparing his model and keep in mind that it should represent the actual conditions.

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CHAPTER 5

DESIGN AND RETROFITTING CONCEPTS

5.1 DUCTILITY

Although bridges may potentially be designed to resist earthquakes elastically, it is generally not economically feasible. It is also inconsistant with the current design philosophy, which relies on energy absorption to avoid total collapse of a bridge in a major earthquake.

This energy absorption in the post-elastic range is commonly handled through the use of a so-called ductility factor, or response modification factor. Ideally, bridge structures should be designed so that the earthquake energy will be dissipated by the individual members acting in a ductile manner, avoiding brittle shear failures. This is, however, not possible in all cases for bridge design, since some of the components may behave in a non-ductile fashion. Since the ductility levels may vary for the individual components of a bridge, reduction of the elastic response spectrums for design may be somewhat misleading and may result in some members being underdesigned. Using elastic design response spectrums to predict the overall structure response and then designing the ductile components to absorb the required energy appears at the present time to be the most rational approach that can be used using elastic dynamic analysis capabilities. This approach is also consistent with the philosophy of the new California Department of Transportation code which attempts to convey to the designer how a bridge would actually behave during an earthquake. It is often helpful to qualify ductility as either being available ductility or required ductility.

The available ductility may be determined either experimentally or analytically. Most of the research done to date deals with the ductility of reinforced concrete beams. Little work has been done on members subjected to bending at various levels of axial load.

Most of the studies conducted to date on ductility as related to bridges have been done at the University of Canterbury, Christchurch, New Zealand. These studies have been conducted on a continuing basis in cooperation with the New Zealand Ministry of Works in Wellington, New Zealand. The most recent applicable studies were initiated by P. D. Leslie [5.1]. Additional studies followed by, B. E. Davey [5.2], A. J. Cameron [5.3], I. R. M. Muro [5.4], and N. G. Heng [5.5]. This overall study is concerned with designing to provide enough ductility in a bridge to avoid collapse by the formation of plastic hinges at predetermined locations.

The work conducted in the initial stages at the University formed the basis for a publication entitled "Ductility of Bridges With Reinforced Concrete Piers" [5.6] written by the New Zealand Ministry of Works to help with the estimation of the available strucure ductility. The materials presented here are based on the work in New Zealand.

The required ductility is either determined by nonlinear dynamic analysis or estimated by considering the results of an elastic dynamic analysis.

Another distinction must also be made between ductility of the section of an individual component of a structure or the overall ductility of a structure.

5.1.1 AVAILABLE DUCTILITY

5.1.1.1 Available Section Ductility

The available section ductility or curvature ductility is defined as:

$$\mu_\phi = \frac{\phi_u}{\phi_y} \qquad (5.1.1)$$

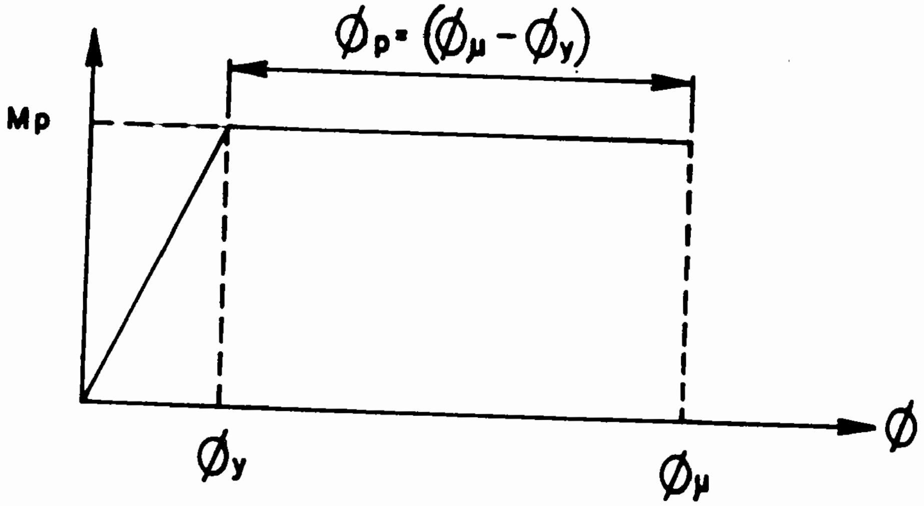
where

μ_ϕ = available section (curvature) ductility
ϕ_u = ultimate curvature of the section
ϕ_y = curvature of the section at yield

This may be represented as shown in Figure 5.1.1 for a constant axial load.

5.1.1.2 Available Structure Ductility

The available structure ductility often referred to as the available displacement ductility can be determined by considering the available section ductilities of the individual components. The available structure ductility is defined as:



AVAILABLE SECTION DUCTILITY

Figure 5.1.1

$$\mu_\Delta = \frac{\Delta u}{\Delta y} \qquad (5.1.2)$$

where

μ_Δ = available structure (displacement) ductility

Δu = ultimate displacement of the structure

Δy = structure displacement at yield

The available structure (displacement) ductility for a simple inverted pendulum structure can be calculated by considering the elastic and post-elastic displacements as follows:

Total displacement

$$\Delta_u = \Delta_p + \Delta_y \qquad (5.1.3)$$

In the elastic range

$$\Delta = \frac{FL^3}{3EI} \qquad (5.1.4)$$

Displacement at yield

$$\Delta_y = \frac{F_y L^3}{3EI} \qquad (5.1.5)$$

The displacement at yield may be expressed as

$$\Delta_y = \frac{M_y L^2}{3EI} \qquad (5.1.6)$$

The post-elastic displacement Δ_p is limited by the available section (curvature) ductility or the ultimate curvature of the section ϕ_u as shown in Figure 5.1.2.

$$\Delta_p = \theta_p \left(L - \frac{h}{2}\right) \qquad (5.1.7)$$

where

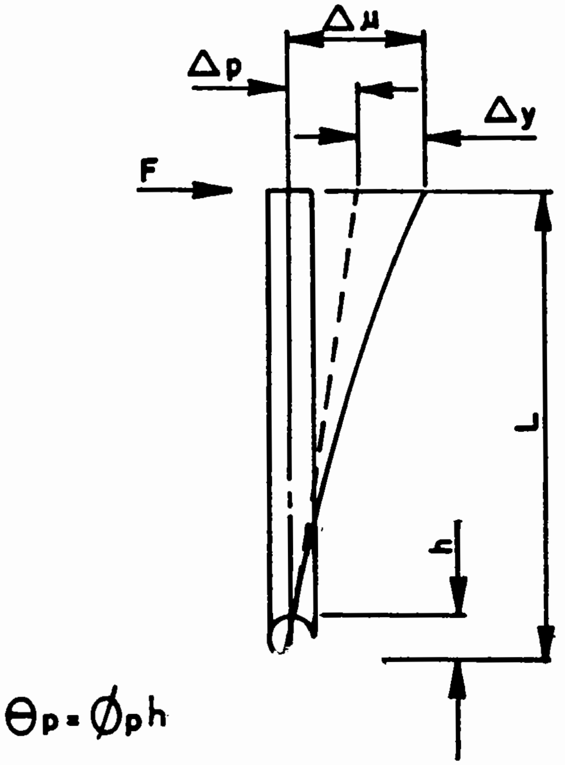
Δ_p = post-elastic displacement

θ_p = total rotation of the plastic hinge

h = length of the plastic hinge

The total rotation of the plastic hinge is given by:

$$\theta_p = h\,\phi_p \qquad (5.1.8)$$



POST ELASTIC DISPLACEMENTS

Figure 5.1.2

Substituting Equation 5.1.8 into 5.1.7 yields

$$\Delta_p = h\phi_p(L-.5h)$$

Knowing

$$\phi_u = \phi_y + \phi_p$$

Yields

$$\Delta_p = h(\phi_u-\phi_y)(L-.5h) \qquad (5.1.9)$$

Rewriting Equation 5.1.3 and substituting in Equation 5.1.9 yields

$$\Delta_u = \Delta_p + \Delta_y$$

$$\Delta_u = h(\phi_u - \phi_y)(L - 0.5h) + \Delta_y \qquad (5.1.10)$$

Having an expression for the ultimate displacement, we can now express the available structure ductility as:

$$\mu_\Delta = \frac{\Delta_u}{\Delta_y} = \frac{h(\phi_u - \phi_y)(L - .5h) + \Delta_y}{\Delta_y}$$

or,

$$\mu_\Delta = 1 + \frac{h(\phi_u - \phi_y)(L - .5h)}{\Delta_y} \qquad (5.1.11)$$

From Equation (5.1.6)

$$\Delta_y = \frac{M_y L^2}{3EI}$$

and assuming from our basic moment-curvature relations

$$\frac{M_p}{EI} = \frac{1}{R_y} = \phi_y$$

Yields

$$\Delta_y = \frac{\phi_y L^2}{3} \qquad (5.1.12)$$

Substituting into Equation (5.1.11) yields

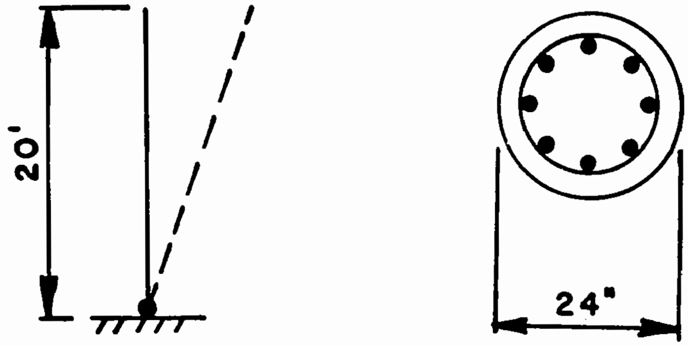
$$\mu_\Delta = 1 + \frac{h(\phi_u - \phi_y)(L - .5h)}{3\,\phi_y L^2}$$

simplifying

$$\mu_\Delta = 1 + \frac{1.5h}{L}\left(2 - \frac{h}{L}\right)(\mu_\phi - 1) \qquad (5.1.13)$$

Thus Equation (5.1.13) gives an expression for the available structure ductility of the simple inverted pendulum in terms of the available section (curvature) ductility, the length of the plastic hinge, (which is generally assumed to be equal to the depth of the member) and the length of the inverted pendulum.

From the Moment vs. Section Ductility graphs shown in Figure 5.1.4 the Section Ductility equals 15 at the onset of strain hardening of the compression steel. Assuming that this is tne maximum desirable section ductility that can be obtained for several cycles of loading from the graph, the following equation is obtained.



SAMPLE CALCULATION OF DUCTILITY

Figure 5.1.3

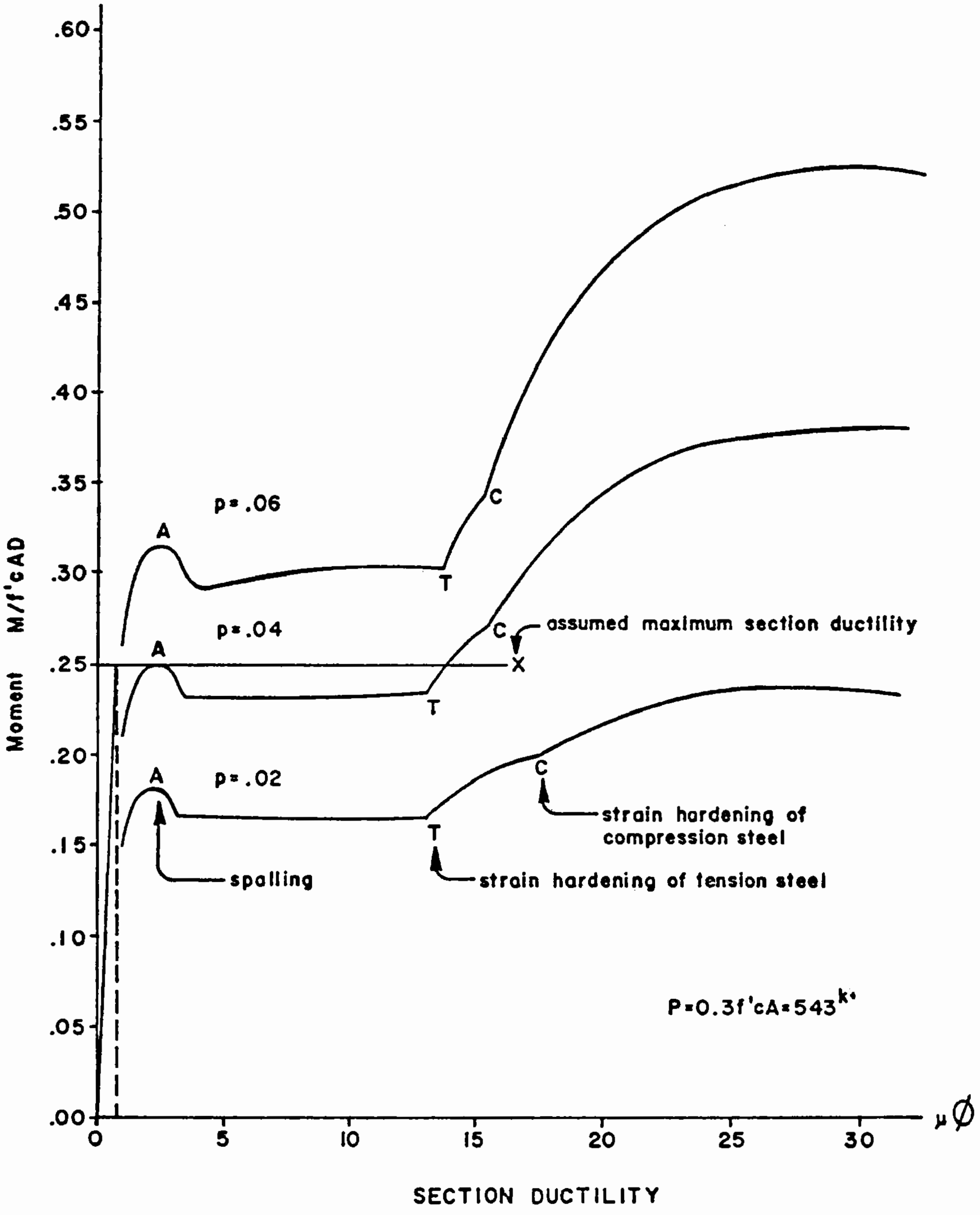
$$\frac{M_p}{f'_c AD} = 0.25$$

From Equation (5.1.13) for μ_ϕ = 15

$$\mu_\Delta = 1 + \frac{1.5h}{L}\left(2 - \frac{h}{L}\right)(\mu_\phi - 1)$$

$$= 1 + \frac{1.5 \times 24}{240}\left(2 - \frac{24}{240}\right)(15-1)$$

$$\mu_\Delta = 1 + 3.9 = 4.9$$



MOMENT VS. SECTION DUCTILITY

Figure 5.1.4

From Equation (5.1.6) the displacement at yield is

$$\Delta_y = \frac{M_p L^2}{3EI} = \frac{10850(240)^2}{3 \times 3000 \times 16300} = 4.26''$$

From Equation (5.1.2)

$$\Delta_u = \mu_\Delta \Delta_y = 5.0 \times 4.26 = 21.3''$$

5.1.2 REQUIRED DUCTILITY

The required structure ductility may be expressed as:

$$\bar{\mu}_\Delta = \frac{\bar{\Delta}_u}{\Delta_y} \qquad (5.1.14)$$

where

$\bar{\mu}_\Delta$ = required structure (displacement) ductility

$\bar{\Delta}_u$ = ultimate required displacement of the structure structure displacement at yield

Consider the simple pendulum structure again subjected to a horizontal ground motion as shown in Figure 5.1.5.

In the elastic range, the structure responds with a load deflection relationship as represented in Figure 5.1.6. Where point b is the maximum response. The area, abc, under the curve represents the potential energy stored at the maximum deflection. This potential energy is converted to kenetic as the mass returns to the neutral position.

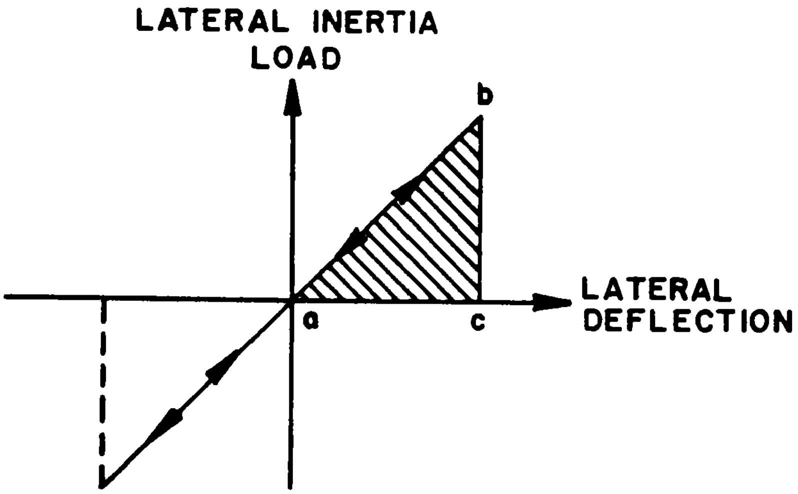
If the oscillator is not strong enough to carry the full elastic response inertia load, a plastic hinge will form as shown in Figure 5.1.7.

The elastic and post-elastic characteristics of this structure are represented by the load deflection curve shown in Figure 5.1.8. When the elastic capacity of the structure is reached, the plastic hinge forms at point d and the deflection response proceeds along line de, and point e represents the maximum or ultimate required displacement of the structure Δ_u.

The potential energy stored at this maximum deflection is represented by the area adef. Note that the force acting on

SIMPLE PENDULUM STRUCTURE

Figure 5.1.5



LOAD VS. DEFLECTION
ELASTIC RANGE

Figure 5.1.6

the structure is limited by the plastic hinge capacity. When the mass returns to the neutral position, the energy converted to kenetic energy is represented by the small triangular area EFG, because the energy represented by the area ADEG is dissipated by the plastic hinge.

The criteria used for determining the required structure ductility factor is dependent on the period of the structure. The following three period ranges and corresponding load reduction factors, R, are generally used for design.

$$R = \frac{\text{Design Load}}{\text{Elastic Response Load}} \qquad (5.1.15)$$

Period Range	Reduction R	Criteria
Short	1	Force
Long	$\frac{1}{\bar{\mu}_\Delta}$	Displacement
Intermediate	$\frac{1}{\sqrt{2\bar{\mu}_\Delta - 1}}$,	Energy

i.) For the short period structure force levels must be maintained, conserving force (acceleration); thus there is no reduction using an elastic analysis.

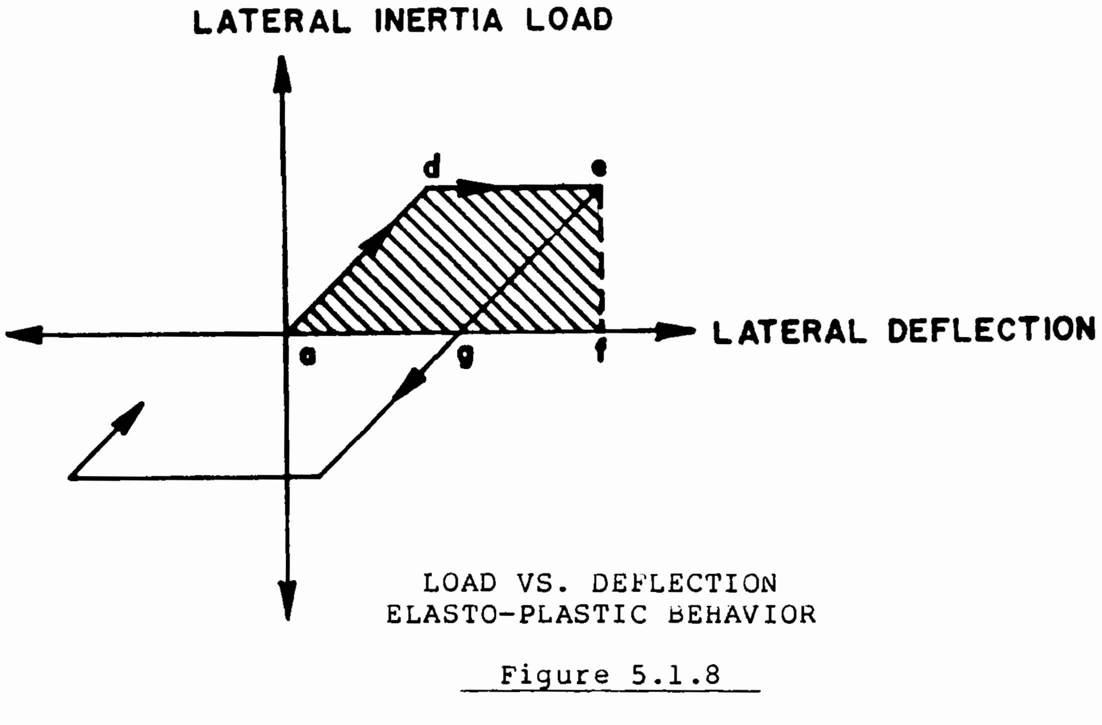
ii.) For the long period structures, the elastoplastic displacements of a structure are assumed to be equal to the elastic displacements. This behavior is represented in Figure 5.1.9.

$$R = \frac{1}{\bar{\mu}_\Delta} \qquad (5.1.16)$$

iii.) For the intermediate period range, energy is conserved and the reduction is based on an equal energy concept. This implies that the potential energy stored in the elastic system at maximum deflection is equal to the energy in the elasto-

SIMPLE PENDULUM STRUCTURE
WITH PLASTIC HINGE

Figure 5.1.7



LOAD VS. DEFLECTION
ELASTO-PLASTIC BEHAVIOR

Figure 5.1.8

plastic system at maximum deflection. This is illustrated in Figure 5.1.10 and requires that area OCD is equal to area OEFG.

From Figure 5.1.10

$$\frac{OA\cdot OD}{2} = \frac{OB\cdot \Delta_y}{2} + (\bar{\Delta}_u - \Delta_y)\,OB$$

or

$$= -\frac{OB\cdot \Delta_y}{2} + \bar{\Delta}_u \cdot OB \qquad (5.1.17)$$

also

$$\frac{OD}{\Delta_y} = \frac{OA}{OB} \quad \text{or} \quad OD = \Delta_y \frac{OA}{OB} \qquad (5.1.18)$$

substituting into Equation 5.1.17 yields

$$\frac{\Delta_y (OA)^2}{2\,OB} = OB(\bar{\Delta}_u - 0.5\Delta_y)$$

or

$$\left(\frac{OB}{OA}\right)^2 = \frac{\Delta_y}{2(\bar{\Delta}_u - .5\Delta_y)} = \frac{1}{2\frac{\bar{\Delta}_u}{\Delta_y} - 1}$$

but

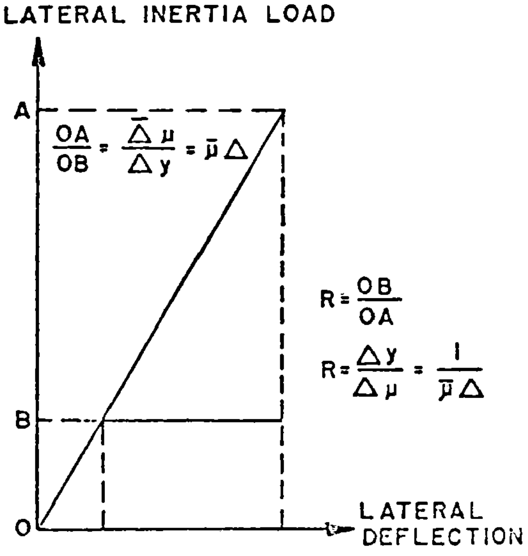
$$R = \frac{OB}{OA}$$

yields

$$R^2 = \frac{1}{2\bar{\mu}_\Delta - 1} \quad \text{and} \quad R = \frac{1}{\sqrt{2\bar{\mu}_\Delta - 1}} \qquad (5.1.19)$$

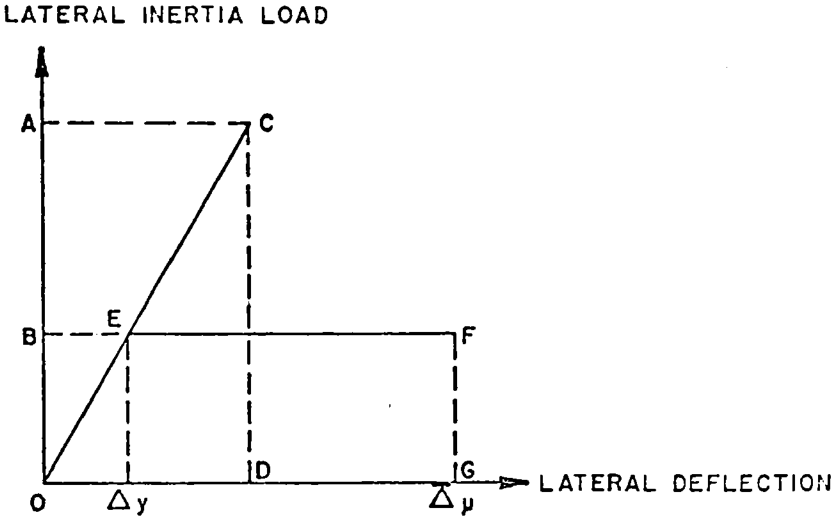
Thus from the above formulations it is possible to determine the ductilities required for the intermediate and long period structures for various desired reductions. These values are tabulated in Table 5.1.1 for Several reduction Factors.

Assume that for the oscillator considered that there was evaluated and found to be a required ductility of 4.0. This would yield a reduction in force of .38 for the given loading.



LOAD VS. DEFLECTION
LONG PERIOD STRUCTURE

Figure 5.1.9



LOAD VS. DEFLECTION
INTERMEDIATE PERIOD STRUCTURES

Figure 5.1.10

Reduction	Long Period	Intermediate Period
R		
0.2	5	13.0
0.25	4	8.5
0.4	2.5	3.63
0.6	1.67	1.89
0.8	1.25	1.28
1.0	1.0	1.0

REQUIRED DUCTILITIES VS. DESIRED REDUCTION

Table 5.1.1

The system has an available ductility of 5 indicating that we have not reached the capacity of the system.

5.2 DESIGN OF DUCTILE MEMBERS

5.2.1 INTRODUCTION

If a bridge is to survive a maximum credible earthquake, it is generally necessary that the columns continue to provide resistance to earthquake induced forces after several cycles of yielding. Ideally, the strength of the structure should not degrade during strong ground motion. Following an earthquake, the bridge should be structurally capable of carrying the traffic for which it was designed.

During the San Fernando Earthquake, bridge columns were pulverized due to the many load reversals at stress levels beyond the yield limit [5.7]. Concrete was broken into small chunks and fell from between the reinforcing bars leaving only the unsupported steel to resist the loads. Obviously a column that fails in this manner will be unable to support a bridge during a major earthquake which may last for over a minute.

Since a column is generally expected to yield in flexure during an earthquake, it follows that concrete in the region of yielding is likely to become highly fractured. Cover concrete, which is unreinforced, will surely fail and spall. Concrete shear resistance will be greatly decreased due to the fracturing. The resulting shear failure and grinding of the concrete that is likely to follow will further degrade the column. Bond between steel and concrete will be decreased and unless sufficient transverse steel is provided to contain the longitudinal reinforcing and the concrete within the core of the column, a failure similar to the one shown in Figure 5.2.1 will occur.

SAN FERNANDO EQ COLUMN FAILURE

Figure 5.2.1

It is clear, that in order to prevent these types of failures, the designer must give careful consideration to the structural details within these zones of severe flexural yielding.

Research designed to increase the understanding of the ductile behavior of reinforced concrete columns has been conducted since the turn of the century [5.8, 5.9]. Although there is still much to be learned about the behavior of a concrete column subjected to earthquake loading, this research has contributed to our present understanding of this behavior and is reflected in the current criteria for seismic design of columns.

5.2.2 DUCTILE FAILURE

Failure of a column must be ductile. That is, it must continue to yield beyond the elastic limit without undergoing a sudden brittle failure. For this reason it is desirable to limit the axial load on a column to insure failure below the balance point on the moment-axial load interaction diagram. Failure in this zone is controlled by the reinforcing steel which is a ductile type failure as opposed to the brittle concrete failure that will occur above the balance point. A steel failure is further assured by limiting the amount of vertical reinforcing steel.

The AASHTO code provisions allows both grade 40 and grade 60 reinforcing steel for earthquake reinforcement. However, there would seem to be advantages to using the milder grade 40 reinforcing. Not only is the grade 40 steel more ductile and less likely to strain harden and fail suddenly, but the steel yields at a lower strain level, so that for a given displacement, more energy will be absorbed. This is important to earthquake resistance. It is also worth noting that a smaller bond stress is required to develop the strength of the bar. This could be important because of the highly fractured concrete that can be expected in the yield zone during an earthquake.

5.2.3 CONFINEMENT

Use of hoop or spiral reinforcement to provide concrete confinement in regions of ductile behavior is important to preventing strength degradation.

The strength of a confined concrete member is classically determined by considering separately the contribution of the concrete core and the concrete cover to the axial force resistance [5.8].

While the presence of the confining reinforcement significantly affects the contribution offered by the concrete in the confined core, it does not become effective until the cover spalls. For this reason, the increase in core concrete strength due to the confining reinforcement should be sufficient to replace the load carrying capacity lost due to spalling of the concrete cover. This loss can be computed as follows:

$$\text{Loss} = A_g \phi f'_c - A_{ch} \phi f'_c = (A_g - A_{ch}) \phi f'_c \qquad (5.2.1)$$

where

f'_c = concrete cylinder breaking strength

A_g = area of the gross section

A_{ch} = area of the core

ϕ = factor to compensate for difference between column and concrete cylinder

The required increase in core stress is therefore given by:

$$\sigma_1 = \left(\frac{A_g}{A_{ch}} - 1 \right) \phi f'_c \qquad (5.2.2)$$

By assuming that the confining reinforcement is stressed to f''_s and taking equilibrium on a section of column as shown in Figure 5.2.2, the confinement pressure is determined to be:

$$f_r = \frac{A_{sh} f''_s}{h_c\, s} \qquad (5.2.3)$$

where

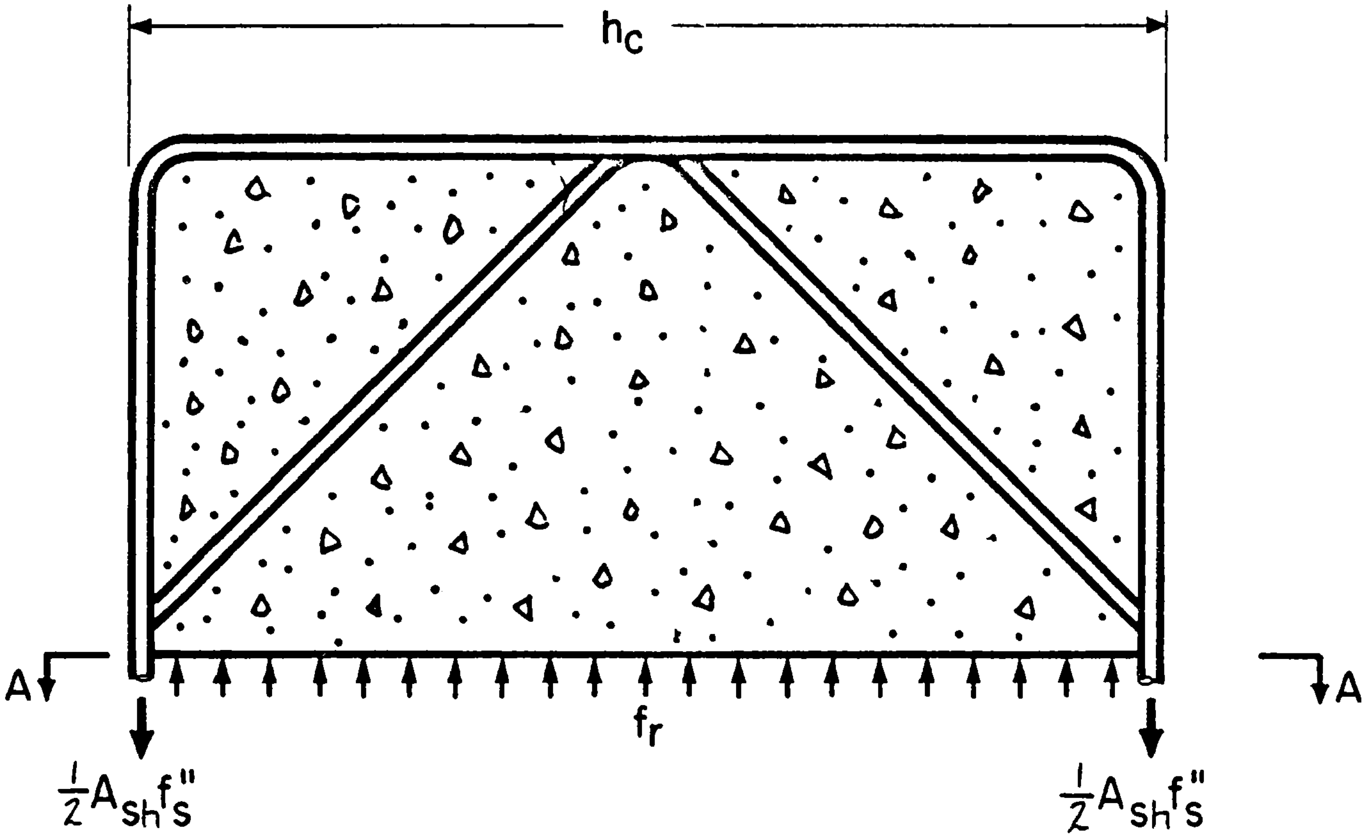
A_{sh} = cross-sectional area of confining reinforcement

f_r = confinement pressure

h_c = width of the concrete core at the section

s = center-to-center spacing of the confining reinforcement along the longitudinal axis of the column

The effectiveness of the confining reinforcement in increasing



COLUMN CONFINEMENT

Figure 5.2.2

the core concrete stress may be written in terms of an effectiveness coefficient, k_o .

$$\sigma_2 = k_o f_r = k_o \frac{A_{sh} f_s''}{h_c s} \tag{5.2.4}$$

Since σ_2 must be greater than or equal to σ_1:

$$k_o \frac{A_{sh} f_s''}{h_c s} \geq \left(\frac{A_g}{A} - 1 \right) \phi f_c'$$

or;

$$A_{sh} \geq \frac{\phi}{k_o} s h_c \frac{f_c'}{f_s''} \left(\frac{A_g}{A_{ch}} - 1 \right) \tag{5.2.5}$$

The Uniform Building Code currently suggests that

$$A_{sh} \geqslant .30 s h_c \frac{f_c'}{f_y}\left(\frac{A_g}{A_{ch}} - 1\right) \qquad (5.2.6)$$

for rectangular hoop reinforcement.

The value .30 reflects the effectiveness of the confinement in increasing longitudinal core concrete stress. For very large sections $(A_g/A_{ch} < 1.4)$, the amount of reinforcing required by the UBC is based on an empirical minimum for confinement and is given by:

$$p'' = .12 \frac{f_c'}{f_y} \qquad (5.2.7)$$

where

p'' = volumetric ratio of spiral reinforcing to concrete core

Although these formulas provide a way of selecting transverse reinforcement for confined concrete, their derivation does not consider the interaction of bending moment or shear. There is some disagreement about whether they are the best formulas for design. For more information on this subject consult "Reinforced Concrete Structures" by R. Park and T. Paulay [5.11].

5.2.4 SHEAR STRENGTH

A column must be able to yield in flexure without experiencing a brittle shear failure. Since yielding of the column in flexure may cause a deterioration of concrete shear capacity, careful consideration must be given to providing adequate shear reinforcement. Since the shear produced in the columns during an earthquake is dependent on the ultimate moment capacity of the columns. The maximum shear that must be resisted by the various columns is given by:

$$V_{uf} = \sum \frac{M_u^T + M_u^B}{L_c} \qquad (5.2.8)$$

where

M_u^T = Ultimate moment capacity at the top of the column.

M_u^B = Ultimate moment capacity at bottom of the column.

Since the main column steel is likely to strain harder during an earthquake, it is advisable to increase the shear force for design purposes to insure ductile failure.

The area of transverse reinforcement required to resist shear A_v, is given by:

$$A_v = \frac{s}{d_c}\left(\frac{V_{uf} - V_c}{\phi f_y}\right) \qquad (5.2.10)$$

where

f_y = yield stress of transverse reinforcement

d_c = dimension of the column core in the direction of the load

s = spacing of the reinforcement

V_c = concrete shear capacity

ϕ = capacity reduction factor for ultimate strength design

The contribution of the concrete to the shear capacity, V_c, varies with the amount of yielding. Judgment should be used in determining the effectiveness of the concrete in shear.

5.2.5 RESTRAINING DEVICES

Prior to the San Fernando earthquake of 1971, resistance to transverse forces at the abutments and intermediate hinges was provided by shear keys, keepers, or other restraining devices of nominal design. For simply supported spans longitudinal force was transferred to the substructure through fixed bearings at one end of the span. In most cases adjacent spans were not tied together. The lack of restraint in this type of design has led to several bearing and structural failures during earthquakes [5.12].

In Japan, earthquakes have caused numerous bridge failures. Many of these failures were the result of inadequate restraint at the bearings. In the Niigata earthquake of 1964, severe structural damage to many bridges was either partially or totally due to this type of bearing failure although liquifaction and permanent support displacements was the cause of many failures. One of the major causes of damage in certain bridges was the lack of restrainers to tie adjacent simple spans together.

The San Fernando earthquake of 1971 pointed out the inability of the substructure to prevent excessive relative longitudinal movement of the superstructure elements in bridges with intermediate expansion joints. Continuous concrete bridges separated at the expansion joints and collapsed. The fact that many bridges sustained severe substructure damage, but remained standing, led to the obvious conclusion that continuous construction provides more earthquake resistance. As a result of this experience, bridge designers began providing restraint to longitudinal movement, usually in the form of high strength steel cables or rods specifically designed with earthquakes in mind.

A test of the effectiveness of these restrainers came in the Guatemala earthquake of 1976 [5.13], when bridges with and without restrainers were subjected to strong shaking. The Rio Agua Caliente Bridge, a fairly modern five span steel plate girder bridge of simple span construction, was damaged during the earthquake. Three of the five spans fell off their bearing supports, whereas only very minor evidence of yielding was observed at the base of the reinforced concrete columns. A nearby railroad bridge of similar construction remained standing, presumably because of the longitudinal restraint provided by the tracks.

The Incienso Bridge, which was built after the San Fernando earthquake of 1971, sustained only minor damage in the Guatemala earthquake of 1976. This bridge was constructed with both hinge and abutment restrainers. Although one of these restrainers failed during the earthquake, their presence resulted in the successful performance of this structure as a whole.

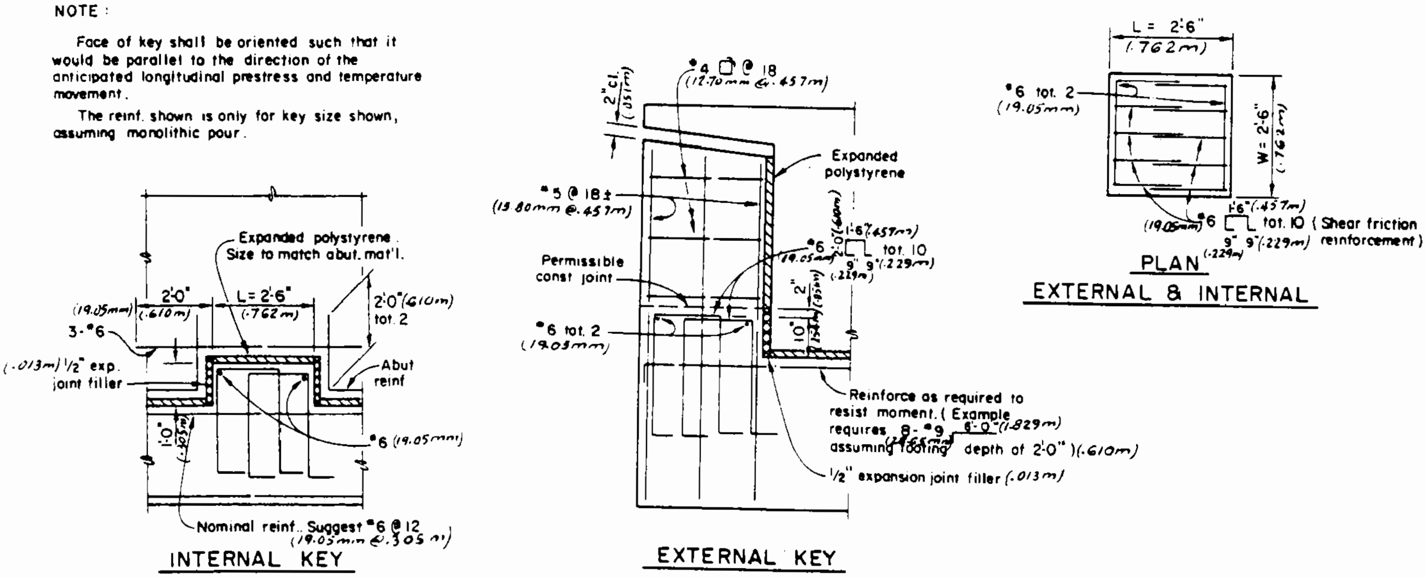
Restraint in the form of shear keys, keepers, cables, etc., has been used almost exclusively for earthquake resistance at bridge abutments and hinges. Restraint represents a cost effective means of mitigating catastrophic failures, but it is generally not effective in absorbing the energy generated by an earthquake. Thus post-elastic behavior of the supporting members must still be relied on for this purpose.

Determination of the forces in longitudinal cable restrainers is difficult if not impossible to predict analytically. This is because of the nonlinearity created by the gaps provided to allow normal movement of the superstructure due to temperature, creep, prestress shortening, etc. These gaps will vary depending on temperature and many other factors. Therefore, for the designer to predict actual service loads during an earthquake presents a problem. For this reason and because restrainers are relatively inexpensive, a simplified formula which is designed to predict the maximum conceivable force is generally used for design. The AASHTO code requires that restrainers be capable of taking a tensile force equal to 25 percent of the deadload of the lightest adjacent section of superstructure less the column shears [5.14].

An alternate, but still conservative, method of predicting force in cable restrainers is to include them in the elastic dynamic analysis as members which can be loaded in both tension and compression. Since the cables currently used in restrainers are considered to be nonductile components, no reduction should be made to the resulting elastic forces for ductility. Past as well as current AASHTO earthquake design response spectra loadings are reduced by a factor to account for ductility and risk. Use of these reduced loadings in determining forces at the bearings is likely to lead to unconservative results since tests have shown restrainers and shear keys do not behave as ductile components. CALTRANS currently uses unreduced earthquake design spectrums loadings, shown in Appendix E.2, to determine resulting component forces and then reduces each of these forces, depending on the ductility of the individual components [5.15]. This approach yields unreduced, more realistic results for restrainer forces.

In the case of abutment shear keys, it has been found that yielding of the columns tends to increase the transverse earthquake load carried at the abutments [5.16]. An approximate method of calculating these forces adopted by the AASHTO code, requires that 25 percent of the "contributing deadload" be applied as a minimum shear force at the abutments. When abutment shear key forces are determined from an elastic analysis, it is recommended that they be increased by a factor of 1.5 to account for column yielding. These keys should be designed to resist the load elastically. Some typical shear key details for concrete bridges are shown in Figure 5.2.3.

As was pointed out before, most restrainers have very little energy dissipation and therfore will not eliminate the need for postelastic behavior in the supporting members. Their main function is to limit the amount of relative displacement occurring between adjacent elements of the structure.


NOTE :

Face of key shall be oriented such that it would be parallel to the direction of the anticipated longitudinal prestress and temperature movement.

The reinf. shown is only for key size shown, assuming monolithic pour.

Note: The use of internal key should be avoided when possible because of the difficulty in repairing them when damaged.

SHEAR KEY DETAIL

Figure 5.2.3

5.2.6 RETROFITTING

By today's standards, most existing highway bridges in this country have not been designed to resist the forces generated during an earthquake. This became evident after the San Fernando Earthquake. Thus, many existing bridges may potentially be damaged or fail if subjected to strong seismic motions. This is clearly undesirable and raises the following questions:

1) Which existing bridges should be retrofitted?

2) What degree of improvement should be made relative to the probability of an earthquake?

Most of the current research and development work to improve the seismic design methodology of bridges is generally most applicable to new construction. Very little attention has been directed specifically toward upgrading the seismic resistance of existing structures. Only two known efforts have been made in connection with retrofitting existing bridges in this country. The Illinois Institute of Technology Research Institute under the sponsorship of the Federal Highway Administration conducted a multiphase research project entitled "Seismic Retrofit Measure For Highway Bridges" [5.17, 5.18], and the California Department of Transportation has, since the San Fernando earthquake, initiated an extensive retrofit program [5.19].

The primary purpose of this section is to introduce some of the design details developed to date in conjunction with the analytical procedures presented in this course for retrofitting existing bridges.

5.2.6.1 Prioritizing Retrofitting Work

In California it was realized immediately after the 1971 earthquake that existing bridges should be retrofitted in order to increase their seismic resistance. A prioritizing system was devised which assigned weighted values to:

1) Type of bearing

2) Width of hinge or bearing seat

3) Restraint of supports

4) Height of structure

5) Type of supports

6) Flexibility of supports

7) Curvature in alignment

8) Probable earthquake intensity

9) Hazard to public on and under structure

10) Disruption to traffice and utilities

11) Danger to building or facilities under the structure

This system worked well for identifying candidate structures for immediate retrofitting. However, the prioritizing number obtained did not always reflect the true relative importance of some structures. In certain circumstances a single factor was important enough to justify a high priority regardless of all other factors. A less important structure rated lower in the number of less important categories, but got a higher overall rating. It was found that any prioritizing system should be subject to an adjustment by good judgment.

There were also practical considerations that did, to some extent, override strict adherence to the prioritizing system. For example, a greater degree of efficiency could be achieved if a number of bridges in one area could be included in a single contract. It was more efficient to prepare plans and let contracts for a few large jobs than a great number of single bridge contracts. A contractor's mobilization costs could be spread out and personnel could be trained and used efficiently on a job with a number of bridges. A single inspector on too small a job would have time to waste unless he could be given other work to do. For efficiency, it was obvious that bridges in a contract should be located reasonably close together.

5.2.6.2 Hinge and Bearing Restrainers

Although retrofitting existing structures will increase their seismic resistance considerably, a designer is limited by economics and the capabilities and feature of the existing facilities . Retrofitting hinges with restrainers can significantly reduce the probability of column failures. Portions of some existing structures have to be strengthened to accommodate the anchorage forces that restrainers require. When hinges are not restrained, segments of a bridge can act independently, and forces in the columns can be significantly greater than if hinge movements are limited.

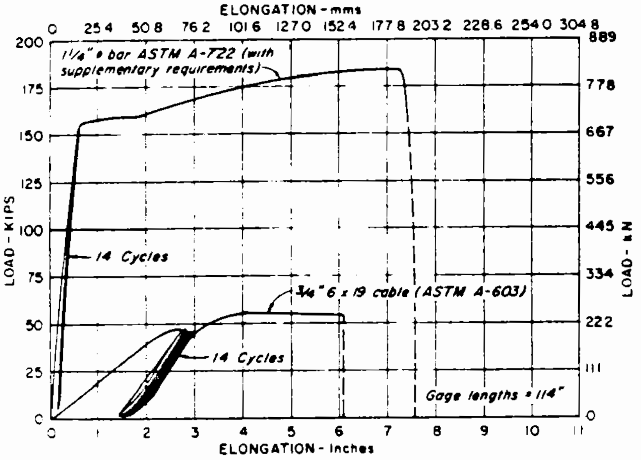
As was mentioned earlier, restrainers should be capable of developing a force equal to at least 25 percent of the weight of the lighter segment of superstructure connected based on working strength design. This rule of thumb is satisfactory for relatively short structures where the influence of the abutment backfill on the superstructure is uncertain. However, dynamic analyses should be made for larger and more complex structures and provisions made for larger forces, if required.

Slightly different assumptions for restrainer arrangements, foundation conditions, column stiffnesses, abutment restraint, linear or non-linear action of the restrainers and columns, etc., can make drastic differences in the results of a dynamic analysis. Compounding these assumptions with the lack of accurate information concerning actual ground movements and distortions at the particular site will lead a practical designer to the conclusion that the seismic analysis of a bridge is a developing art rather than an exact science. Therefore, analysis results should be tempered with judgment.

California has used 3/4" preformed 6 x 19 galvanized cables (ASTM Designation a-603) with a minimum breaking strength of 23 tons (205 kN) as the basic unit for its restraining device. Swaged end fittings are used that are required to develop the minimum breaking strength of the cable. This type of cable and end anchorage have been used in highway barrier systems for many years. They are being tested on a regular basis and have an excellent performance record. 1-1/4" diameter galvanized ASTM A-722 (with supplementary requirements) steel bars that have a specified minimum elongation of seven percent measured in 10-bar diameters are also being used.

If restrainers are permitted to yield, greater joint openings and column deflections will be realized. Once either type of restrainer is stretched beyond its elastic limit it obviously will not assist in closing the joint to its normal position. By using a ductility factor of one for restrainers, the forces, in the restrainer will generally not exceed yield stress. Many older bridges which are being retrofitted have shear key that are inadequate for keeping the two sides of the hinge aligned longitudinally if the structure is subjected to seismic shaking. Since a transverse shearing action at the hinge could cause the rods to fail and become ineffective in tension, supplemental solid mild steel rods are installed through the hinge in order to provide additional shear resistance.

California has conducted a number of tests of 3/4" cables and 1-1/4" diameter bars to compare their qualities as restrainers. Figure 5.2.4 shows the stress-strain relationship of specimens tensioned from near zero stress to specified minimum yield stress (assumed to be 0.85fy for cables) for 14 cycles and then to failure.



EXPERIMENTAL STRESS/STRAIN CURVES
RESTRAINERS

Figure 5.2.4

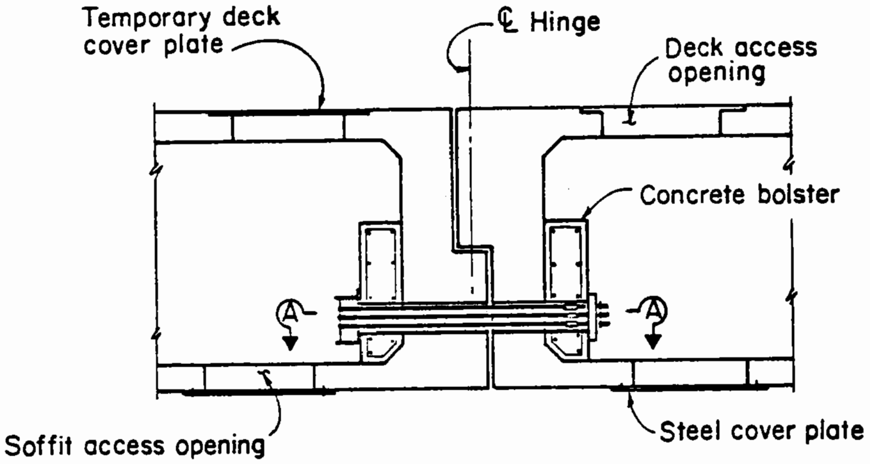
5.2.6.3 Restrainer Details

Figure 5.2.5 shows a commonly used detail for retrofitting hinges of existing concrete box girder bridges. The concrete bolsters are generally required to spread out the concentrated forces of the restrainers so that they don't destroy the hinge diaphragms.

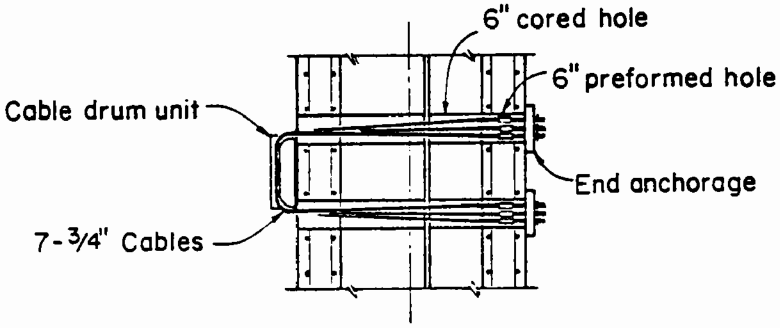
Figure 5.2.6 is a modification of the concept shown in Figure 5.2.5. It is generally restricted to hinges and end support of shorter span T-beam bridges where the restraining force requirements are considerably lower.

The detail shown in Figure 5.2.7 can be used where the diaphragms are not capable of being adequately strengthened and where it would have been less desirable to attach restrainers directly to the girder stems. In this case it may be necessary to place the cable anchorages far enough from the ends of the deck slab to prevent pulling ends out of the spans.

Variations of Figure 5.2.8 have been used in a number of instances where drop-in spans could be expected to fall if



SECTION THRU HINGE



SECTION A-A

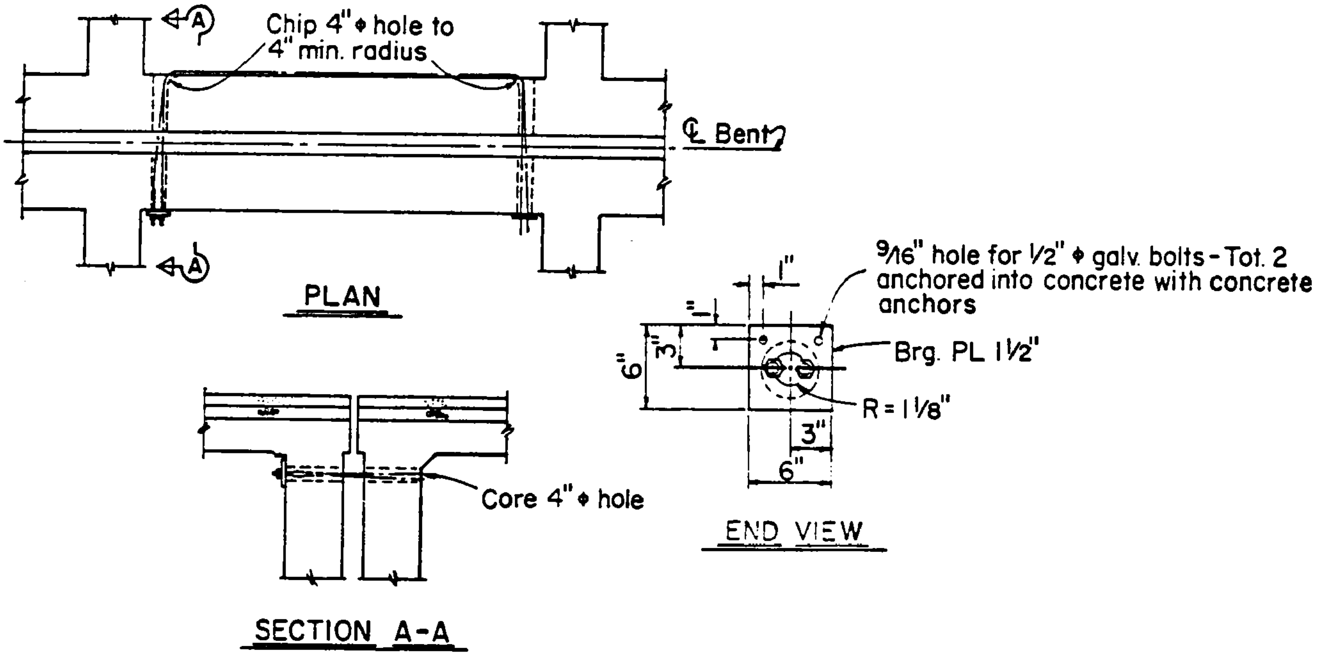
HINGE RETROFIT DETAIL
CONCRETE BOX GIRDER

Figure 5.2.5

the structure were shaken in an earthquake If the hinge seats are very narrow and the cable very long, additional cables might be required in order to limit the amount of stretching under seismic loading. This method is uneconomical in very long spans.

An installation using high strength rods is illustrated in Figure 5.2.9. Cables could also be used in this scheme.

Figure 5.2.10 shows a commonly used detail for restraining



PLAN

SECTION A-A

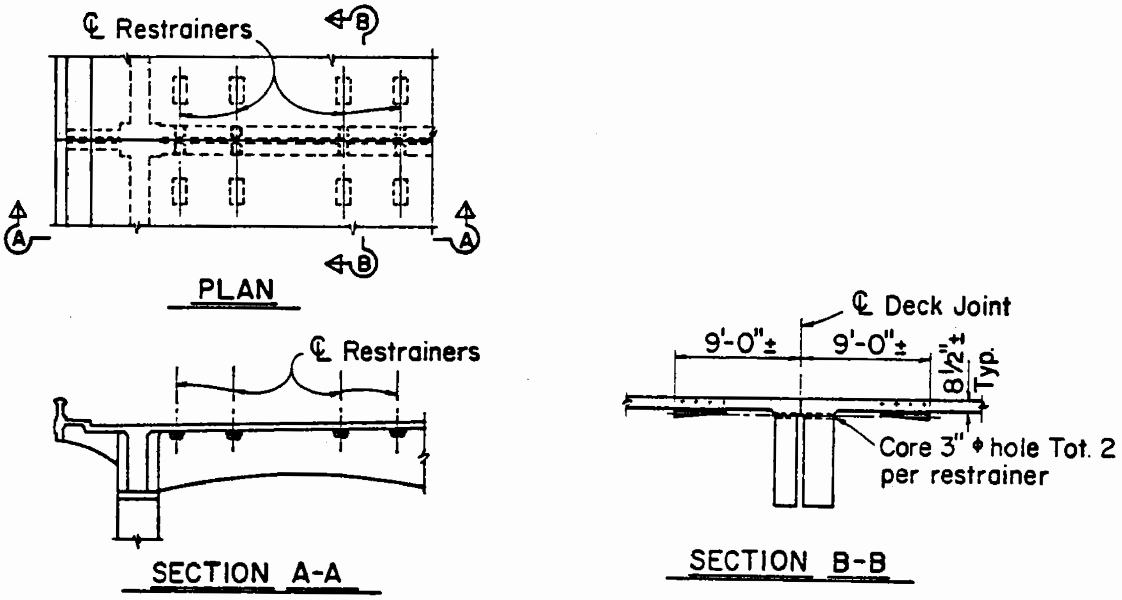
END VIEW

HINGE RETROFIT DETAIL
CONCRETE T BEAM

Figure 5.2.6

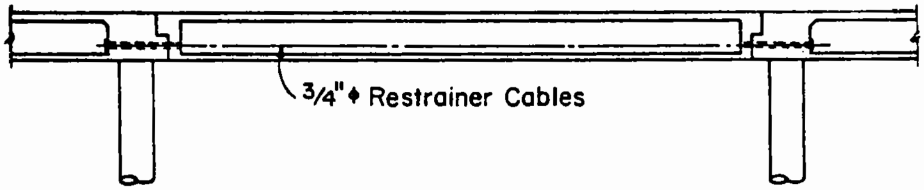
steel girders which are in line with each other. When girders in adjacent spans are offset, transverse beams are attached to the bottom girder flange which are used for anchoring the restrainer cables as shown in Figure 5.2.11. Figure 5.2.12 illustrates a method of attaching the ends of steel girders directly to the supporting concrete bents.

The restrainers illustrated above are only a few of the many type we have used to date. Each bridge has its own peculiarities and requires special attention and details.



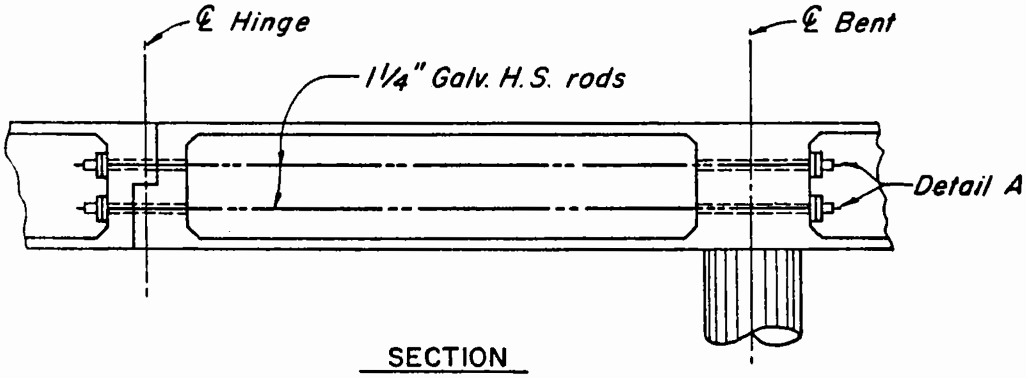
HINGE RETROFIT DETAIL
TIED TO DECK

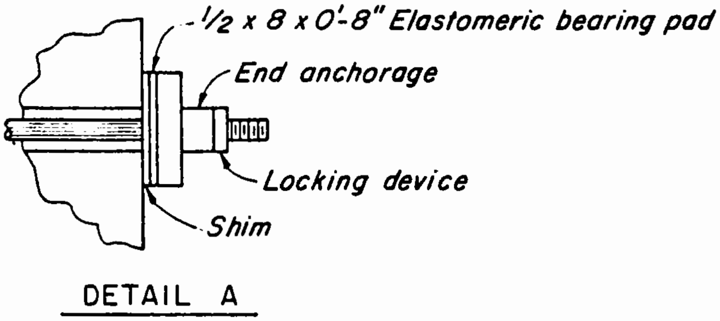
Figure 5.2.7



RESTRAINER RETROFIT
SUSPENDED SPAN

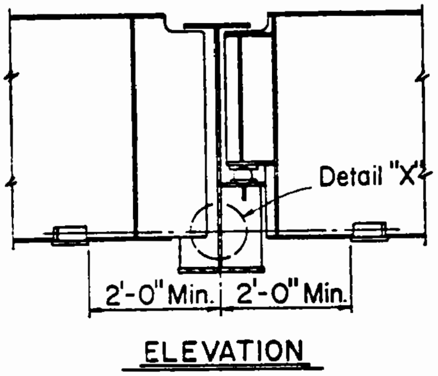
Figure 5.2.8

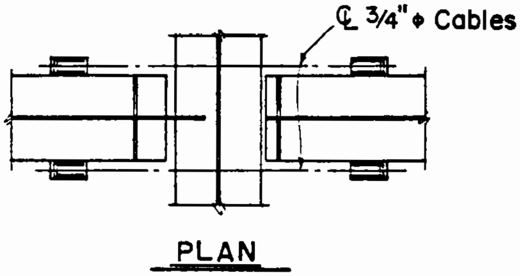




HIGH STRENGTH ROD RESTRAINER

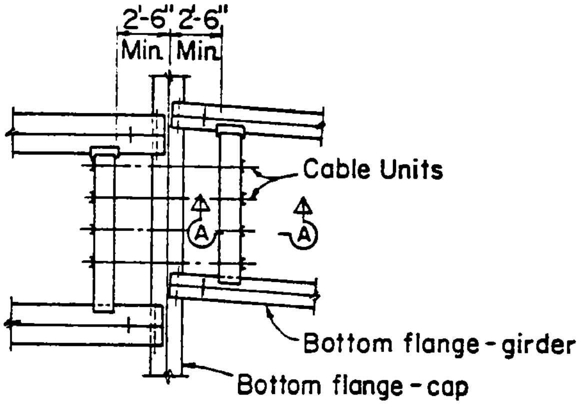
Figure 5.2.9



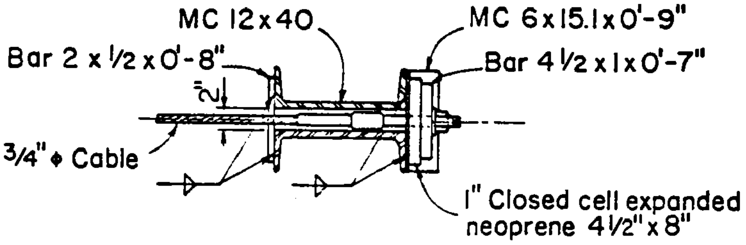


RESTRAINER DETAIL
ALIGNED STEEL GIRDERS

Figure 5.2.10



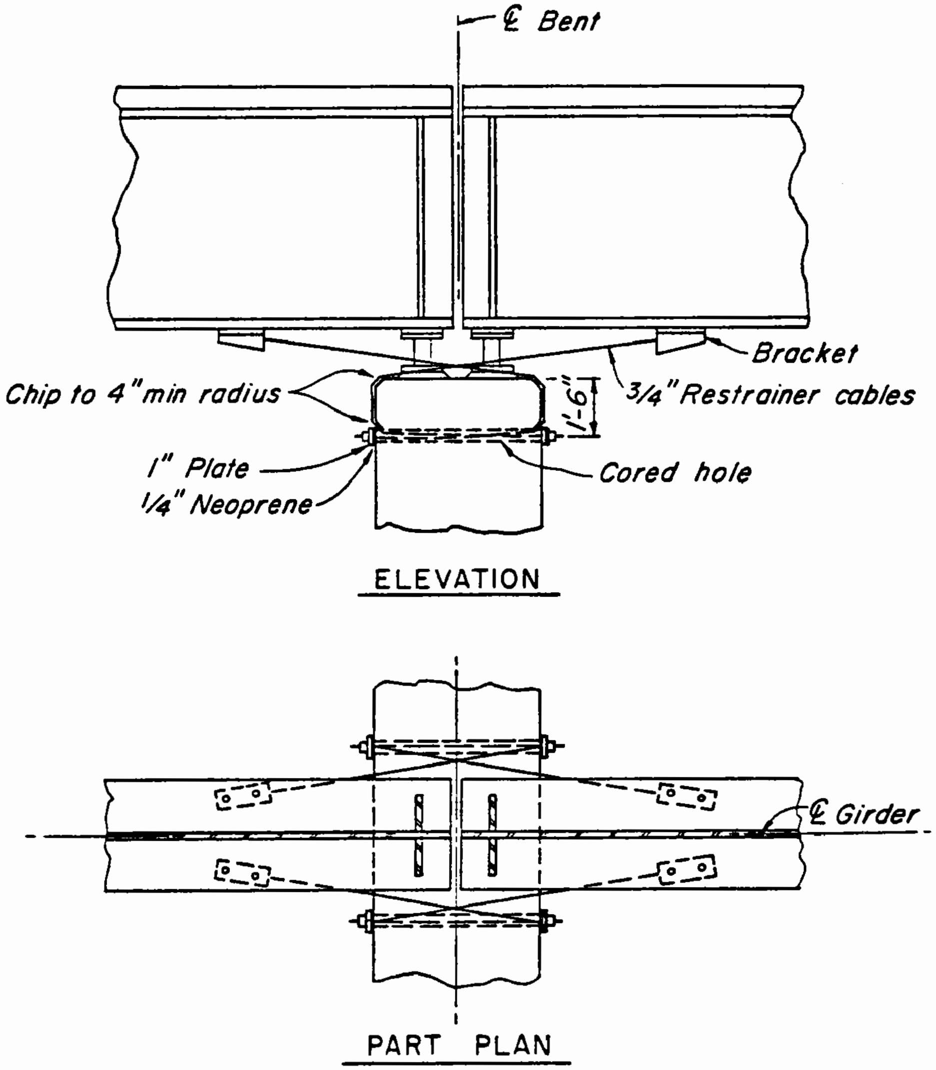
PLAN



SECTION A-A

RESTRAINER DETAIL
UNALIGNED STEEL GIRDERS

Figure 5.2.11



RESTRAINER DETAIL
TIED TO PIER

Figure 5.2.12

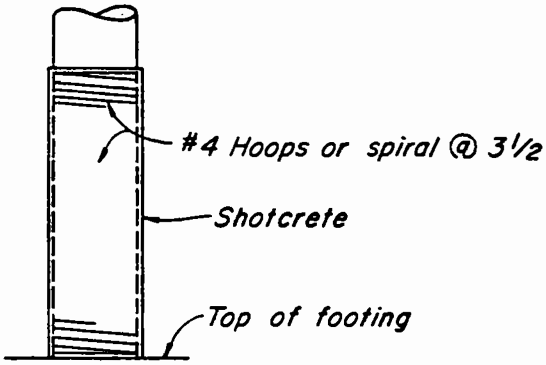
5.2.6.4 Retrofitting Columns

The second greatest weakness of pre-1971 structures pointed out by the San Fernando earthquake was that the reinforcing steel ties in columns did not provide adequate confinement of the concrete. Bridges with single column bents are particularly vulnerable. Since the restraining of the superstructure at hinges and bearings was judged to be a more serious problem, and providing that restraint alleviated the seriousness of the column deficiency, more can be obtained for the money by retrofitting the hinges and bearings first. Methods of retrofitting columns to make them more earthquake resistant are being investigated.

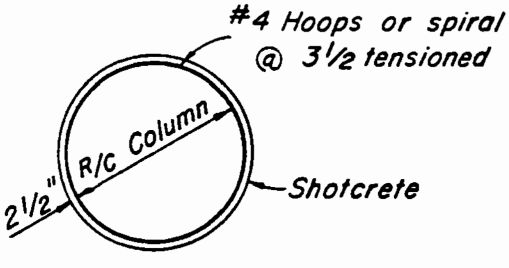
Figure 5.2.13 illustrates reinforcing steel hoops that are prestressed on the outer face of the column which is then covered with shotcrete. The device shown in Figure 5.2.14 was especially designed for this purpose. It is basically a turnbuckle that develops the strength of the reinforcing steel and place an initial pre-stress in the hoop.

The column retrofitting method shown in Figure 5.2.15 consists of wrapping a column with tensioned prestressing wire and applying a protective coat of shotcrete.

Figure 5.2.16 illustrates a method that consists of welding a steel shell around an existing column and filling the space between the shell and column with grout. Weathered steel can be used for achieving an architectural effect, if desired, or ordinary steel can be used and painted. In this case, since it is undesirable to have failure below the ground level, the footing capacity should be investigated to insure it will handle the increased column capacity provided by the longitudinal effects of the steel shell. A typical detail that could be used to retrofit a footing is shown in Figure 5.2.17



ELEVATION



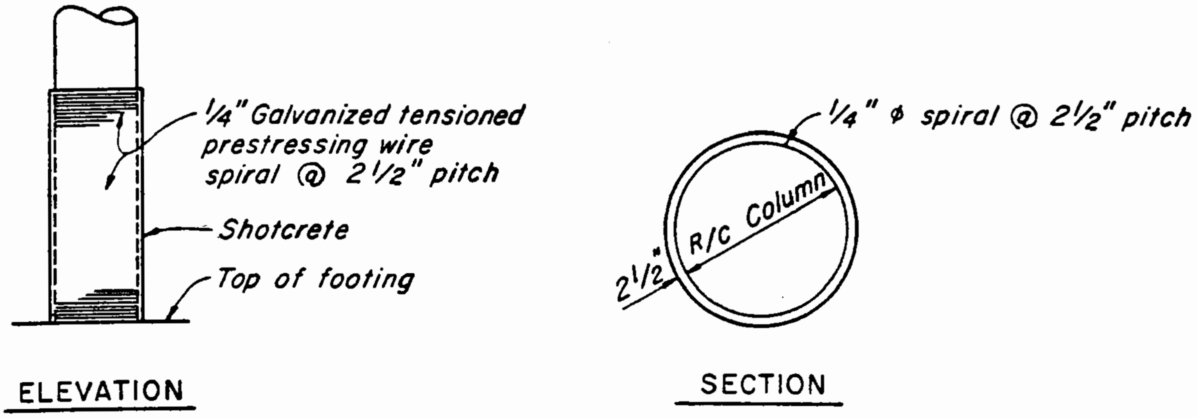
SECTION

COLUMN RETROFIT
STEEL HOOPS

Figure 5.2.13

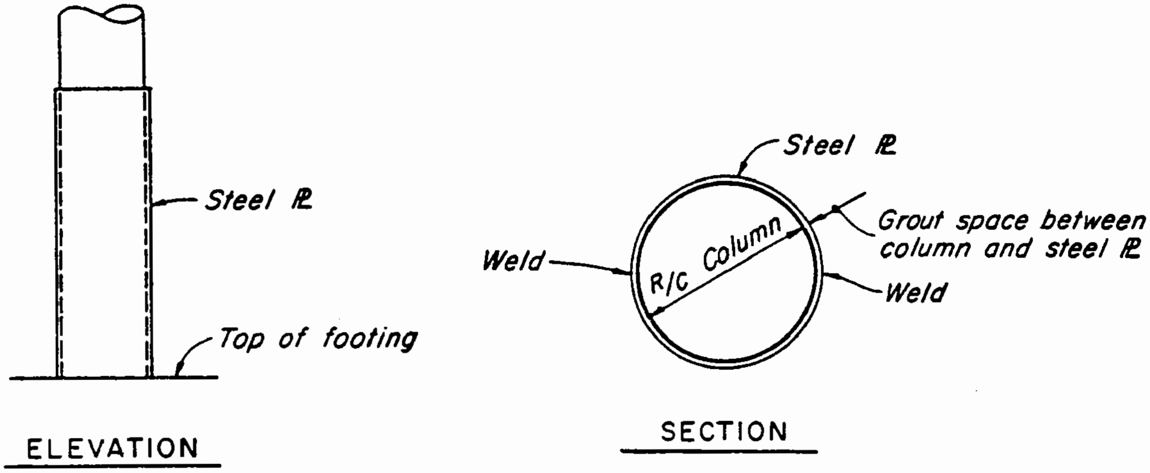
TURNBUCKLE FOR RETROFITTING COLUMNS

Figure 5.2.14



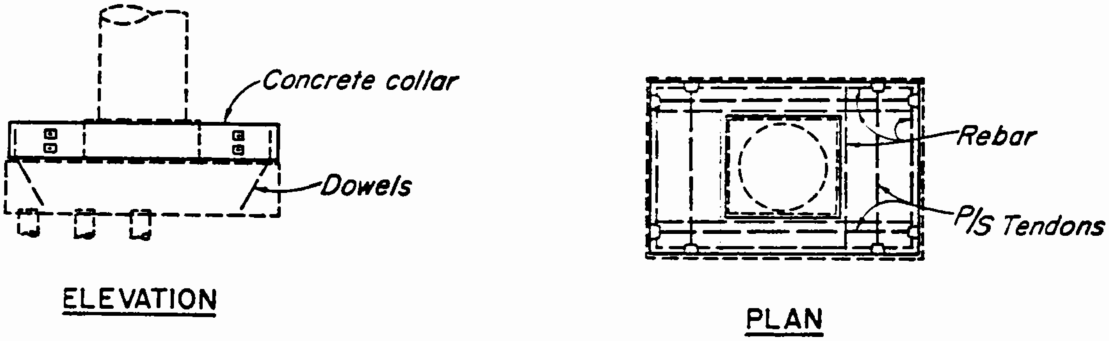
COLUMN RETROFIT
PRESTRESS WIRE

Figure 5.2.15



COLUMN RETROFIT
STEEL SHELL

Figure 5.2.16



FOOTING RETROFIT

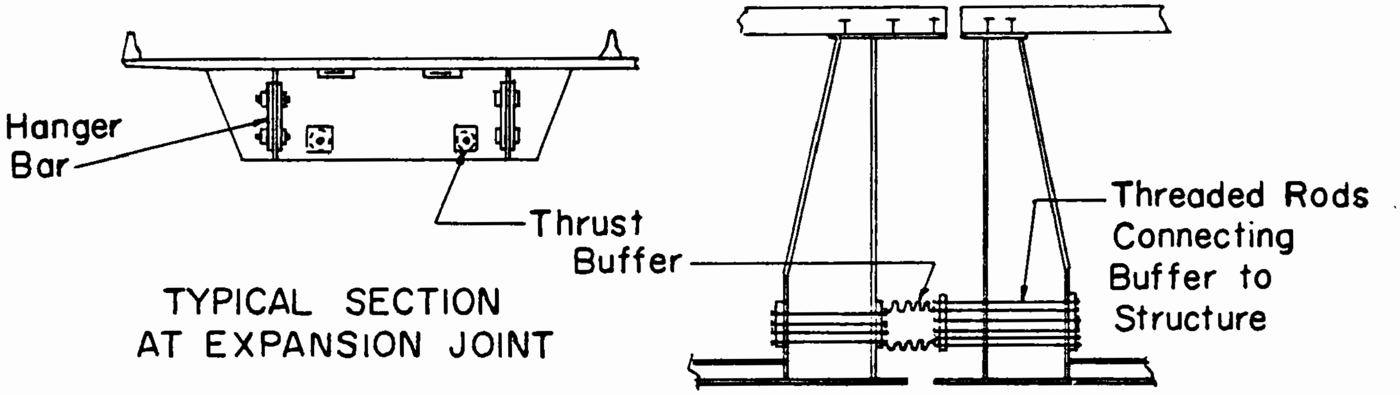
Figure 5.2.17

5.3 RECENT DESIGN INNOVATIONS

Some recent innovative attempts have been made to increase the earthquake resistance of bridge structures through use of the principles of restraint, energy dissipation and isolation. A few cases are described in the paragraphs that follow.

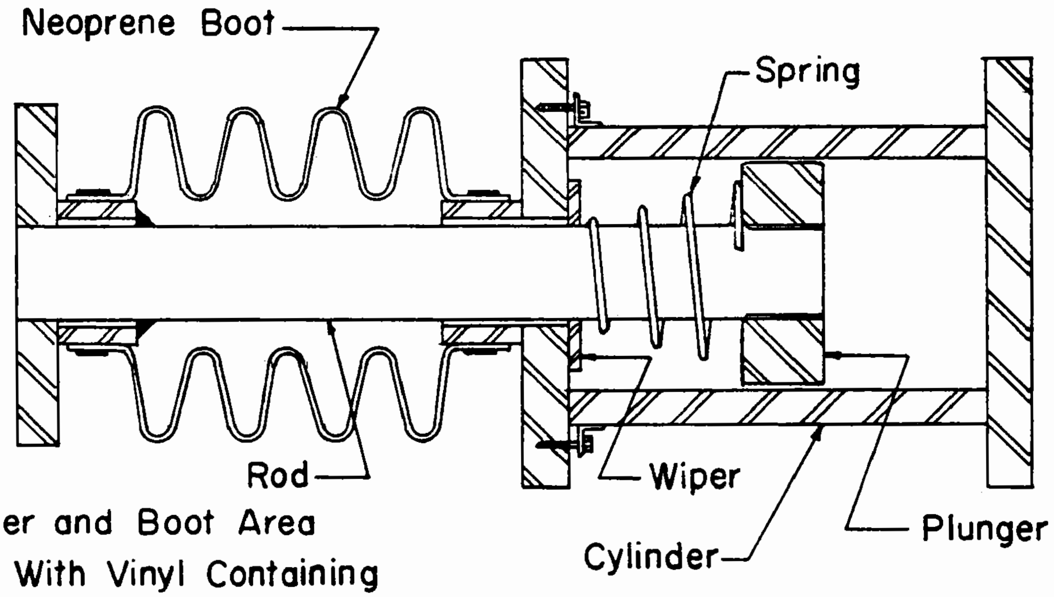
The Dumbarton Bridge main channel crossing to be constructed at the southern tip of San Francisco Bay will be located in a highly seismic zone midway between the San Andreas and Hayward Faults. This 3150 foot steel girder composite deck bridge is designed with two intermediate expansion joint hinges. The expansion joints are fitted with thrust buffers as shown in Figure 5.3.1. These buffers are designed to provide for normal temperature superstructure movements. However, during an earthquake the buffers lock up and transmit forces as restrainer devices [5.20].

This principle has also been employed on three bridges in Japan [5.21]. On the Tokyo Expressway, special oil dampers were used at the superstructure-pier connections to provide for normal movement. Lock-up will occur during earthquakes to transmit longitudinal forces to the piers. On the Ohtagawa Railroad Bridge this same behavior was achieved by using the earthquake stopper devices shown in Figure 5.3.2 instead of oil dampers. The viscous material shown in the depressed section of the pier will flow around the movable post attached to the superstructure for slow movements and resist rapid movement during an earthquake. This device also has the potential for energy dissipation if the movable post yields. A third scheme, also employed on the



TYPICAL SECTION
AT EXPANSION JOINT

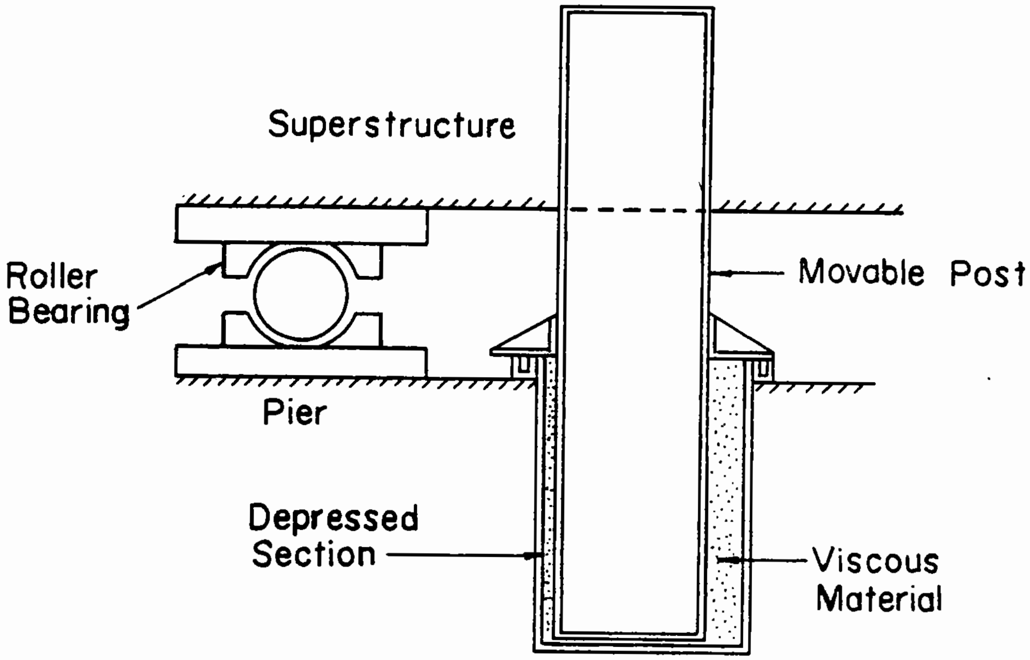
SECTION OF HINGE
AT THRUST BUFFER



Cylinder and Boot Area
Filled With Vinyl Containing
Methyl Silicone Gum.

THRUST BUFFER DETAIL FOR
NEW DUMBARTON BRIDGE

Figure 5.3.1

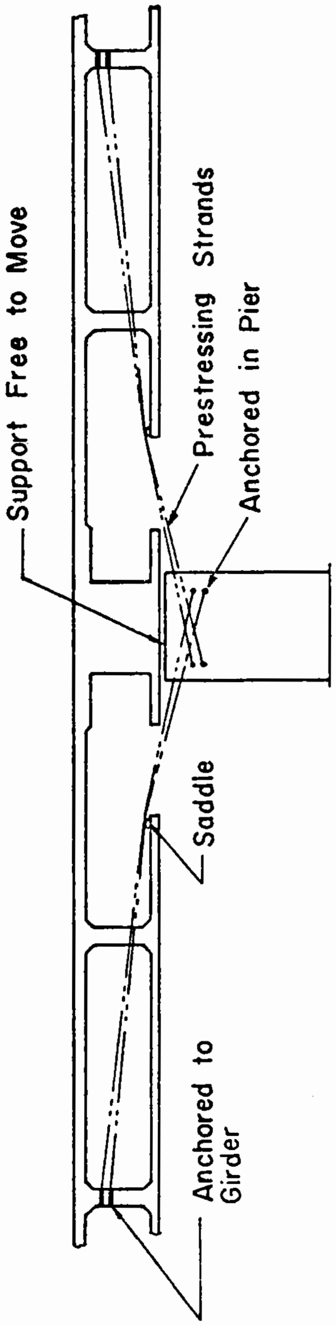


JAPANESE EARTHQUAKE STOPPER
OHTAGAWA RAILWAY BRIDGE

Figure 5.3.2

Tokyo Expressway, uses prestress strands as shown in Figure 5.3.3 to resist earthquake loads. The strands, which are connected in series to the columns, are flexible enough to allow for normal superstructure movement. During an earthquake they provide restraint plus some energy dissipation by sliding over specially designed saddles.

The San Joaquin River Bridges, a steel girder composite deck bridge, near Antioch, California, and the Napa River Bridge in Napa County, California, a lightweight prestressed concrete bridge, employ a somewhat different concept for earthquake design in that displacements during an earthquake are not fully restrained. The bearings are permitted to move with rubber bumpers, as shown in Figure 5.3.4, provided to reduce impact



JAPANESE S. U. DAMPERS
TOKYO EXPRESSWAY

Figure 5.3.3

forces as adjacent members collide. Another large structure, the Denny Creek Bridge in Washington, also uses this approach.

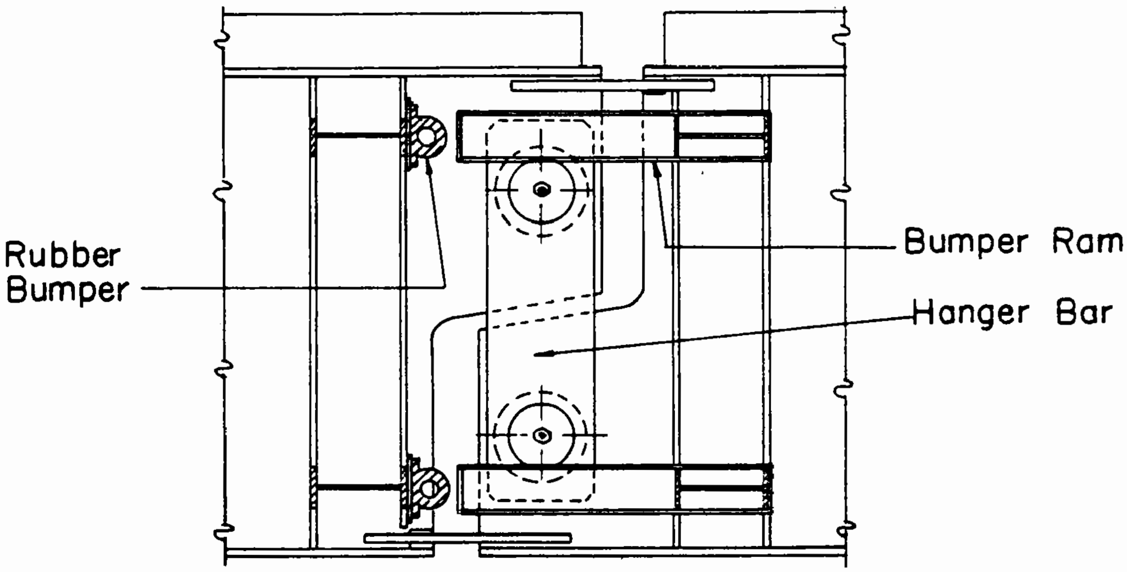
The failure of a number of concrete box girder bridges during the San Fernando earthquake demonstrated the vulnerability of conventional superstructure hinges as shown in Figure 5.3.5a. The welded steel hinge assembly [5.22] shown in Figure 5.3.5b was designed to provide a greater range of movement and to eliminate the possibility of loss of support. Restrainers and/or energy dissipators could be easily installed on this hinge. This hinge concept has been used in the East Connector Overcrossing in Bakersfield, California, and the Route 4/242 Interchange in Concord, California. The hinges are currently functioning effectively as expansion joints although the cost for the initial attempt to use this detail was high.

Because of the large amount of energy generated in a strong motion earthquake, it is usually necessary for a structure to dissipate some of this energy by some form of yielding in order to remain standing. If special auxiliary devices can be provided that will function as energy dissipators, the need for post--elastic behavior in other structural load carrying components could be curtailed or eliminated. The gap necessary to provide for the free movement at bearings would render the energy dissipating devices ineffective at these locations, unless some type of motion induced arrester is employed. Since energy dissipators require deformation to be effective, they are best suited for use at "fixed" bearings which would move only during an earthquake.

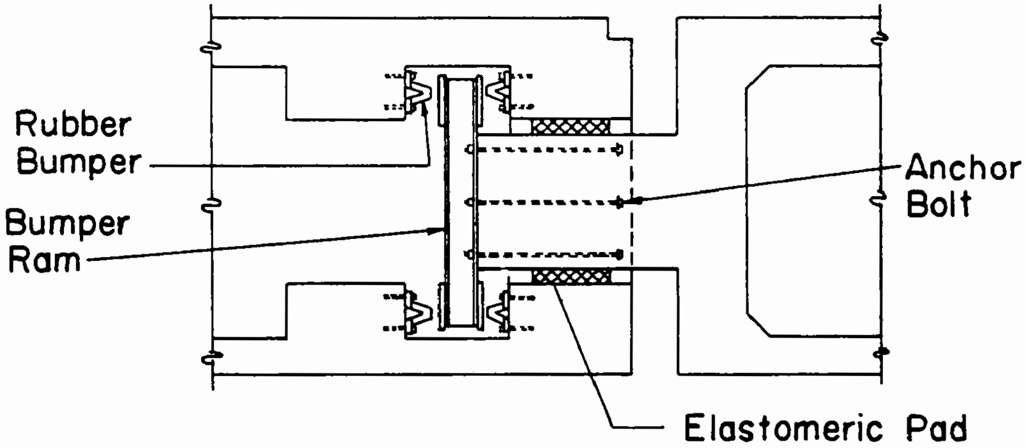
Several attempts have been made to use special energy dissipation devices on bridges in New Zealand [5.23, 5.24]. Several different types of energy dissipation devices have been used. Devices which rely on the plastic deformation of steel shapes are designed to function as hysteretic dampers. An energy dissipator which relies on the extrusion of lead through a restricted orifice was also installed on bridges in New Zealand. This device takes advantage of the low recrystallization temperature of lead. The various types of energy dissipation devices are shown in Figure 5.3.6 [5.25, 5.26].

These devices are most commonly used at superstructure/substructure connections such as abutments or the top of piers. One cleaver application of these devices was in the base of a two column bridge bent which was designed to "walk" during an earthquake. As each of the two columns is alternately subjected to tensile forces due to overturning, the energy dissipators in the base will yield, thus absorbing energy and reducing the dynamic response.

If the natural frequency of a structure can be decreased to a value significantly below the predominant frequency of an earth-



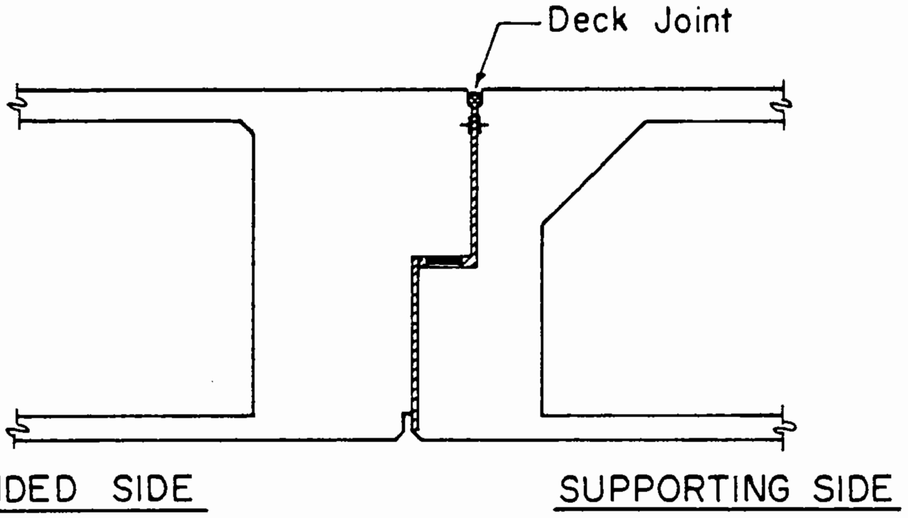
SAN JOAQUIN RIVER BRIDGE



NAPA RIVER BRIDGE

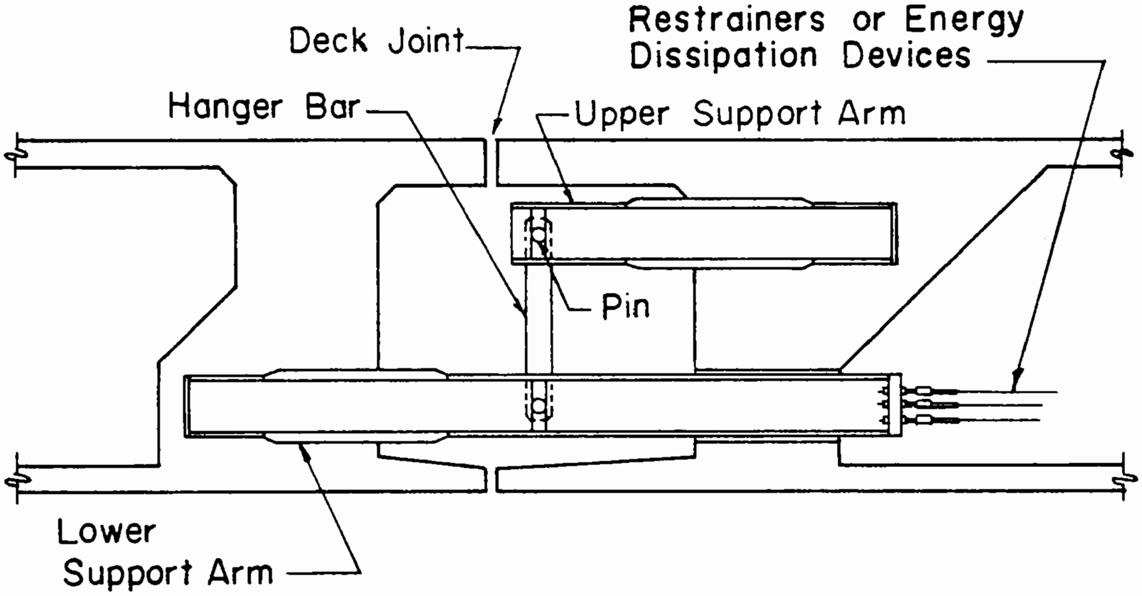
EXPANSION JOINT HINGES WITH
RUBBER BUMPERS

Figure 5.3.4



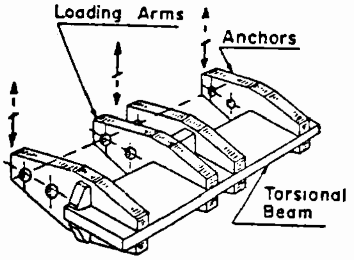
TYPICAL CONCRETE BOX GIRDER HINGE

Figure 5.3.5a

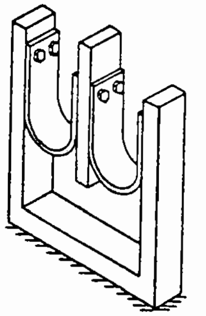


STEEL HINGE ASSEMBLY

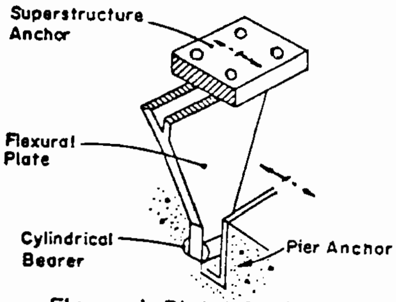
Figure 5.3.5b



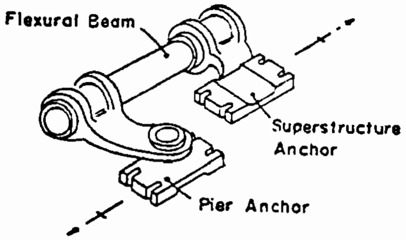
Torsional Beam Device



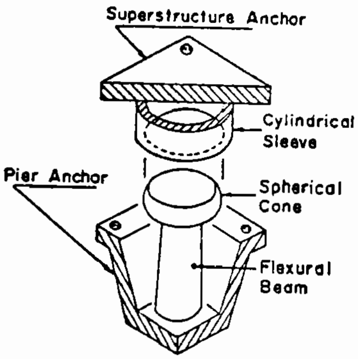
Rolling - Bending Device



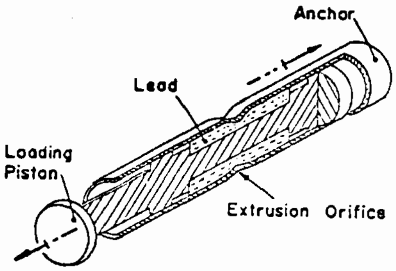
Flexural Plate Device



Flexural Beam Device



Flexural Beam Device
(Omnidirectional Action)



Lead Extrusion Device

NEW ZEALAND ENERGY
DISSIPATION DEVICES

Figure 5.3.6

quake, the forces induced by the earthquake will be reduced. In addition the number of oscillations into the high force range will be decreased. This usually is done at the expense of increased displacements. Because of adjacent roadways it may be undesirable to allow for this movement in bridge structures. For this reason, isolation devices used in bridges have required the supplemental use of restrainers or energy dissipators to limit or decrease relative movement. When separate devices such as restrainers or energy dissipators are used to resist earthquake loadings, a certain amount of isolation can be designed into the structure by making the supporting members more flexible. Isolation can also be achieved through the use of flexible mountings connected in series with the substructure elements.

In bridges, flexible mountings may result in excessive movement under other loads such as wind, liveload, etc. This can be remedied in other types of structures by using "fuses" which give way under strong loadings such as earthquakes [5.27, 5.28].

The Walnut Creek Bridge to be located on the Concord fault in California was considered for design using the isolation principles suggested by Ikonomou [5.28]. The structure consists of a four span concrete box girder with no intermediate expansion joints. The structure was to be supported on low friction teflon bearings with an auxiliary rubber isolating member vulcanized to the superstructure and one of the abutments. Undesirable superstructure movement due to normal loading would have been prevented by a concrete "fuse" designed to shear in an earthquake. During design, however, it was found that structure movements during an earthquake would be large, and the design concept has since been abandoned for economic reasons.

The Feather River Bridge to be constructed near the City of Oroville, California employs the principle of isolation for a major earthquake. The bridge is supported on stiff piers which were dictated by architectural considerations. Teflon "pot bearings" separate the piers from the superstructure and are fitted with lateral restraints to prevent transverse movement for wind loadings and moderate earthquakes. A major earthquake will cause the lateral restraints to fail thus freeing the structure in the transverse direction at the piers. The earthquake force will be taken by the abutments and a single continuous pier, but the reduced structural period will result in lower total seismic forces.

These cases illustrate how the principles of restraint, energy dissipation, and isolation were specifically applied to increase structure earthquake resistance. These principles are important considerations in earthquake design whether or not special devices are used. The designer should keep them in mind when designing a structure. For example, massive stiff piers may result in unnecessarily high seismic loads. More flexible piers could result in better economy. Design of piers for seismic loads that are unreduced for ductility (response modification factors) will eliminate the advantage of energy dissipation due to yielding and will probably result in excessive shear forces on the pier. Energy dissipation due to friction between adjacent components, movement of foundations through soil, internal molecular friction during deformation, etc., is generally present and accounted for by a damping factor. The designer should be aware of any unusual situations that could increase or decrease the potential of energy dissipation.

5.4 RESEARCH

A condiderable amount of research has been done or is planned to study the effects of isolation and energy dissipation on structural response.

Isolation of a structure [5.29, 5.30, 5.31] through the use of rubber mountings was first used to minimize the effects of noise and groundborne vibrations from sources such as traffic. In 1972, the Malaysian Rubber Producers' Research Association entered into a contract with Atkins Research and Development, a British firm, to study the effect of an earthquake on a building with and without rubber bearings. Computer analysis showed that dynamic force levels could be reduced considerably. This project eventually led to a scale model test on the shaking table at the Richmond Field Station of the Earthquake Engineering Research Center of the University of California, Berkeley. The results of these tests generally confirmed the findings of the analytical study. Movement of the building on such bearings due to loads such as wind, would require that stabilizing "fuses" be included in any practical design. This approach would present a problem in designing bridge bearings which must accommodate movement due to temperature, creep, etc.

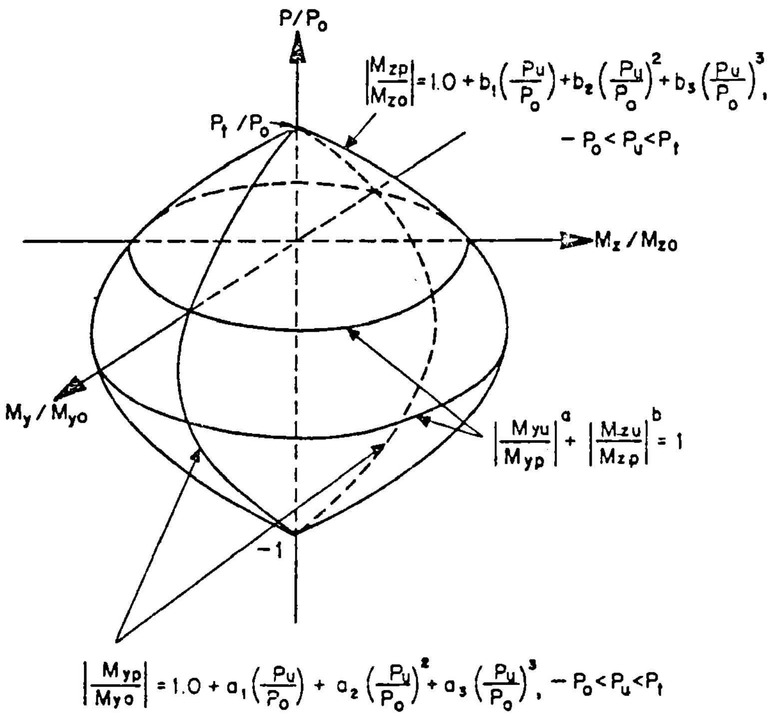
A practical method of using energy absorption devices in parallel with isolating bearings was suggested by Skinner, et al [5.27]. In applying this method to bridge structures, these devices would be placed in locations where fixed bearings would normally be used.

A problem exists in using energy dissipators at bridge expansion bearings. In order to accommodate the required relative movement, gaps must usually be provided to allow free movement before the devices are engaged. A recently published parameter study [5.16] to determine the effects of non-linear response on earthquake forces was conducted at the University of California, Berkeley, under Phase 6 of an FHWA sponsored research project. This study showed that restrainer cables which are fitted with similar gaps would be unstressed if certain actual bridges were subjected to strong earthquake motion. Energy absorbers with similar gaps would therefore be ineffective for the cases studied.

Nuclear power plants have eliminated the need for gaps by employing velocity induced arresters or "snubbers" similar to the devices used on the Dumbarton Bridge and the bridges in Japan. These devices are expensive hydraulic systems which require extensive maintenance and are thus not cost-effective.

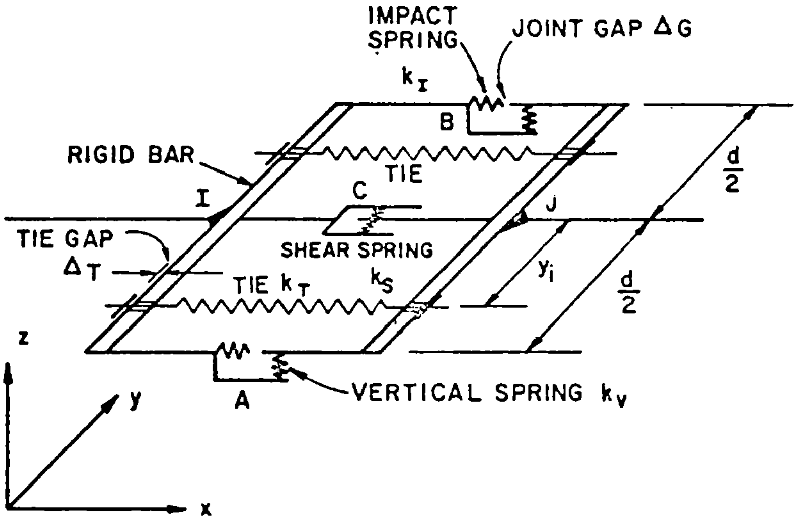
Research is currently underway at the University of California to develop solid state energy absorbers to replace expensive and troublesome "snubber" devices currently used in powerplants. These energy absorbers will employ a form of TRIP (TRansformed Inducted Plasticity) steel developed by metallurgists at the university. Analytical studies to determine the effect of using this steel for energy absorption in bridge expansion joint hinges is currently planned at the University of California in a research project sponsored by the California Department of Transportation.

The analytical tools to be used in the above project were developed as part of an FHWA sponsored research project conducted by the University of California entitled "An Investigation of the Effectiveness of Existing Bridge Design Methodology in Providing Adequate Structural Resistance to Seismic Disturbances." The computer program NEABS (Nonlinear Earthquake Analysis of Bridge Systems) was developed to consider the effect of non-linear behavior in the columns, expansion joints, and at the supports [5.32, 5.33]. Columns are considered to behave in an ideally elasto-plastic manner with yielding occurring on the yield surface as shown in Figure 5.4.1. A schematic diagram of the expansion joint idealization is shown in Figure 5.4.2. Correlative investigations on theoretical and experimental behavior of this element were carried out in Phase 5 of this project [5.33]. After modification of the element, good correlation was achieved. This computer program was also used in the previously mentioned parameter study [5.16]. The elements currently available in NEABS may be used to simulate a wide variety of bridge structural types and support conditions and represent a powerful research tool for future studies.



NEABS COLUMN YIELD SURFACE

Figure 5.4.1



NEABS NONLINEAR EXPANSION JOINT ELEMENT

Figure 5.4.2

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CHAPTER 6

ADVANCED TOPICS FOR SEISMIC DESIGN OF BRIDGES

6.1 INTRODUCTION

The AASTHO code provides for seismic design by the equivalent static force method for simple structures, and response spectrum and dynamic analysis for more complex structures [6.1]. CALTRANS, now a leader in the seismic design of bridges, is currently using response spectrum analysis on a large number of their structures since they have found the equivalent static force method yields unreliable results in many cases [6.2]. In light of the CALTRANS experience, it is probable that the trend nationwide will be toward increased use of the response spectrum analysis technique for bridges. Although the response spectrum method generally appears to be satisfactory for the seismic design of most bridges, there are limits to its applicability that the bridge designer should realize.

The first shortcoming of the response spectrum approach is that the time domain has been removed. Since maximum modal responses do not occur simultaneously it is necessary to use a statistical combination of modal responses such as root mean square in order to obtain realistic design loads. The actual combination of modal response depends on several factors related to the type of structure and the nature of the actual ground motion. Therefore, the arbitrary use of a statistical approach to replace the effects of the removed time domain may not yield realistic results in certain cases.

Another deficiency in the response spectrum approach is that the duration of shaking is not accounted for by the spectrum. The major effect of duration will be on the stiffness and strength loss once the member begins yielding.

The response spectrum has an advantage for design in that it can envelope the effects of several possible earthquake ground motions. Techniques have been developed to generate a time history of ground motion that will result in a smoothed design type response spectrum. However, several different time histories can satisfy this same requirement, each giving a different combination of modal response for each different structural component of the bridge. Therefore, considering the unpredictability of the actual ground motion, the response spectrum technique which employs a statistical approach may be the most rational method available for most bridges. More research is needed, however, to identify the best statistical method(s) for the combination of modal results.

A particular problem arises when two structural modes of vibration of a given bridge have periods that are very close together and occur near the peak of the response spectrum. This problem is further complicated if the modes in question include simultaneous response in more than one orthogonal direction. The contributing modes may respond in phase in one of the directions and out of phase in the other(s), making it almost impossible to predict forces. Since column design is based on force interaction, the forces in all directions are important. For this case without further research into the improvement of response spectrum modal combination, the designer may be forced to use a sophisticated time history analysis. Such an analysis will require careful selection of a family of time history loading(s).

Perhaps a more serious shortcoming of the response spectrum technique as well as the elastic time history analysis is the absence of direct consideration of post-elastic and nonlinear behavior. The complex nonlinear behavior that occurs in bridges subjected to earthquakes is currently accounted for by reducing the results from a linear analysis by an assumed ductility factor. This does not account for the redistribution of forces due to nonlinear behavior nor does it predict the areas of maximum ductility demand.

Accurately predicting the response of complex bridge structures to strong earthquake motions requires the use of sophisticated nonlinear dynamic analysis computer programs not generally available to the bridge design engineer.

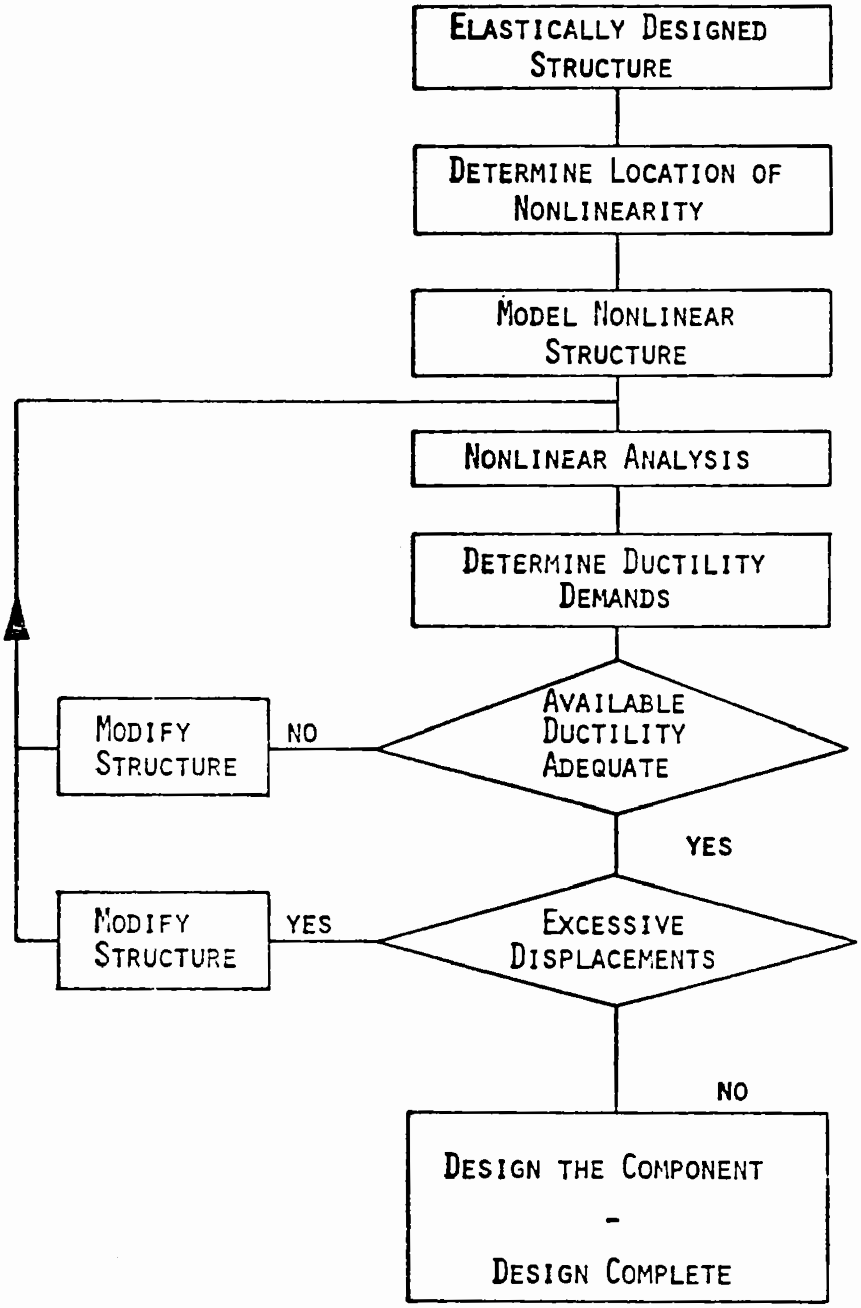
The ultimate design methodology would employ a nonlinear analysis of structural reponse to earthquake loading. While there are still many obstacles to the use of this approach, the following factors could accelerate the adoption of this procedure on a semi-regular basis.

1) The availability of reliable computerized procedures at a relatively low cost.

2) The limitations inherent in a linear-elastic approach.

3) A better understanding by the profession of the factors affecting dynamically induced forces.

A suggested nonlinear seismic design process is shown in Figure 6.1.1. The procedure would start with a elastically designed structure. From the elastic design, the existence and probable location of nonlinear behavior would be determined. The structure would then be modeled with nonlinear elements, if necessary, and an analysis would be performed. Ductility demands would be determined from the resulting nonlinear deformation.



SUGGESTED NONLINEAR SEISMIC DESIGN PROCESS

Figure 6.1.1

These would be compared with the available ductility of the individual components comprising the structure. Excessive displacements at points such as bearing seats sould also be checked. If ductility or displacement values were exceeded, the structure would then be modified and reanalyzed.

The utilization of dynamic and response spectrum analysis techniques for seismic design of bridges is just beginning to become a reality on a national level. Continued progress in this area depends on general nationwide acceptance and understanding of these techniques at the design level and continued participation of the practicing engineer in the development of new techniques.

6.2 COMPARISON OF RESULTS FROM THREE TYPES OF ANALYSIS

6.2.1 INTRODUCTION

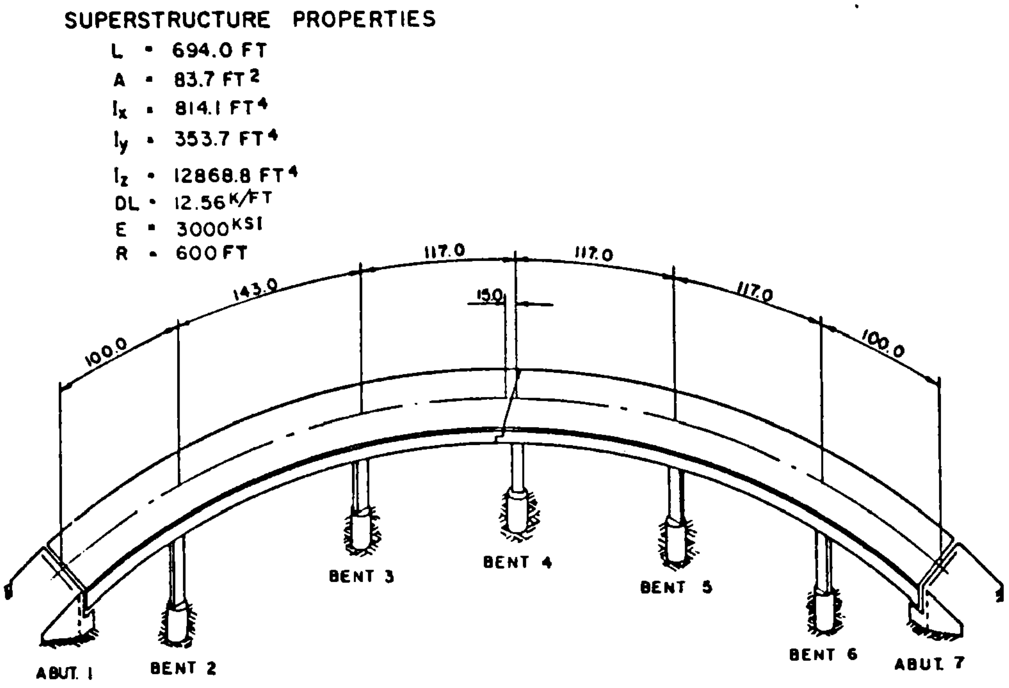
A comparison of three different types of analyses of the Route 80 Onramp Undercrossing shown in Figure 6.2.1 will give an indication of the differences that can be expected from each of these techniques [6.3]. A response spectrum analysis, a linear time history analysis and a nonlinear time history dynamic analysis were performed on the structure.

The linear analysis capabilities of STRUDL were used to perform the response spectrum and linear time history analyses. STRUDL is a well-known general purpose computer program for static and dynamic analysis of linearly elastic structural systems. The McAuto proprietory version was used along with bridge structure generating capabilities.

The nonlinear time history analysis was performed by NEABS (Nonlinear Earthquake Anlysis of Bridge Systems) [6.4, 6.5], a computer program developed as part of an intensive research effort at the University of California to improve the seismic design methodology of bridges. This computer program uses a step-by-step integration procedure which assumes piecewise linear behavior over each increment of time. The acceleration may be assumed to either be constant or vary linearly over each increment of time. The linear acceleration method was used for this study. Loading was input as rigid support accelerations. The program has the following linear and nonlinear element types:

1) Linear elastic truss elements

2) Linear elastic and elasto-plastic straight beam elements



ROUTE 80 ONRAMP UNDERCROSSING

Figure 6.2.1

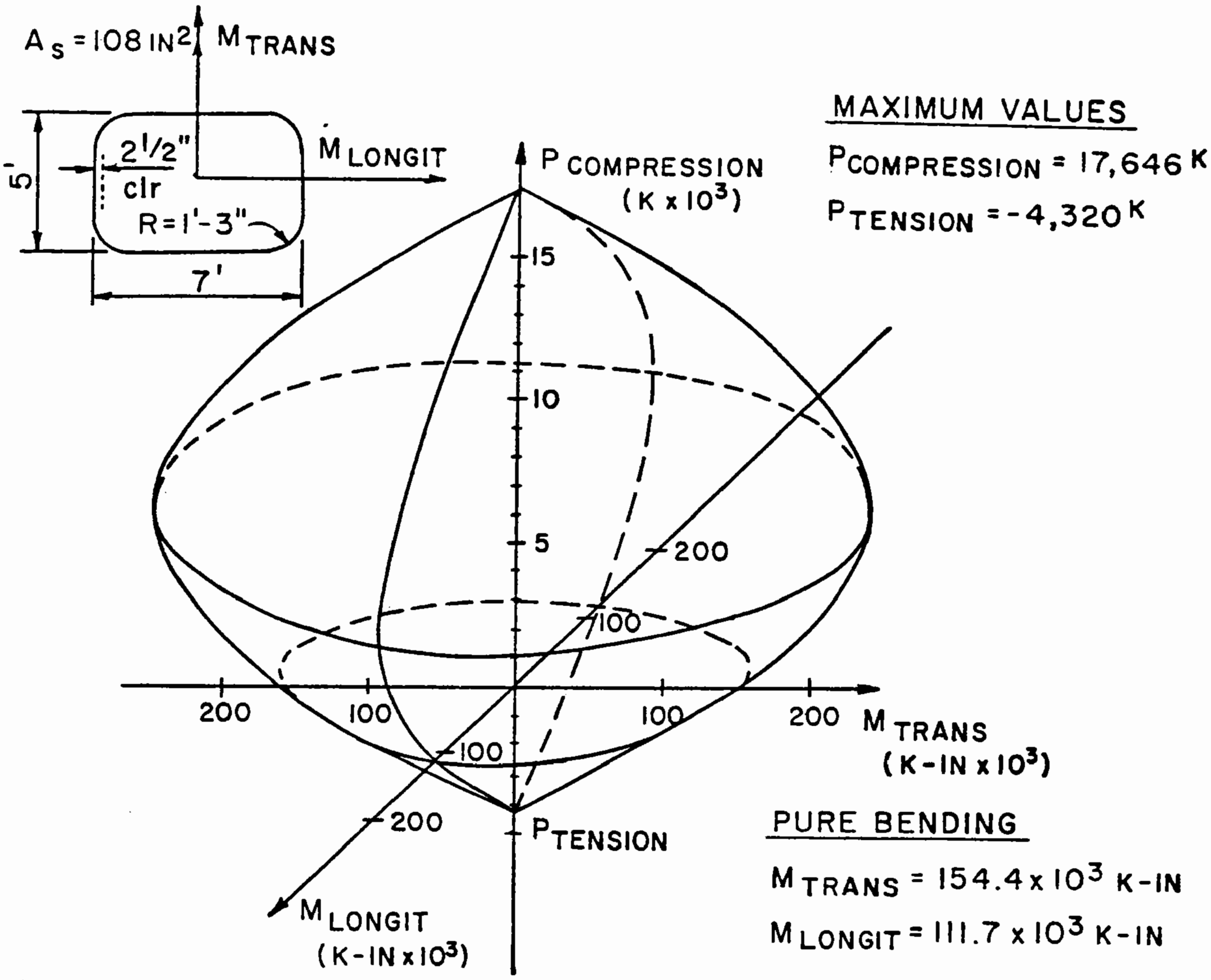
3) Linear elastic circularly curved beam elements

4) Linear elastic and bi-linear boundary spring elements

5) Linear and nonlinear expansion joint elements

The two nonlinear conditions modeled for this study were the yielding of the single column bents, and the nonlinearity of the expansion joint hinges.

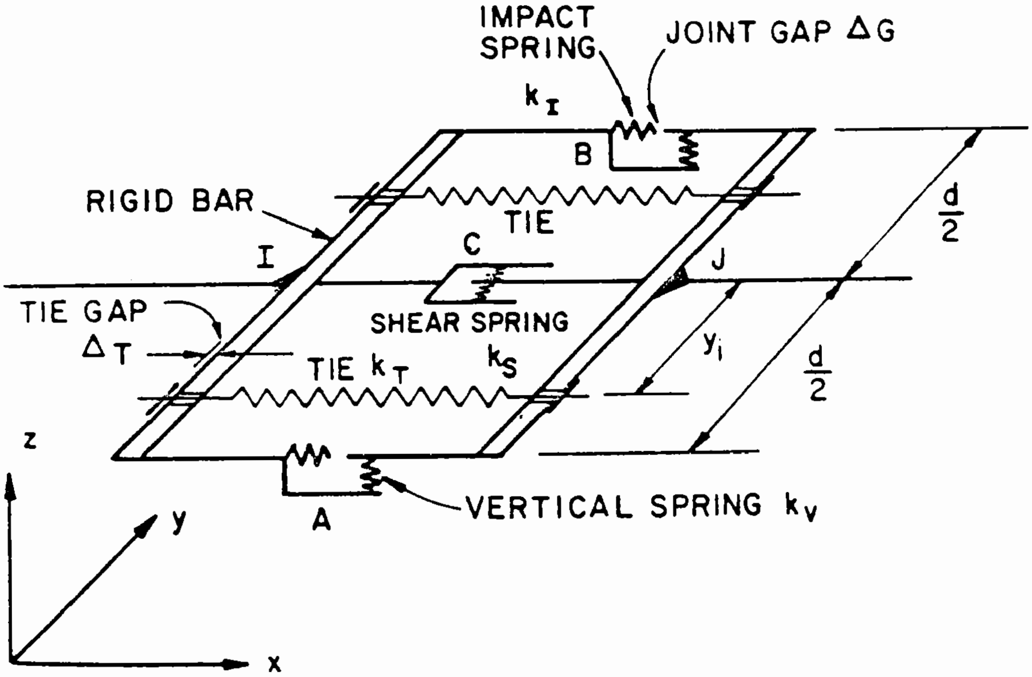
The yielding of columns is limited to axial and flexural yielding along an interaction yield surface. The yield surface for Route 80 Onramp U.C. column is shown in Figure 6.2.2. The parameters which define this surface were calculated using a separate computer program called YIELD [6.4]. The ultimate capacity of the column in shear is considered to be infinite.



ROUTE 80 U.C. BENT 4 INTERACTION SURFACE

Figure 6.2.2

The nonlinear behavior of the expansion joint hinges are modelled using the expansion joint element shown in Figure 6.2.3. In this expansion joint hinge, the restrainers are assumed inactive until movement at the joint is sufficient to overcome the gaps which are normally placed in the restrainer anchorages to allow for normal operation of the expansion joint. When the restrainers are activated, they behave in an ideally elasto-plastic manner. Closure of the hinge is limited by impact springs which become active after a seat gap is taken up. This represents banging on impact of the two adjacent superstructure sections. The effect of bearing pads



NEABS HINGE IDEALIZATION

Figure 6.2.3

is also included in the expansion joint element. The vertical and shear stiffness of the pads can be represented by springs. Sliding of the pads occurs when the specified coefficient of friction at the pad/concrete interface is overcome.

The linear dynamic analysis program BSAP (Bridge Structural Analysis Program) [6.4] has an input format similar to NEABS. To check the NEABS model, a BSAP analysis was run using input data identical to that used by NEABS for joint coordinate member incidences and elastic member properties. The results of a frequency analysis on BSAP were then compared with STRUDL frequency results as a means of spotting gross errors in the input prior to attempting a nonlinear analysis.

6.2.2 SEISMIC EXCITATION

Rigid support motion was assumed for all of the bridges. Ground motion accelerations developed by Seed and Idress for a simulated 8+ Richter magnitude earthquake were used. The response spectrum used was generated on STRUDL, using the above mentioned ground history motion as input. The ground motion was applied in two horizontal directions. The longitudinal motion was directed parallel to a straight line between the abutments, and the transverse motion applied perpendicular to it.

Because of the costs involved, the nonlinear analysis was run for only the first 20 seconds of ground motion. Using the knowledge gained from prior elastic analyses, this appeared to include the most critical portions of the record. With the three types of analyses considered and ground motion in two perpendicular directions, the total number of cases examined amounted to 6. These cases are numbered for convenience. The numbering scheme is shown in Table 6.2.1.

CASE	DIRECTION OF EXCITATION	ANALYSIS TYPE
1	Transverse	Response Spectrum
2	Longitudinal	Response Spectrum
3	Transverse	Linear Time History
4	Longitudinal	Linear Time History
5	Transverse	Nonlinear Time History
6	Longitudinal	Nonlinear Time History

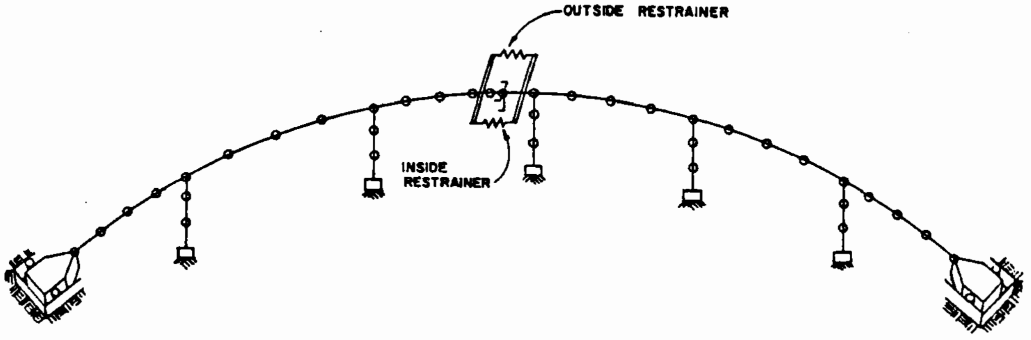
CASE NUMBERS

Table 6.2.1

6.2.3 MODELING

The bridge deck and columns were modeled with space frame members. The inertial effects of the structure are modeled by lumping the mass at the quarter points of the bridge deck and the third points of the columns. Only translational inertia effects are considered.

For simplicity, the base of each column was assumed fixed at the footing. A typical structure idealization showing the location of lumped masses is shown in Figure 6.2.4.



RTE 80 STRUCTURE IDEALIZATION

Figure 6.2.4

A structure generation program was used to develop the bridge idealization for STRUDL. Freedom of movement at the abutments and intermediate expansion hinges is idealized, in the generated model, by member end releases. To avoid losing the inertia effects at the joints adjacent to member end releases, short members are inserted at these locations. This is done to insure that the superstructure mass is lumped on the proper portion of the superstructure. The curved portion of the deck is modeled with straight space frame members since STRUDL does not have curved members.

The expansion joint hinge idealization for STRUDL is modeled by releasing member axial forces, and transverse and longitudinal bending moments at the hinge. The effect of restrainers is represented by placing transversely eccentric space frame members between both sections of the superstructure. This idealization assumes no gap and that both tension and compression can occur in the restrainers.

The basic assemblage of members for BSAP and NEABS is similar to that used for STRUDL with a few exceptions. First, the curved superstructure is represented by circularly curved beam members. Secondly, the freedom at the abutments and the expansion joint hinge are modeled with special support elements and expansion joint elements. These elements make it unnecessary to use short space frame members to insure proper lumping of the mass.

The NEABS expansion joint element has several nonlinear parameters that must be input. Design values for tie and seat gaps were used. In actuality, these values will vary with time depending on temperature, shrinkage, etc. Cable restrainer stiffnesses were calculated assuming an effective Youngs modulus of 13,700 kips per square inch. The yield force in a typical 3/4 inch restrainer was taken as 30.6 kips. The shear stiffness of elastomeric bearing pads was calculated based on an assumed shear modulus of 135 psi. The coefficient of sliding friction was assumed to be 0.4. For the purposes of modeling impacting of the superstructure, the impact spring was assumed to have the axial stiffness of the shortest adjacent section of the superstructure.

Nonlinear column elements were used at locations where column yielding might be expected. Nonlinear columns were modeled on NEABS by using parameters calculated from results obtained from a separate column analysis computer program YIELD.

6.2.4 RESULTS OF ANALYSIS

The results from the three different analyses are summarized in the tables which follow. The following is a brief description and discussion of the results given.

6.2.4.1 Structure Period and Participation Factors

The structure period for the first modes of vibration were determined using both STRUDL and BSAP. The participation factors, which are taken from the STRUDL program, are defined as

$$(PF) = [\phi]^T[M][N] \qquad (6.2.1)$$

where:

$[\phi]^T$ = the transpose of the matrix of eigenvectors normalized with respect to unit mass

$[M]$ = the system mass matrix

$[N]$ = the rigid body vectors which relate the motion at each joint to the support motion

Participation factors are useful in determining the relative participation of each mode for a shock in a given direction.

The structure periods shown in Table 6.2.2 indicate the first 10 modes of vibration are concentrated in a range from 0.40 to 0.22 seconds. The participation factors, together with the mode shape plots for the second and third modes shown in Figure 6.2.5, indicate that there is simultaneous motion in the two horizontal directions. The periods of the two modes differ by only 0.004 seconds and result in near peak response for the SI8+ earthquake, thus indicating that both these modes will contribute substantially to the total response of this bridge. The signs of the participation factors for the second and third modes of vibration indicate out of phase modal responses due to a transverse excitation.

6.2.4.2 Deadload Reactions

The deadload reactions at the base of the columns are calculated by the NEABS program prior to the nonlinear dynamic analysis. The structure is analyzed as a space frame to determine deadload member forces. These values are used internally by NEABS since the effect of deadload must be considered in determining nonlinear response. For an elastic analysis, this is not necessary, however. Therefore, in order to make a meaningful comparison of results, it was necessary to add deadload member forces to the earthquake member forces derived from an elastic analysis. The deadload reactions and corresponding moments are given at the base of the column in the local coordinate system. The longitudinal and transverse directions correspond to directions perpendicular and radial to the superstructure respectively. Deadload forces are shown in Table 6.2.3.

Mode	Period (Sec) STRUDL	BSAP	Participation Factor X (Long.)	Y (Vert.)	Z (Trans.)
1	0.399	0.398	1.6	26.8	28.7
2	0.371	0.371	83.3	-0.8	-75.3
3	0.367	0.367	55.6	-6.5	115.2
4	0.340	0.340	-66.5	0.2	3.2
5	0.309	0.309	-30.4	4.5	-3.3
6	0.294	0.294	73.6	0.3	2.6
7	0.261	0.261	-3.9	-9.5	-5.1
8	0.240	0.239	4.3	14.6	-34.4
9	0.234	0.233	-15.8	-22.7	4.0
10	0.221	0.222	17.0	-74.8	-4.8

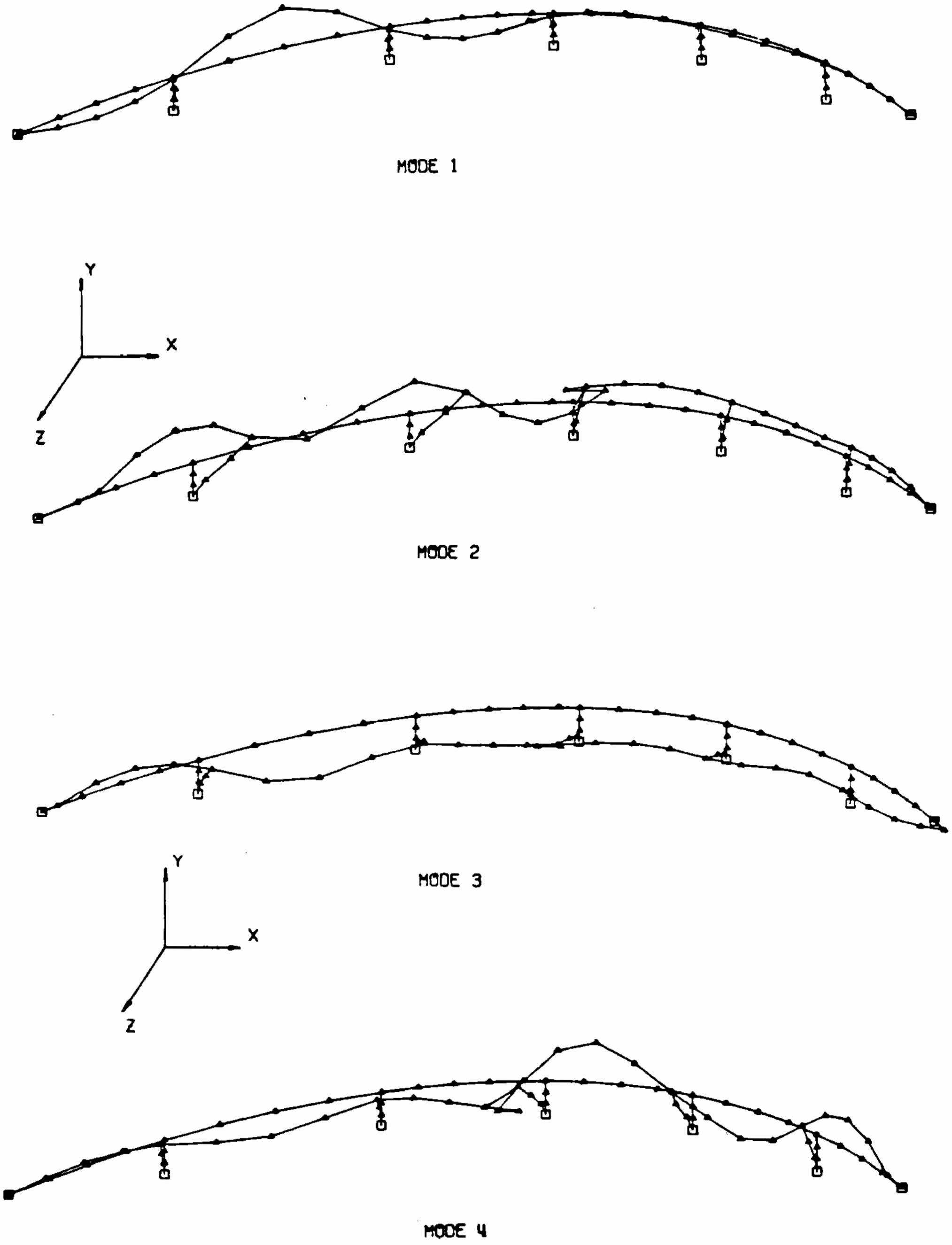
STRUCTURE PERIOD AND PARTICIPATION FACTORS

Table 6.2.2

6.2.4.3 Maximum Column Base Moment and Corresponding Shear

The maximum column base moments and shears are compared for three types of analyses. Deadload moments and shears have been added to the results for the elastic analyses. The yield moments shown on the chart were taken from the column interaction yield surface for an axial load equal to the deadload reaction.

The moments and shears are given in the local coordinate system. The earthquake ground accelerations are applied in the global coordinate system.



RTE 80 ONRAMP U.C.
MODES 1 TO 4

Figure 6.2.5

Location	Axial Force (kips)	Trans. Shear (kips)	Long. Shear (kips)	Torsional Moment (kip-ft)	Long. Moment (kip-ft)	Trans. Moment (kip-ft)
Bent 1	0	3	418	-247	0	0
Bent 2	1802	20	161	-6	-1377	210
Bent 3	1827	27	-161	-4	1160	332
Bent 4	1389	8	172	9	-950	281
Bent 5	1618	22	-136	14	1484	283
Bent 6	1633	9	-37	8	622	71
Bent 7	0	3	418	-247	0	0

DEADLOAD FORCES AT THE SUPPORTS

Table 6.2.3

The maximum transverse and longitudinal bending moments recorded for a single time history analysis do not necessarily occur at the same time. The values shown are the individual maximum force component that occurred during the time history analysis. These values correspond to the response spectrum RMS values generally used for design. Although this is somewhat conservative, these forces would envelope the maximum loading case.

The maximum local or rotational ductility demands tabulated in Table 6.2.4 were calculated using the same basic approach used by Tseng and Penzien [6.4]. The column flexural yield rotations and corresponding assumed plastic hinge lengths that were used in these calculations are shown in Table 6.2.5.

Transverse Excitation - The maximum bending moments and corresponding shears at the base of the column due to a

Location	Maximum Nonlinear Rotational Distortion		Maximum Rotational Ductility Demand	
	Trans. Shock (Rad.x10-3)	Long. Shock (Rad.x10-3)	Trans. Shock	Long. Shock
Bent 2	1.337	3.176	1.79	2.86
Bent 3	2.456	3.238	2.50	2.89
Bent 4	3.588	1.781	3.20	2.04
Bent 5	2.625	2.155	2.58	2.26
Bent 6	.989	1.729	1.59	2.01

MAXIMUM LOCAL BENDING DUCTILITY DEMANDS

Table 6.2.4

transverse shock are reported in Table 6.2.6. In all column the response spectrum RMS results for the transverse bending moments are less than the linear time history analysis results. The differences range from 21 percent at Bent 2, to 11 percent at Bent 6. These differences are a result of replacing the effects of the time domain with a statistical averaging technique.

The differences between the response spectrum and linear time history results for longitudinal moment are somewhat erratic. The response spectrum results for the longitudinal moment are generally greater, except at Bent 3 where they are 63 percent less. The maximum difference occurs at Bent 2 where the response spectrum predicts a maximum moment 94 percent larger than the linear time history analysis. Since there is no consistent pattern to this variation, the possibility of obtaining better results by using another statistical means of combining the response spectrum modal results is somewhat remote.

The nonlinear time history analysis results reported in Table 6.2.6 indicates that yielding will occur in all the columns due to transverse motion. The time history plot of

Direction	I (ft)	M (k-ft)	h (ft)	Y (rad. x 10)
Long. (Y)	72.9	12,000	7.0	1.715
Trans. (2)	142.9	16,000	5.0	1.633

COLUMN FLEXURAL YIELD ROTATIONS

Table 6.2.5

nonlinear transverse deformations at the base of Bent 4 is shown in Figure 6.2.6. This plot shows a large amount of yielding taking place.

The maximum rotational ductility demand of 3.2 reported at Bent 4 is far below the ductility generally considered available in a reinforced concrete column. With the degree of cyclic yielding taking place, however, it is likely that considerable structural damage will occur with a resulting degradation in column stiffness. This raises a question about the validity of maximum ductility demand as a measure of a structure's ability to withstand damage from seismic loadings.

The linear time history analysis results indicate that a ductility reduction factor of between 3 and 4 applied to the elastic moments would have resulted in a similar column design for this seismic loading. With the amount of cyclic yielding that occurred in Case 5, however, it is doubtful that these columns would have performed satisfactorily during this earthquake. It should be noted that one of the main reasons for the extensive cyclic yielding of this structure was its relatively short period range which results in a greater number of nonlinear excursions.

Longitudinal Excitation - The maximum bending moments and corresponding shears at the base of the columns due to longitudinal shock are shown in Table 6.2.7. The response spectrum RMS results for longitudinal moment are approximately 30 percent less than those predicted by a linear time history analysis. However, the RMS transverse moments are in some cases several times greater than the

Bent No.	Direction of Shear and Moment	Yield Moment*	CASE NO.		
			1 (R.S.)	3 (L.T.H.)	5 (N.T.H.)
2	Trans.	16,457	22689 # (1040) +	28802 (1312)	14941 (688)
	Long.	12,002	31253 (2306	17395 (1388)	9396 (762)
3	Trans.	16,497	44593 (1978)	55713 (2484)	16754 (727)
	Long.	12,033	3670 (2671)	8420 (672)	4514 (404)
4	Trans.	16,748	53254 (2368)	62014 (2721)	16536 (791)
	Long.	11,465	6117 (660)	2167 (328)	2867 (222)
5	Trans.	16,148	44185 (1907)	51336 (2208)	16329 (708)
	Long.	11,768	11855 (903)	10597 (801)	5335 (708)
6	Trans.	16,175	24186 (1029)	27032 (1133)	14405 (605)
	Long.	11,789	14528 (964)	14641 (979)	7843 (562)

*Moment Corresponding to Deadload
#Maximum Column Base Moment (kip-ft)
+Maximum Column Base Shear (kips)

MAXIMUM COLUMN MOMENTS AND CORRESPONDING SHEARS
(TRANSVERSE EXCITATION)

Table 6.2.6

linear time history results. In Bent 4, for example, the response spectrum analysis predicts a transverse moment nearly twice the longitudinal moment. The time history results, however, indicate this moment will be considerably smaller. This structure, because of its close periods and coupling of response in the global directions, is particularly unsuited for analysis by the response spectrum method.

The relatively uniform maximum rotational ductility demands shown in Table 6.2.4 indicate relatively uniform participation of all the columns due to longitudinal ground motion. Notice that the two columns which support an approximately equal weight section to the left of the hinge have a greater ductility demand than the three columns to the right. The plot in Figure 6.2.7 shows a much more desirable history of yielding at Bent 4 than was experienced for transverse motion. Notice that, for this case, where columns of equal stiffness participate equally in resisting the load, the maximum rotational ductility demands are related to the elastic moment reductions at the columns. Unfortunately, this uniformity of response is not typical of bridge structures.

6.2.4.4 Maximum Transverse Force In The Shear Keys

The accurate prediction of abutment and hinge shear forces that occur during an earthquake is somewhat more critical in that these forces must be resisted by nonductile components such as shear keys. The problem of obtaining realistic forces at these locations using a linear analysis is compounded by the fact that yielding in the ductile components results in a redistribution of the earthquake resisting forces to the stiffer nonductile components. Transverse motion is generally the governing case for design.

The maximum shear key forces are tabulated in Table 6.2.8. The response spectrum analysis predicts forces which are slightly less than those predicted by the time history analysis. The nonlinear results, however, are larger than the time history by 46 percent and 28 percent at Abutments 1 and 7 respectively. This increase in shear force can be attributed to yielding in the columns and redistribution of the earthquake resisting forces to the abutments.

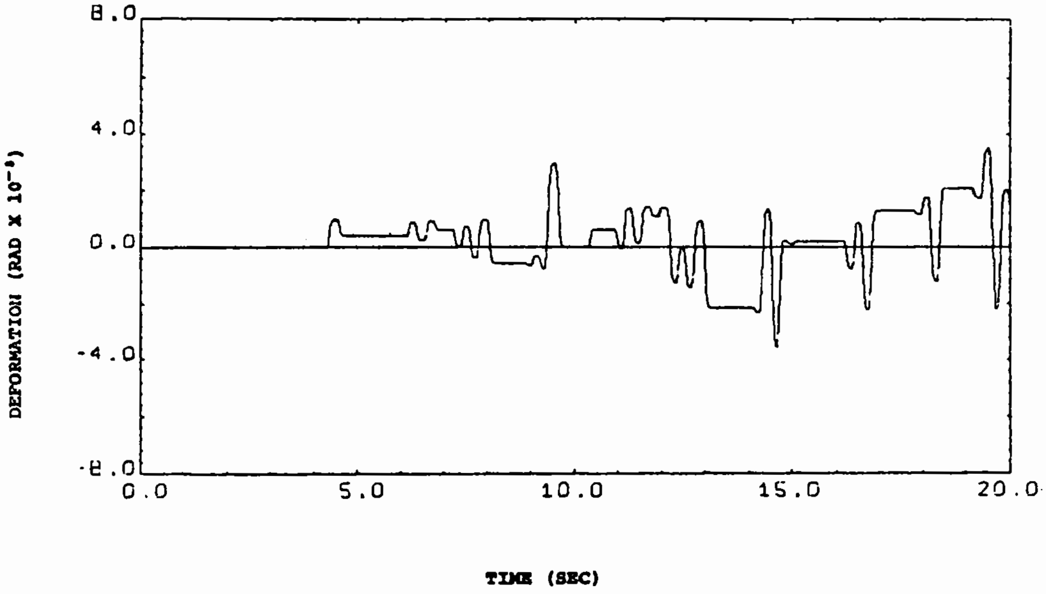
The shear force at the intermediate hinge, as determined by the response spectrum analysis, is less than that obtained from the linear time history analysis by about 14 percent. The nonlinear analysis, however, indicates a reduction in force level by a factor of 2.6. This corresponds to an

				CASE NO.	
Bent No.	Direction of Shear and Moment	Yield Moment*	2 (R.S.)	4 (L.T.H.)	6 (N.T.H.)
2	Trans.	16,457	15,468# (733)+	9773 (519)	6457 (330)
	Long.	12,002	33,216 (2469)	35739 (2662)	12331 (1019)
3	Trans.	16,497	26,998 (1221)	9112 (471)	6724 (301)
	Long.	12,033	31,398 (2349)	37207 (2807)	12293 (1031)
4	Trans.	15,748	29,440 (1292)	7626 (339)	3727 (162)
	Long.	11,465	16,721 (1333)	23334 (1716)	11980 (929)
5	Trans.	16,148	25,902 (1140)	15317 (727)	9102 (428)
	Long.	11,768	21,956 (1826)	28932 (2258)	11975 (428)
6	Trans.	16,175	15,139 (663)	11640 (548)	7657 (347)
	Long.	11,789	16,802 (1180)	22250 (1477)	12107 (318)

*Moment Corresponding to Deadload
#Maximum Column Base Moment (kip-ft)
+Maximum Column Base Shear (kips)

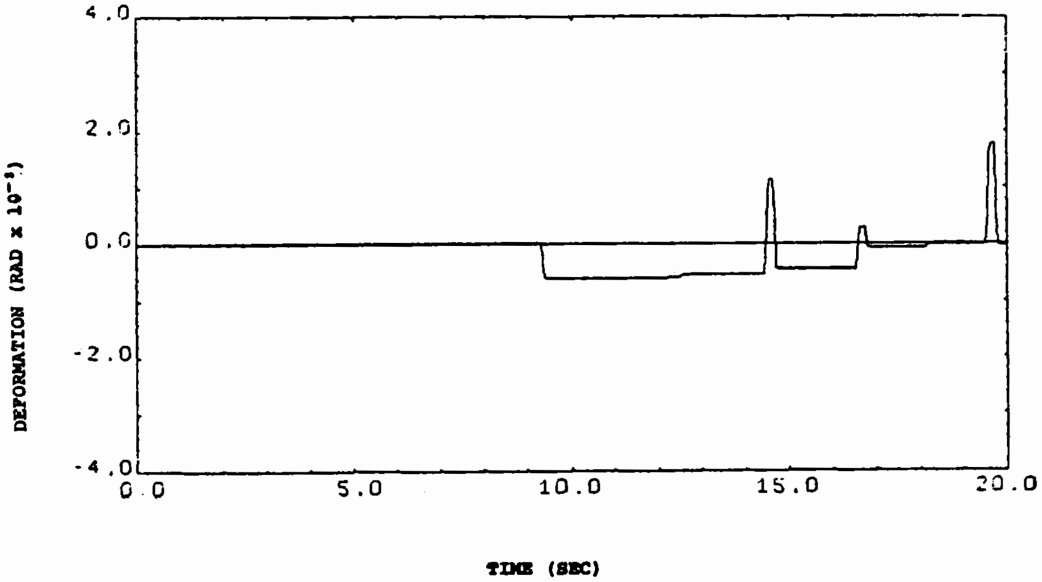
MAXIMUM COLUMN MOMENTS AND CORRESPONDING SHEARS

Table 6.2.7



TRANSVERSE NONLINEAR COLUMN ROTATIONAL DEFORMATIONS
DUE TO TRANSVERSE EXCITATION

Figure 6.2.6



LONGITUDINAL NONLINEAR COLUMN ROTATIONAL DEFORMATIONS
DUE TO LONGITUDINAL EXCITATION

Figure 6.2.7

	LOCATION		
CASE	Abutment 1 (kips)	Abutment 7 (kips)	Hinge Span 3 (kips)
1	234	330	803
3	260	336	936
5	379	430	363
2	188	265	475
4	252	316	229
6	178	240	

MAXIMUM TRANSVERSE FORCE FOR SHEAR KEYS
(TRANSVERSE AND LONGITUDINAL SHOCKS)

Table 6.2.8

overall reduction in shear force of approximately 2.5 for the structure.

The tabulated shear forces due to the longitudinal shocks are less than those obtained for the transverse shock and, consequently, would not govern for design, but do, however, indicate that there is coupling in this case.

6.2.4.5 Maximum Deck Displacements

Transverse Excitation - The maximum deck displacements due to a transverse shock (Cases 1, 3 and 5) are shown in Table 6.2.9. The response spectrum displacements at the bents in the transverse direction are consistently less than the values obtained from the linear time history analysis. This is because three modes, with closely spaced periods, are all responding to this transverse excitation. It is very likely that the peak modal responses will occur simultaneously. The RMS value of the modal responses will,

Location	Global Direction	Case No. 1 (R.S.)	 3 (L.T.H.)	 5 (N.T.H.)
Abut. 1	Trans.	.0665	.0373	.0284
	Long.	.1014	.0555	.0435
Bent 2	Trans.	.0763	.1012	.0637
	Long.	.1146	.0236	.0242
Bent 3	Trans.	.1293	.1711	.1100
	Long.	.1307	.0088	.0243
Bent 4	Trans.	.1623	.1903	.1390
	Long.	.0109	.0068	.0224
Bent 5	Trans.	.1503	.1730	.1090
	Long.	.0116	.0039	.0217
Bent 6	Trans.	.1032	.1134	.0747
	Long.	.0258	.0201	.0238
Abut. 7	Trans.	.0403	.0408	.0296
	Long.	.0597	.0603	.0453

MAXIMUM BRIDGE DECK DISPLACEMENTS IN FEET
DUE TO TRANSVERSE SHOCK

Table 6.2.9

therefore, yield low displacements. For longitudinal displacement the response spectrum RMS values vary drastically from the linear time history results. This is also due to the small period differences and the degree of coupling between the longitudinal and transverse directions in this bridge.

The nonlinear deck displacements in the transverse direction are 27 percent to 37 percent less than the linear time history displacements. This reduction in displacement is caused by the reduction in response due to energy dissipation in the columns. The displacements that occur during the first excursion into the nonlinear range at 4.40 seconds exceed the values reported by the linear analysis at the same time. This is due to the increased initial deformation caused by yielding. With succeeding reversals in the direction of the ground acceleration, however, energy dissipation and reduction in the elastic restoring forces occurs in the column, thus reducing the maximum response of the structure.

It will be noted that there is a small but significant increase in the longitudinal displacements at Bents 3, 4 and 5. This occurs because these three bents have significant deadload moments due to unbalanced span lengths and the presence of a hinge. During initial yielding of these bents, rotational deformations occur which tend to relieve the longitudinal deadload moments. This would occur even for relatively small earthquake longitudinal moment components. Once this yielding has occurred, subsequent column yielding will be entirely due to earthquake forces. Thus the rotational deformations percipitated by high longitudinal deadload moments cause permanent nonlinear deformations in the structure resulting in a biased seismic response in the longitudinal direction. This results in the increased longitudinal displacements.

Longitudinal Excitation - The maximum deck displacements due to a longitudinal shock (Cases 2, 4 and 6), are shown in Table 6.2.10. The longitudinal displacement values for the response spectrum are also consistently less than those for the linear time history. The difference is less to the left of the hinge where mode 2 dominates the longitudinal motion. For transverse displacements the response spectrum analysis does not at all agree with the linear time history. This is due to the close fundamental periods and out of phase transverse participation between the second and third modes of vibration.

The nonlinear displacements in the longitudinal direction are also less than the displacements resulting from a linear time history analysis. The nonlinear displacements

Location	Global Direction	Case No. 2 (R.S.)	 4 (L.T.H.)	 6 (N.T.H.)
Abut 1	Trans.	.0702	.0731	.0609
	Long.	.1067	.1119	.0934
Bent 2	Trans.	.0770	.0551	.0367
	Long.	.1074	.1273	.1066
Bent 3	Trans.	.0880	.0274	.0156
	Long.	.1132	.1390	.1165
Bent 4	Trans.	.0895	.0218	.0115
	Long.	.0686	.0960	.0870
Bent 5	Trans.	.0846	.0425	.0270
	Long.	.0676	.0960	.0870
Bent 6	Trans.	.0632	.0393	.0347
	Long.	.0637	.0919	.0820
Abut. 7	Trans.	.0417	.0554	.0481
	Long.	.0632	.0845	.0737

MAXIMUM BRIDGE DECK DISPLACEMENTS IN FEET
DUE TO LONGITUDINAL EXCITATION

Table 6.2.10

on the left side of the hinge at Bents 2 and 3 are reduced more than those on the right of the hinge at Bents 4 to 6. This is due partially to increased yielding and energy dissipation on the left side and partially to the approximately equal but opposite deadload moments which results in less biased movement in the bents to the left of the hinge. The unbalanced deadload column moments to the right of the hinge have a tendency to magnify the nonlinear displacements at these columns because of the biased response previously mentioned.

6.2.4.6 Maximum Hinge Movements and Restrainer Forces

The assumptions inherent in the elastic analysis approach currently used for design limits the modelling capabilities at the intermediate expansion joint hinge. The idealizations used are approximate in that the restrainer unit takes compression as well as tension and the gaps provided for temperature movements are ignored. The banging effects caused by closing of the seat gap and yielding of the tie bar cannot be incorporated into the model. The assumptions imposed by the limitations of an elastic analysis have been of major concern to the bridge designer. This concern results because of uncertainties about the overall response of the structure and the localized effects on the restrainer units. Basically, the designer's viewpoint has been that the assumptions inherent in this approach will not have a significant effect on the overall response and will yield results for the restrainer unit forces that are approximate but conservative.

The maximum hinge separations and corresponding restrainer forces due to both longitudinal and transverse shocks are recorded in Table 6.2.11. The response spectrum results for both hinge movement and restrainer forces are 6 to 10 times larger than the elastic time history results for the transverse shock. These large differences are due to the out of phase response that occurs between the second and third modes of vibration. Contrary to this, however, the results for the longitudinal motion agree quite well. The results agree within 10 percent for both the inside and outside restrainer units. All three analyses of longitudinal motion, which generally controls restrainer designs, results in restrainer forces which are less than the 542 kip force obtained using 25 percent of the deadload or the adjacent superstructure section as required by the AASHTO code provisions.

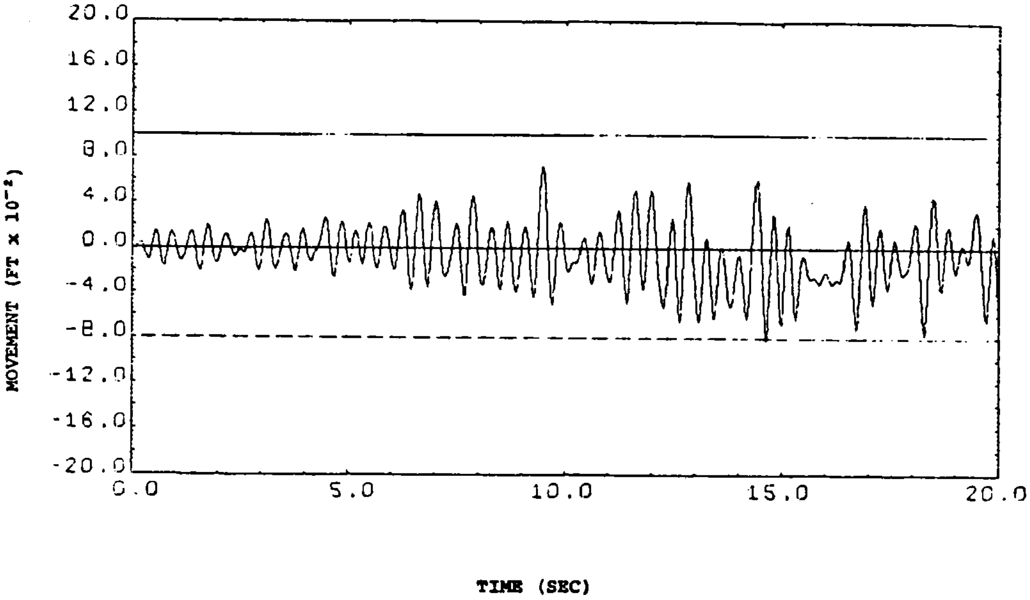
The nonlinear analysis yielded no restrainer bar forces indicating that the temperature gap of 0.1 foot in the restrainer was not taken up due to either the transverse or longitudinal motions. Time history plots of the nonlinear expansion joint movements at the right and left edge of deck for both the transverse and longitudinal motions are shown in Figures 6.2.8 and 6.2.9 respectively. The tie gap and seat gap are superimposed on the plots. The plots indicate that the hinge movements are not quite large enough to stress the restrainers. The seat gaps closed only a few times resulting in a small amount of banging

	Max. Hinge Movement (ft)		Max. Retrainer Force (kips)	
Case	Inside Unit (Rt)	Outside Unit (Lt)	Inside Unit (Rt)	Outside Unit (Lt)
	.1224	.1307	149	159
3	.0123	.0220	15	27
5	.0432	.0481	0	0
2	.1249	.1315	152	160
4	.1124	.1158	137	141
6	.0824	.0836	0	0

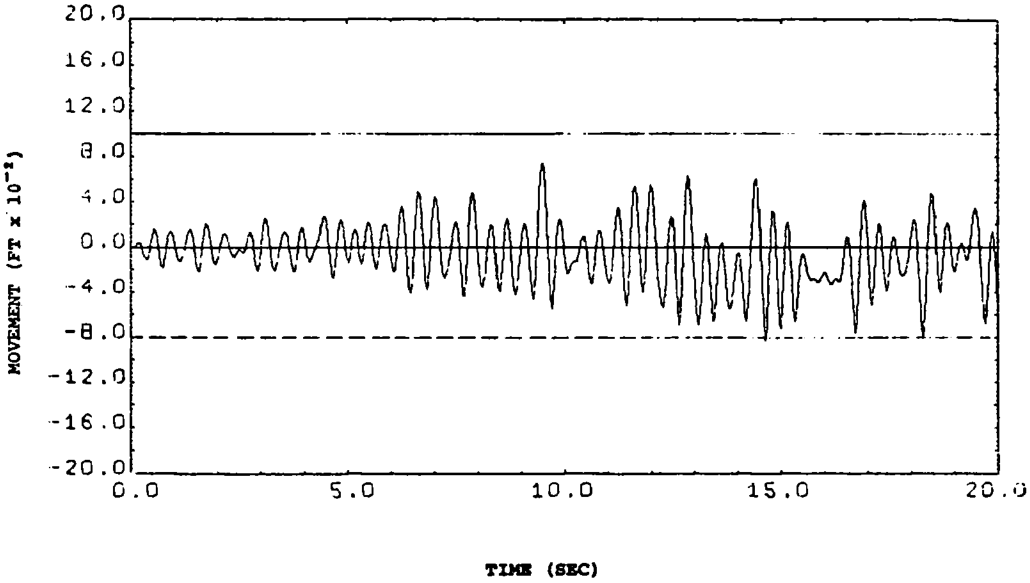
MAXIMUM HINGE SEPARATIONS AND RESTRAINER FORCES

Table 6.2.11

action at the hinge. The plots of expansion joint movement due to transverse excitation indicates the tendency of the hinge to open due to the nonlinear behavior occurring in the bridge. The biased movement would be more pronounced with additional column yielding or sliding of the expansion joint bearing. Considering the number of excursions of the columns into the nonlinear range and the probable degredation in column stiffness, forces in the restrainer would probably occur in a real case.



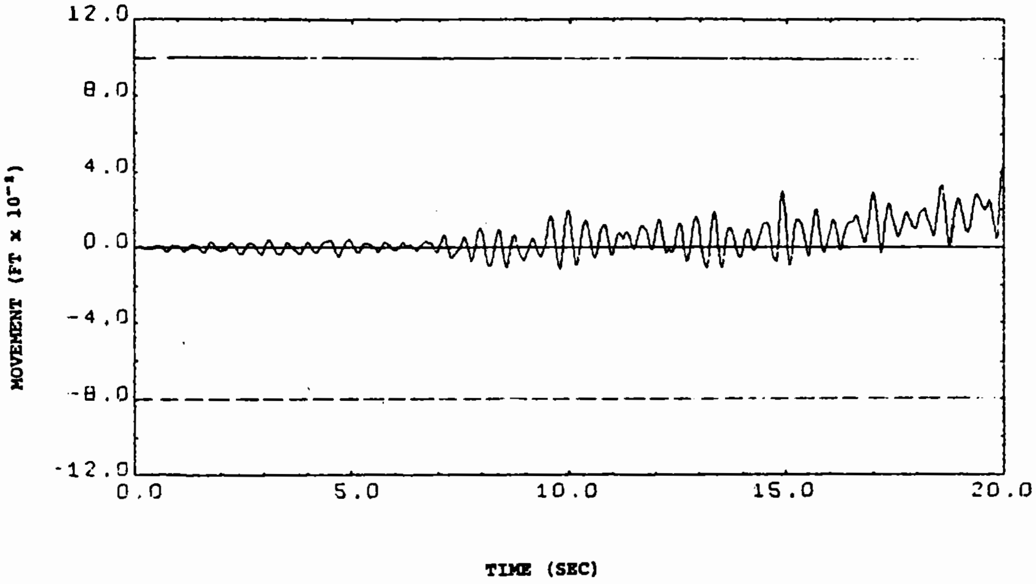
Expansion Joint Movement at Right Edge of Deck - Span 3 Joint



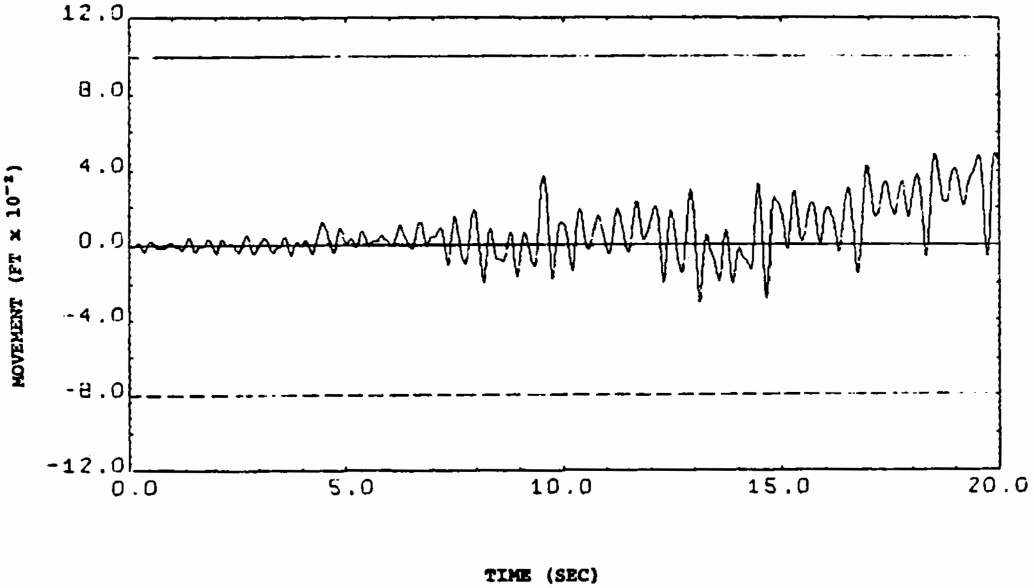
Expansion Joint Movement at Left Edge of Deck - Span 3 Joint

MOVEMENT AT HINGE (RIGHT AND LEFT)
DUE TO LONGITUDINAL EXCITATION

Figure 6.2.8



Expansion Joint Movement at Right Edge of Deck - Span 3 Joint



Expansion Joint Movement at Left Edge of Deck - Span 3 Joint

MOVEMENT AT HINGE (RIGHT AND LEFT EDGE)
DUE TO TRANSVERSE SHOCK

Figure 6.2.9

6.3 EVALUATION OF FOUNDATION BEHAVIOR

6.3.1 INTRODUCTION

This section is included to present the bridge designer with a procedure for calculating spring coefficient and damping for pile groups and spread footings [6.6 thru 6.10]. The techniques presented herein are considered by the authors to be the most suitable for bridge foundation and consistent with the degree of complexity, economy, definition and state of knowledge of associated variables in the overall seismic analysis.

Bridges, like other structures, derive their support from a foundation through which any earthquake motion is transmitted. Generally, a bridge is supported on relatively flexible column or bent as compared to the foundation and may be assumed to be supported on a rigid foundation. Then a seismic motion used as an input forcing function is applied to the fixed base of the structure as a rigid body input motion with all the supports experiencing the same motion. This method of analysis is generally referred to as a noninteractive system. However, completely at the other end of the spectrum in foundation modeling is the fully interactive system which also includes the effect of the structure response on the foundation.

This fully interactive system must be modelled as a continuous system employing a network of finite elements to characterize the complete soil-structure system. The input forcing functions are input at the element boundaries as bedrock motion. This system accounts for the feedback of the structure response to the soil medium as well as the compliance of the foundation. Although not fully understood from a practical point of view, the effects of non-rigid support motion produced by traveling seismic waves may be included in such an idealization.

A methodology, between these two extremes, which satisfies compatability and equilibrium at the structure interface and accounts for the foundation compliance is called the half space method. This method is presented for spread footings and also, in an extended form, to include pile foundations. Using this method, the foundation effects can be determined independently, without considering the support motions. The resulting system now including the flexibility of the foundation can be analyzed using the conventional dynamic analysis techniques. The natural frequencies, mode shapes and response of the foundation will be reflected in the overall structural system.

6.3.2 STIFFNESS COEFFICIENTS FOR INDIVIDUAL PILES

Initially, the horizontal and vertical stiffness of the individual piles will be determined. Batter piles are not considered in this presentation. The following assumptions are made with respect to the individual pile behavior:

1) There is no variation in the pile properties along the length of the pile.

2) The axial soil load transfer can be represented by a simple function.

3) The piles can be expressed as linear springs being equal in tension and compression.

4) The load transfer to the soil can be expressed as a linear-elastic relationship.

6.3.2.1 Vertical Stiffness

The vertical stiffness of an individual pile is a function of the transfer mechanism by which the load is transferred to the soil. This pile-soil interaction is a complex phenomenon and approximated for analysis purposes in one of the three ways as represented in Figure 6.3.1. These include point bearing, constant skin friction and linear skin friction.

Case A The vertical stiffness of a point bearing pile as shown in Figure 6.3.1a is given by:

$$k_v = \frac{AE}{L} \qquad (6.3.1)$$

where

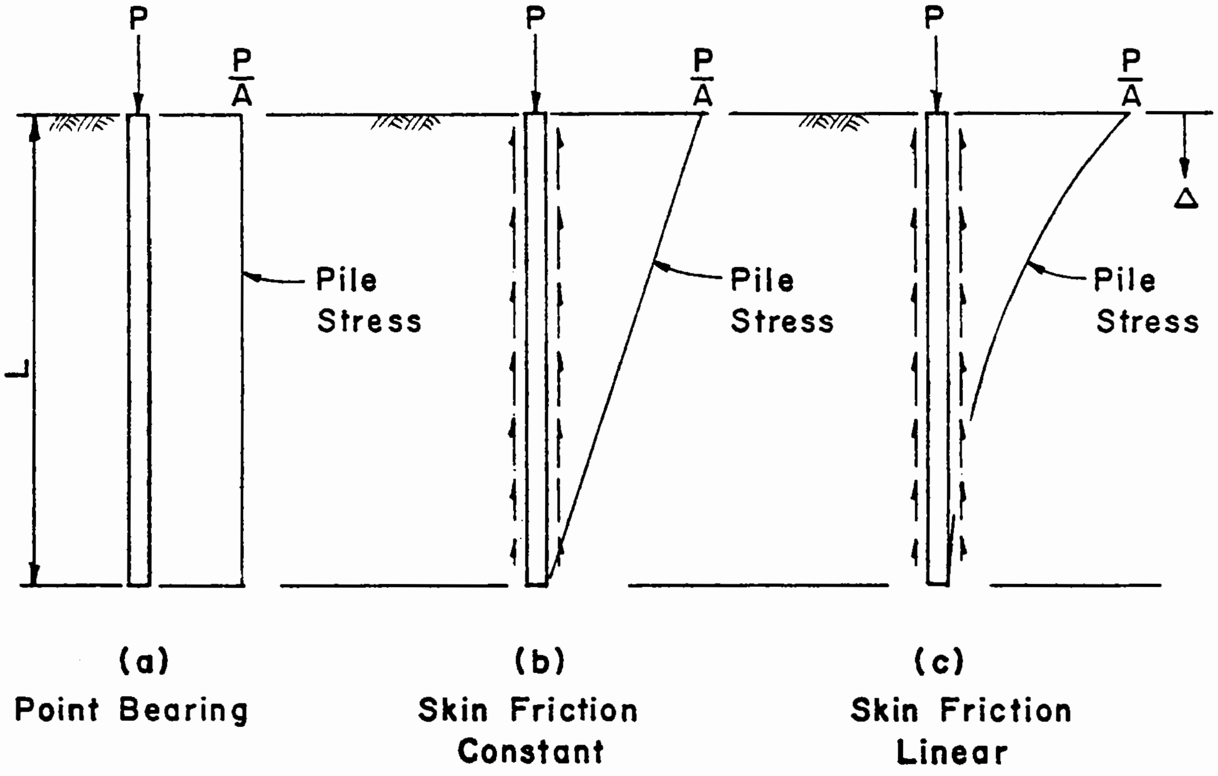
A = Cross sectional area of the pile

E = Young's modulus of the pile

L = Pile length

Case B The vertical stiffness of pile having a constant skin friction along the pile as illustrated in Figure 6.3.1b is given by:

$$k_v = \frac{2AE}{L} \qquad (6.3.2)$$



(a)
Point Bearing

(b)
Skin Friction
Constant

(c)
Skin Friction
Linear

PILE TO SOIL LOAD TRANSFER

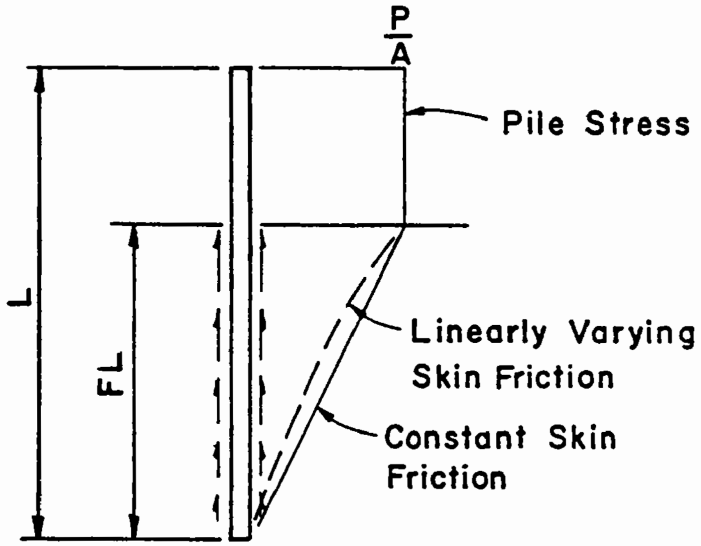
Figure 6.3.1

Case C The vertical stiffness of a pile with a linearly varying skin friction along the pile as shown in Figure 6.3.1c is given by the following expression:

$$k_v = \frac{3AE}{L} \tag{6.3.3}$$

The expression for an end bearing pile is given by Equation 6.3.1. The constant skin friction case expressed mathematically by Equation 6.3.2 is recommended for soft to medium stiff cohesive soils. For these soils, the modulus increases with depth. Case C is used to represent the situation of a relatively stiff soil as compared to the pile, this is represented by Case C having the linearly varying skin friction. This is expressed mathematically by Equation 6.3.3.

Occasionally a pile may transfer the load to the soil for only a portion of its length as shown in Figure 6.3.2.



PARTIAL TRANSFER OF LOAD TO SOIL

Figure 6.3.2

Thus only a fraction of the pile is effective in the load transfer mechanism. The expression for the constant and linearly varying skin frictions are expressed in Equations 6.3.4 and 6.3.5 respectively.

$$k_v = \frac{AE}{(1+\frac{F}{2})L} \qquad (6.3.4)$$

$$k_v = \frac{AE}{(1+\frac{F}{3})L} \qquad (6.3.5)$$

where

F = the fraction of the pile embedded in the soil

Note that in these formulations the piles are assumed to behave the same in tension and compression. This is generally a valid assumption as long as the piles remain in compression due to deadload, however, the designer should keep in mind that a pile subject to uplift may have a slightly different behavior.

6.3.2.2 Lateral Stiffness

The basic technique used in the development of the lateral stiffness of a pile is based on a beam on an elastic foundation. The response quantity needed to formulate a stiffness coefficient is the lateral deflection at the top of the pile. The lateral deflection is based on the spring rate along the length of the pile as disccussed for the two cases below:

Case A This case is based on the pile acting as a long beam on an elastic foundation with the foundation spring rate, k_s, constant along the length of the pile. This case represents the stiff to hard clays.

The lateral stiffness coefficient is given by the following expression:

$$k_L = \frac{1.414\, k_s^{.75} (EI)^{.25}}{B} \qquad (6.3.6)$$

where

k_s = $\frac{k_{v1}}{1.5}$ = Subgrade modulus for a long slender pile

k_{v1} = Coefficient of vertical subgrade reaction for a one foot square plate (pcf) which may be empirically determined from Curve 1 in Figure 6.3.3 based on unconfined compressive strength of clay.

B = Deflection coefficient,

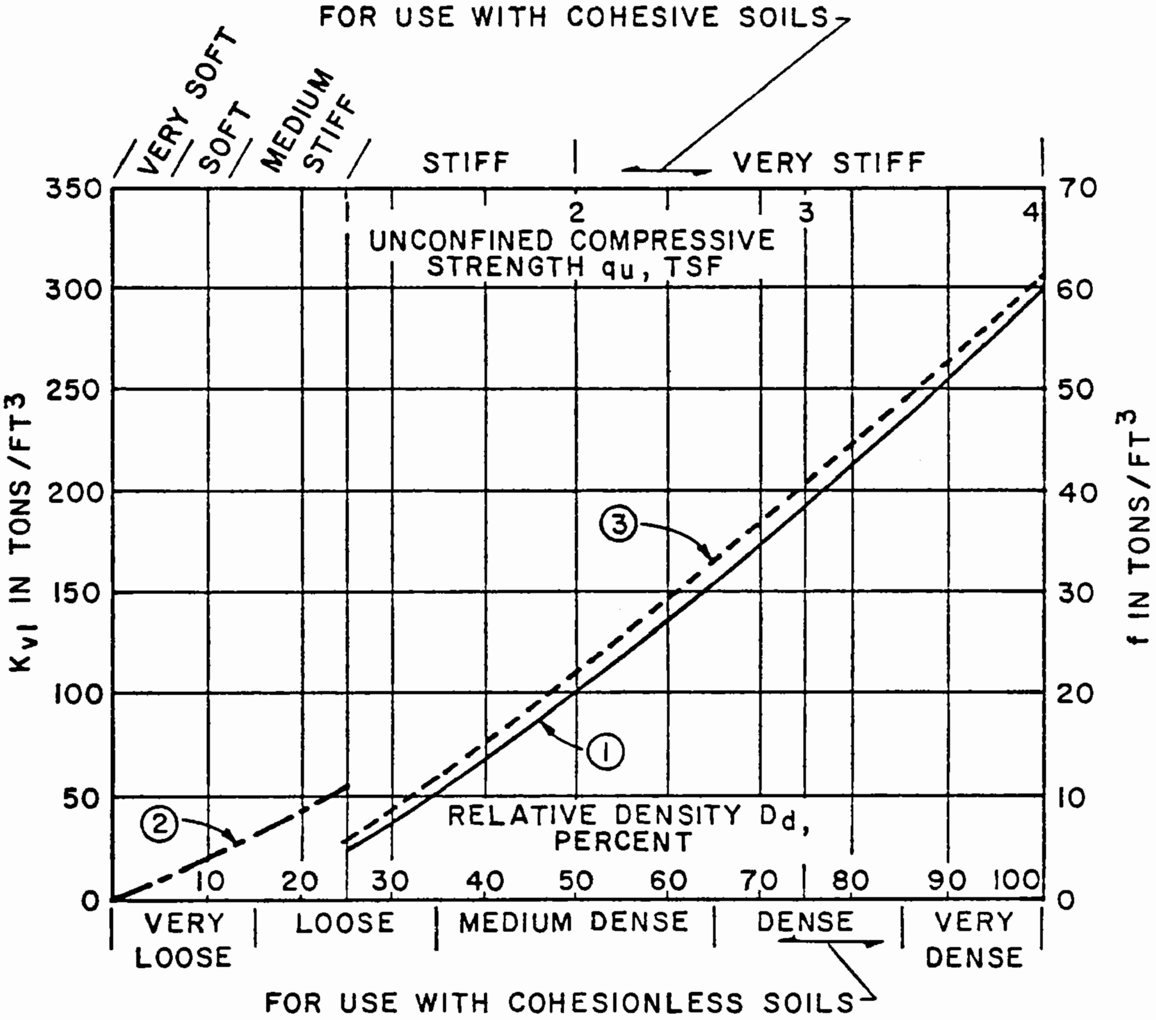
= 1.0 (fixed head pile)

= 2.0 (free head pile)

E = Young's modulus of the pile

I = Moment of inertia of the pile

Case B This case is based on the assumption that the pile acts as a beam on an elastic foundation with the foundation spring rate being a function of the distance below the surface. The expression for this case is derived from the



CURVE ① - K_{vl} VERSUS q_u FOR COHESIVE SOILS

CURVE ② - f VERSUS q_u FOR COHESIVE SOILS

CURVE ③ - f VERSUS D_d FOR COHESIONLESS SOILS

MODULUS OF VERTICAL SUBGRADE REACTION

COEFFICIENT OF VARIATION OF SOIL MODULUS OF ELASTICITY WITH DEPTH

Figure 6.3.3

curves by Reese and Matlock [6.9, 6.11, 6.12] for the condition of the pile length greater than 5 times the diameter, but for practical purposes, may be used for a pile length as short as three times the diameter. The soil types that are included in Case B are the very soft to medium stiff clays as well as the cohesionless soils.

The lateral stiffness coefficient is given by the following expression:

$$k_L = \frac{f^{.6}(EI)^{.4}}{A_\delta} \tag{6.3.7}$$

where

f = Constant of modulus variation with depth Z

A_δ = Deflection coefficient

= 0.93 for a fixed head (for L>5T)

= 2.4 for a free head (for L>5T)

Figure 6.3.3, which was determined empirically, may be used to estimate the constant of modulus variation. Curve 2 is used for very soft to medium stiff clay and Curve 3 is used for cohesionless soils. The constant of modulus variation with depth is also given by:

$$f = \frac{k_h}{z} \tag{6.3.8}$$

where

k_h = coefficient of horizontal subgrade reaction

The deflection coefficient A_δ is a function of the pile length and the coefficient T which is given by the following expressions:

$$T = \left(\frac{EI}{f}\right)^{.2} \tag{6.3.9}$$

6.3.3 STIFFNESS COEFFICIENTS FOR FOOTING FOUNDATIONS

The traditional assumption of a rigid foundation resting on an elastic, homogeneous, isotropic, and semi-infinite body is used. With this assumption, the theory of elasticity provides solutions for the response of the foundations due to vertical, horizontal, rocking, or torsional mode of simple harmonic excitation. The stiffness coefficient of the foundation is then represented by the linear spring constant between the applied load or moment and the resulting linear or angular displacement of the supporting soil. The factors affecting this spring constant are the shear modulus of elasticity, G, the Poisson's ratio, , the size and shape of the foundation, and the direction of rotation.

6.3.3.1 Vertical Vibration

The spring constant of a circular footing due to vertical mode of vibration is given by:

$$k_z = \frac{4Gr_o}{1-\nu} \tag{6.3.20}$$

where

r_o = the radius of the circular footing.

If the footing is rectangular in shape, then the spring constant is computed by:

$$k_z = \frac{\beta_z G}{1-\nu}\sqrt{A} \tag{6.3.21}$$

where

A = the area of the rectangular footing

β_z = shape factor

where the shape factor depends on the length to width ratio (L/W) as follows:

L/W	1.00	1.50	2.00	3.00	5.00	10.00
β_z	2.12	2.14	2.18	2.26	2.44	2.82

Table 6.3.1

The equation for circular footing may be applied to foundations of arbitrary shape, including rectangular, provided that an equivalent radius is computed by,

$$r_e = \sqrt{\frac{A}{\pi}} \tag{6.3.22}$$

in which A is the area of the actual footing. The spring constant, k_z, is computed by Eq. (6.3.20) by substituting r_e for r_o. The error involved in this approximation will be less than 10 percent with the length to width ratio of not more than 5.

6.3.3.2 Horizontal Vibration

It is assumed that the footing is resting on the surface of elastic half space without embedment. Then the stiffness coefficient of the circular rigid footing against horizontal sliding is given by:

$$k_x = \frac{32(1-\nu)Gr_o}{7-8\nu} \tag{6.3.23}$$

For the rectangular footing, the value of k_x is computed by,

$$k_x = \beta_x (1-\nu) G \sqrt{A} \tag{6.3.24}$$

where

β_x = shape factor whose value is approximately 2.0 for the length to width ratio of 5 or less.

For a foundation of arbitrary shape, Equation 6.3.22 may be used to compute the equivalent radius for the substitution of the radius value in Equation 6.3.23.

6.3.3.3 Rocking Vibration

The spring constant of circular footing for rocking vibration is given by:

$$k_\psi = \frac{8Gr_o^3}{3(1-\nu)} \quad (6.3.25)$$

where the value of k_ψ is also the product of the coefficient of non-uniform compressibility of supporting soil due to the external rocking moment and the moment of inertia of the foundation area with respect to the axis of rotation.

For rectangular foundation with sides 2c x 2d where the axis of rotation is in the direction parallel to 2c, the spring constant, k_ψ , is evaluated using the expression,

$$k_\psi = \beta_\psi \frac{8Gcd^2}{1-\nu} \quad (6.3.26)$$

where

β_ψ = the shape factor whose value depends on the ratio of d/c as shown in Table 6.3.2.

d/c	0.20	0.50	1.00	2.00	4.00	6.00	8.00
β_ψ	0.40	0.45	0.50	0.60	0.80	0.95	1.10

SHAPE FACTOR VS. D/C RATIO

Table 6.3.2

The expression given for a circular footing in Equation (6.3.25) may be applied to rectangular foundations or foundations with arbitrary shape. The equivalent radius r_e for rectangular foundation may be computed by:

$$r_e = \sqrt[4]{\frac{16cd^3}{3\pi}} \quad (6.3.27)$$

Therefore, it is obvious that the value of spring constant, k_ψ, for rocking vibration depends on shear modulus and Poisson's ratio of the soil, size and shape of the foundation, and the moment of inertia of the foundation with respect to the axis of rotation.

6.3.3.4 Torsional Vibration

The torsional vibration considered here is the movement of a rigid circular footing around a vertical axis through the center of the contact area due to torque T_θ. The relation between this torque and the resulting angular rotation θ is represented by the spring constant k_θ as:

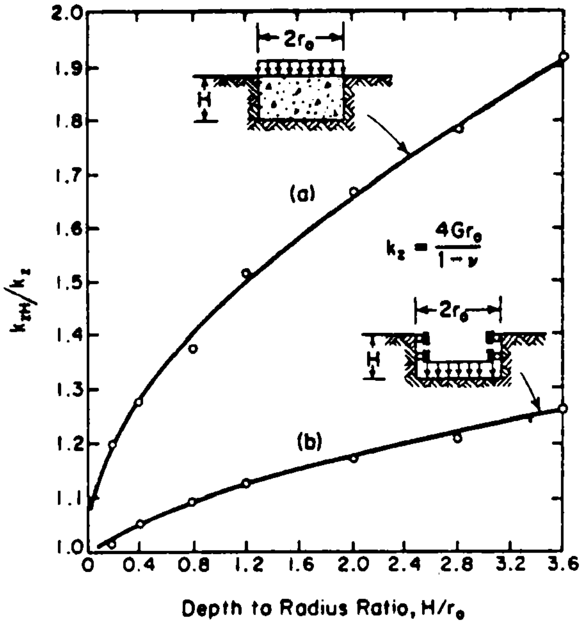
$$k_\theta = \frac{16}{3} G r_0^3 \qquad (6.3.28)$$

For a rectangular footing with sides 2c and 2d subjected to torsional vibration around a vertical axis passing through the centroid of the foundation area, an equivalent radius may be computed for substitution into the above equation as follows:

$$r_e = \sqrt[4]{\frac{16cd(c^2+d^2)}{6\pi}} \qquad (6.3.29)$$

6.3.3.5 Embedment Factor

The embedment of foundation provides effective increase in the stiffness coefficient. This effect is more pronounced as shown in curve A, Figure 6.3.4 in which the circular rigid foundation adheres to the soil along the vertical surface of contact as well as bearing at the base. Curve B represents the increase in stiffness of foundation when the vertical contact is considered frictionless. Figure 6.3.4 is based on the study by Kaldjaan (1969) for vertical vibration of rigid circular footings. The effect of embedment should be more significant for other modes of vibration. Therefore, the same ratio of increase in stiffness coefficient should be applied for other modes of vibrations including horizontal, rotational and torsional.



EFFECT OF EMBEDMENT ON SPRING CONSTANTS

Figure 6.3.4

6.3.4 SOIL DAMPING

Damping is used to represent the effect of energy dissipating nature of foundation vibration in a lumped mass-spring-dashpot system. In such a system, the damping force is defined as the product of a damping constant, c, and the velocity of the motion. The damping ratio, D , which is also called as damping factor, is defined as the ratio of this damping constant to the critical damping constant as follows:

$$D = \frac{c}{C_c} \qquad (6.3.30)$$

in which the critical damping constant, C_c , is computed by,

$$C_c = 2\sqrt{km} \qquad (6.3.31)$$

where

k = the spring constant of the soil-foundation system

m = the effective mass of the foundation

There are two types of damping in the soil foundation system during vibration. One is the geometrical damping that is due to the loss of energy through propagation of elastic waves away from the foundation and the other called internal damping which is energy loss due to the inelastic, viscous, and hysteretic effects of soils. The methods of computing the damping constants for various modes of vibrations are described below.

6.3.4.1 Geometrical Damping

Based on Richart, Hall, and Woods [6.7], the geometrical damping ratio for rigid circular foundation may be expressed in terms of mass ratio, β , as follows:

Mode of Vibration	Mass Ratio β	Damping Ratio D
Vertical	$\beta_z = \frac{(1-\nu)m}{4\rho r_o^3}$	$D_z = \frac{0.425}{\sqrt{\beta_z}}$
Horizontal	$\beta_x = \frac{(7-8\nu)m}{32(1-\nu)\rho r_o^3}$	$D_x = \frac{0.288}{\sqrt{\beta_x}}$
Rocking	$\beta_\psi = \frac{3(1-\nu)I_\psi}{8\rho r_o^5}$	$D_\psi = \frac{0.15}{(1+\beta_\psi)\sqrt{\beta_\psi}}$
Torsional	$\beta_\theta = \frac{I_\theta}{\rho r_o^5}$	$D_\theta = \frac{0.5}{1+2\beta_\theta}$

Table 6.3.3

Where:

ρ = mass density of soil = $\frac{\gamma}{g}$

γ = soil density

g = the gravitational acceleration

r_o = the radius of the rigid circular foundation

m = effective mass of the foundation

ν = Poisson's ratio of soil

I_ψ = mass moment of inertia of the foundation with respect to the axis of rotation through the base

I_θ = mass moment of inertia of the foundation with respect to the vertical axis through the center of gravity

If the shape of foundation is not circular, an equivalent radius may be computed using Equations (6.3.22), (6.3.27) and (6.3.29) for substitution of radius r_o in the above Table.

6.3.4.2 Internal Damping

The internal damping of soil depends on the soil type and the strain amplitude. Generally speaking, this damping effect is relatively small as compared to geometrical damping effect in translatory vibrations (vertical and horizontal), but is about the same order of magnitude in rotationary vibrations. The experimental data indicate that this internal damping ratio is approximately 0.05 (Richard, Hall and Woods) which should be added to the damping ratio computed for geometrical damping to give the total damping ratio.

6.3.4.3 Embedment Effect

Embedment of foundation increases the overall damping effect. For circular footings with vertical vibration, the increase in damping ratio may be estimated for depth of embedment up to the radius by the following relationship based on Lysmer and Kuhlemeyer (1969).

$$D \approx D_o \left(1 + 0.9 \frac{H}{r_o}\right) \qquad (6.3.32)$$

where

D_o = the damping ratio without embedment

H = the depth of embedment

Although no experimental data are available at this time, it is reasonable to assure that the increase in damping ratio due to embedment for other modes of vibration is at least of the same order of magnitude as that of the vertical vibration.

6.3.5 ELASTIC SOIL CONSTANTS

The elastic soil constants that are used in the dynamic analysis of foundations are the shear modulus, G, and the Poisson's ratio, ν . These constants are determined indirectly from geophysical measurements of wave velocities. The non-homogeneous, anisotropic nature of earth makes these determinations very difficult. The geophysical measurement techniques generally used include seismic refraction survey, downhole or uphole survey, cross-hole survey and the surface wave method. These methods result in determination of the compression wave velocity, shear wave velocity, and Rayleigh wave velocity. The elastic constants are then computed from these wave velocities.

6.3.5.1 Poisson's Ratio

The Poisson's ratio may be computed from,

$$\nu = \frac{0.5 R^2 - 1}{R^2 - 1} \qquad (6.3.33)$$

where

R = the ratio of compressive wave velocity to shear wave velocity.

The theoretical range of the Poisson's ratio is from 0 to 1/2 with the value for in-place soil generally varying from 0.25 to 0.45. For saturated, soft soil, the Poisson's ratio may become very close to 1/2. Generally, the Poisson's ratio varies from 0.25 to 0.35 for cohesionless soils and from about 0.35 to 0.45 for cohesive soils. It has been found that for design purposes, little error is introduced if Poisson's ratio is assumed as 0.33 for cohesionless soils and 0.4 for cohesive soils.

6.3.5.2 Shear Modulus, G

The shear modulus, G, is computed by,

$$G = \rho V_s^2 \tag{6.3.34}$$

where

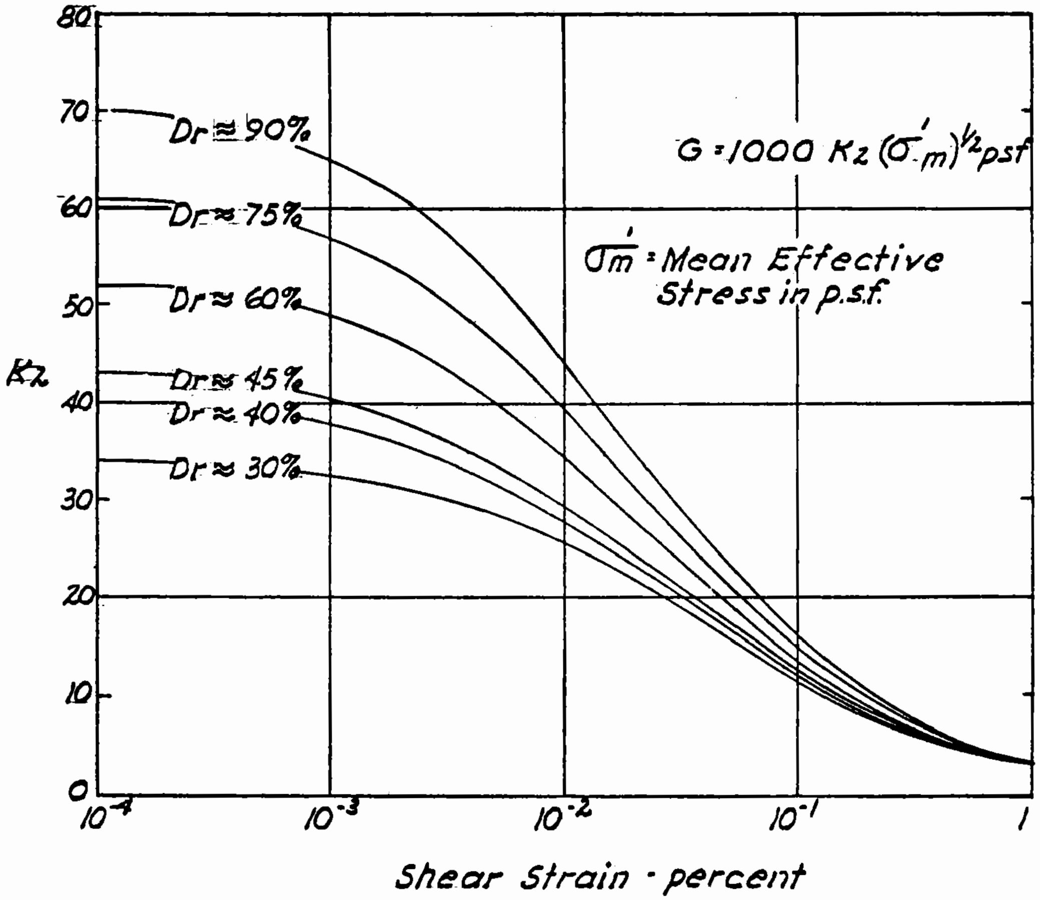
ρ = mass density of soil

V_s = shear wave velocity

when the surface wave method is used in a geophysical survey, the Rayleigh wave velocity is determined. Since the shear wave velocity is only slightly greater than the Rayleigh wave velocity, this measured Rayleigh wave velocity is used in the above equation to compute the value of G.

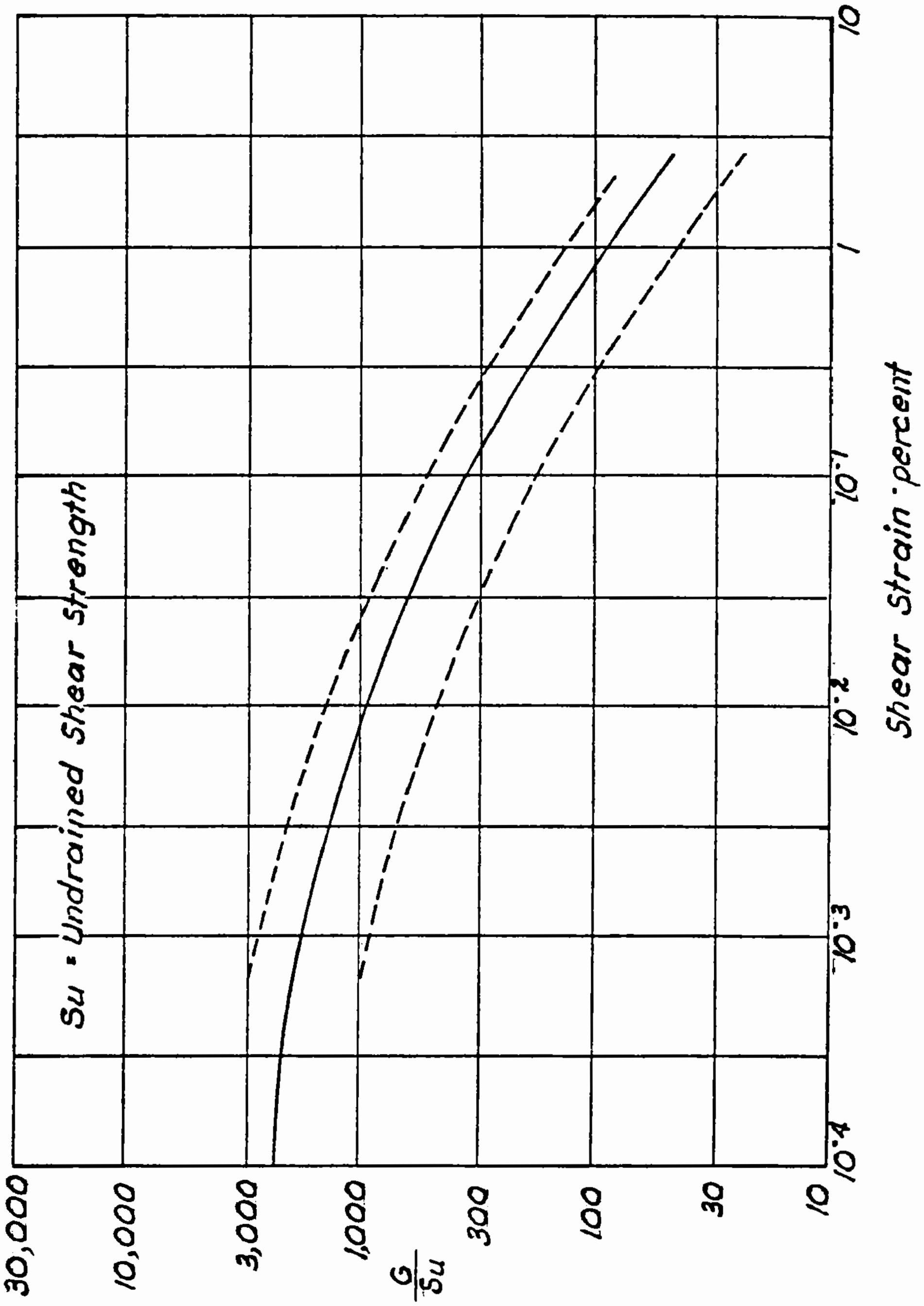
6.3.5.3 Effects of Strain Amplitude

The strain amplitude is the most important factor affecting the elastic soil constants. Increase in strain level will result in the decrease of wave velocity and therefore, the shear modulus. If the elastic constants are determined based on the geophysical survey data, these values should be considered as valid only at small strain amplitude of about .0001 percent which is the level of strain amplitude generated in seismic survey techniques. Therefore, these constants should be corrected to reflect the actual strain amplitude experienced by soil during the earthquake vibrations. The relationship between shear modulus and strain amplitude for cohesionless soils and saturated clays are suggested by Seed and Idriss [6.13] as shown in Figure 6.3.5 and Figure 6.3.6. In Figure 6.3.5, the shear modulus of sands for different relative densities are shown in terms of effective average stress and strain amplitudes in percent. Therefore, this figure may also be used to compute the shear modulus based on relative density of sands which are commonly correlated to Standard Penetration Test data as shown in Figure 6.3.7 [6.14]. For saturated clays or cohesive soils, Figure 6.3.6 shows the



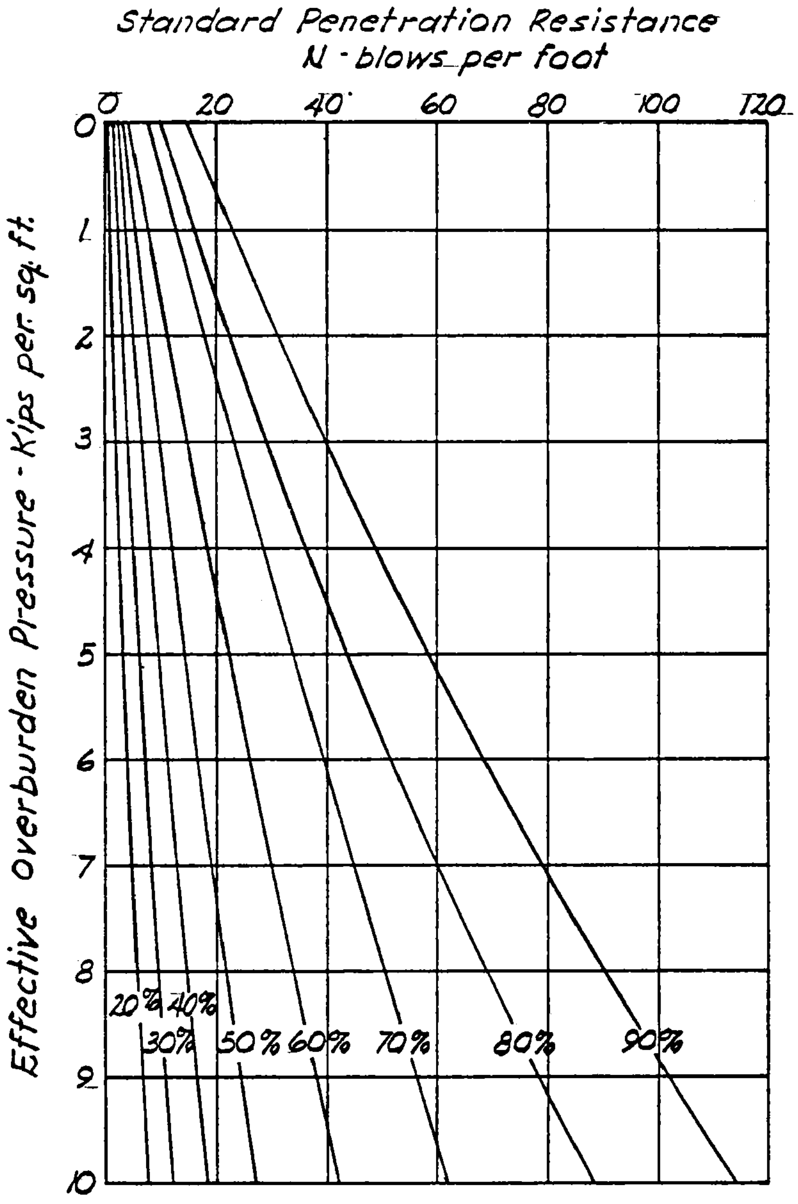
SHEAR MODULUS VS. STRAIN AMPLITUDE AS A FUNCTION OF RELATIVE DENSITY FOR COHESIONLESS SOILS

Figure 6.3.5



SHEAR MODULUS VS. STRAIN AMPLITUDE
SATURATED CLAYS

Figure 6.3.6



RELATIONSHIP BETWEEN STANDARD PENETRATION RESISTANCE, RELATIVE DENSITY AND EFFECTIVE OVERBURDEN PRESSURE

Figure 6.3.7

relationship of shear modulus versus strain amplitude as a function of the undrained shear strength of the soil. The undrained shear strength of saturated cohesive soils may be obtained using laboratory unconfined compression tests on undisturbed soil sample, from in-situ shear tests, or based on Standard Penetration Test data.

6.4 REFERENCES

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6.2 "Bridge Planning Design Manual," Vol. 1, Sect. 2-16, April, 1977, California Department of Transportation, Office of Structures.

6.3 Imbsen, R.A., Nutt, R.V., and Penzien, J., "Seismic Response of Bridges - Case Studies," Report No. DBC/EERC - 78/14, Earthquake Engineering Research Center, University of California, Berkeley, June, 1978.

6.4 Tseng, W.S., and Penzien, J., "Analytical Investigations of the Seismic Response of Long Multiple Span Highway Bridges," Report No. EERC 73-12, Earthquake Engineering Research Center, University of California, Berkeley, June, 1973.

6.5 Kawashima, K., and Penzien, J., "Correlative Investigations on Theoretical and Experimental Dynamic Behavior of a Model Bridge Structure," Report No. EERC 76-26, Earthquake Engineering Research Center, University of California, Berkeley, July 1976.

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6.7 Richard, F. E., Hall, J.R., and Words, R.D., "Vibrations of Soils and Foundations," Prentice - Hall, Inc., 1970.

6.8 Margolin, M.N., "A Simplified Analysis For Three Dimensional Pile Groups," American Wood Preservers Institute, Guideline No.P5, 1975.

6.9 Reese, L.C., and Matlock, H., "Nondimensional Solutions For Laterally Loaded Piles With Soil Modulus Proportioned to Depth," Eighth Texas Conference on Soil Mechanics and Foundation Engineering, University of Texas, Austin, 1956.

6.10 Arthur, D.F. et al., "Evaluation of Methods for Seismic Analysis of Nuclear Fuel reprocessing and Fabrication Facilities," Lawrence Livermore Laboratory, University of California, Livermore, UCRL-52159, February, 1978.

6.11 Matlock, H., and Reese, L.C., "Generalized Solutions For Laterally Loaded Piles," Journal - Soil Mechanics and Foundations, ASCE, October, 1960.

6.12 Naval Facilities Engineering Command, Department of the Navy, "Design Manual - Soil Mechanics Foundations, and Earth Structures," NAVFAC DA-7, March, 1971, pp 7-13-16 and 7-13-17.

6.13 Seed, H.B., and Idriss, I.M., "Soil Moduli and Damping Factors for Dynamic Response Analysis," Earthquake Engineering Research Center, Report No. 70-10, U.C. Berkeley, 1970.

6.14 Gibbs, H.J., and Holtz, W.G., "Research on Determining the Density of Sand by Spoon Penetration Tests," Proceedings Fourth International Conference on Soil Mechanics and Foundation Engineering, Vol. 1, 1957.

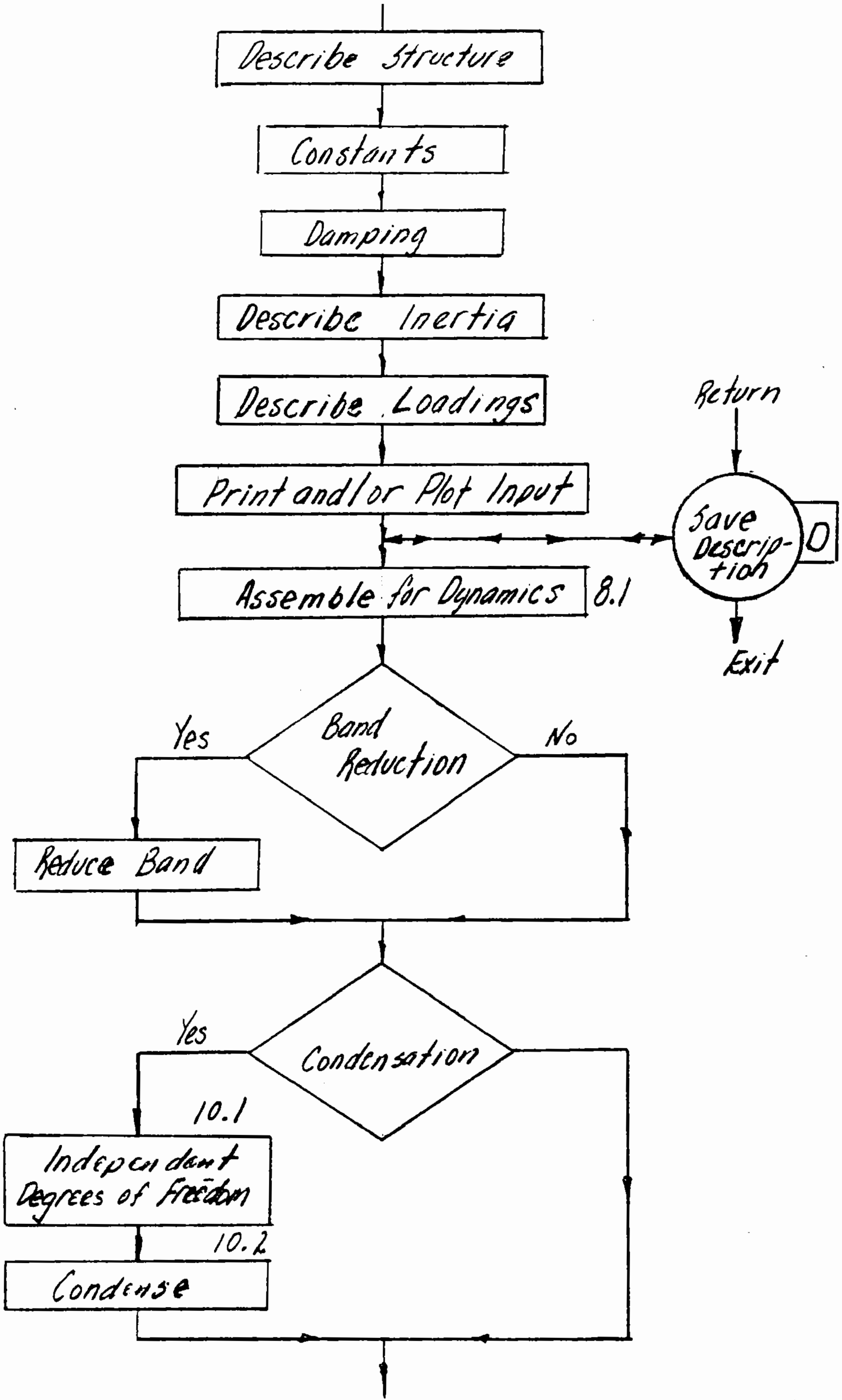
Appendix A.1

STRUDL/DYNAL SEISMIC ANALYSIS

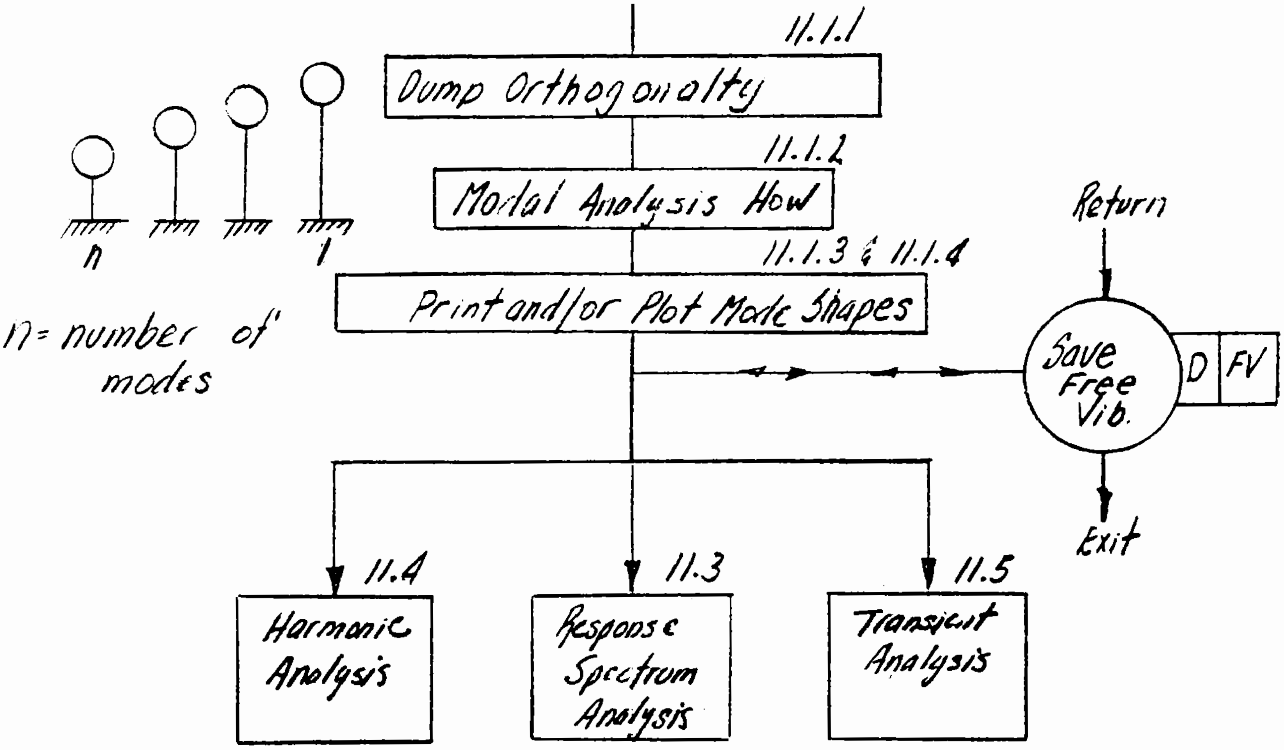
ICES STRUDL, (STRUctural Design Language), is a large general purpose structural analysis and design computer system. The code is designed to allow the user to define his problem in terms that are familiar to the structural engineer. This means that the user need not be familiar with computer programing. It is important, however, that the user understand the command structure and the way the computer will interpret the commands to solve a typical problem. On the following pages is some information that will aid the user in solving a seismic analysis problem.

A STRUDL/DYNAL Dynamic Analysis Flowchart is included which describes a sequence of necessary and optional solution steps that can be taken to use STRUDL for a dynamic analysis. A suggested outline of STRUDL commands for a response spectrum dynamic analysis is also included which is referenced to the McAuto STRUDL manual.

Several example problems illustrating the use of STRUDL for dynamic analysis are included in Appendix A.2 to A.7.



STRUDL/DYNAL DYNAMIC ANALYSIS FLOWCHART



STRUDL/DYNAL DYNAMIC ANALYSIS FLOWCHART

SUGGESTED OUTLINE OF STRUDL COMMANDS
FOR RESPONSE SPECTRUM DYNAMIC ANALYSIS

COMMAND		REFERENCE
STRUDL "name"		
* ACTIVE JOINTS ..,..,..		(Vol. 1, P. 112)
UNITS		
JOINT COORDINATES		(Vol. 1, P. 43)
.		
* MESH COORDINATES		McAUTO SECTION 6.2.3.1
TYPE SPACE FRAME		
.		
MEMBER INCIDENCES		(Vol. 1, P. 57)
.		
* MESH INCIDENCES		McAUTO SECTION 6.2.3.2
.		
* TYPE SPACE TRUSS	(Use for Hinge Restrainer Overlay)	(Vol. 1, P. 38)
* MEMBER INCIDENCES		
* MESH INCIDENCES		
.		
* MEMBER ECCENTRICITIES		(Vol. 1, P. 65)
.		
MEMBER PROPERTIES		(Vol. 1, P. 71)
.		
CONSTANTS		(Vol. 1, P. 79)

COMMAND	REFERENCE
E . . . & DENSITY . . .	
* BETA . . .	(Vol. 1, P. 27 & 79)
.	
.	
.	
MEMBER RELEASES	(Vol. 1, P. 59)
.	
.	
.	
JOINT RELEASES	(Vol. 1, P. 49)
.	
JOINT RESTRAINTS	McAUTO SECTION 6.1.2.5
.	
.	
INERTIA OF JOINTS LUMPED	McAUTO SECTION 6.1.5.1
DAMPING PERCENTS . . ., . . .	McAUTO SECTION 11.2.1
SHOCK SPECTRUM LOAD 1 ' '	McAUTO SECTION 7.2.2
SPECTRUM TRANSLATION Z FILE ' ' FACTOR . . .	
SHOCK SPECTRUM LOAD 2 ' '	
.	
.	
.	
END OF DYNAMIC LOADING	
PRINT DATA	(Vol. 1, P. 120)

COMMAND	REFERENCE
ASSEMBLE FOR DYNAMICS (REDUCE BAND)	McAUTO SECTION 8.1
INDEPENDENT DEGREES OF FREEDOM SELECT	McAUTO SECTION 10.1
CONDENSE DYNAMIC MATRICES	McAUTO SECTION 10.2
* DUMP ORTHOGANALITY	McAUTO SECTION 11.1.1
MODAL ANALYSIS HOW . . .	McAUTO SECTION 11.1.2
LIST DYNAMIC NORMALIZED EIGENVECTORS	McAUTO SECTION 11.1.4
* LIST DYNAMIC PARTICIPATION FACTORS	McAUTO SECTION 11.1.5
SHOCK SPECTRUM ANALYSIS LOADS 1, ...	McAUTO SECTION 11.3.2
LIST DYNAMIC SYSTEM SHOCK SPECTRUM - RESULTS RMS PRMS - FORCES DISPLACEMENTS DISTORTIONS	McAUTO SECTION 11.3.3.1 (SECTION 16.3.1)
* LIST DYNAMIC MODAL SHOCK SPECTRUM - RESULTS FORCE DISPLACEMENT	McAUTO SECTION 11.3.3.1 (SECTION 16.3.1)
PLOT DEVICE PLOTTER	McAUTO APPENDIX I SECTION A
.	
.	
.	
DISPLAY THREE DIMENSIONS	McAUTO APPENDIX I SECTION B.3.3
.	
.	
.	
ACTIVE MODES ALL BUT	McAUTO SECTION 11.2.2
DISPLAY MODE SHAPE OVERLAY	McAUTO APPENDIX I SECTION B.4
.	
.	
.	
PLOT FINISH	

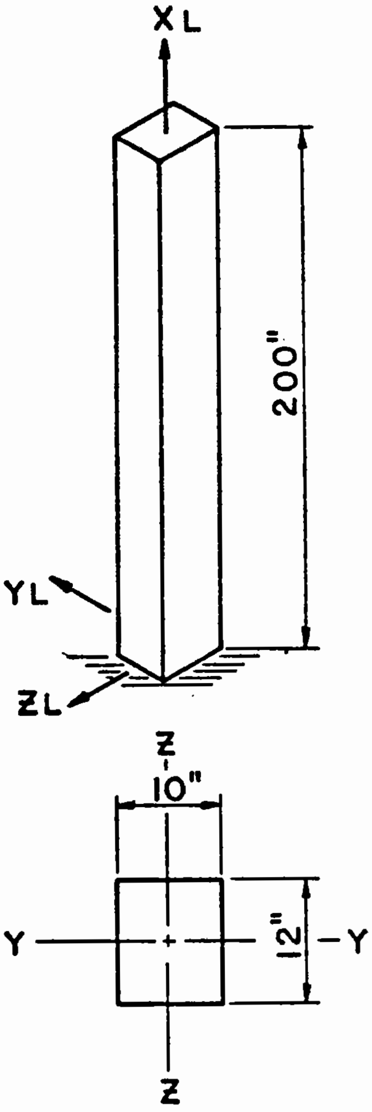
* OPTIONAL

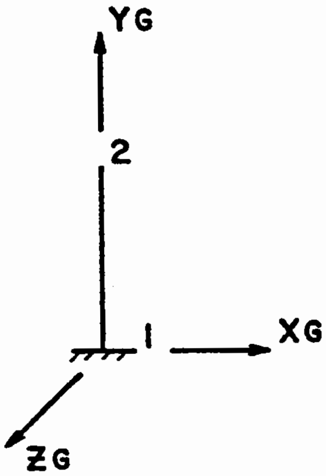
Appendix A.2

SAMPLE PROBLEM 1

DETERMINATION OF MODE SHAPES AND FREQUENCIES

PROBLEM STATEMENT: Determine the mode shapes and frequencies for the simple cantilever structure of uniform mass that is shown in the figure below. Assume that the mass is lumped at the beginning and end of the member. The properties of the member shown below were selected so the elements of the matrix could easily be calculated by hand to compare with intermediate prints from the computer solution.

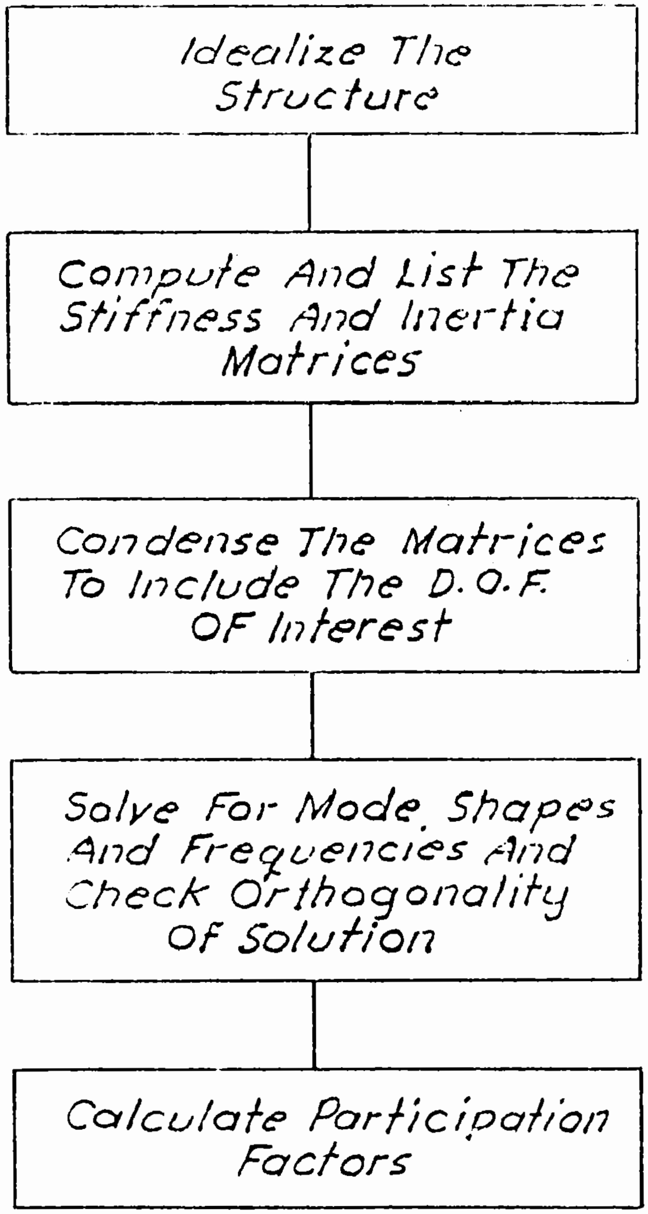




PROPERTIES:

A = 120 in
I = 600 in
I = 1440 in
I = 1000 in
MASS DENSITY = 1 lb mass
3
in
E = 3,000,000 psi

FLOWCHART OF PROBLEM SOLUTION



80/80 INPUT CARD IMAGES

```
STRUDL 'PROB 1' 'BASIC PRINCIPLES IN DYNAMICS'
$
$                    *****************************************
$                    ***                                   ***
$                    *** SEISMIC DESIGN OF HIGHWAY BRIDGES ***
$                    ***                                   ***
$                    *****************************************
$
$    SPONSORED BY:
$
$         FEDERAL HIGHWAY ADMINISTRATION
$
$    PRESENTED BY:
$
$         ENGINEERING COMPUTER CORPERATION
$         601 UNIVERSITY AVE.  SUITE 213
$         SACRAMENTO, CALIFORNIA
$
$    PHONE: (916) 922-9316
$
$    INSTRUCTORS:
$
$         ROY A. IMBSEN   -  PRINCIPLE INVESTIGATOR
$         RICHARD V. NUTT
$         JAMES GATES
$
$
$    THIS PROBLEM ILLUSTRATES HOW THE MATRICES ARE GENERATED AND
$    MANIPULATED AND HOW THE EIGENVALUES AND EIGENVECTORS ARE
$    CALCULATED FOR A SIMPLE VERTICAL CANTILEVER STRUCTURE OF
$    UNIFORM MASS.
$
$    THE SECTION NUMBERS LISTED TO THE RIGHT OF THE COMMANDS
$    ARE THE REFERENCE SECTION IN THE MCAUTO MANUAL.
$
$
$   *****************************************************************
$   ***              SPECIFY SYSTEM OF UNITS                      ***
$   *****************************************************************
$
UNITS INCHES POUNDS LBM                      $ *** SECTION 4.2.1
$
$   *****************************************************************
$   ***              STRUCTURE DESCRIPTION                        ***
$   *****************************************************************
$
TYPE SPACE FRAME                             $ *** SECTION 6.1.1
JOINT COORDINATES                            $ *** SECTION 6.1.2.1
1              SUPPORT
2 0.0 200.0
MEMBER INCIDENCES                            $ *** SECTION 6.1.3.1
1 1 2
MEMBER PROPERTIES PRISMATIC                  $ *** SECTION 6.1.3.6.1
1 AX 120. IX 600. IY 1440. IZ 1000.
CONSTANTS E 3.E6 ALL                         $ *** SECTION 6.1.4
          DENSITY 1. ALL
INERTIA OF JOINTS LUMPED                     $ *** SECTION 6.1.5.1
$
$   *****************************************************************
$   ***           GENERATION OF INERTIA MATRIX                    ***
```

```
$   ******************************************************************
$
ASSEMBLE FOR DYNAMICS                        $ *** SECTION 8.1
$
$   ******************************************************************
$   ***                       LIST MATRICES                       ***
$   ******************************************************************
$
LIST DYNAMIC STIFFNESS INERTIA MATRIX        $ *** SECTION 16.3.2
$
$   ******************************************************************
$   ***                    CONDENSE MATRICES                      ***
$   ******************************************************************
$
INDEPENDENT DEGREES OF FREEDOM SELECT        $ *** SECTION 10.1
CONDENSE DYNAMIC MATRICES PUNCH              $ *** SECTION 10.2
$
$   ******************************************************************
$   ***   PERFORM MODAL ANALYSIS AND PRINT ORTHOGONALITY VALUES  ***
$   ******************************************************************
$
DUMP ORTHOGINALITY                           $ *** SECTION 11.1.1
MODAL ANALYSIS HOW                           $ *** SECTION 11.1.2
$
$   ******************************************************************
$   ***       LIST EIGENVECTORS AND PARTICIPATION FACTORS         ***
$   ******************************************************************
$
LIST DYNAMIC NORMALIZED MODES BY JOINT       $ *** SECTION 11.1.4
LIST DYNAMIC PARTICIPATION FACTORS           $ *** SECTION 11.1.5
FINISH
```

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                                                                                                        PAGE -         2

STRUDL 'PROB 1' 'BASIC PRINCIPLES IN DYNAMICS'
          **************************************************
          *                                                *
          *       MCAUTO STRUDL*          RELEASE 4.0      *
          *       MCAUTO STRUDL DYNAL*    RELEASE 6.0      *
          *       MCAUTO STRUDL PLOTS     RELEASE 3.0      *
          *       MCAUTO STRUDL DANOS     RELEASE 2.0      *
          *                                                *
          * (*JOINTLY OWNED BY MCAUTO AND MULTISYSTEMS,INC) *
          *                                                *
          *           TIME 11.23.16, 10/11/78              *
          *                                                *
          *         DATA POOL SIZE   30640 BYTES           *
          *                                                *
          **************************************************

$
$                    ****************************************
$                    ***                                  ***
$                    *** SEISMIC DESIGN OF HIGHWAY BRIDGES ***
$                    ***                                  ***
$                    ****************************************
$
$    SPONSORED BY:
$
$         FEDERAL HIGHWAY ADMINISTRATION
$
$    PRESENTED BY:
$
$         ENGINEERING COMPUTER CORPERATION
$         601 UNIVERSITY AVE.  SUITE 213
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$         SACRAMENTO, CALIFORNIA
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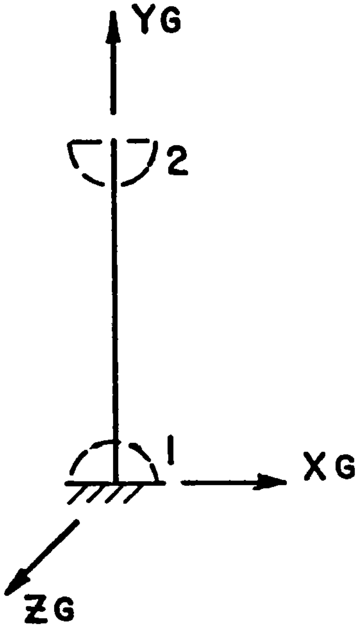
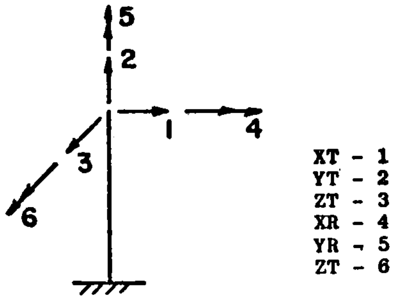
$    CALCULATED FOR A SIMPLE VERTICAL CANTILEVER STRUCTURE OF
$    UNIFORM MASS.
$
$    THE SECTION NUMBERS LISTED TO THE RIGHT OF THE COMMANDS
$    ARE THE REFERENCE SECTION IN THE MCAUTO MANUAL.
$
$
$   ****************************************************************
$   ***                     SPECIFY SYSTEM OF UNITS                ***
$   ****************************************************************
$
UNITS INCHES POUNDS LBM                     $ *** SECTION 4.2.1
```

```
$
$   ****************************************************************
$   ***                 STRUCTURE DESCRIPTION                   ***
$   ****************************************************************
$
TYPE SPACE FRAME                             $ *** SECTION 6.1.1
JOINT COORDINATES                            $ *** SECTION 6.1.2.1
1               SUPPORT
2 0.0 200.0
MEMBER INCIDENCES                            $ *** SECTION 6.1.3.1
1 1 2
MEMBER PROPERTIES PRISMATIC                  $ *** SECTION 6.1.3.6.1
1 AX 120. IX 600. IY 1440. IZ 1000.
CONSTANTS E 3.E6 ALL                         $ *** SECTION 6.1.4
          DENSITY 1. ALL
INERTIA OF JOINTS LUMPED                     $ *** SECTION 6.1.5.1
$
$   ****************************************************************
$   ***              GENERATION OF INERTIA MATRIX               ***
$   ****************************************************************
$
ASSEMBLE FOR DYNAMICS                        $ *** SECTION 8.1

**** STRUDL MESSAGE - BANDWIDTH STATISTICS ARE AS FOLLOWS :
                      THE MAXIMUM BANDWIDTH IS     0
                      THE AVERAGE BANDWIDTH IS     0.0
                      THE STANDARD DEVIATION IS    0.0
```



ASSEMBLE FOR DYNAMICS



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                                                                                                       PAGE -         5

                                                      CORRESPONDENCE TABLE
   JOINT ID      XT      YT      ZT      XR      YR      ZR          JOINT ID      XT      YT      ZT      XR      YR      ZR

   1                                                               2              1       2       3       4       5       6

$
$   ****************************************************************
$   ***                      LIST MATRICES                      ***
$   ****************************************************************
$
LIST DYNAMIC STIFFNESS INERTIA MATRIX      $ *** SECTION 16.3.2
```

```
                                                                                                    PAGE -        6

                                      ***** SYSTEM INERTIA MATRIX ( OUTPUT IN INTERNAL UNITS ) *****
                                           ORDER =      6
(MISSING ELEMENTS ALL ZERO)

      1    0.31084919D+02      2    0.31084919D+02      3    0.31084919D+02
```

System Mass (Inertia) Matrix

$$[M] = \begin{array}{c} \begin{array}{cccccc} \text{XT} & \text{YT} & \text{ZT} & \text{XR} & \text{YR} & \text{ZR} \end{array} \\ \left[\begin{array}{ccc|ccc} \frac{mAL}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{mAL}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{mAL}{2} & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

```
                    ***** SYSTEM STIFFNESS MATRIX ( OUTPUT IN INTERNAL UNITS ) *****

                          ORDER =      6

(MISSING ELEMENTS ALL ZERO)

ROW
  1      1    0.45000000D+04
ROW
  2      2    0.18000000D+07
ROW
  3      3    0.64800000D+04
ROW
  4      3   -0.64800000D+06      4    0.86400000D+08
ROW
  5      5    0.36000000D+07
ROW
  6      1    0.45000000D+06      6    0.60000000D+08
```

$$[K] = \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array}\left[\begin{array}{ccc|ccc} \mathrm{XT} & \mathrm{YT} & \mathrm{ZT} & \mathrm{XR} & \mathrm{YR} & \mathrm{ZR} \\ \frac{12EI_z}{L^3} & 0 & 0 & 0 & 0 & \frac{6EI_z}{L^2} \\ 0 & \frac{AE}{L} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{12EI_y}{L^3} & \frac{-6EI_y}{L^2} & 0 & 0 \\ \hline 0 & 0 & \frac{-6EI_y}{L^2} & \frac{4EI_y}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{GI_x}{L} & 0 \\ \frac{6EI_z}{L^2} & 0 & 0 & 0 & 0 & \frac{4EI_z}{L} \end{array}\right]$$

```
$   ****************************************************************
$   ***                  CONDENSE MATRICES                       ***
$   ****************************************************************
$
INDEPENDENT DEGREES OF FREEDOM SELECT      $ *** SECTION 10.1
CONDENSE DYNAMIC MATRICES PUNCH            $ *** SECTION 10.2

                                              CONDENSATION CORRESPONDENCE TABLE
                                                  INDEPENDENT COORDINATES

   JOINT ID      XT      YT      ZT      XR      YR      ZR        JOINT ID      XT      YT      ZT      XR      YR      ZR

   2              1       2       3

                                              CONDENSATION CORRESPONDENCE TABLE
                                                   DEPENDENT COORDINATES

   JOINT ID      XT      YT      ZT      XR      YR      ZR        JOINT ID      XT      YT      ZT      XR      YR      ZR

   2                                      1       2       3


**** STRUDL MESSAGE - THE ICES SYSTEM WILL TEMPORARILY BE ROLLED OUT
                      WHILE CONDENSATION IS PERFORMED.


 ROLL-OUT OF THE ICES SYSTEM COMPLETED ; CONDENSATION INITIATED
```

$$\left[\begin{array}{c|c} M_{ii} & M_{id} \\ \hline M_{di} & M_{dd} \end{array}\right] \left\{\begin{array}{c} \ddot{V}_i \\ \hline \ddot{V}_d \end{array}\right\} + \left[\begin{array}{c|c} K_{ii} & K_{id} \\ \hline K_{di} & K_{dd} \end{array}\right] \left\{\begin{array}{c} V_i \\ \hline V_d \end{array}\right\} = \left\{\begin{array}{c} 0 \\ - \\ 0 \end{array}\right\}$$

$$\left[\begin{array}{c|c} M_{ii} & 0 \\ \hline 0 & 0 \end{array}\right] \left\{\begin{array}{c} \ddot{V}_i \\ \hline \ddot{V}_d \end{array}\right\} + \left[\begin{array}{c|c} K_{ii} & K_{id} \\ \hline K_{di} & K_{dd} \end{array}\right] \left\{\begin{array}{c} V_i \\ \hline V_d \end{array}\right\} = \left\{\begin{array}{c} 0 \\ - \\ 0 \end{array}\right\}$$

CONDENSE DYNAMICS MATRICES

$$\left[M_{ii} \right] \left\{ \ddot{V}_i \right\} + \left[K_{ii} \right] \left\{ V_i \right\} + \left[K_{id} \right] \left\{ V_d \right\} = 0$$

$$\left[K_{di} \right] \left\{ V_i \right\} + \left[K_{dd} \right] \left\{ V_d \right\} = 0$$

Solving for $\left\{ V_d \right\}$ yields

$$\left\{ V_d \right\} = - \left[K_{dd} \right]^{-1} \left[K_{di} \right] \left\{ V_i \right\}$$

Substituting for V_d yields

$$\left[M_{ii} \right] \left\{ \ddot{V}_i \right\} + \left[K_{ii} \right] \left\{ V_i \right\} - \left[K_{id} \right] \left[K_{dd} \right]^{-1} \left[K_{di} \right] \left\{ V_i \right\} = 0$$

$$\left[M_{ii} \right] \left\{ \ddot{V}_i \right\} + \left(\left[K_{ii} \right] - \left[K_{id} \right] \left[K_{dd} \right]^{-1} \left[K_{di} \right] \right) \left\{ V_i \right\} = 0$$

or $$\left[M_{ii} \right] \left\{ \ddot{V}_i \right\} + \left[\bar{K}_{ii} \right] \left\{ V_i \right\} = 0$$

Where $\left[\bar{K}_{ii} \right]$ is the condensed stiffness matrix

$$\left[\bar{K}_{ii} \right] = \left[K_{ii} \right] - \left[K_{id} \right] \left[K_{dd} \right]^{-1} \left[K_{di} \right]$$

Solving for $\begin{bmatrix} K_{id} \end{bmatrix} \begin{bmatrix} K_{dd} \end{bmatrix}^{-1} \begin{bmatrix} K_{di} \end{bmatrix}$

$$\begin{bmatrix} 0 & 0 & \frac{6EI_z}{L^2} \\ 0 & 0 & 0 \\ \frac{-6EI_y}{L^2} & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{L}{4EI_y} & 0 & 0 \\ 0 & \frac{L}{GI_x} & 0 \\ 0 & 0 & \frac{L}{4EI_z} \end{bmatrix} \begin{bmatrix} 0 & 0 & \frac{-6EI_y}{L^2} \\ 0 & 0 & 0 \\ \frac{6EI_z}{L^2} & 0 & 0 \end{bmatrix}$$

Yields after matrix multiplication:

$$\begin{bmatrix} K_{id} \end{bmatrix} \begin{bmatrix} K_{dd} \end{bmatrix}^{-1} \begin{bmatrix} K_{di} \end{bmatrix} = \begin{bmatrix} \frac{9EI_z}{L^3} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{9EI_z}{L^3} \end{bmatrix}$$

Substituting

$$\begin{bmatrix} K_{ii} \end{bmatrix} = \begin{bmatrix} K_{ii} \end{bmatrix} - \begin{bmatrix} K_{id} \end{bmatrix} \begin{bmatrix} K_{dd} \end{bmatrix}^{-1} \begin{bmatrix} K_{di} \end{bmatrix}$$

Yields

$$\begin{bmatrix} K_{ii} \end{bmatrix} = \begin{bmatrix} \frac{12EI_z}{L^3} - \frac{9EI_z}{L^3} & 0 & 0 \\ 0 & \frac{AE}{L} & 0 \\ 0 & 0 & \frac{12EI_y}{L^3} - \frac{9EI_y}{L^3} \end{bmatrix}$$

$$\begin{bmatrix} K_{ii} \end{bmatrix} = \begin{bmatrix} \frac{3EI_z}{L^3} & 0 & 0 \\ 0 & \frac{AE}{L} & 0 \\ 0 & 0 & \frac{3EI_y}{L^3} \end{bmatrix}$$

$$\begin{bmatrix} M_{ii} \end{bmatrix} \begin{Bmatrix} \ddot{V}_i \end{Bmatrix} + \begin{bmatrix} K_{ii} \end{bmatrix} \begin{Bmatrix} V_i \end{Bmatrix} = 0$$

$$\begin{bmatrix} \frac{mAL}{2} & 0 & 0 \\ 0 & \frac{mAL}{2} & 0 \\ 0 & 0 & \frac{mAL}{2} \end{bmatrix} \begin{bmatrix} \ddot{V}_x \\ \ddot{V}_y \\ \ddot{V}_z \end{bmatrix} + \begin{bmatrix} \frac{3EI_z}{L^3} & 0 & 0 \\ 0 & \frac{AE}{L} & 0 \\ 0 & 0 & \frac{3EI_y}{L^3} \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{mAL}{2} \ddot{V}_x + \frac{3EI_z}{L^3} V_x = 0; \text{ or } \ddot{V}_x + \frac{6EI_z}{mAL^4} V_x = 0$$

$$\frac{mAL}{2} \ddot{V}_y + \frac{AE}{L} V_y = 0; \text{ or } \ddot{V}_y + \frac{2E}{mL^2} V_y = 0$$

$$\frac{mAL}{2} \ddot{V}_z + \frac{3EI_y}{L^3} V_z = 0; \text{ or } \ddot{V}_z + \frac{6EI_y}{mAL^4} V_z = 0$$

Now in the same form as Equation 2.3.3

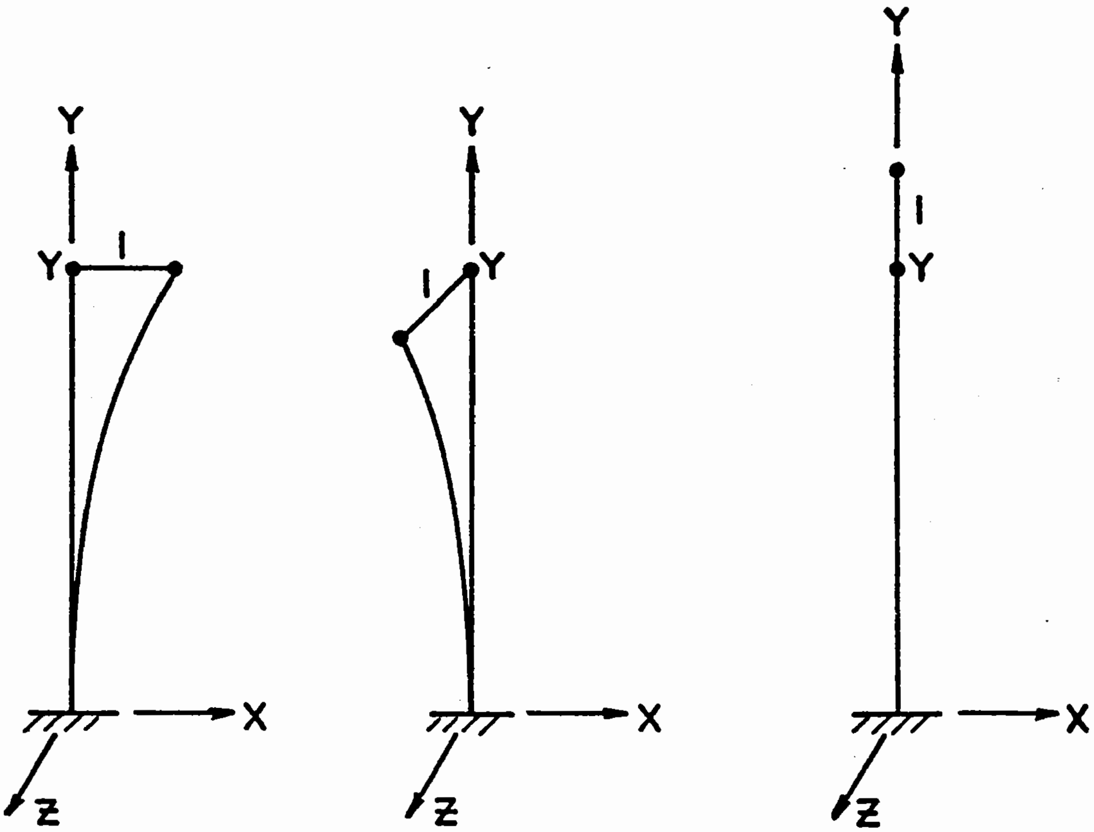
$$\ddot{V} + \omega^2 V = 0$$

$$\omega_x^{\,2} = \frac{6EI_z}{mAL^4} = 36.19$$

$$\omega_y^{\,2} = \frac{2E}{mL^2} = 57906$$

$$\omega_z^{\,2} = \frac{6EI_y}{mAL^4} = 52.12$$

	X	Y	Z
ω^2 (eigenvalue - rad^2/sec^2)	36.19	57906	52.12
f (frequency - rad/sec)	6.02	241	7.22
T (period - sections)	1.04	.026	.87
Mode	1	3	2



Mode	1	2	3
Eigenvector	1	2	3

```
                                                                                 PAGE -      2


STRUDL

      ***************************************************
      *                                                 *
      *       MCAUTO STRUDL*         RELEASE 4.0        *
      *       MCAUTO STRUDL DYNAL*   RELEASE 6.0        *
      *       MCAUTO STRUDL PLOTS    RELEASE 3.0        *
      *       MCAUTO STRUDL DANOS    RELEASE 2.0        *
      *                                                 *
      * (*JOINTLY OWNED BY MCAUTO AND MULTISYSTEMS,INC) *
      *                                                 *
      *            TIME 18.48.21, 11/27/78              *
      *                                                 *
      *          DATA POOL SIZE   30640 BYTES           *
      *                                                 *
      ***************************************************


ROLL-IN OF THE ICES SYSTEM COMPLETED ; CONDENSATION ACCOMPLISHED




                                                                                 PAGE -      3



$

$   ****************************************************************

$   ***   PERFORM MODAL ANALYSIS AND PRINT ORTHOGONALITY VALUES   ***

$   ****************************************************************

$

DUMP ORTHOGINALITY                         $ *** SECTION 11.1.1

MODAL ANALYSIS HOW                         $ *** SECTION 11.1.2

**** STRUDL  MESSAGE  - ALL EIGENVALUES ARE COMPUTED.
                        EIGENVECTORS COMPUTED FOR FIRST      3 EIGENVALUES
```

```
  ****************************
  *RESULTS OF LATEST ANALYSIS*
  ****************************

 PROBLEM - PROB 1     TITLE - BASIC PRINCIPLES IN DYNAMICS

 ACTIVE UNITS INCH LB   RAD  FAHR SEC  LBM



   EIGENVALUES

/--MODE--/--EIGENVALUE--/------FREQUENCY------/---------PERIOD--------/

     1     0.361912D+02       0.601591D+01          0.104443D+01
     2     0.521153D+02       0.721909D+01          0.870357D+00
     3     0.579059D+05       0.240636D+03          0.261107D-01
```

DUMP ORTHOGONALITY

(Modes Have Been Normalized to Unit Generalized Mass)

SUM = (MODE (I)) TRANS * INERTIA * MODE (J)

$$[M]^* = [\emptyset]^T [M][\emptyset] = [I]$$

Consider the first mode for the cantilever beam

$$M_i^* = [\emptyset]_1^T [M][\emptyset]_1 = 1$$

$$M_1^* = \emptyset_1^2 M_1 + \emptyset_2^2 M_2 + \emptyset_3^2 M_3 = 1$$

Since $\emptyset_2 = \emptyset_3 = 0$ for the first mode in this case

$$M_1^* = 1 = \emptyset_1^2 M_1 \quad \text{or} \quad \emptyset_1 = \sqrt{\frac{1}{M_1}}$$

$$\emptyset_1 = \sqrt{\frac{1}{31.08}} = .17931$$

$$[\emptyset]_1 = \begin{bmatrix} \emptyset_1 \\ \emptyset_2 \\ \emptyset_3 \end{bmatrix}_1 = \begin{bmatrix} .17931 \\ 0 \\ 0 \end{bmatrix}_1$$

First mode eigenvector normalized w.r.t. unit mass.

SUM = (MODE (I)) TRANS * INTERTIA * MODE (J)

$$\text{SUM} = [\emptyset]_I^T [M][\emptyset]_J = 0$$

```
                                                                                    PAGE -        5

                                        ********** ORTHOGONALITY CHECK **********

                                   MODES HAVE BEEN NORMALIZED TO UNIT GENERALIZED MASS

                                        SUM=(MODE(I))TRANS*INERTIA*MODE(J))

              I     J       SUM             I     J        SUM              I     J       SUM              I     J       SUM

              1     2   1.263D-40           2     3   2.367D-30
$
$   ***************************************************************
$   ***       LIST EIGENVECTORS AND PARTICIPATION FACTORS       ***
$   ***************************************************************
$
LIST DYNAMIC NORMALIZED MODES BY JOINT      $ *** SECTION 11.1.4
```

```
                                                                                              PAGE -      6

   ***************************
   *RESULTS OF LATEST ANALYSIS*
   ***************************

  PROBLEM - PROB 1     TITLE - BASIC PRINCIPLES IN DYNAMICS

  ACTIVE UNITS INCH LB   RAD  FAHR SEC  LBM


    NORMALIZED EIGENVECTORS


     MODE    1    MAXIMUM VALUE IS   1.793598E-01 AT JOINT 2        IN DIRECTION DISP X

JOINT            /-----------------DISPLACEMENT-----------------//------------------ROTATION------------------/
                         X DISP.          Y DISP.          Z DISP.          X ROT.          Y ROT.          Z ROT.

1         GLOBAL        0.0              0.0              0.0              0.0              0.0             0.0
2         GLOBAL        1.0000000       -0.0000000        0.0000000        0.0000000        0.0            -0.0075000

     MODE    2    MAXIMUM VALUE IS   1.793598E-01 AT JOINT 2        IN DIRECTION DISP Z

JOINT            /-----------------DISPLACEMENT-----------------//------------------ROTATION------------------/
                         X DISP.          Y DISP.          Z DISP.          X ROT.          Y ROT.          Z ROT.

1         GLOBAL        0.0              0.0              0.0              0.0              0.0             0.0
2         GLOBAL       -0.0000000       -0.0000000        1.0000000        0.0075000        0.0             0.0000000

     MODE    3    MAXIMUM VALUE IS   1.793598E-01 AT JOINT 2        IN DIRECTION DISP Y

JOINT            /-----------------DISPLACEMENT-----------------//------------------ROTATION------------------/
                         X DISP.          Y DISP.          Z DISP.          X ROT.          Y ROT.          Z ROT.

1         GLOBAL        0.0              0.0              0.0              0.0              0.0             0.0
2         GLOBAL        0.0000000        1.0000000        0.0000000        0.0000000        0.0            -0.0000000
LIST DYNAMIC PARTICIPATION FACTORS          $ *** SECTION 11.1.5
```

```
                                                                                                   PAGE -          7

     ***************************
     *RESULTS OF LATEST ANALYSIS*
     ***************************

    PROBLEM - PROB 1      TITLE - BASIC PRINCIPLES IN DYNAMICS
                   PARTICPATION FACTORS -(OUTPUT IN INTERNAL UNITS)


  MODE          /-----------------DISPLACEMENT-----------------//------------------ROTATION-------------------/
                            X DISP.         Y DISP.         Z DISP.         X ROT.          Y ROT.          Z ROT.

     1                   5.5753851      -0.0000000       0.0000000       0.0             0.0             0.0
     2                  -0.0000000      -0.0000000       5.5753851       0.0             0.0             0.0
     3                   0.0000000       5.5753851       0.0000000       0.0             0.0             0.0

  FINISH
```

$$\begin{bmatrix} PF \end{bmatrix} = \begin{bmatrix} \emptyset \end{bmatrix}^T \begin{bmatrix} M \end{bmatrix}$$

Consider the participation factor for the first mode

$$\begin{bmatrix} PF \end{bmatrix} = \begin{bmatrix} \emptyset_1 & \emptyset_2 & \emptyset_3 \end{bmatrix} \qquad \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix}$$

Since $\emptyset_2 = \emptyset_3 = 0$

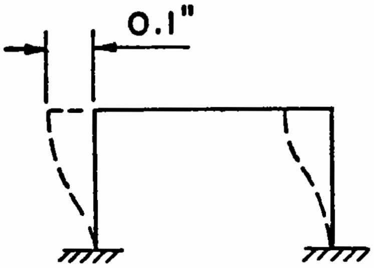
$$PF_1 = .17931 \times 31.085$$

Appendix A.3

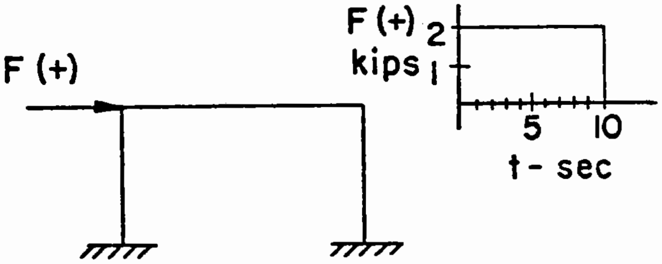
SAMPLE PROBLEM 2

DETERMINATION OF THE EFFECTS OF STRUCTURAL DAMPING

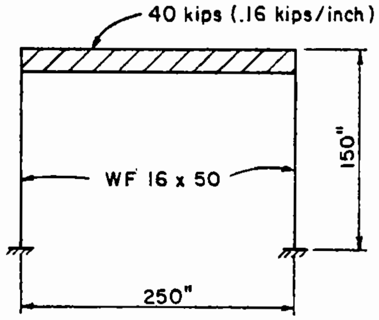
PROBLEM STATEMENT: Determine the free vibration characteristics and the effect of various levels of structural damping on the displacement response of a simple single story frame structure shown in the figure below. Assume the mass of the columns to be small and the bulk of the mass to be concentrated on the first floor. Consider the following two loading conditions:



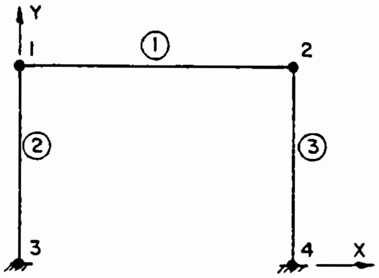
A. Initial displacement and release of top story.



B. Suddenly applied lateral load at top story.



SCHEMATIC OF STRUCTURE



STRUCTURE IDEALIZATION

MEMBER PROPERTIES
UNITS: KIPS AND INCHES

	1	2 AND 3
A	23.0	14.7
I	1050000.0	657.0

Calculation of Member Properties :

For member 1, the first story girder, the density must be selected since STRUDL uses density and area to calculate the mass of a member. In this case, the mass will be considered concentrated at joints 1 & 2. The default units for mass in McAuto's version of STRUDL is pound mass (ie the mass present per pound of weight). Therefore

$$.16 \text{ kips}/\text{in} = 160 \text{ pound mass}/\text{in}$$

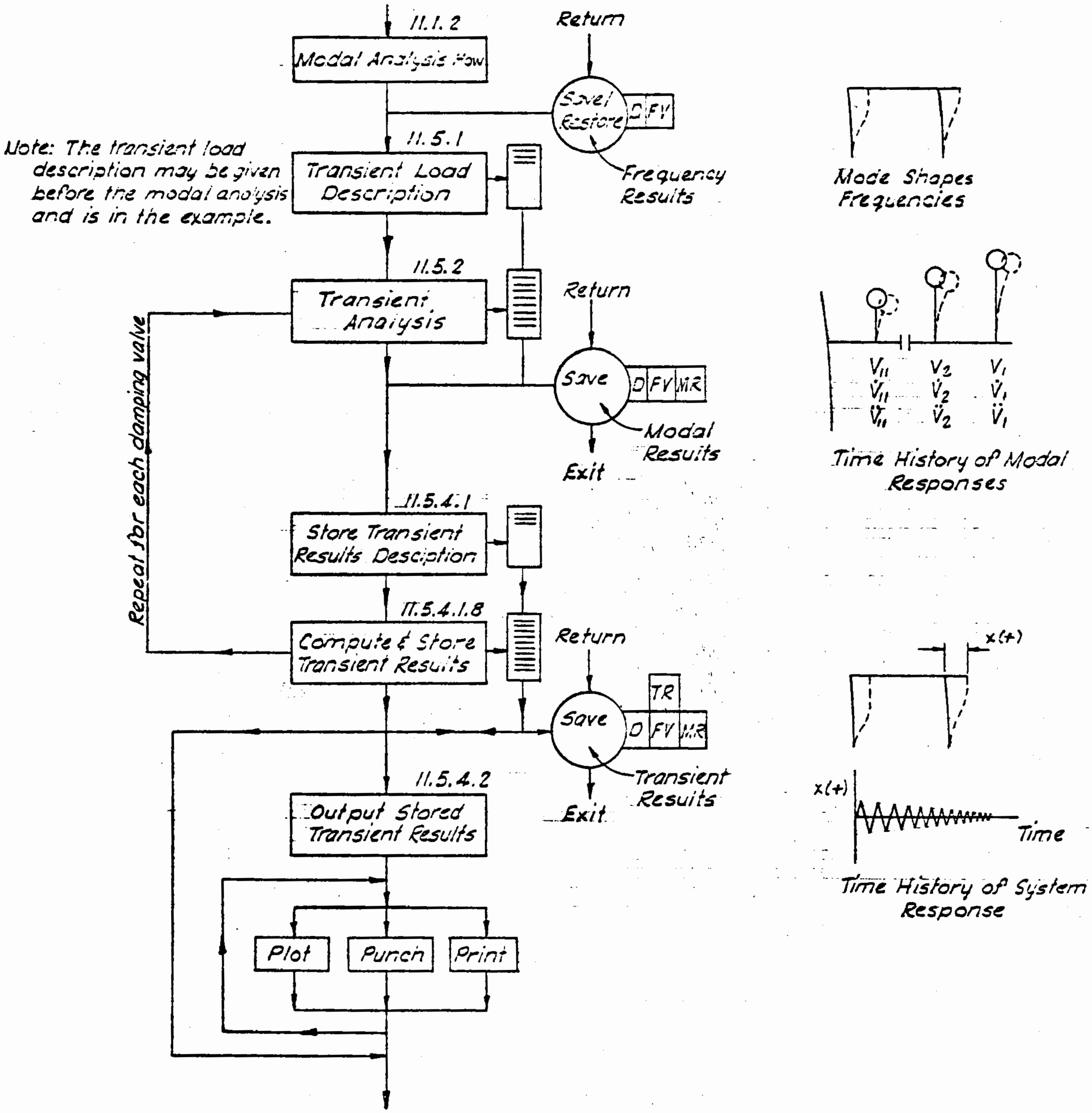
Since

$$A = 23 \text{ in}^2$$

The mass density should be

$$\rho = 160/23 = 6.95652 \frac{\text{pound mass}}{\text{in}^3}$$

PROBLEM SOLUTION FLOW CHART



Note: The transient load description may be given before the modal analysis and is in the example.

80/80 INPUT CARD IMAGES

```
STRUDL 'PROB 2' 'PRINCIPLES OF DAMPING'
$
$                    *****************************************
$                    ***                                   ***
$                    *** SEISMIC DESIGN OF HIGHWAY BRIDGES ***
$                    ***                                   ***
$                    *****************************************
$
$    SPONSORED BY:
$
$         FEDERAL HIGHWAY ADMINISTRATION
$
$    PRESENTED BY:
$
$         ENGINEERING COMPUTER CORPERATION
$         601 UNIVERSITY AVE.  SUITE 213
$         SACRAMENTO, CALIFORNIA
$
$    PHONE: (916) 922-9316
$
$    INSTRUCTORS:
$
$         ROY A. IMBSEN  -  PRINCIPLE INVESTIGATOR
$         RICHARD V. NUTT
$         JAMES GATES
$
$
$    THIS PROBLEM ILLUSTRATES THE EFFECTS OF STRUCTURAL DAMPING ON
$    THE RESPONSE OF A SIMPLE STRUCTURE SUBJECTED TO THE FOLLOWING
$    LOADS:
$
$               A.  INITIAL DISPLACEMENT
$               B.  SUDDENLY APPLIED FORCE AT TOP OF FRAME
$
$
$   ***************************************************************
$   ***                     STRUCTURE DESCRIPTION               ***
$   ***************************************************************
$
UNITS KIPS INCHES
TYPE PLANE FRAME
JOINT COORDINATES
    1       0.0       150.0
    2     250.0       150.0
    3       0.0         0.0   SUPPORT
    4     250.0         0.0   SUPPORT
MEMBER INCIDENCES
1 1 2
2 3 1
3 4 2
MEMBER PROPERTIES
    1 AX 23.0 IZ 1050000.0
    2 3 AX 14.7 IZ 657.0
CONSTANTS
    E 30000. ALL
    DENSITY 0.000001 ALL BUT 6.95652 1
INERTIA OF JOINTS LUMPED
$
$   ***************************************************************
$   ***                TRANSIENT LOADING DISCRIPTIONS           ***
```

```
$   *****************************************************************
$
TRANSIENT LOAD 1 'INITIAL DISPLACEMENT AND RELEASE'   $ *** SECTION 7.4.2.1
INITIAL CONDITIONS JOINT DISPLACEMENT                  $ *** SECTION 7.4.2.6
1 2 XT -0.1
INTEGRATE FROM 0.0 TO 5.0 AT DELTA 0.02                $ *** SECTION 7.4.2.9
END OF DYNAMIC LOADING                                 $ *** SECTION 7.4.2.10
TRANSIENT LOAD 2 'SUDDENLY APPLIED LOAD'
JOINT LOADS                                            $ *** SECTION 7.4.2.2
    1 FORCE X FILE 'SUDDEN' 2.0 0.0 2.0 10.0
INTEGRATE FROM 0.0 TO 5.0 AT DELTA 0.02
END OF DYNAMIC LOAD
$
$   *****************************************************************
$   ***                        PRINT INPUT DATA                   ***
$   *****************************************************************
$
PRINT DYNAMIC DATA                                     $ *** SECTION 16.1.1
$
$   *****************************************************************
$   ***                    PERFORM MODAL ANALYSIS                 ***
$   ***                      AND SAVE RESULTS                     ***
$   *****************************************************************
$
ASSEMBLE FOR DYNAMICS
DUMP ORTHOGINALITY
MODAL ANALYSIS HOW
SAVE 'FREEVIB'                                         $ *** SECTION 4.1.2
$
$   *****************************************************************
$   ***         LIST EIGENVECTORS AND PARTICIPATION FACTORS       ***
$   *****************************************************************
$
LIST DYNAMIC NORMALIZED EIGENVECTORS
LIST DYNAMIC PARTICIPATION FACTORS
$
$   *****************************************************************
$   ***         PERFORM TRANSIENT ANALYSIS FOR LOADS 1 & 2        ***
$   ***                                                           ***
$   ***        DEFINE RESULTS TO BE STORED IN OUTPUT FILE         ***
$   *****************************************************************
$
$
$   *****************************************************************
$                          *** DAMPING = 1% ***
$   *****************************************************************
$
DAMPING PERCENT 1.0    6                               $ *** SECTION 11.2.1
TRANSIENT ANALYSIS LOAD 1 2                            $ *** SECTION 11.5.2
STORE TRANSIENT SET IDENT 'DAMP1'                      $ *** SECTION 11.5.4.1.1
USE MODAL RESULTS OF TRANSIENT ANALYSIS 1 2            $ *** SECTION 11.5.4.1.2
JOINT DISPLACEMENT                                     $ *** SECTION 11.5.4.1.3
    1  TRAN X
END OF STORE TRANSIENT                                 $ *** SECTION 11.5.4.1.7
COMPUTE AND STORE TRANSIENT SET ID 'DAMP1'             $ *** SECTION 11.5.4.1.8
$
$   *****************************************************************
$                          *** DAMPING = 2% ***
$   *****************************************************************
$
```

```
DAMPING PERCENT 2.0  6
TRANSIENT ANALYSIS LOAD 1 2
STORE TRANSIENT SET IDENT 'DAMP2'
USE MODAL RESULTS OF TRANSIENT ANALYSIS 1 2
JOINT DISPLACEMENT
    1 TRANS X
END OF STORE TRANSIENT
COMPUTE AND STORE TRANSIENT SET ID 'DAMP2'
$
$   ***************************************************************
$                        *** DAMPING = 3% ***
$   ***************************************************************
$
DAMPING PERCENT 3.0  6
TRANSIENT ANALYSIS LOAD 1 2
STORE TRANSIENT SET IDENT 'DAMP3'
USE MODAL RESULTS OF TRANSIENT ANALYSIS 1 2
JOINT DISPLACEMENT
    1 TRANS X
END OF STORE TRANSIENT
COMPUTE AND STORE TRANSIENT SET ID 'DAMP3'
$
$   ***************************************************************
$                        *** DAMPING = 5% ***
$   ***************************************************************
$
DAMPING PERCENT 5.0  6
TRANSIENT ANALYSIS LOAD 1 2
STORE TRANSIENT SET IDENT 'DAMP5'
USE MODAL RESULTS OF TRANSIENT ANALYSIS 1 2
JOINT DISPLACEMENT
    1 TRANS X
END OF STORE TRANSIENT
COMPUTE AND STORE TRANSIENT SET ID 'DAMP5'
$
$   ***************************************************************
$                        *** DAMPING = 10% ***
$   ***************************************************************
$
DAMPING PERCENT 10.0  6
TRANSIENT ANALYSIS LOAD 1 2
STORE TRANSIENT SET IDENT 'DAMP10'
USE MODAL RESULTS OF TRANSIENT ANALYSIS 1 2
JOINT DISPLACEMENT
    1 TRANS X
END OF STORE TRANSIENT
COMPUTE AND STORE TRANSIENT SET ID 'DAMP10'
$
$   ***************************************************************
$   ***          OUTPUT RESULTS IN THE FORM OF PLOTS            ***
$   ***************************************************************
$
OUTPUT STORED TRANSIENT RESULTS                   $ *** SECTION 11.5.4.2.1
PLOT 1 ON                                         $ *** SECTION 11.5.4.2.2
ABSCISSA OF .5                                    $ *** SECTION 11.5.4.2.3
ORDINATE ON INDIVIDUAL BASIS                      $ *** SECTION 11.5.4.2.4
USE STORE TRANSIENT SET ID 'DAMP1'                $ *** SECTION 11.5.4.2.5
USE TRANS 1 2                                     $ *** SECTION 11.5.4.2.6
JOINT DISPLACEMENT                                $ *** SECTION 11.5.4.2.7
     1 TRANS X
```

```
END OF SUBSET
USE STORE TRANSIENT SET ID 'DAMP2'
USE TRANS 1 2
JOINT DISPLACEMENT
      1 TRANS X
END OF SUBSET
USE STORE TRANSIENT SET ID 'DAMP3'
USE TRANS 1 2
JOINT DISPLACEMENT
      1 TRANS X
END OF SUBSET
USE STORE TRANSIENT SET ID 'DAMP5'
USE TRANS 1 2
JOINT DISPLACEMENT
      1 TRANS X
END OF SUBSET
USE STORE TRANSIENT SET ID 'DAMP10'
USE TRANS 1 2
JOINT DISPLACEMENT
      1 TRANS X
END OF SUBSET
END
FINISH
```

```
                                                                                              PAGE -         10

   ****************************
   *RESULTS OF LATEST ANALYSIS*
   ****************************

  PROBLEM - PROB 2      TITLE - PRINCIPLES OF DAMPING

  ACTIVE UNITS INCH KIPS RAD   FAHR SEC   LBM



    EIGENVALUES

/--MODE--/--EIGENVALUE--/------FREQUENCY------/--------PERIOD--------/

     1       0.134024D+04       0.366093D+02          0.171628D+00
     2       0.567478D+05       0.238218D+03          0.263758D-01
     3       0.574083D+05       0.239600D+03          0.262236D-01
     4       0.107897D+06       0.328477D+03          0.191282D-01
     5       0.487424D+12       0.698158D+06          0.899966D-05
     6       0.146024D+13       0.120841D+07          0.519957D-05
```

```
                                   ********** ORTHOGONALITY CHECK **********

                              MODES HAVE BEEN NORMALIZED TO UNIT GENERALIZED MASS

                                       SUM=(MODE(I))TRANS*INERTIA*MODE(J))

              I     J      SUM          I    J        SUM             I    J      SUM             I    J       SUM


              1     2   4.028D-18       2     3  1.789D-14            3     4  -2.849D-13           4     5  9.442D-18
              5     6  -5.551D-17

SAVE 'FREEVIB'                                $ *** SECTION 4.1.2
****NON-DESTRUCTIVE SAVE SUCCESSFUL FOR FILE FREEVIB  USING MAP SAVESTRU
****ON NOV 15, 1978 AT 21.32.45  -  THIS FILE HAS BEEN SAVED    1 TIME(S)

$

$   *****************************************************************

$   ***         LIST EIGENVECTORS AND PARTICIPATION FACTORS      ***

$   *****************************************************************

$

LIST DYNAMIC NORMALIZED EIGENVECTORS
```

```
                                                                                                         PAGE -       12
    ***************************
    *RESULTS OF LATEST ANALYSIS*
    ***************************

   PROBLEM - PROB 2     TITLE - PRINCIPLES OF DAMPING

   ACTIVE UNITS INCH KIPS RAD  FAHR SEC  LBM



     NORMALIZED EIGENVECTORS

      MODE    1    MAXIMUM VALUE IS   9.822911E-02 AT JOINT 2        IN DIRECTION DISP X

  JOINT            /-----------------DISPLACEMENT-----------------//-------------------ROTATION-------------------/
                            X DISP.          Y DISP.          Z DISP.          X ROT.          Y ROT.          Z ROT.

  1        GLOBAL        1.0000000        0.0144684                                                        -0.0001226
  2        GLOBAL        1.0000000       -0.0144684                                                        -0.0001226
  3        GLOBAL        0.0              0.0                                                               0.0
  4        GLOBAL        0.0              0.0                                                               0.0

      MODE    2    MAXIMUM VALUE IS   9.823942E-02 AT JOINT 1        IN DIRECTION DISP Y

  JOINT            /-----------------DISPLACEMENT-----------------//-------------------ROTATION-------------------/
                            X DISP.          Y DISP.          Z DISP.          X ROT.          Y ROT.          Z ROT.

  1        GLOBAL       -0.0000000        1.0000000                                                        -0.0000000
  2        GLOBAL       -0.0000000        0.9999999                                                        -0.0000000
  3        GLOBAL        0.0              0.0                                                               0.0
  4        GLOBAL        0.0              0.0                                                               0.0

      MODE    3    MAXIMUM VALUE IS  -9.822911E-02 AT JOINT 2        IN DIRECTION DISP Y

  JOINT            /-----------------DISPLACEMENT-----------------//-------------------ROTATION-------------------/
                            X DISP.          Y DISP.          Z DISP.          X ROT.          Y ROT.          Z ROT.

  1        GLOBAL        0.0144684       -0.9999999                                                         0.0079943
  2        GLOBAL        0.0144684        1.0000000                                                         0.0079943
  3        GLOBAL        0.0              0.0                                                               0.0
  4        GLOBAL        0.0              0.0                                                               0.0
```

```
     MODE    4    MAXIMUM VALUE IS   9.823942E-02 AT JOINT 1        IN DIRECTION DISP X

JOINT            /-----------------DISPLACEMENT-----------------//-------------------ROTATION-------------------/
                           X DISP.         Y DISP.         Z DISP.          X ROT.          Y ROT.          Z ROT.

1         GLOBAL         1.0000000      -0.0000000                                                     -0.0000208
2         GLOBAL        -1.0000000       0.0000000                                                      0.0000208
3         GLOBAL         0.0             0.0                                                            0.0
4         GLOBAL         0.0             0.0                                                            0.0

     MODE    5    MAXIMUM VALUE IS  -9.823943E-01 AT JOINT 1        IN DIRECTION ROT Z

JOINT            /-----------------DISPLACEMENT-----------------//-------------------ROTATION-------------------/
                           X DISP.         Y DISP.         Z DISP.          X ROT.          Y ROT.          Z ROT.

1         GLOBAL         0.0000002      -0.0000000                                                      1.0000000
2         GLOBAL        -0.0000002      -0.0000000                                                     -1.0000000
3         GLOBAL         0.0             0.0                                                            0.0
4         GLOBAL         0.0             0.0                                                            0.0

     MODE    6    MAXIMUM VALUE IS  -9.823940E-01 AT JOINT 2        IN DIRECTION ROT Z

JOINT            /-----------------DISPLACEMENT-----------------//-------------------ROTATION-------------------/
                           X DISP.         Y DISP.         Z DISP.          X ROT.          Y ROT.          Z ROT.

1         GLOBAL         0.0000001       0.0000799                                                      1.0000000
2         GLOBAL         0.0000001      -0.0000799                                                      1.0000000
3         GLOBAL         0.0             0.0                                                            0.0
4         GLOBAL         0.0             0.0                                                            0.0

LIST DYNAMIC PARTICIPATION FACTORS
```

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   ****************************
   *RESULTS OF LATEST ANALYSIS*
   ****************************

  PROBLEM - PROB 2     TITLE - PRINCIPLES OF DAMPING
             PARTICPATION FACTORS -(OUTPUT IN INTERNAL UNITS)


MODE          /-----------------DISPLACEMENT-----------------//------------------ROTATION-------------------/
                        X DISP.        Y DISP.         Z DISP.         X ROT.          Y ROT.          Z ROT.

    1                 10.1781464      -0.0000000       0.0             0.0             0.0             0.0
    2                 -0.0000000      10.1792116       0.0             0.0             0.0             0.0
    3                 -0.1472614      -0.0000004       0.0             0.0             0.0             0.0
    4                  0.0000000      -0.0000000       0.0             0.0             0.0             0.0
    5                 -0.0000000       0.0000000       0.0             0.0             0.0             0.0
    6                 -0.0000071       0.0000000       0.0             0.0             0.0             0.0

$

$   ****************************************************************

$   ***          PERFORM TRANSIENT ANALYSIS FOR LOADS 1 & 2       ***

$   ***                                                           ***

$   ***         DEFINE RESULTS TO BE STORED IN OUTPUT FILE        ***

$   ****************************************************************

$

$

$   ****************************************************************

$                        *** DAMPING = 1% ***

$   ****************************************************************

$

DAMPING PERCENT 1.0   6                          $ *** SECTION 11.2.1

TRANSIENT ANALYSIS LOAD 1 2                      $ *** SECTION 11.5.2

**** STRUDL WARNING - NO INITIAL CONDITIONS SPECIFIED FOR TRANSIENT LOAD 2
                      ZEROES ASSUMED.
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                                                                                                    PAGE -       19

     ****************************
     *RESULTS OF LATEST ANALYSIS*
     ****************************

    PROBLEM - PROB 2     TITLE - PRINCIPLES OF DAMPING

    ACTIVE UNITS INCH KIPS RAD  FAHR SEC  LBM

                                              TRANSIENT LOAD 1

                                   TIME HISTORY FOR JOINT  1          XT   RELATIVE DISPLACEMENT

         TIME           VALUE            TIME          VALUE            TIME          VALUE            TIME           VALUE

    0.0             -9.999985E-02    2.000000E-02  -7.448071E-02    4.000000E-02  -1.143372E-02    6.000000E-02    5.648961E-02
    7.999998E-02     9.467757E-02    9.999996E-02   8.415711E-02    1.199999E-01   3.097441E-02    1.400000E-01   -3.721519E-02
    1.600000E-01    -8.548629E-02    1.799999E-01  -8.954120E-02    2.000000E-01  -4.796456E-02    2.200000E-01    1.740095E-02
    2.399999E-01     7.295024E-02    2.600000E-01   9.058219E-02    2.800000E-01   6.187847E-02    3.000000E-01    2.098825E-03
    3.200000E-01    -5.789188E-02    3.400000E-01  -8.754832E-02    3.600000E-01  -7.221413E-02    3.800000E-01   -2.036018E-02
    4.000000E-01     4.108898E-02    4.200000E-01   8.073783E-02    4.400000E-01   7.873809E-02    4.600000E-01    3.670361E-02
    4.800000E-01    -2.340498E-02    4.999999E-01  -7.073271E-02    5.200000E-01  -8.137321E-02    5.400000E-01   -5.045014E-02
    5.599999E-01     5.687665E-03    5.800000E-01   5.811133E-02    6.000000E-01   8.020920E-02    6.199999E-01    6.117689E-02
    6.400000E-01     1.128795E-02    6.600000E-01  -4.362414E-02    6.799999E-01  -7.553720E-02    7.000000E-01   -6.854558E-02
    7.200000E-01    -2.677927E-02    7.399999E-01   2.800154E-02    7.600000E-01   6.773907E-02    7.800000E-01    7.243520E-02
    8.000000E-01     4.020494E-02    8.200000E-01  -1.201862E-02    8.400000E-01  -5.736766E-02    8.600000E-01   -7.286298E-02
    8.800000E-01    -5.105708E-02    9.000000E-01  -3.591646E-03    9.200000E-01   4.501021E-02    9.400000E-01    7.000548E-02
    9.600000E-01     5.901749E-02    9.800000E-01   1.815669E-02    9.999999E-01  -3.135055E-02    1.020000E+00   -6.418395E-02
    1.040000E+00    -6.387913E-02    1.059999E+00  -3.107614E-02    1.080000E+00   1.706086E-02    1.099999E+00    5.581791E-02
    1.120000E+00     6.561112E-02    1.139999E+00   4.187608E-02    1.160000E+00  -2.823782E-03    1.179999E+00   -4.543669E-02
    1.200000E+00    -6.430864E-02    1.219999E+00  -5.018374E-02    1.240000E+00  -1.073031E-02    1.259999E+00    3.360935E-02
    1.280000E+00     6.020411E-02    1.299999E+00   5.577859E-02    1.320000E+00   2.303408E-02    1.339999E+00   -2.095550E-02
    1.360000E+00    -5.364173E-02    1.379999E+00  -5.855732E-02    1.400000E+00  -3.360613E-02    1.419999E+00    8.081682E-03
    1.440000E+00     4.505184E-02    1.459999E+00   5.856125E-02    1.480000E+00   4.207572E-02    1.499999E+00    4.419487E-03
    1.520000E+00    -3.493784E-02    1.540000E+00  -5.594469E-02    1.559999E+00  -4.817964E-02    1.580000E+00   -1.601256E-02
    1.599999E+00     2.383473E-02    1.620000E+00   5.097538E-02    1.639999E+00   5.178340E-02    1.660000E+00    2.622985E-02
    1.679999E+00    -1.229904E-02    1.700000E+00  -4.400917E-02    1.719999E+00  -5.286468E-02    1.740000E+00   -3.469238E-02
    1.759999E+00     8.682667E-04    1.780000E+00   3.547049E-02    1.799999E+00   5.152128E-02    1.820000E+00    4.112257E-02
    1.839999E+00     9.950094E-03    1.860000E+00  -2.583199E-02    1.879999E+00  -4.794939E-02    1.900000E+00   -4.534701E-02
    1.919999E+00    -1.970851E-02    1.940000E+00   1.558576E-02    1.959999E+00   4.243662E-02    1.980000E+00    4.730381E-02
    1.999999E+00     2.803010E-02    2.020000E+00  -5.226158E-03    2.040000E+00  -3.533950E-02    2.059999E+00   -4.703302E-02
    2.080000E+00    -3.462463E-02    2.099999E+00  -4.778411E-03    2.120000E+00   2.706553E-02    2.139999E+00    4.467471E-02
    2.160000E+00     3.929564E-02    2.179999E+00   1.399963E-02    2.200000E+00  -1.805206E-02    2.219999E+00   -4.045217E-02
    2.240000E+00    -4.194320E-02    2.259999E+00  -2.207042E-02    2.280000E+00   8.743748E-03    2.299999E+00    3.466095E-02
    2.320000E+00     4.256460E-02    2.339999E+00   2.869482E-02    2.360000E+00   4.257308E-04    2.379999E+00   -2.764894E-02
    2.400000E+00    -4.124672E-02    2.419999E+00  -3.365907E-02    2.440000E+00  -9.054773E-03    2.459999E+00    1.979932E-02
    2.480000E+00     3.815937E-02    2.499999E+00   3.683557E-02    2.520000E+00   1.678771E-02    2.540000E+00   -1.151085E-02
    2.559999E+00    -3.354087E-02    2.580000E+00  -3.818386E-02    2.599999E+00  -2.333006E-02    2.620000E+00    3.179035E-03
    2.639999E+00     2.768519E-02    2.660000E+00   3.774785E-02    2.679999E+00   2.845670E-02    2.700000E+00    4.820827E-03
    2.719999E+00    -2.092442E-02    2.740000E+00  -3.564831E-02    2.759999E+00  -3.201904E-02    2.780000E+00   -1.215006E-02
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                                                                                          PAGE -       20

     TIME          VALUE            TIME          VALUE            TIME          VALUE            TIME          VALUE

 2.799999E+00   1.361249E-02    2.820000E+00   3.207402E-02    2.839999E+00   3.394670E-02    2.860000E+00   1.851884E-02
 2.879999E+00  -6.107777E-03    2.900000E+00  -2.726853E-02    2.919999E+00  -3.424647E-02    2.940000E+00  -2.369713E-02
 2.959999E+00  -1.243169E-03    2.980000E+00   2.151696E-02    2.999999E+00   3.299761E-02    3.020000E+00   2.752095E-02
 3.040000E+00   8.120257E-03    3.059999E+00  -1.513027E-02    3.080000E+00  -3.034398E-02    3.099999E+00  -2.989643E-02
 3.120000E+00  -1.424298E-02    3.139999E+00   8.430269E-03    3.160000E+00   2.648429E-02    3.179999E+00   3.079967E-02
 3.200000E+00   1.938033E-02    3.219999E+00  -1.734420E-03    3.240000E+00  -2.165968E-02    3.259999E+00  -3.027407E-02
 3.280000E+00  -2.335868E-02    3.299999E+00  -4.658066E-03    3.320000E+00   1.614089E-02    3.339999E+00   2.842442E-02
 3.360000E+00   2.606599E-02    3.379999E+00   1.047853E-02    3.400000E+00  -1.021418E-02    3.419999E+00  -2.540875E-02
 3.440000E+00  -2.745353E-02    3.459999E+00  -1.549923E-02    3.480000E+00   4.167866E-03    3.499999E+00   2.142830E-02
 3.520000E+00   2.753432E-02    3.540000E+00   1.954110E-02    3.559999E+00   1.720870E-03    3.580000E+00  -1.671595E-02
 3.599999E+00  -2.637908E-02    3.620000E+00  -2.247909E-02    3.639999E+00  -7.197615E-03    3.660000E+00   1.152411E-02
 3.679999E+00   2.410980E-02    3.700000E+00   2.424458E-02    3.719999E+00   1.204088E-02    3.740000E+00  -6.112255E-03
 3.759999E+00  -2.089136E-02    3.780000E+00  -2.482534E-02    3.799999E+00  -1.607009E-02    3.820000E+00   7.349318E-04
 3.839999E+00   1.692170E-02    3.860000E+00   2.426278E-02    3.879999E+00   1.915136E-02    3.900000E+00   4.369333E-03
 3.919999E+00  -1.242107E-02    3.940000E+00  -2.264712E-02    3.959999E+00  -2.120082E-02    3.980000E+00  -8.987796E-03
 3.999999E+00   7.621001E-03    4.020000E+00   2.011051E-02    4.040000E+00   2.218550E-02    4.059999E+00   1.294154E-02
 4.080000E+00  -2.753173E-03    4.099999E+00  -1.681881E-02    4.120000E+00  -2.212196E-02    4.139999E+00  -1.609155E-02
 4.160000E+00  -1.960840E-03    4.179999E+00   1.296219E-02    4.200000E+00   2.107266E-02    4.219999E+00   1.834268E-02
 4.240000E+00   6.318968E-03    4.259999E+00  -8.745275E-03    4.280000E+00  -1.914062E-02    4.299999E+00  -1.964540E-02
 4.320000E+00  -1.014655E-02    4.339999E+00   4.377153E-03    4.360000E+00   1.646246E-02    4.379999E+00   1.999540E-02
 4.400000E+00   1.330257E-02    4.419999E+00  -6.178065E-05    4.440000E+00  -1.320042E-02    4.459999E+00  -1.943121E-02
 4.480000E+00  -1.568412E-02    4.499999E+00  -4.010841E-03    4.520000E+00   9.533603E-03    4.540000E+00   1.803010E-02
 4.559999E+00   1.722872E-02    4.580000E+00   7.672384E-03    4.599999E+00  -5.649015E-03    4.620000E+00  -1.590234E-02
 4.639999E+00  -1.791492E-02    4.660000E+00  -1.078248E-02    4.679999E+00   1.732794E-03    4.700000E+00   1.318442E-02
 4.719999E+00   1.776087E-02    4.740000E+00   1.323340E-02    4.759999E+00   2.038123E-03    4.780000E+00  -1.003148E-02
 4.799999E+00  -1.682125E-02    4.820000E+00  -1.495305E-02    4.839999E+00  -5.503349E-03    4.860000E+00   6.609283E-03
 4.879999E+00   1.518279E-02    4.900000E+00   1.590611E-02    4.919999E+00   8.525304E-03    4.940000E+00  -3.086193E-03
 4.959999E+00  -1.295853E-02    4.980000E+00  -1.609357E-02    4.999999E+00  -1.099405E-02

MAXIMUM  9.467757E-02  TIME  7.999998E-02  MINIMUM -9.999985E-02  TIME  0.0
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****************************
*RESULTS OF LATEST ANALYSIS*
****************************

PROBLEM - PROB 2     TITLE - PRINCIPLES OF DAMPING

ACTIVE UNITS INCH KIPS RAD  FAHR SEC  LBM

                                         TRANSIENT LOAD 2

                              TIME HISTORY FOR JOINT  1           XT   RELATIVE DISPLACEMENT

     TIME           VALUE            TIME            VALUE            TIME            VALUE            TIME            VALUE

0.0             0.0             2.000000E-02    3.690201E-03    4.000000E-02    1.279574E-02    6.000000E-02    2.261560E-02
7.959998E-02    2.815725E-02    9.999996E-02    2.667951E-02    1.199999E-01    1.905299E-02    1.400000E-01    9.263225E-03
1.600000E-01    2.336554E-03    1.799999E-01    1.766435E-03    2.000000E-01    7.757548E-03    2.200000E-01    1.716937E-02
2.399999E-01    2.516308E-02    2.600000E-01    2.768888E-02    2.800000E-01    2.353566E-02    3.000000E-01    1.490667E-02
3.200000E-01    6.249789E-03    3.400000E-01    1.962295E-03    3.600000E-01    4.154753E-03    3.800000E-01    1.161080E-02
4.000000E-01    2.045493E-02    4.200000E-01    2.616341E-02    4.400000E-01    2.587553E-02    4.600000E-01    1.982495E-02
4.800000E-01    1.117565E-02    4.999999E-01    4.369132E-03    5.200000E-01    2.845502E-03    5.400000E-01    7.307082E-03
5.599999E-01    1.540056E-02    5.800000E-01    2.295857E-02    6.000000E-01    2.614627E-02    6.199999E-01    2.340764E-02
6.400000E-01    1.622393E-02    6.600000E-01    8.315876E-03    6.799999E-01    3.717913E-03    7.000000E-01    4.720800E-03
7.200000E-01    1.073151E-02    7.399999E-01    1.861708E-02    7.600000E-01    2.433594E-02    7.800000E-01    2.500771E-02
8.000000E-01    2.036215E-02    8.200000E-01    1.283884E-02    8.400000E-01    6.306585E-03    8.600000E-01    4.074052E-03
8.800000E-01    7.214349E-03    9.000000E-01    1.405147E-02    9.200000E-01    2.105309E-02    9.400000E-01    2.465476E-02
9.600000E-01    2.307406E-02    9.800000E-01    1.719156E-02    9.999999E-01    1.006394E-02    1.020000E+00    5.336985E-03
1.040000E+00    5.381774E-03    1.059999E+00    1.010671E-02    1.080000E+00    1.703992E-02    1.099999E+00    2.262161E-02
1.120000E+00    2.403127E-02    1.139999E+00    2.061205E-02    1.160000E+00    1.417368E-02    1.179999E+00    8.035723E-03
1.200000E+00    5.316596E-03    1.219999E+00    7.349726E-03    1.240000E+00    1.303108E-02    1.259999E+00    1.941638E-02
1.280000E+00    2.324606E-02    1.299999E+00    2.260840E-02    1.320000E+00    1.789272E-02    1.339999E+00    1.155781E-02
1.360000E+00    6.850749E-03    1.379999E+00    6.143142E-03    1.400000E+00    9.737093E-03    1.419999E+00    1.574155E-02
1.440000E+00    2.106642E-02    1.459999E+00    2.301229E-02    1.480000E+00    2.063832E-02    1.499999E+00    1.521540E-02
1.520000E+00    9.547338E-03    1.540000E+00    6.521836E-03    1.559999E+00    7.639967E-03    1.580000E+00    1.227251E-02
1.599999E+00    1.801109E-02    1.620000E+00    2.191953E-02    1.639999E+00    2.203559E-02    1.660000E+00    1.835522E-02
1.679999E+00    1.280623E-02    1.700000E+00    8.239217E-03    1.719999E+00    6.963711E-03    1.740000E+00    9.580843E-03
1.759999E+00    1.470231E-02    1.780000E+00    1.968573E-02    1.799999E+00    2.199737E-02    1.820000E+00    2.049988E-02
1.839999E+00    1.601063E-02    1.860000E+00    1.085749E-02    1.879999E+00    7.672235E-03    1.900000E+00    8.047100E-03
1.919999E+00    1.173962E-02    1.940000E+00    1.682271E-02    1.959999E+00    2.068971E-02    1.980000E+00    2.139065E-02
1.999999E+00    1.861488E-02    2.020000E+00    1.382533E-02    2.040000E+00    9.488389E-03    2.059999E+00    7.804222E-03
2.080000E+00    9.591211E-03    2.099999E+00    1.388959E-02    2.120000E+00    1.847565E-02    2.139999E+00    2.101164E-02
2.160000E+00    2.023694E-02    2.179999E+00    1.659387E-02    2.200000E+00    1.197783E-02    2.219999E+00    8.751798E-03
2.240000E+00    8.537084E-03    2.259999E+00    1.139917E-02    2.280000E+00    1.583702E-02    2.299999E+00    1.956959E-02
2.320000E+00    2.070788E-02    2.339999E+00    1.871041E-02    2.360000E+00    1.463916E-02    2.379999E+00    1.059588E-02
2.400000E+00    8.637551E-03    2.419999E+00    9.730320E-03    2.440000E+00    1.327379E-02    2.459999E+00    1.742930E-02
2.480000E+00    2.007346E-02    2.499999E+00    1.988280E-02    2.520000E+00    1.699553E-02    2.540000E+00    1.292001E-02
2.559999E+00    9.747252E-03    2.580000E+00    9.078573E-03    2.599999E+00    1.121780E-02    2.620000E+00    1.503560E-02
2.639999E+00    1.856493E-02    2.660000E+00    2.001414E-02    2.679999E+00    1.867605E-02    2.700000E+00    1.527206E-02
2.719999E+00    1.156427E-02    2.740000E+00    9.443764E-03    2.759999E+00    9.966455E-03    2.780000E+00    1.282796E-02
```

```
      TIME           VALUE           TIME           VALUE           TIME           VALUE           TIME           VALUE

  2.799999E+00   1.653824E-02   2.820000E+00   1.919705E-02   2.839999E+00   1.946675E-02   2.860000E+00   1.724485E-02
  2.879999E+00   1.369816E-02   2.900000E+00   1.065062E-02   2.919999E+00   9.645667E-03   2.940000E+00   1.116497E-02
  2.959999E+00   1.439875E-02   2.980000E+00   1.767662E-02   2.999999E+00   1.933004E-02   3.020000E+00   1.854130E-02
  3.040000E+00   1.574725E-02   3.059999E+00   1.239874E-02   3.080000E+00   1.020768E-02   3.099999E+00   1.027214E-02
  3.120000E+00   1.252653E-02   3.139999E+00   1.579189E-02   3.160000E+00   1.839201E-02   3.179999E+00   1.901350E-02
  3.200000E+00   1.736891E-02   3.219999E+00   1.432800E-02   3.240000E+00   1.145839E-02   3.259999E+00   1.021776E-02
  3.280000E+00   1.121371E-02   3.299999E+00   1.390694E-02   3.320000E+00   1.690237E-02   3.339999E+00   1.867143E-02
  3.360000E+00   1.833177E-02   3.379999E+00   1.608688E-02   3.400000E+00   1.310675E-02   3.419999E+00   1.091845E-02
  3.440000E+00   1.062397E-02   3.459999E+00   1.234560E-02   3.480000E+00   1.517803E-02   3.499999E+00   1.766385E-02
  3.520000E+00   1.854323E-02   3.540000E+00   1.739207E-02   3.559999E+00   1.482562E-02   3.580000E+00   1.217038E-02
  3.599999E+00   1.077871E-02   3.620000E+00   1.134038E-02   3.639999E+00   1.354119E-02   3.660000E+00   1.623747E-02
  3.679999E+00   1.805004E-02   3.700000E+00   1.806945E-02   3.719999E+00   1.631190E-02   3.740000E+00   1.369750E-02
  3.759999E+00   1.156904E-02   3.780000E+00   1.100248E-02   3.799999E+00   1.226339E-02   3.820000E+00   1.468363E-02
  3.839999E+00   1.701482E-02   3.860000E+00   1.807207E-02   3.879999E+00   1.733594E-02   3.900000E+00   1.520705E-02
  3.919999E+00   1.278891E-02   3.940000E+00   1.131618E-02   3.959999E+00   1.152447E-02   3.980000E+00   1.328338E-02
  3.999999E+00   1.567535E-02   4.020000E+00   1.747407E-02   4.040000E+00   1.777291E-02   4.059999E+00   1.644160E-02
  4.080000E+00   1.418127E-02   4.099999E+00   1.215556E-02   4.120000E+00   1.139181E-02   4.139999E+00   1.226031E-02
  4.160000E+00   1.429538E-02   4.179999E+00   1.644458E-02   4.200000E+00   1.761263E-02   4.219999E+00   1.721947E-02
  4.240000E+00   1.548783E-02   4.259999E+00   1.331830E-02   4.280000E+00   1.182118E-02   4.299999E+00   1.174848E-02
  4.320000E+00   1.311649E-02   4.339999E+00   1.520817E-02   4.360000E+00   1.694868E-02   4.379999E+00   1.745749E-02
  4.400000E+00   1.649360E-02   4.419999E+00   1.456889E-02   4.440000E+00   1.267668E-02   4.459999E+00   1.177933E-02
  4.480000E+00   1.231898E-02   4.499999E+00   1.400014E-02   4.520000E+00   1.595080E-02   4.540000E+00   1.717445E-02
  4.559999E+00   1.705904E-02   4.580000E+00   1.568275E-02   4.599999E+00   1.376422E-02   4.620000E+00   1.228755E-02
  4.639999E+00   1.199770E-02   4.660000E+00   1.302490E-02   4.679999E+00   1.482734E-02   4.700000E+00   1.647659E-02
  4.719999E+00   1.713568E-02   4.740000E+00   1.648363E-02   4.759999E+00   1.487131E-02   4.780000E+00   1.313306E-02
  4.799999E+00   1.215521E-02   4.820000E+00   1.242427E-02   4.839999E+00   1.378520E-02   4.860000E+00   1.552964E-02
  4.879999E+00   1.676438E-02   4.900000E+00   1.686855E-02   4.919999E+00   1.580558E-02   4.940000E+00   1.413332E-02
  4.959999E+00   1.271151E-02   4.980000E+00   1.226001E-02   4.999999E+00   1.299443E-02

MAXIMUM  2.815725E-02  TIME  7.999998E-02  MINIMUM  0.0          TIME  0.0

USE STORE TRANSIENT SET ID 'DAMP2'

USE TRANS 1 2

JOINT DISPLACEMENT

     1 TRANS X

END OF SUBSET
```

```
TRANSIENT LOAD 1

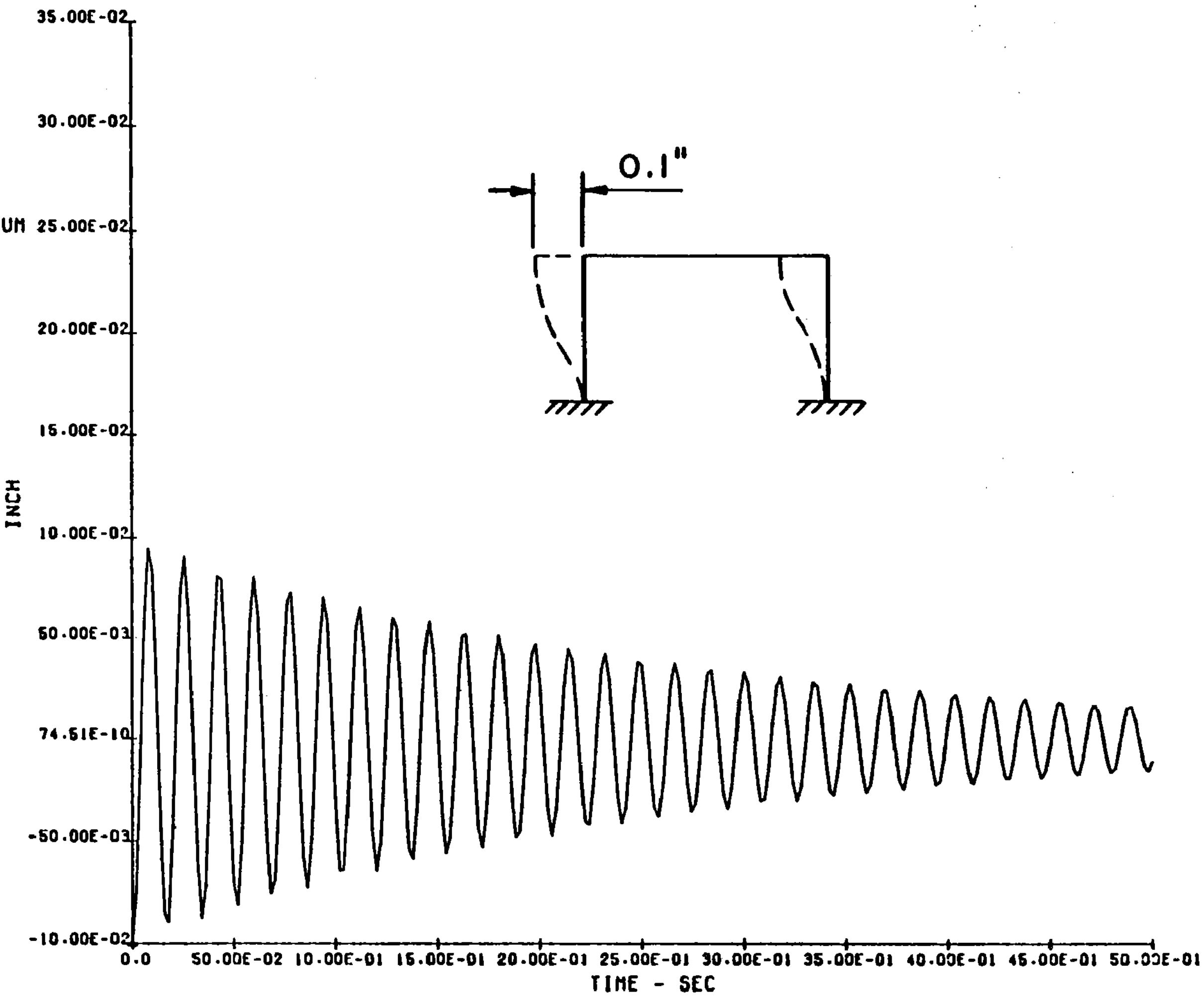
TIME HISTORY FOR JOINT  1
   XT   RELATIVE DISPLACEMENT

PARAMETER    SYMBOL    MINIMUM      MAXIMUM

                 ▲    -99.99E-03    94.67E-03
```

Damping Percent = 1

```
MIN X    00.00E+02
MAX X    49.99E-01
```



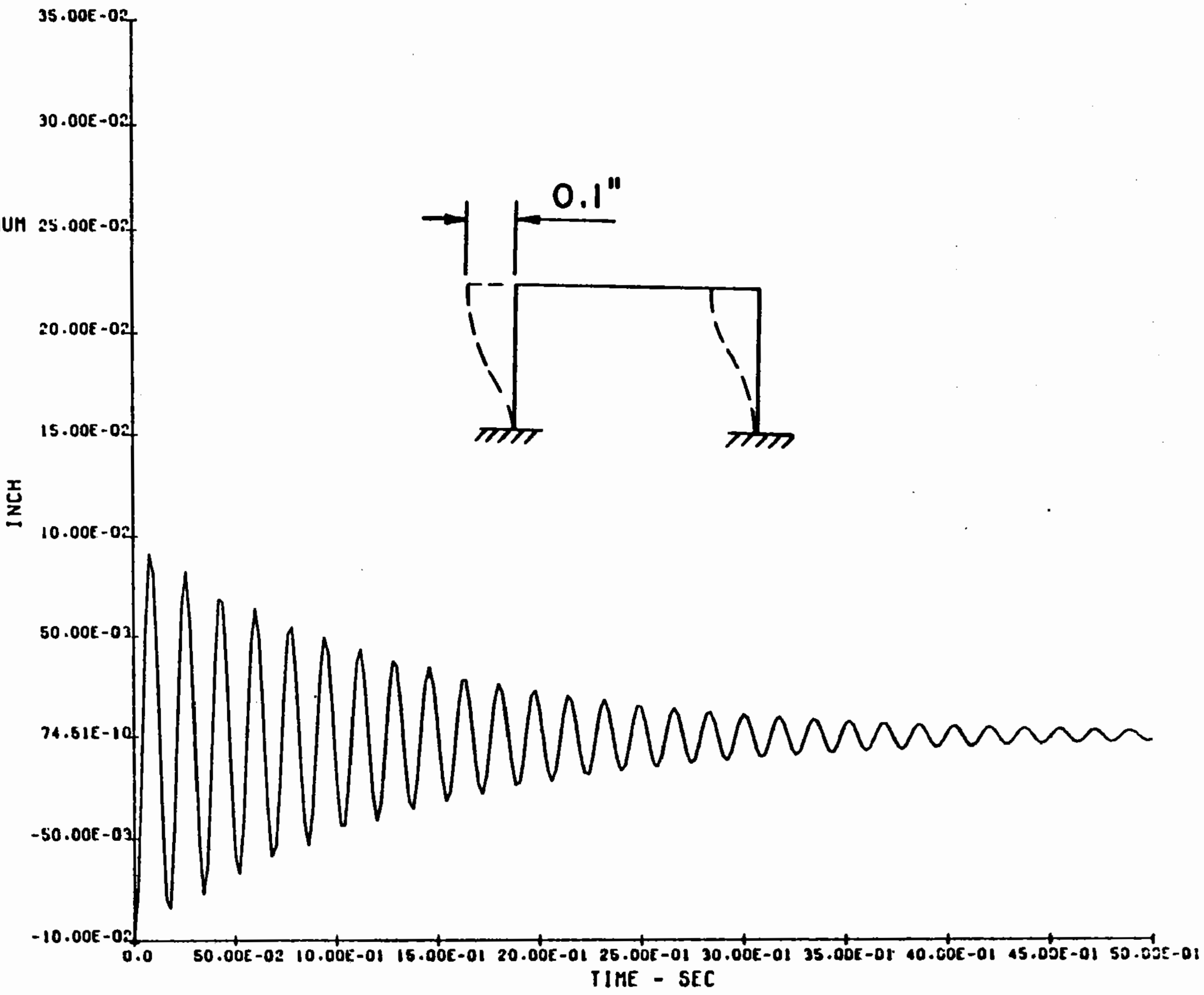
TRANSIENT LOAD 1

TIME HISTORY FOR JOINT 1
XT RELATIVE DISPLACEMENT

PARAMETER	SYMBOL	MINIMUM	MAXIMUM
	Δ	-99.99E-03	91.73E-03

Damping Percent = 2

MIN X 00.00E+02
MAX X 49.99E-01



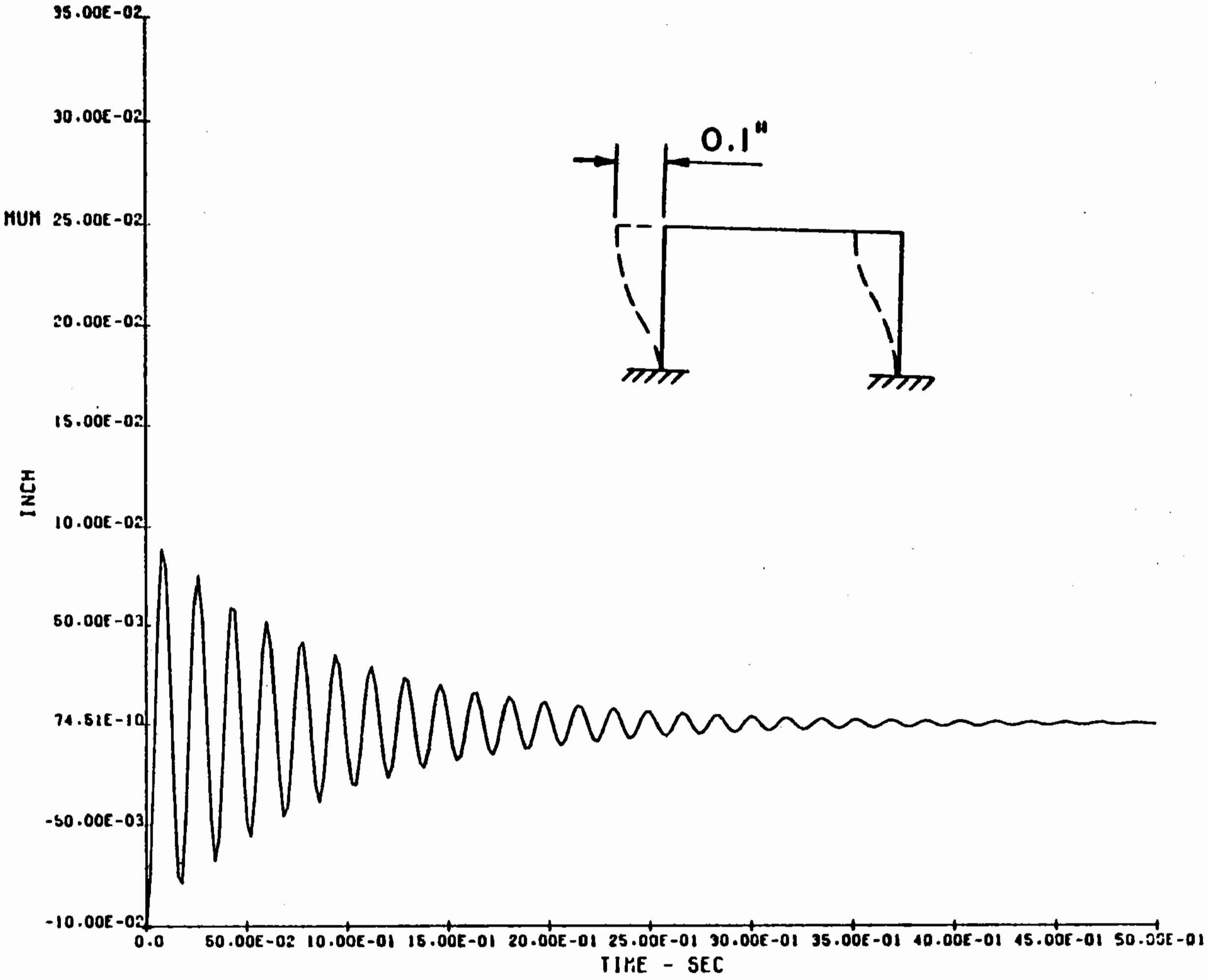
TRANSIENT LOAD 1

TIME HISTORY FOR JOINT 1
X1 RELATIVE DISPLACEMENT

PARAMETER	SYMBOL	MINIMUM	MAXIMUM
	Δ	-99.99E-03	88.88E-03

Damping Percent = 3

MIN X 00.00E+02
MAX X 49.93E-01



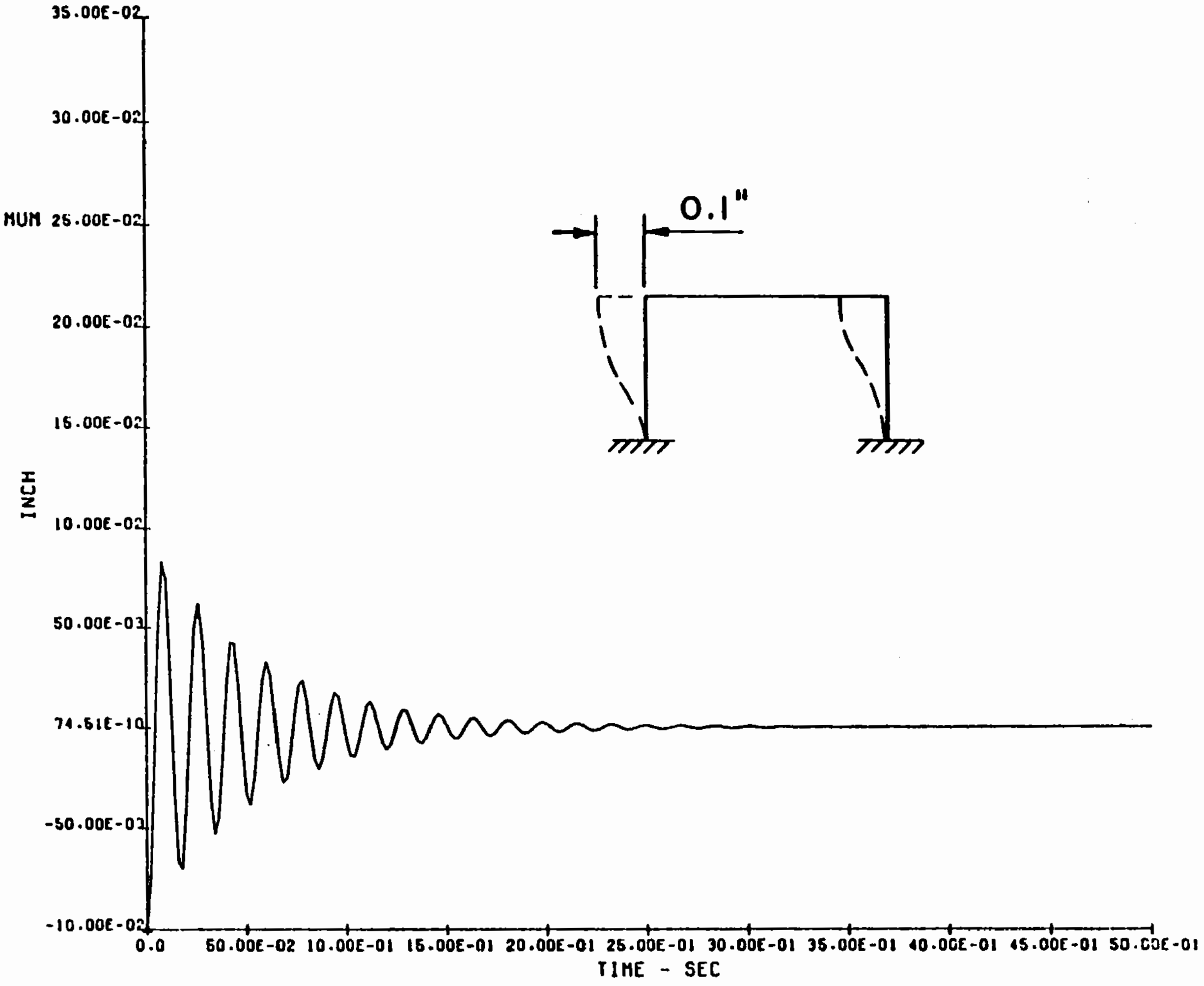
TRANSIENT LOAD 1

TIME HISTORY FOR JOINT 1
XT RELATIVE DISPLACEMENT

PARAMETER	SYMBOL	MINIMUM	MAXIMUM
	▵	-99.99E-03	83.40E-03

Damping Percent = 5

MIN X 00.00E+02
MAX X 49.99E-01



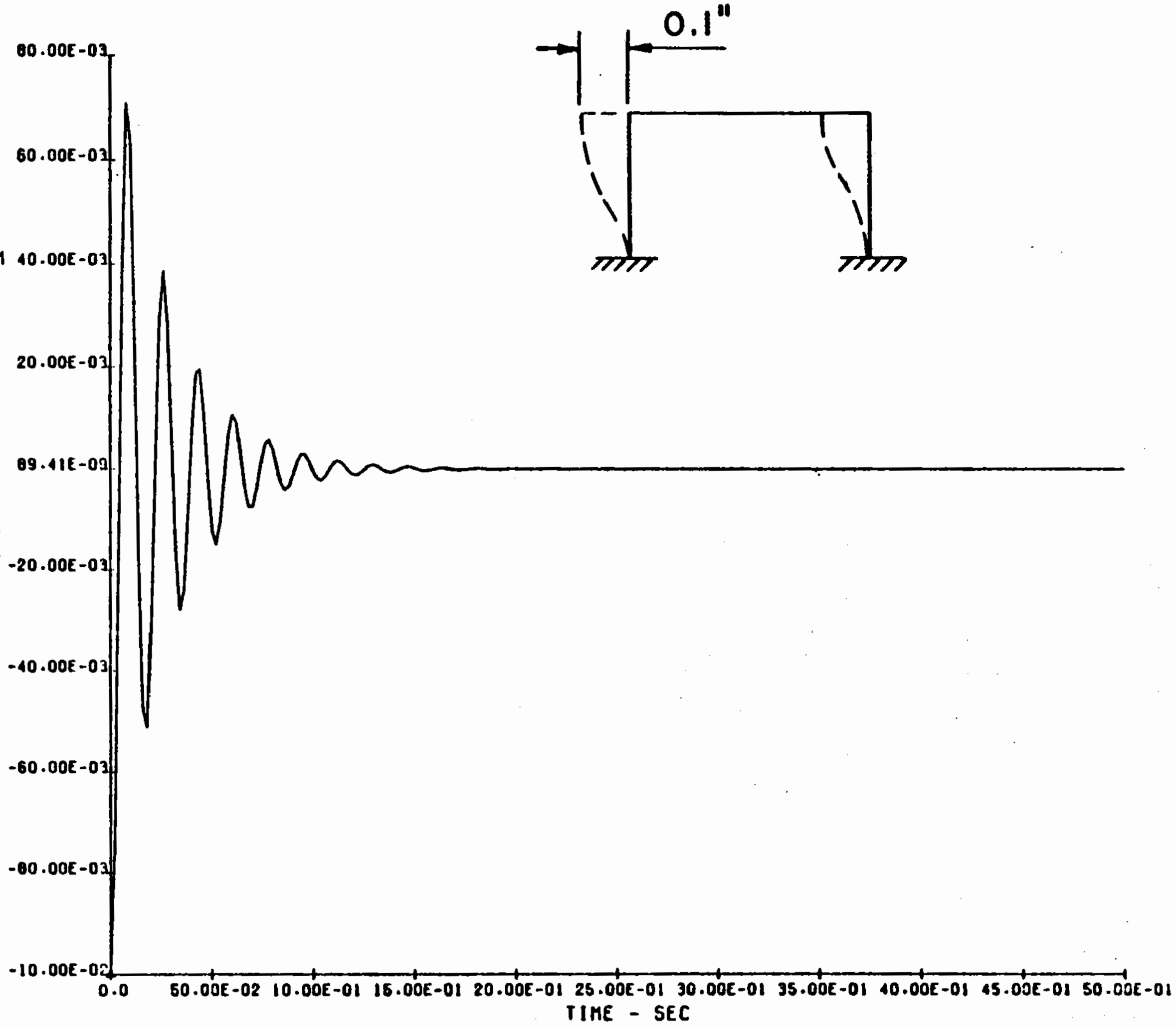
TRANSIENT LOAD 1

TIME HISTORY FOR JOINT 1
XT RELATIVE DISPLACEMENT

PARAMETER	SYMBOL	MINIMUM	MAXIMUM
	△	-99.99E-03	70.97E-03

Damping Percent = 10

MIN X 00.00E+02
MAX X 49.99E-01



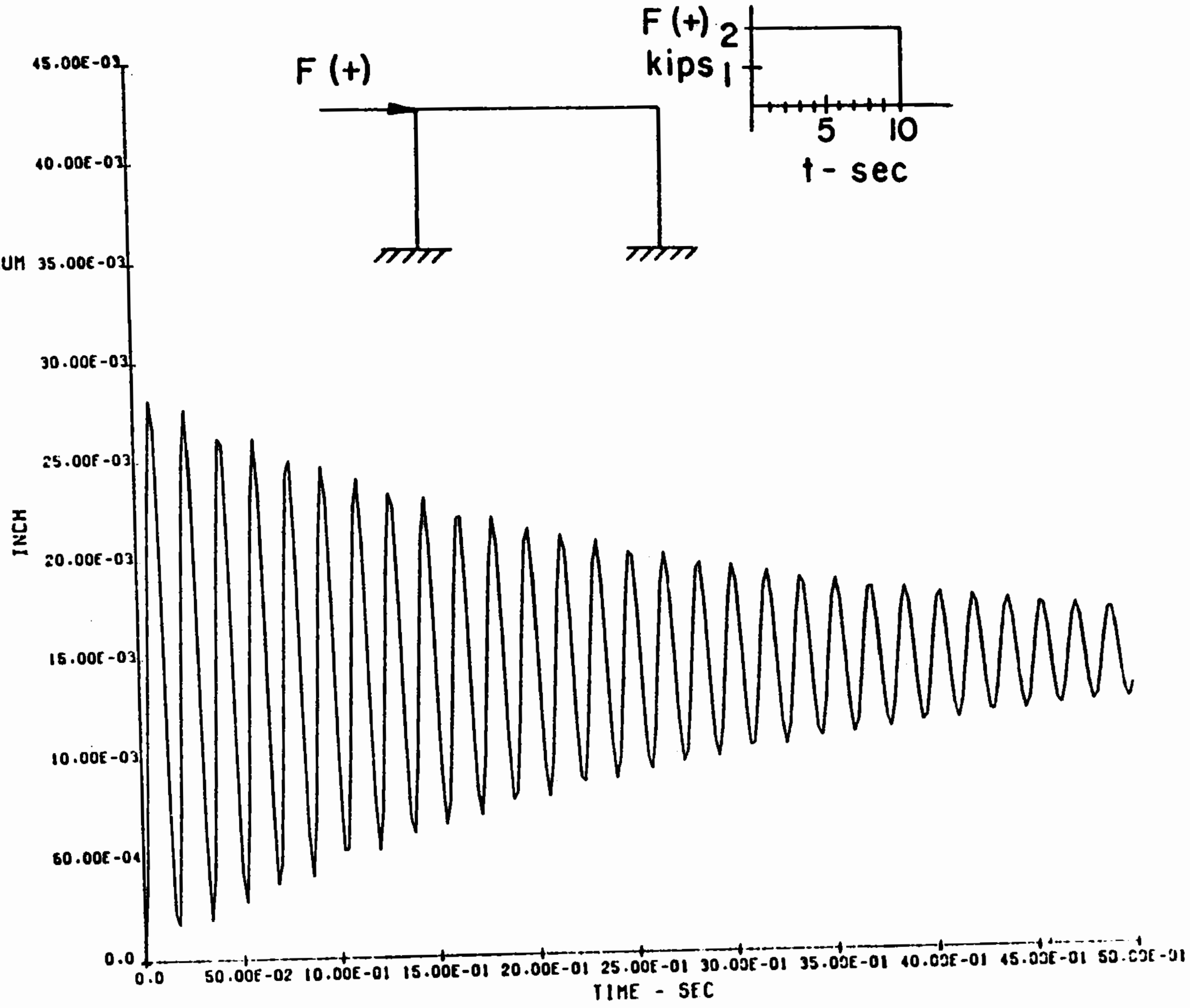
TRANSIENT LOAD 2

TIME HISTORY FOR JOINT 1
XT RELATIVE DISPLACEMENT

PARAMETER	SYMBOL	MINIMUM	MAXIMUM
	△	00.00E+02	28.15E-03

Damping Percent = 1

MIN X 00.00E+02
MAX X 49.99E-01



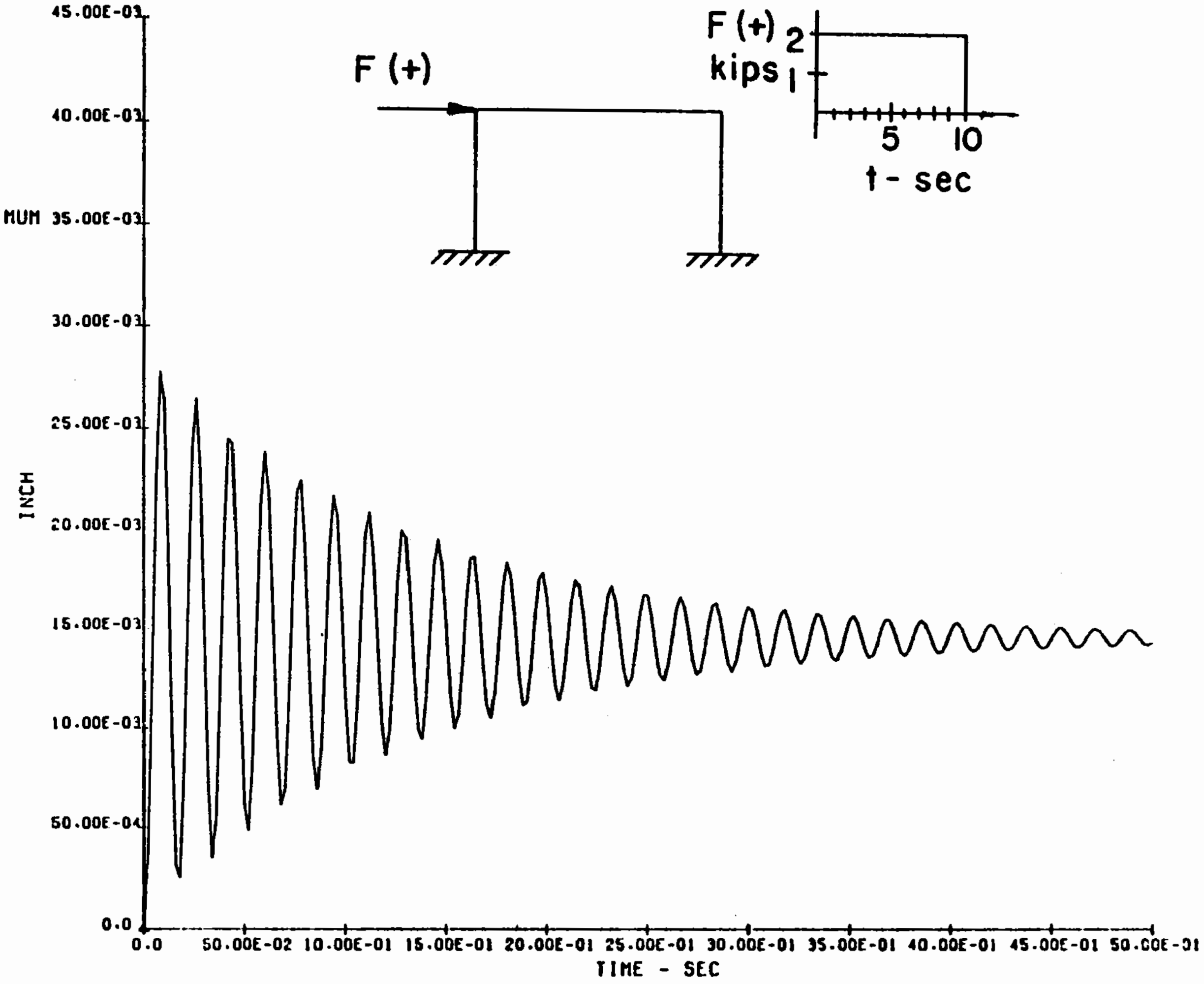
TRANSIENT LOAD 2

TIME HISTORY FOR JOINT 1
XT RELATIVE DISPLACEMENT

PARAMETER	SYMBOL	MINIMUM	MAXIMUM
	△	00.00E+02	27.74E-03

Damping Percent = 2

MIN X 00.00E+02
MAX X 49.99E-01



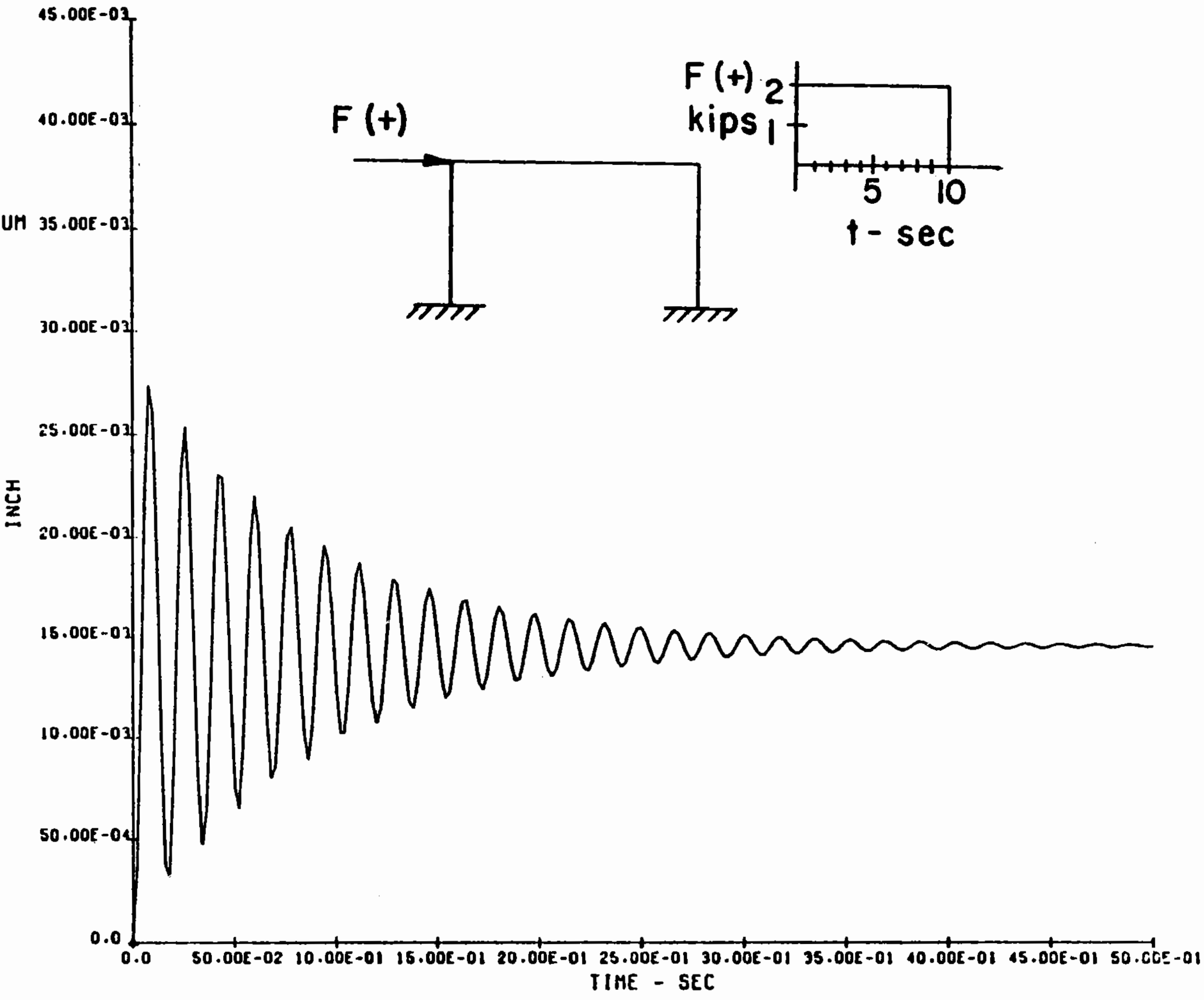
TRANSIENT LOAD 2

TIME HISTORY FOR JOINT 1
XT RELATIVE DISPLACEMENT

PARAMETER	SYMBOL	MINIMUM	MAXIMUM
	△	00.00E+02	27.34E-03

Damping Percent = 3

MIN X 00.00E+02
MAX X 49.99E-01



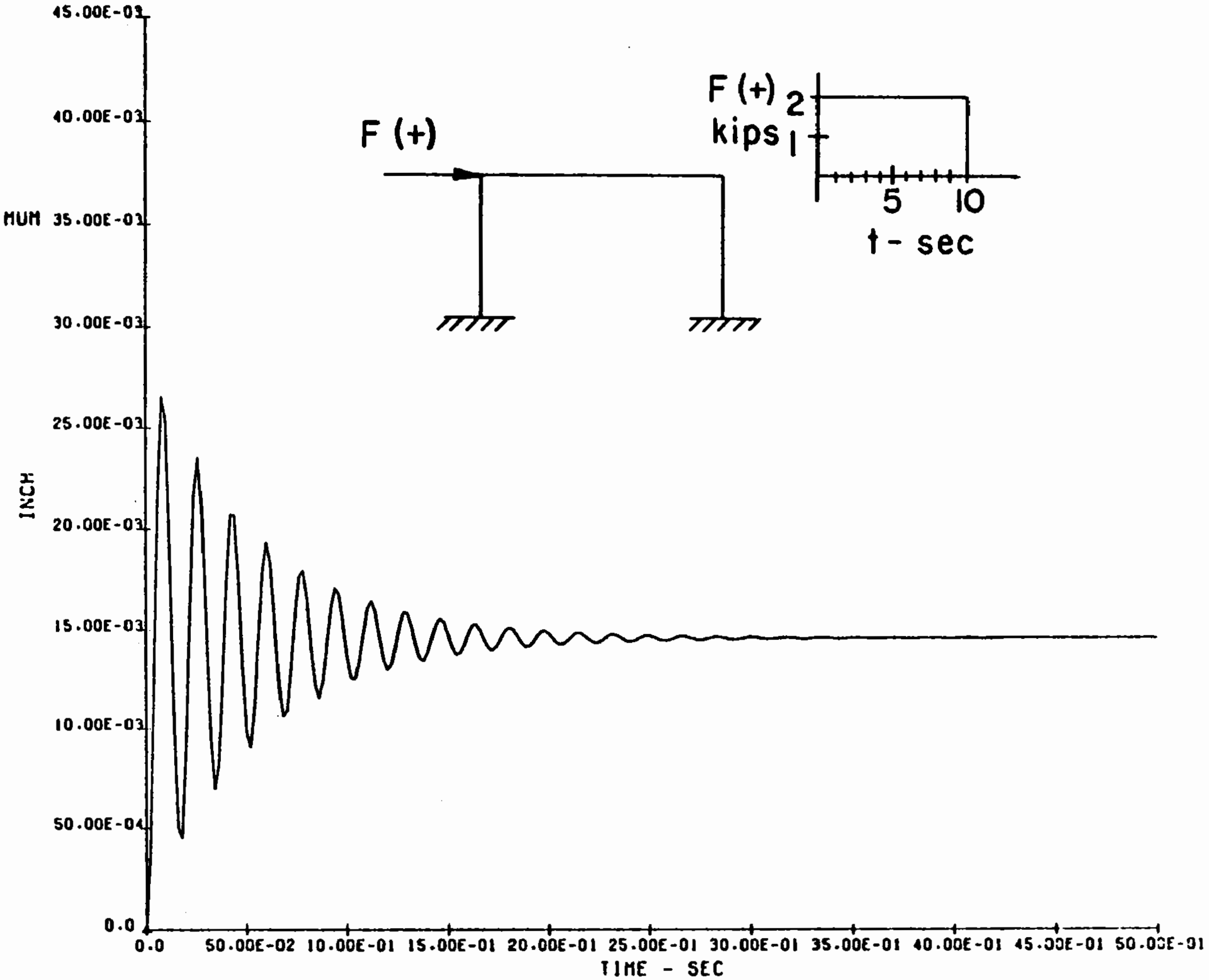
TRANSIENT LOAD 2

TIME HISTORY FOR JOINT 1
X1 RELATIVE DISPLACEMENT

PARAMETER	SYMBOL	MINIMUM	MAXIMUM
	△	00.00E+02	26.56E-03

Damping Percent = 5

MIN X 00.00E+02
MAX X 49.93E-01



```
TRANSIENT LOAD 2

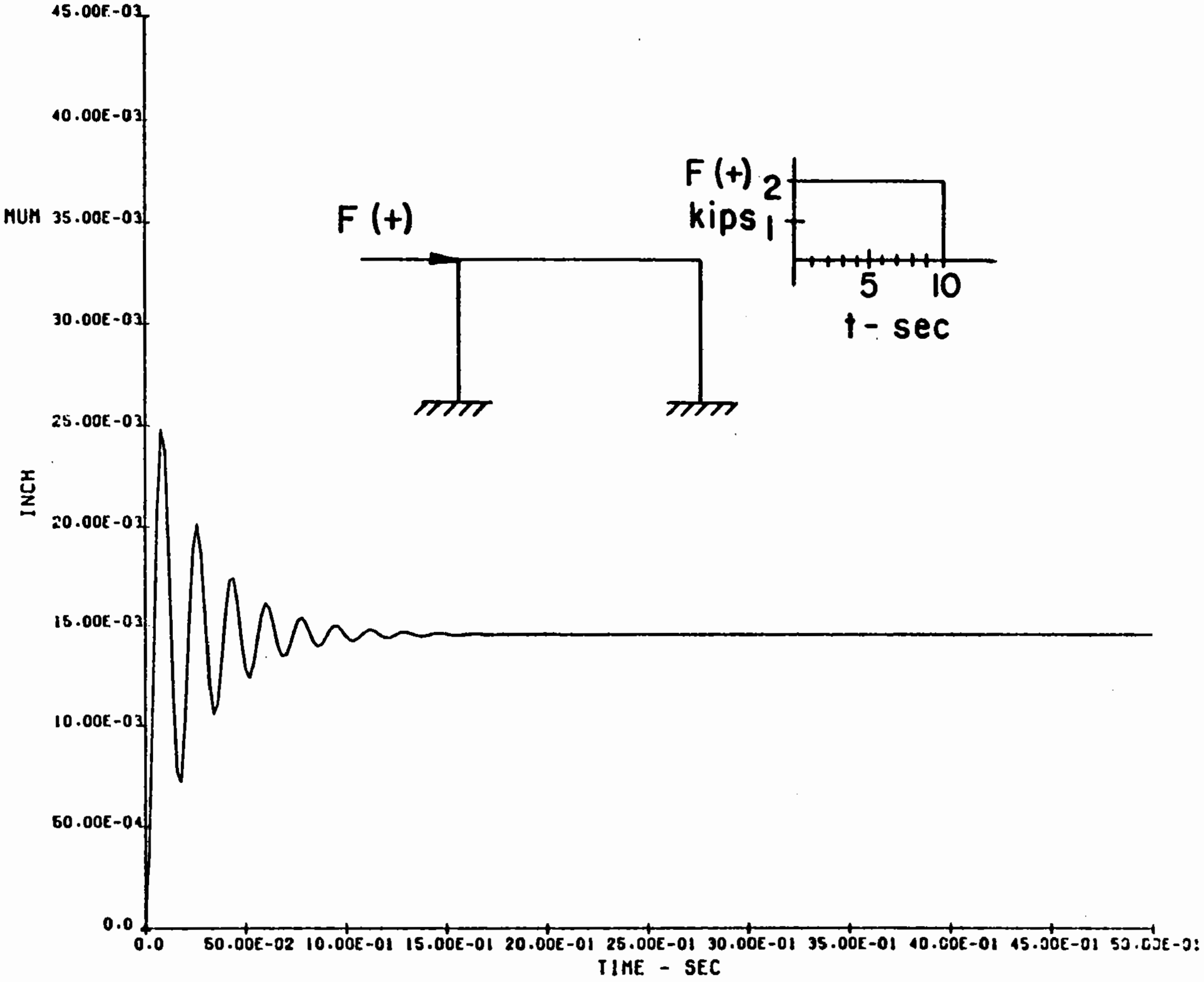
TIME HISTORY FOR JOINT  1
   XT   RELATIVE DISPLACEMENT

PARAMETER    SYMBOL    MINIMUM       MAXIMUM

               △      00.00E+02     24.79E-03
```

Damping Percent = 10

```
MIN X   00.00E+02
MAX X   49.99E-01
```

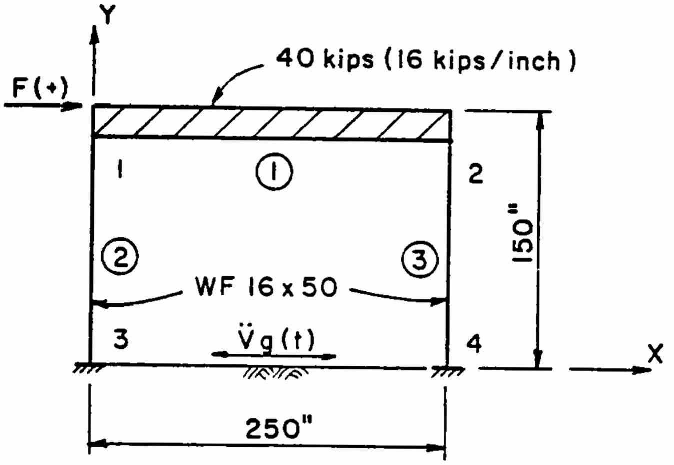


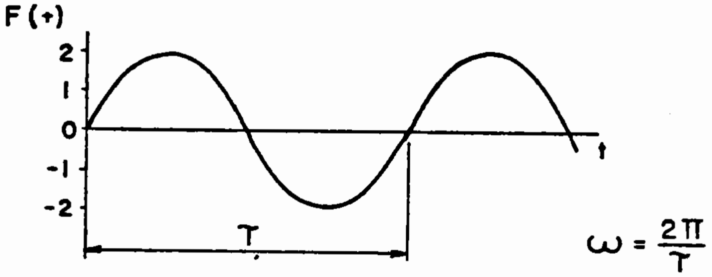
Appendix A.4

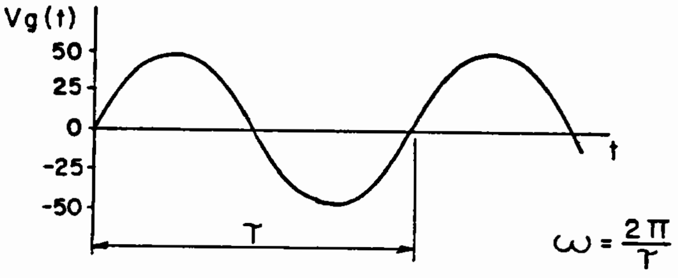
SAMPLE PROBLEM 3

DETERMINATION OF DYNAMIC MAGNIFICATION

PROBLEM STATEMENT: Determine the effect that varying the frequency of excitation has on the base shear of the structure analyzed in SAMPLE PROBLEM 2. Consider both a harmonic forcing function at the top of the frame and harmonic support motion. Vary the frequency about the natural frequency of the structure which is 36.6 RAD/SEC.



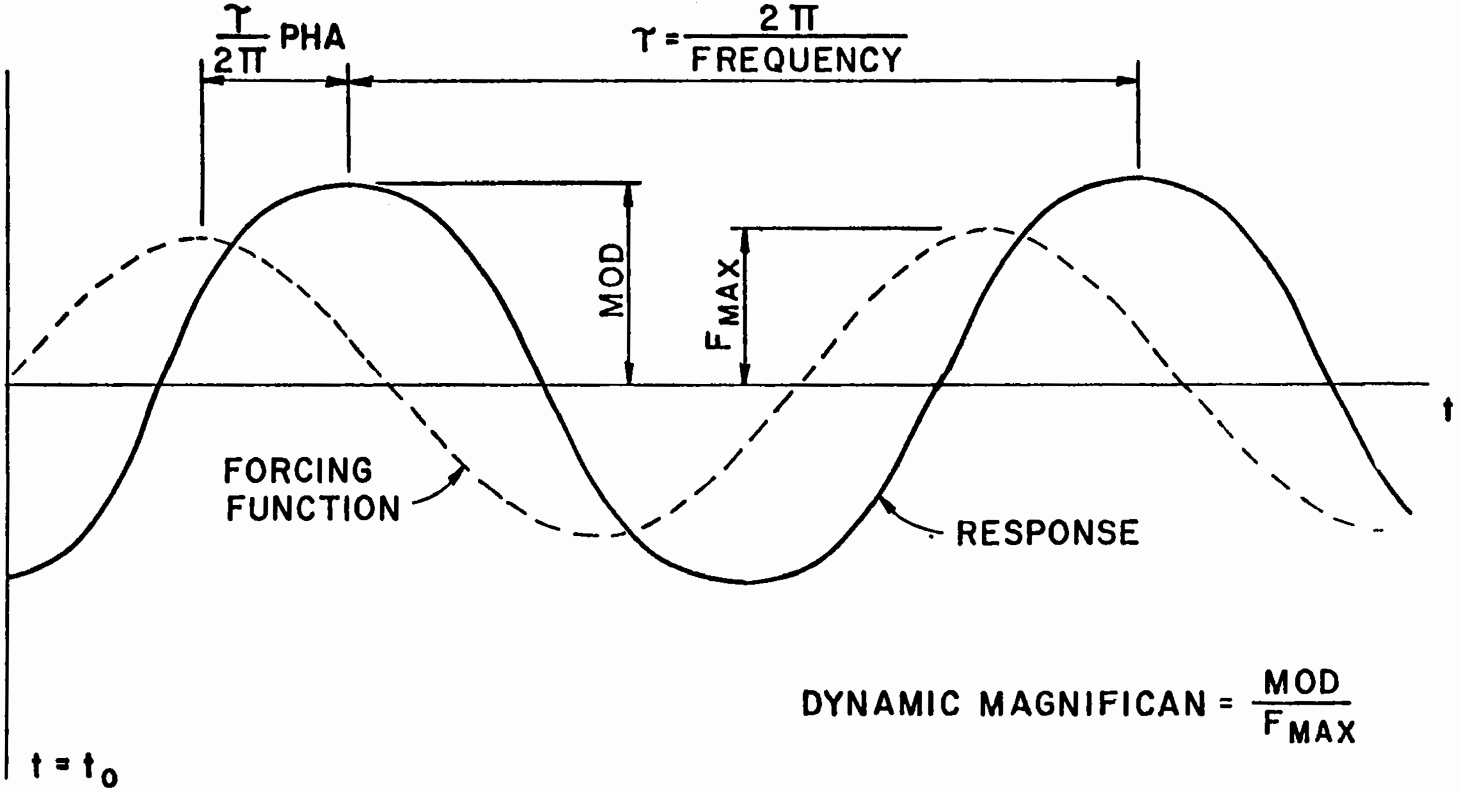




```
$STRUDL 'PROB 3' 'PRINCIPLES OF DYNAMIC MAGNIFICATION'
$
$                     *****************************************
$                     ***                                   ***
$                     *** SEISMIC DESIGN OF HIGHWAY BRIDGES ***
$                     ***                                   ***
$                     *****************************************
$
$    SPONSORED BY:
$
$         FEDERAL HIGHWAY ADMINISTRATION
$
$    PRESENTED BY:
$
$         ENGINEERING COMPUTER CORPERATION
$         601 UNIVERSITY AVE.  SUITE 213
$         SACRAMENTO, CALIFORNIA
$
$    PHONE: (916) 922-9316
$
$    INSTRUCTORS:
$
$         ROY A. IMBSEN  -  PRINCIPLE INVESTIGATOR
$         RICHARD V. NUTT
$         JAMES GATES
$
$
$    THIS PROBLEM ILLUSTRATES THE EFFECTS OF EXCITATION FREQUENCY ON
$    THE RESPONSE OF A SIMPLE STRUCTURE SUBJECTED TO THE FOLLOWING
$    LOADS
$
$               A.  HARMONIC FORCING FUNCTION AT TOP OF FRAME
$                   1.  FREQ. =  5 RAD/SEC
$                   2.  FREQ. =  15 RAD/SEC
$                   3.  FREQ. =  20 RAD/SEC
$                   4.  FREQ. =  25 RAD/SEC
$                   5.  FREQ. =  30 RAD/SEC
$                   6.  FREQ. =  35 RAD/SEC
$                   7.  FREQ. =  40 RAD/SEC
$                   8.  FREQ. =  45 RAD/SEC
$                   9.  FREQ. =  55 RAD/SEC
$               B.  HARMONIC SUPPORT MOTION
$                   1.  FREQ. =  5 RAD/SEC
$                   2.  FREQ. =  15 RAD/SEC
$                   3.  FREQ. =  20 RAD/SEC
$                   4.  FREQ. =  25 RAD/SEC
$                   5.  FREQ. =  30 RAD/SEC
$                   6.  FREQ. =  35 RAD/SEC
$                   7.  FREQ. =  40 RAD/SEC
$                   8.  FREQ. =  45 RAD/SEC
$                   9.  FREQ. =  55 RAD/SEC
$
$
$   *******************************************************************
$   ***      RESTORE THE FREE VIBRATION SOLUTION FROM PROBLEM 2     ***
$   *******************************************************************
$
STRUDL RESTORE 'FREEVIB'                                $ *** SECTION 4.1.1.2
$
$   *******************************************************************
```

```
$                       *** SET DAMPING = 5% ***
$   *****************************************************************
$
DAMPING PERCENT 5.0  6
$
$   *****************************************************************
$   ***                 DEFINE HARMONIC LOADINGS                  ***
$   *****************************************************************
$
HARMONIC LOADING 3 'FREQUENCY =  5 RAD/SEC'      $ *** SECTION 11.4.1.1
FREQUENCY  5.0                                   $ *** SECTION 11.4.1.2
JOINT LOADS                                      $ *** SECTION 11.4.1.3
    1 FORCE X AMP 2.0
END OF DYNAMIC LOAD                              $ *** SECTION 11.4.1.6
HARMONIC LOADING 4 'FREQUENCY =  15 RAD/SEC'
FREQUENCY  15.0
JOINT LOADS
    1 FORCE X AMP 2.0
END OF DYNAMIC LOAD
HARMONIC LOADING 5 'FREQUENCY =  20 RAD/SEC'
FREQUENCY  20.0
JOINT LOADS
    1 FORCE X AMP 2.0
END OF DYNAMIC LOAD
HARMONIC LOADING 6 'FREQUENCY =  25 RAD/SEC'
FREQUENCY  25.0
JOINT LOADS
    1 FORCE X AMP 2.0
END OF DYNAMIC LOAD
HARMONIC LOADING 7 'FREQUENCY =  30 RAD/SEC'
FREQUENCY  30.0
JOINT LOADS
    1 FORCE X AMP 2.0
END OF DYNAMIC LOAD
HARMONIC LOADING 8 'FREQUENCY =  35 RAD/SEC'
FREQUENCY  35.0
JOINT LOADS
    1 FORCE X AMP 2.0
END OF DYNAMIC LOAD
HARMONIC LOADING 9 'FREQUENCY =  40 RAD/SEC'
FREQUENCY  40.0
JOINT LOADS
    1 FORCE X AMP 2.0
END OF DYNAMIC LOAD
HARMONIC LOADING 10 'FREQUENCY =  45 RAD/SEC'
FREQUENCY  45.0
JOINT LOADS
    1 FORCE X AMP 2.0
END OF DYNAMIC LOAD
HARMONIC LOADING 11 'FREQUENCY =  55 RAD/SEC'
FREQUENCY  55.0
JOINT LOADS
    1 FORCE X AMP 2.0
END OF DYNAMIC LOAD
HARMONIC LOADING 12 '5 RAD/SEC HARMONIC SUPPORT MOTION'
FREQUENCY  5.0
SUPPORT ACCELERATION TRANSLATION X -             $ *** SECTION 11.4.1.4
     AMPLITUDE 50.0
END OF DYNAMIC LOADING
HARMONIC LOADING 13 '15 RAD/SEC HARMONIC SUPPORT MOTION'
```

```
FREQUENCY  15.0
SUPPORT ACCELERATION TRANSLATION X -
     AMPLITUDE 50.0
END OF DYNAMIC LOADING
HARMONIC LOADING 14 '20 RAD/SEC HARMONIC SUPPORT MOTION'
FREQUENCY  20.0
SUPPORT ACCELERATION TRANSLATION X -
     AMPLITUDE 50.0
END OF DYNAMIC LOADING
HARMONIC LOADING 15 '25 RAD/SEC HARMONIC SUPPORT MOTION'
FREQUENCY  25.0
SUPPORT ACCELERATION TRANSLATION X -
     AMPLITUDE 50.0
END OF DYNAMIC LOADING
HARMONIC LOADING 16 '30 RAD/SEC HARMONIC SUPPORT MOTION'
FREQUENCY  30.0
SUPPORT ACCELERATION TRANSLATION X -
     AMPLITUDE 50.0
END OF DYNAMIC LOADING
HARMONIC LOADING 17 '35 RAD/SEC HARMONIC SUPPORT MOTION'
FREQUENCY  35.0
SUPPORT ACCELERATION TRANSLATION X -
     AMPLITUDE 50.0
END OF DYNAMIC LOADING
HARMONIC LOADING 18 '40 RAD/SEC HARMONIC SUPPORT MOTION'
FREQUENCY  40.0
SUPPORT ACCELERATION TRANSLATION X -
     AMPLITUDE 50.0
END OF DYNAMIC LOADING
HARMONIC LOADING 19 '45 RAD/SEC HARMONIC SUPPORT MOTION'
FREQUENCY  45.0
SUPPORT ACCELERATION TRANSLATION X -
     AMPLITUDE 50.0
END OF DYNAMIC LOADING
HARMONIC LOADING 20 '55 RAD/SEC HARMONIC SUPPORT MOTION'
FREQUENCY  55.0
SUPPORT ACCELERATION TRANSLATION X -
     AMPLITUDE 50.0
END OF DYNAMIC LOADING
$
$   ****************************************************************
$   ***          PERFORM HARMONIC ANALYSIS OF LOAD 3 THRU 20       ***
$   ****************************************************************
$
HARMONIC ANALYSIS LOADS 3 TO 20                      $ *** SECTION 11.4.2
$
$   ****************************************************************
$   ***                       LIST RESULTS                         ***
$   ****************************************************************
$
LIST DYNAMIC SYSTEM HARMONIC RESULTS MODULUS -  $ *** SECTION 11.4.3
FORCE MEMBER 2 3
LIST DYNAMIC SYSTEM HARMONIC RESULTS MODULUS -
DISPLACEMENTS JOINT 1 2
FINISH
```



TYPICAL STEADY STATE RESPONSE TO HARMONIC FORCING FUNCTION

```
                                                                                                    PAGE -      10

   ****************************
   *RESULTS OF LATEST ANALYSIS*
   ****************************

  PROBLEM - PROB 2     TITLE - PRINCIPLES OF DAMPING

  ACTIVE UNITS INCH KIPS RAD  FAHR SEC  LBM

**** STRUDL WARNING - LOAD 1        IS INCONSISTENT WITH OUTPUT REQUEST - OUTPUT IS NOT PERFORMED

**** STRUDL WARNING - LOAD 2        IS INCONSISTENT WITH OUTPUT REQUEST - OUTPUT IS NOT PERFORMED

/--LOAD--/--MEMB--/--FREQ--/--TYPE--/--NODE--/-----------------------------------------------------------------------------------------
                   (CY/TIM)                          AXIAL          SHEAR Y        SHEAR Z        TORSION       MOMENT Y       MOMENT Z


 3        2          0.80   MOD   FOR 3        0.6097602E+00 0.1031423E+01                                                0.7758842E+02
                                      1        0.6097602E+00 0.1031423E+01                                                0.7712492E+02
                            PHA   FOR 3        0.3127399E+01-0.1376359E-01                                               -0.1376475E-01
                                      1       -0.1419393E-01 0.3127830E+01                                               -0.1376243E-01
                                                             MAXIMUM MODULUS VALUES
                            MOD   FOR 3        0.6097602E+00 0.1031423E+01                                                0.7758842E+02
                                      1        0.6097602E+00 0.1031423E+01                                                0.7712492E+02
                                                      FREQUENCY VALUES (CYC/TIM) AT MAXIMA
                            MOD   FOR 3        0.7957742E+00 0.7957742E+00                                                0.7957742E+00
                                      1        0.7957742E+00 0.7957742E+00                                                0.7957742E+00

          3          0.80   MOD   FOR 4        0.6097602E+00 0.1006385E+01                                                0.7570961E+02
                                      2        0.6097602E+00 0.1006385E+01                                                0.7524805E+02
                            PHA   FOR 4       -0.1419393E-01-0.1406816E-01                                               -0.1406857E-01
                                      2        0.3127399E+01 0.3127525E+01                                               -0.1406775E-01
                                                             MAXIMUM MODULUS VALUES
                            MOD   FOR 4        0.6097602E+00 0.1006385E+01                                                0.7570961E+02
                                      2        0.6097602E+00 0.1006385E+01                                                0.7524805E+02
                                                      FREQUENCY VALUES (CYC/TIM) AT MAXIMA
                            MOD   FOR 4        0.7957742E+00 0.7957742E+00                                                0.7957742E+00
                                      2        0.7957742E+00 0.7957742E+00                                                0.7957742E+00

/--LOAD--/--MEMB--/--FREQ--/--TYPE--/--NODE--/-----------------------------------------------------------------------------------------
                   (CY/TIM)                          AXIAL          SHEAR Y        SHEAR Z        TORSION       MOMENT Y       MOMENT Z


 4        2          2.39   MOD   FOR 3        0.7208192E+00 0.1212784E+01                                                0.9123254E+02
                                      1        0.7208192E+00 0.1212784E+01                                                0.9068492E+02
                            PHA   FOR 3        0.3091539E+01-0.4873095E-01                                               -0.4873455E-01
                                      1       -0.5005414E-01 0.3092862E+01                                               -0.4872736E-01
                                                             MAXIMUM MODULUS VALUES
                            MOD   FOR 3        0.7208192E+00 0.1212784E+01                                                0.9123254E+02
                                      1        0.7208192E+00 0.1212784E+01                                                0.9068492E+02
                                                      FREQUENCY VALUES (CYC/TIM) AT MAXIMA
```

```
/--LOAD--/--MEMB--/--FREQ--/--TYPE--/--NODE--/------------------------------------------------------------------------------------------
                    (CY/TIM)                        AXIAL          SHEAR Y        SHEAR Z        TORSION       MOMENT Y       MOMENT Z

                          MOD   FOR 3         0.2387322E+01 0.2387322E+01                                                 0.2387322E+01
                                    1         0.2387322E+01 0.2387322E+01                                                 0.2387322E+01

              3      2.39 MOD   FOR 4         0.7208192E+00 0.1187722E+01                                                 0.8935193E+02
                                    2         0.7208192E+00 0.1187722E+01                                                 0.8880630E+02
                          PHA   FOR 4        -0.5005414E-01-0.4966327E-01                                                -0.4966458E-01
                                    2         0.3091539E+01 0.3091930E+01                                                -0.4966195E-01
                                                            MAXIMUM MODULUS VALUES
                          MOD   FOR 4         0.7208192E+00 0.1187722E+01                                                 0.8935193E+02
                                    2         0.7208192E+00 0.1187722E+01                                                 0.8880630E+02
                                                     FREQUENCY VALUES (CYC/TIM) AT MAXIMA
                          MOD   FOR 4         0.2387322E+01 0.2387322E+01                                                 0.2387322E+01
                                    2         0.2387322E+01 0.2387322E+01                                                 0.2387322E+01

/--LOAD--/--MEMB--/--FREQ--/--TYPE--/--NODE--/------------------------------------------------------------------------------------------
                    (CY/TIM)                        AXIAL          SHEAR Y        SHEAR Z        TORSION       MOMENT Y       MOMENT Z


      5       2      3.18 MOD   FOR 3         0.8560480E+00 0.1433562E+01                                                 0.1078421E+03
                                    1         0.8560480E+00 0.1433562E+01                                                 0.1071921E+03
                          PHA   FOR 3         0.3062714E+01-0.7707804E-01                                                -0.7708293E-01
                                    1        -0.7888007E-01 0.3064515E+01                                                -0.7707310E-01
                                                            MAXIMUM MODULUS VALUES
                          MOD   FOR 3         0.8560480E+00 0.1433562E+01                                                 0.1078421E+03
                                    1         0.8560480E+00 0.1433562E+01                                                 0.1071921E+03
                                                     FREQUENCY VALUES (CYC/TIM) AT MAXIMA
                          MOD   FOR 3         0.3183096E+01 0.3183096E+01                                                 0.3183096E+01
                                    1         0.3183096E+01 0.3183096E+01                                                 0.3183096E+01

              3      3.18 MOD   FOR 4         0.8560480E+00 0.1408500E+01                                                 0.1059615E+03
                                    2         0.8560480E+00 0.1408500E+01                                                 0.1053134E+03
                          PHA   FOR 4        -0.7888007E-01-0.7834297E-01                                                -0.7834476E-01
                                    2         0.3062714E+01 0.3063251E+01                                                -0.7834125E-01
                                                            MAXIMUM MODULUS VALUES
                          MOD   FOR 4         0.8560480E+00 0.1408500E+01                                                 0.1059615E+03
                                    2         0.8560480E+00 0.1408500E+01                                                 0.1053134E+03
                                                     FREQUENCY VALUES (CYC/TIM) AT MAXIMA
                          MOD   FOR 4         0.3183096E+01 0.3183096E+01                                                 0.3183096E+01
                                    2         0.3183096E+01 0.3183096E+01                                                 0.3183096E+01

/--LOAD--/--MEMB--/--FREQ--/--TYPE--/--NODE--/------------------------------------------------------------------------------------------
                    (CY/TIM)                        AXIAL          SHEAR Y        SHEAR Z        TORSION       MOMENT Y       MOMENT Z


              2      3.98 MOD   FOR 3         0.1124050E+01 0.1870994E+01                                                 0.1407511E+03
                                    1         0.1124050E+01 0.1870994E+01                                                 0.1398980E+03
```

```
/--LOAD--/--MEMB--/--FREQ--/--TYPE--/--NODE--/-------------------------------------------------------------------------------------
                    (CY/TIM)                          AXIAL         SHEAR Y        SHEAR Z        TORSION       MOMENT Y       MOMENT Z

                                PHA   FOR 3         0.3012826E+01-0.1264534E+00                                              -0.1264597E+00
                                            1      -0.1287673E+00 0.3015141E+01                                              -0.1264471E+00
                                                                  MAXIMUM MODULUS VALUES
                                MOD   FOR 3         0.1124050E+01 0.1870994E+01                                               0.1407511E+03
                                            1       0.1124050E+01 0.1870994E+01                                               0.1398980E+03
                                                           FREQUENCY VALUES (CYC/TIM) AT MAXIMA
                                MOD   FOR 3         0.3978870E+01 0.3978870E+01                                               0.3978870E+01
                                            1       0.3978870E+01 0.3978870E+01                                               0.3978870E+01

                3         3.98  MOD   FOR 4         0.1124050E+01 0.1845994E+01                                               0.1388751E+03
                                            2       0.1124050E+01 0.1845994E+01                                               0.1380239E+03
                                PHA   FOR 4        -0.1287673E+00-0.1280701E+00                                              -0.1280723E+00
                                            2       0.3012826E+01 0.3013523E+01                                              -0.1280678E+00
                                                                  MAXIMUM MODULUS VALUES
                                MOD   FOR 4         0.1124050E+01 0.1845994E+01                                               0.1388751E+03
                                            2       0.1124050E+01 0.1845994E+01                                               0.1380239E+03
                                                           FREQUENCY VALUES (CYC/TIM) AT MAXIMA
                                MOD   FOR 4         0.3978870E+01 0.3978870E+01                                               0.3978870E+01
                                            2       0.3978870E+01 0.3978870E+01                                               0.3978870E+01

/--LOAD--/--MEMB--/--FREQ--/--TYPE--/--NODE--/-------------------------------------------------------------------------------------
                    (CY/TIM)                          AXIAL         SHEAR Y        SHEAR Z        TORSION       MOMENT Y       MOMENT Z


       7        2         4.77  MOD   FOR 3         0.1795016E+01 0.2965660E+01                                               0.2231051E+03
                                            1       0.1795016E+01 0.2965660E+01                                               0.2217438E+03
                                PHA   FOR 3         0.2895254E+01-0.2434724E+00                                              -0.2434803E+00
                                            1      -0.2463396E+00 0.2898121E+01                                              -0.2434644E+00
                                                                  MAXIMUM MODULUS VALUES
                                MOD   FOR 3         0.1795016E+01 0.2965660E+01                                               0.2231051E+03
                                            1       0.1795016E+01 0.2965660E+01                                               0.2217438E+03
                                                           FREQUENCY VALUES (CYC/TIM) AT MAXIMA
                                MOD   FOR 3         0.4774645E+01 0.4774645E+01                                               0.4774645E+01
                                            1       0.4774645E+01 0.4774645E+01                                               0.4774645E+01

                3         4.77  MOD   FOR 4         0.1795016E+01 0.2941112E+01                                               0.2212630E+03
                                            2       0.1795016E+01 0.2941112E+01                                               0.2199037E+03
                                PHA   FOR 4        -0.2463396E+00-0.2454646E+00                                              -0.2454677E+00
                                            2       0.2895254E+01 0.2896129E+01                                              -0.2454618E+00
                                                                  MAXIMUM MODULUS VALUES
                                MOD   FOR 4         0.1795016E+01 0.2941112E+01                                               0.2212630E+03
                                            2       0.1795016E+01 0.2941112E+01                                               0.2199037E+03
                                                           FREQUENCY VALUES (CYC/TIM) AT MAXIMA
                                MOD   FOR 4         0.4774645E+01 0.4774645E+01                                               0.4774645E+01
```

```
   ****************************
   *RESULTS OF LATEST ANALYSIS*
   ****************************

  PROBLEM - PROB 2     TITLE - PRINCIPLES OF DAMPING

  ACTIVE UNITS INCH KIPS RAD  FAHR SEC  LBM

**** STRUDL WARNING - LOAD 1        IS INCONSISTENT WITH OUTPUT REQUEST - OUTPUT IS NOT PERFORMED

**** STRUDL WARNING - LOAD 2        IS INCONSISTENT WITH OUTPUT REQUEST - OUTPUT IS NOT PERFORMED

/--LOAD--/---JOINT----/--FREQ--/----TYPE-----/---------------------------------------------------------------------------------/
                               (CY/TIM)                     X TRANS.      Y TRANS.      Z TRANS.       X ROT.        Y ROT.        Z ROT.


 3        1        GLO      0.80 REL DISP  MOD      0.0148501     0.0002074                                                 0.0000018
                                           PHA     -0.0137670    -0.0141939                                                 3.1274424
                                                          MAXIMUM MODULUS VALUES
                                REL DISP  MOD      0.0148501     0.0002074                                                 0.0000018
                                                   FREQUENCY VALUES (CYC/TIM) AT MAXIMA
                                REL DISP  MOD      0.7957745     0.7957745                                                 0.7957745
          2        GLO      0.80 REL DISP  MOD      0.0144922     0.0002074                                                 0.0000018
                                           PHA     -0.0140694     3.1273994                                                 3.1273890
                                                          MAXIMUM MODULUS VALUES
                                REL DISP  MOD      0.0144922     0.0002074                                                 0.0000018
                                                   FREQUENCY VALUES (CYC/TIM) AT MAXIMA
                                REL DISP  MOD      0.7957745     0.7957745                                                 0.7957745
/--LOAD--/---JOINT----/--FREQ--/----TYPE-----/---------------------------------------------------------------------------------/
                               (CY/TIM)                     X TRANS.      Y TRANS.      Z TRANS.       X ROT.        Y ROT.        Z ROT.


 4        1        GLO      2.39 REL DISP  MOD      0.0174620     0.0002452                                                 0.0000021
                                           PHA     -0.0487416    -0.0500542                                                 3.0916700
                                                          MAXIMUM MODULUS VALUES
                                REL DISP  MOD      0.0174620     0.0002452                                                 0.0000021
                                                   FREQUENCY VALUES (CYC/TIM) AT MAXIMA
                                REL DISP  MOD      2.3873234     2.3873234                                                 2.3873234
          2        GLO      2.39 REL DISP  MOD      0.0171038     0.0002452                                                 0.0000021
                                           PHA     -0.0496671     3.0915394                                                 3.0915070
                                                          MAXIMUM MODULUS VALUES
                                REL DISP  MOD      0.0171038     0.0002452                                                 0.0000021
                                                   FREQUENCY VALUES (CYC/TIM) AT MAXIMA
```

```
/--LOAD--/---JOINT----/--FREQ--/----TYPE-----/-----------------------------------------------------------------------------------/
                          (CY/TIM)                   X TRANS.      Y TRANS.      Z TRANS.       X ROT.        Y ROT.        Z ROT.

                                   REL DISP  MOD     2.3873234     2.3873234                                                 2.3873234

  5          1        GLO     3.18 REL DISP  MOD     0.0206416     0.0002912                                                 0.0000025
                                             PHA    -0.0770926    -0.0788801                                                 3.0628901
                                                           MAXIMUM MODULUS VALUES
                                   REL DISP  MOD     0.0206416     0.0002912                                                 0.0000025
                                                    FREQUENCY VALUES (CYC/TIM) AT MAXIMA
                                   REL DISP  MOD     3.1830978     3.1830978                                                 3.1830978
             2        GLO     3.18 REL DISP  MOD     0.0202834     0.0002912                                                 0.0000025
                                             PHA    -0.0783482     3.0627136                                                 3.0626707
                                                           MAXIMUM MODULUS VALUES
                                   REL DISP  MOD     0.0202834     0.0002912                                                 0.0000025
                                                    FREQUENCY VALUES (CYC/TIM) AT MAXIMA
                                   REL DISP  MOD     3.1830978     3.1830978                                                 3.1830978

/--LOAD--/---JOINT----/--FREQ--/----TYPE-----/-----------------------------------------------------------------------------------/
                          (CY/TIM)                   X TRANS.      Y TRANS.      Z TRANS.       X ROT.        Y ROT.        Z ROT.


  6          1        GLO     3.98 REL DISP  MOD     0.0269414     0.0003823                                                 0.0000032
                                             PHA    -0.1264723    -0.1287672                                                 3.0130510
                                                           MAXIMUM MODULUS VALUES
                                   REL DISP  MOD     0.0269414     0.0003823                                                 0.0000032
                                                    FREQUENCY VALUES (CYC/TIM) AT MAXIMA
                                   REL DISP  MOD     3.9788723     3.9788723                                                 3.9788723
             2        GLO     3.98 REL DISP  MOD     0.0265841     0.0003823                                                 0.0000032
                                             PHA    -0.1280769     3.0128260                                                 3.0127726
                                                           MAXIMUM MODULUS VALUES
                                   REL DISP  MOD     0.0265841     0.0003823                                                 0.0000032
                                                    FREQUENCY VALUES (CYC/TIM) AT MAXIMA
                                   REL DISP  MOD     3.9788723     3.9788723                                                 3.9788723

/--LOAD--/---JOINT----/--FREQ--/----TYPE-----/-----------------------------------------------------------------------------------/
                          (CY/TIM)                   X TRANS.      Y TRANS.      Z TRANS.       X ROT.        Y ROT.        Z ROT.


  7          1        GLO     4.77 REL DISP  MOD     0.0427067     0.0006106                                                 0.0000052
                                             PHA    -0.2434958    -0.2463397                                                 2.8955307
                                                           MAXIMUM MODULUS VALUES
                                   REL DISP  MOD     0.0427067     0.0006106                                                 0.0000052
                                                    FREQUENCY VALUES (CYC/TIM) AT MAXIMA
                                   REL DISP  MOD     4.7746468     4.7746468                                                 4.7746468
             2        GLO     4.77 REL DISP  MOD     0.0423559     0.0006106                                                 0.0000052
                                             PHA    -0.2454733     2.8952541                                                 2.8951893
                                                           MAXIMUM MODULUS VALUES
                                   REL DISP  MOD     0.0423559     0.0006106                                                 0.0000052
```

```
/--LOAD--/---JOINT----/--FREQ--/----TYPE-----/-------------------------------------------------------------------------------------/
                       (CY/TIM)                     X TRANS.      Y TRANS.      Z TRANS.         X ROT.        Y ROT.        Z ROT.

                                                      FREQUENCY VALUES (CYC/TIM) AT MAXIMA
                                REL DISP  MOD      4.7746468     4.7746468                                                 4.7746468

/--LOAD--/---JOINT----/--FREQ--/----TYPE-----/-------------------------------------------------------------------------------------/
                       (CY/TIM)                     X TRANS.      Y TRANS.      Z TRANS.         X ROT.        Y ROT.        Z ROT.

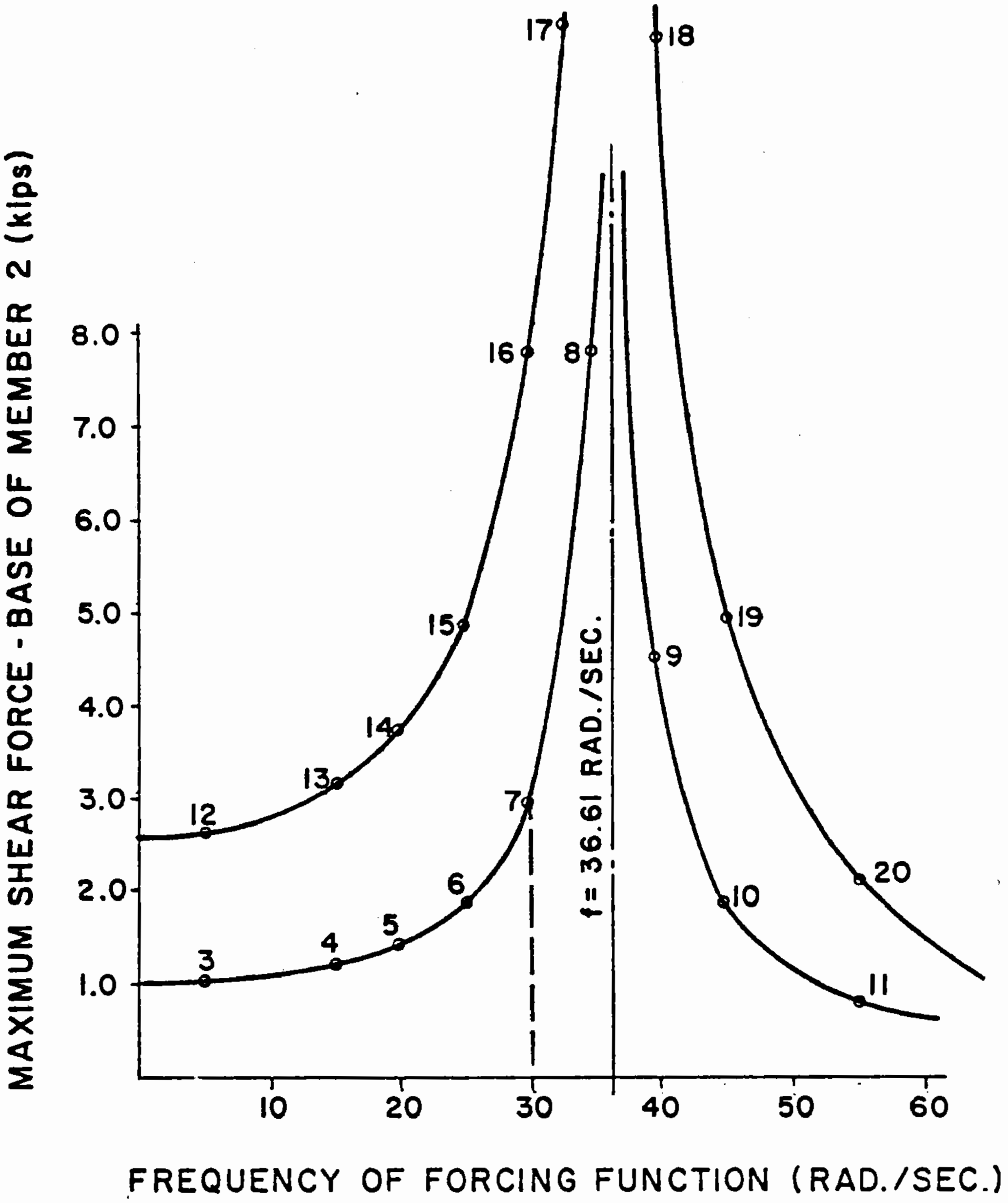

 8        1        GLO     5.57 REL DISP  MOD      0.1121024     0.0016168                                                 0.0000137
                                          PHA     -0.8371218    -0.8405648                                                 2.3013592
                                                             MAXIMUM MODULUS VALUES
                                REL DISP  MOD      0.1121024     0.0016168                                                 0.0000137
                                                      FREQUENCY VALUES (CYC/TIM) AT MAXIMA
                                REL DISP  MOD      5.5704212     5.570421£                                                 5.5704212
          2        GLO     5.57 REL DISP  MOD      0.1118575     0.0016168                                                 0.0000137
                                          PHA     -0.8395014     2.3010283                                                 2.3009539
                                                             MAXIMUM MODULUS VALUES
                                REL DISP  MOD      0.1118575     0.0016168                                                 0.0000137
                                                      FREQUENCY VALUES (CYC/TIM) AT MAXIMA
                                REL DISP  MOD      5.5704212     5.5704212                                                 5.5704212

/--LOAD--/---JOINT----/--FREQ--/----TYPE-----/-------------------------------------------------------------------------------------/
                       (CY/TIM)                     X TRANS.      Y TRANS.      Z TRANS.         X ROT.        Y ROT.        Z ROT.


 9        1        GLO     6.37 REL DISP  MOD      0.0645596     0.0009407                                                 0.0000080
                                          PHA      3.6563416     3.6522388                                                 0.5110344
                                                             MAXIMUM MODULUS VALUES
                                REL DISP  MOD      0.0645596     0.0009407                                                 0.0000080
                                                      FREQUENCY VALUES (CYC/TIM) AT MAXIMA
                                REL DISP  MOD      6.3661957     6.3661957                                                 6.3661957
          2        GLO     6.37 REL DISP  MOD      0.0648738     0.0009407                                                 0.0000080
                                          PHA      3.6535254     0.5106456                                                 0.5105606
                                                             MAXIMUM MODULUS VALUES
                                REL DISP  MOD      0.0648738     0.0009407                                                 0.0000080
                                                      FREQUENCY VALUES (CYC/TIM) AT MAXIMA
                                REL DISP  MOD      6.3661957     6.3661957                                                 6.3661957

/--LOAD--/---JOINT----/--FREQ--/----TYPE-----/-------------------------------------------------------------------------------------/
                       (CY/TIM)                     X TRANS.      Y TRANS.      Z TRANS.         X ROT.        Y ROT.        Z ROT.

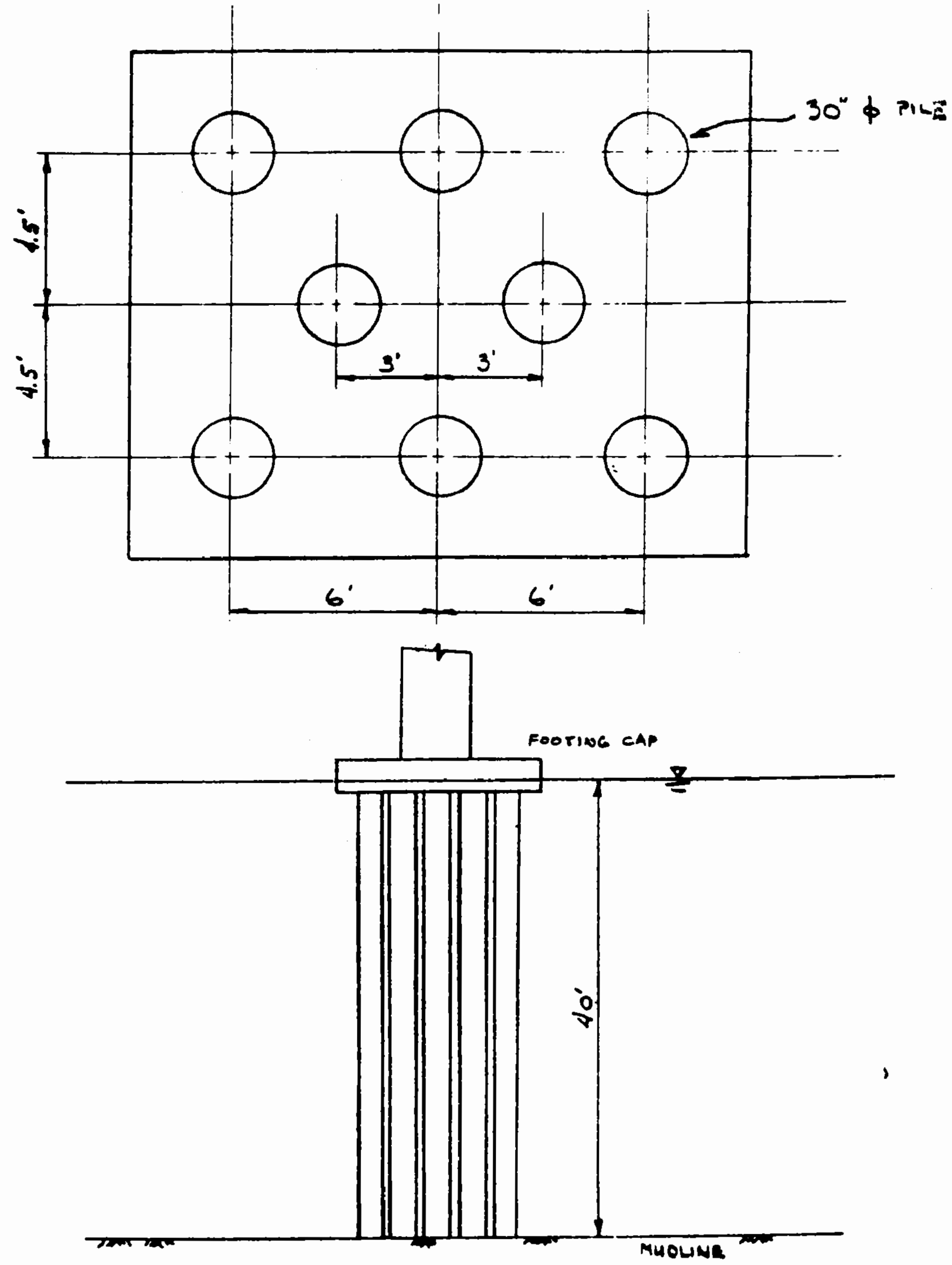

 10       1        GLO     7.16 REL DISP  MOD      0.0272237     0.0004013                                                 0.0000034
                                          PHA      3.3793488     3.3745165                                                 0.2333737
                                                             MAXIMUM MODULUS VALUES
                                REL DISP  MOD      0.0272237     0.0004013                                                 0.0000034
                                                      FREQUENCY VALUES (CYC/TIM) AT MAXIMA
```

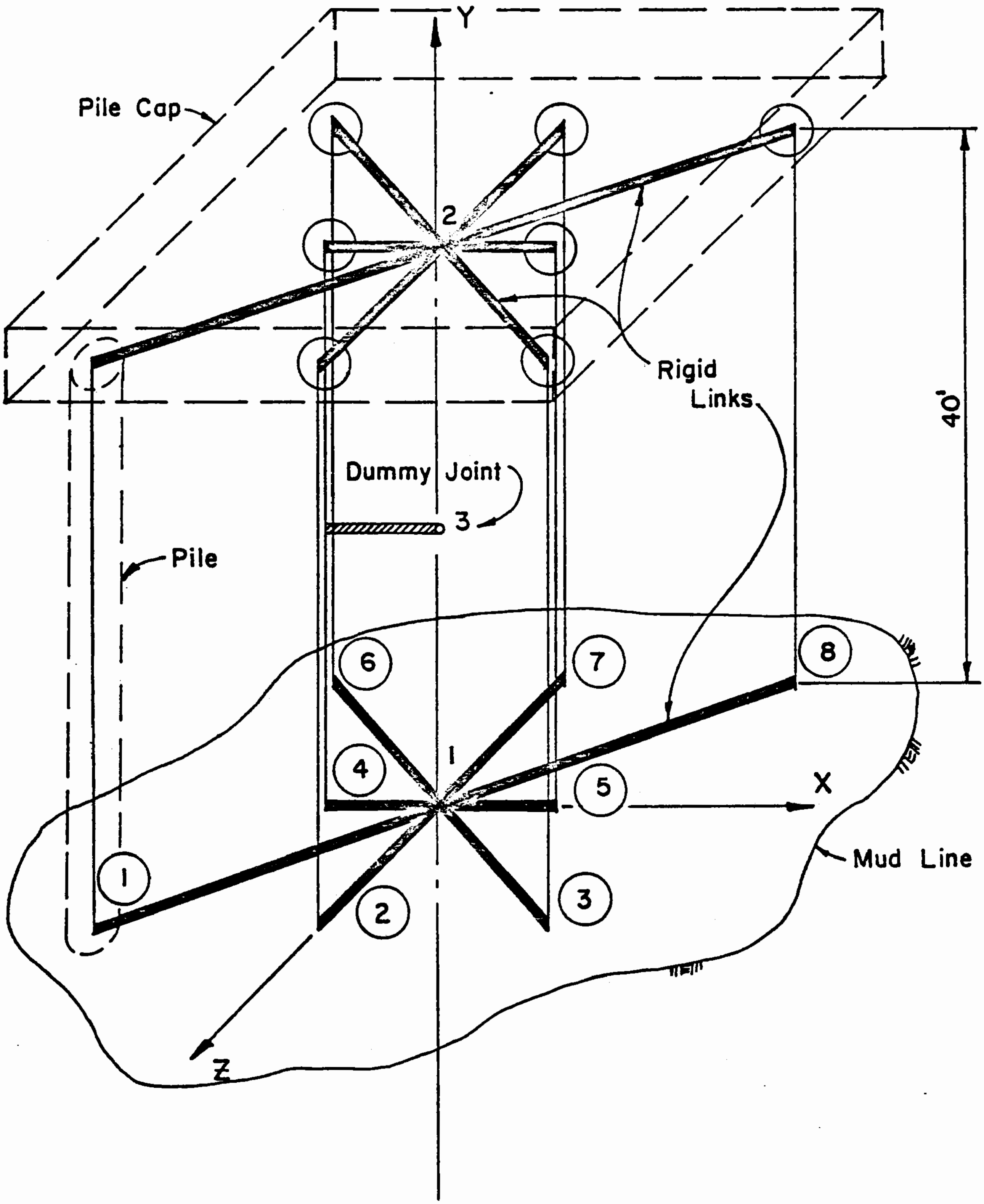

MAXIMUM SHEAR FORCE - BASE OF MEMBER 2 (kips)
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
10
20
30
40
50
60
FREQUENCY OF FORCING FUNCTION (RAD./SEC.)
f = 36.61 RAD./SEC.
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

Appendix A.5

SAMPLE PROBLEM 7

Using the member eccentricities command of STRUDL, develop the stiffness matrix for the pile group shown in the sketch below. Assume the footing to act as a rigid diaphram, and the piles to be fixed at the mudline. Tabulate the stiffness matrices to be used for both the zero length member and the super element.





FOOTING IDEALIZATION

80/80 INPUT CARD IMAGES

```
STRUDL 'PROB. 7' 'PILE FOOTING ELEMENT'
$
$                    *****************************************
$                    ***                                   ***
$                    *** SEISMIC DESIGN OF HIGHWAY BRIDGES ***
$                    ***                                   ***
$                    *****************************************
$
$    SPONSORED BY:
$
$         FEDERAL HIGHWAY ADMINISTRATION
$
$    PRESENTED BY:
$
$         ENGINEERING COMPUTER CORPERATION
$         601 UNIVERSITY AVE.  SUITE 213
$         SACRAMENTO, CALIFORNIA
$
$    PHONE: (916) 922-9316
$
$    INSTRUCTORS:
$
$         ROY A. IMBSEN - PRINCIPAL INVESTIGATOR
$         RICHARD V. NUTT
$         JAMES GATES
$
$
$
$    THIS PROBLEM ILLUSTRATES HOW THE STIFFNESS AND MASS MATRICES
$    ARE CREATED FOR A GROUP OF PILES.  THE STIFFNESS MATRICES TO
$    BE USED FOR BOTH ZERO LENGTH MEMBERS AND SUPERELEMENTS ARE
$    TABULATED.
$
$   *****************************************************************
$   ***                  STRUCTURE DESCRIPTION                    ***
$   *****************************************************************
$
UNITS KIPS FEET LBM
TYPE SPACE FRAME
$
$   *****************************************************************
$   ***         DEFINE A JOINT AT EACH END OF THE PILE GROUP      ***
$   *****************************************************************
$
JOINT COORDINATES
1 Y 0.0  SUPPORT
2 Y 40.0
$
$   *****************************************************************
$   ***                CREATE A MEMBER FOR EACH PILE              ***
$   *****************************************************************
$
MEMBER INCIDENCES
1 1 2
2 1 2
3 1 2
4 1 2
5 1 2
6 1 2
7 1 2
```

```
8 1 2
$
$   ******************************************************************
$   ***            USE MEMBER ECCENTRICITIES TO PROVIDE            ***
$   ***       RIGID LINKS TO THE INDIVIDUAL PILE LOCATIONS         ***
$   ******************************************************************
$
MEMBER ECCENTRICITIES                             $ *** SECTION 6.1.3.4
1 GLOBAL START X -6.0 Z  4.5 END X -6.0 Z  4.5
2 GLOBAL START X  0.0 Z  4.5 END X  0.0 Z  4.5
3 GLOBAL START X  6.0 Z  4.5 END X  6.0 Z  4.5
4 GLOBAL START X -3.0 Z  0.0 END X -3.0 Z  0.0
5 GLOBAL START X  3.0 Z  0.0 END X  3.0 Z  0.0
6 GLOBAL START X -6.0 Z -4.5 END X -6.0 Z -4.5
7 GLOBAL START X  0.0 Z -4.5 END X  0.0 Z -4.5
8 GLOBAL START X  6.0 Z -4.5 END X  6.0 Z -4.5
MEMBER 1 TO 8 PROPERTIES PRIS AX 4.91 IX 3.83 IY 1.92 IZ 1.92
CONSTANTS
    E 432000. ALL
    DENSITY 150. ALL
PRINT DATA
INERTIA OF JOINTS LUMPED
$
$   ******************************************************************
$   ***   CREATE THE MATRICES FOR THE ENTIRE PILE GROUP AT JT 2   ***
$   ******************************************************************
$
ASSEMBLE FOR DYNAMICS
$
$   ******************************************************************
$   ***            PRINT THE STIFFNES S AND INERTIA MATRICES       ***
$   ******************************************************************
$
LIST INDEPENDENT STIFFNESS MATRIX                 $ *** SECTION 16.3.2
LIST INDEPENDENT INERTIA MATRIX                   $ *** SECTION 16.3.2
$
$   ******************************************************************
$   ***        INSERT A DUMMY JOINT SO A CONDENSATION CAN BE       ***
$   ***      PERFORMED PRIOR TO REQUESTING THAT A SUPERELEMENT     ***
$   ***                      BE PUNCHED OUT                        ***
$   ******************************************************************
$
JOINT 3 COORDINATES  0.  20.
CHANGES
JOINT 1 COORDINATE FREE
MEMBER 4 GOES FROM 1 TO 3
ADDITIONS
MEMBER 9 GOES FROM 3 TO 2
MEMBER 9 PROPERTIES PRISMATIC AX 4.91 IX 3.83 IY 1.92 IZ 1.92
MEMBER ECCENTRICITIES
9 GLOBAL START X -3.0 Z 0.0 END X -3.0 Z 0.0
$
$   ******************************************************************
$   ***          SPECIFY THAT JOINTS 1 & 2 WILL BE TAKEN AS        ***
$   ***               INDEPENDENT IN THE CONDENSATION.             ***
$   ******************************************************************
$
INDEPENDENT DEGREES OF FREEDOM JOINT
1 2 DISPLACEMENT ALL ROTATION ALL
ASSEMBLE FOR DYNAMICS
```

```
$
$   ****************************************************************
$   ***     LIST UNCONDENCED STIFFNESS AND INERTIA MATRICES.     ***
$   ****************************************************************
$
LIST DYNAMIC STIFFNESS INERTIA MATRIX
CONDENSE DYNAMIC MATRICES
$
$   ****************************************************************
$   ***   LIST CONDENCED MATRICES (THE JCL ASSIGNS PUNCH TO THE  ***
$   ***  PRINTER FOR THIS PROBLEM).  NOTE THAT THE ENTIRE MATRIX ***
$   ***  WOULD BE INPUT FOR THE SUPERELEMENT (MCAUTO 6.1.3.7.1). ***
$   ***   ONLY THE 6 X 6 SUBMATRIX 2 2 WOULD BE USED FOR A ZERO  ***
$   ***   LENGTH MEMBER AT THE FOOTING CAP (MCAUTO 6.1.3.6.4).   ***
$   ****************************************************************
$
PUNCH CONDENCED SUPERELEMENT 'PILEGP'       $ *** SECTION 16.4.3
LIST DYNAMIC TRANSFORMATION MATRIX          $ *** SECTION 10.3
FINISH
```

```
                                                                                                              PAGE -   10
INERTIA OF JOINTS LUMPED

$

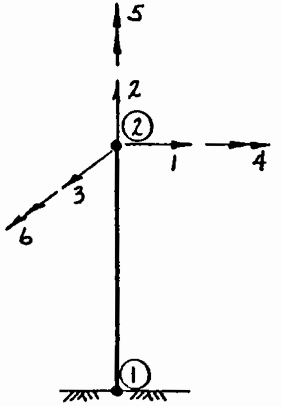
$    CREATE THE MATRICES FOR THE ENTIRE PILE GROUP AT JT 2

$

ASSEMBLE FOR DYNAMICS


**** STRUDL MESSAGE - BANDWIDTH STATISTICS ARE AS FOLLOWS :

                             THE MAXIMUM BANDWIDTH IS     0
                             THE AVERAGE BANDWIDTH IS     0.0
                             THE STANDARD DEVIATION IS    0.0
```



```
                                                     CORRESPONDENCE TABLE

    JOINT ID       XT       YT       ZT       XR       YR       ZR          JOINT ID       XT       YT       ZT       XR       YR       ZR

    1                                                                       2              1        2        3        4        5        6


$

$    PRINT THE STIFFNESS AND INERTIA MATRICES FOR THE PILE GROUP

$

LIST INDEPENDENT STIFFNESS MATRIX
```

```
                    ***** SYSTEM STIFFNESS MATRIX ( OUTPUT IN INTERNAL UNITS ) *****

                          ORDER =      6

 (MISSING ELEMENTS ALL ZERO)

 ROW
   1      1    0.10367992D 06
 ROW
   2      2    0.35351978D 08
 ROW
   3      3    0.10367992D 06
 ROW
   4      3   -0.24883180D 08      4    0.852773930 11
 ROW
   5      1   -0.26193447D-09      5    0.21174556D 10
 ROW
   6      1    0.24883180D 08      5   -0.81956387D-07      6    0.11104899D 12

LIST INDEPENDENT INERTIA MATRIX
```

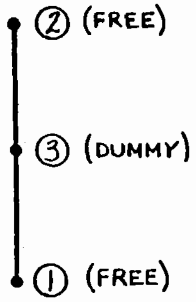
$$K_{SYSTEM} = \begin{array}{c|cccccc|} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 1.04\times10^{5} & 0 & 0 & 0 & 0 & 2.49\times10^{7} \\ 2 & 0 & 3.54\times10^{7} & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1.04\times10^{5} & -2.49\times10^{7} & 0 & 0 \\ 4 & 0 & 0 & -2.49\times10^{7} & 8.53\times10^{10} & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 2.12\times10^{9} & 0 \\ 6 & 2.49\times10^{7} & 0 & 0 & 0 & 0 & 1.11\times10^{11} \\ \hline \end{array}$$

```
                    ***** SYSTEM INERTIA MATRIX ( OUTPUT IN INTERNAL UNITS ) *****
                                      ORDER =     6

 (MISSING ELEMENTS ALL ZERO)
       1   0.30525368D 03     2   0.30525368D 03     3   0.30525368D 03

$
$    INSERT A DUMMY JOINT SO A CONDENSATION CAN BE
$    PERFORMED PRIOR TO REQUESTING THAT A SUPERELEMENT
$    BE PUNCHED OUT
$
JOINT 3 COORDINATES  0.  20.
CHANGES
JOINT 1 COORDINATE FREE
MEMBER 4 GOES FROM 1 TO 3
ADDITIONS
MEMBER 9 GOES FROM 3 TO 2
MEMBER 9 PROPERTIES PRISMATIC AX 4.91 IX 3.83 IY 1.92 IZ 1.92
MEMBER ECCENTRICITIES
9 GLOBAL START X -3.0 Z 0.0 END X -3.0 Z 0.0
$
$    SPECIFY THAT JOINTS 1 & 2 WILL BE TAKEN AS
$    INDEPENDENT IN THE CONDENSATION.
$
INDEPENDENT DEGREES OF FREEDOM JOINT
1 2 DISPLACEMENT ALL ROTATION ALL
ASSEMBLE FOR DYNAMICS
```

$$m_{SYSTEM} = \begin{bmatrix} 305 & 0 & 0 & 0 & 0 & 0 \\ 0 & 305 & 0 & 0 & 0 & 0 \\ 0 & 0 & 305 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



```
**** STRUDL MESSAGE - BANDWIDTH STATISTICS ARE AS FOLLOWS :

                        THE MAXIMUM BANDWIDTH IS     2 AND OCCURS AT JOINT 3
                        THE AVERAGE BANDWIDTH IS     1.00
                        THE STANDARD DEVIATION IS    0.82


                                            CORRESPONDENCE TABLE

  JOINT ID        XT      YT      ZT      XR      YR      ZR         JOINT ID      XT      YT      ZT      XR      YR      ZR

  1                1       2       3       4       5       6       2               7       8       9      10      11      12
  3               13      14      15      16      17      18

$

$    LIST UNCONDENCED STIFFNESS AND INERTIA MATRICES.

$

LIST DYNAMIC STIFFNESS INERTIA MATRIX
```

```
                    ***** SYSTEM INERTIA MATRIX ( OUTPUT IN INTERNAL UNITS ) *****

                           ORDER =     18

(MISSING ELEMENTS ALL ZERO)

        1   0.28617532D 03      2   0.28617532D 03      3   0.28617532D 03      7   0.28617532D 03      8   0.28617532D 03
        9   0.28617532D 03     13   0.38156710D 02     14   0.38156710D 02     15   0.38156710D 02
```

```
                              ***** SYSTEM STIFFNESS MATRIX ( OUTPUT IN INTERNAL UNITS ) *****

                                      ORDER =     18

 (MISSING ELEMENTS ALL ZERO)

 ROW
   1     1   0.19439985D 06
 ROW
   2     2   0.39770975D 08
 ROW
   3     3   0.19439985D 06
 ROW
   4     3   0.34214373D 08     4   0.86272721D 11
 ROW
   5     1  -0.26193447D-09     3   0.32659174D 07     4   0.33592293D 09     5   0.24335757D 10
 ROW
   6     1  -0.34214373D 08     2  -0.15908390D 09     5   0.96857548D-07     6   0.11777133D 12
 ROW
   7     1  -0.90719928D 05     5   0.26193447D-09     6   0.21772783D 08     7   0.19439985D 06
 ROW
   8     2  -0.30932981D 08     6  -0.15908390D 09     8   0.39770975D 08
 ROW
   9     3  -0.90719928D 05     4  -0.21772783D 08     5   0.46655963D 06     9   0.19439985D 06
 ROW
  10     3   0.21772783D 08     4  -0.73331131D 11     5  -0.11197431D 09     9  -0.34214373D 08    10   0.86272721D 11
 ROW
  11     1   0.26193447D-09     3   0.46655963D 06     4   0.11197431D 09     5  -0.19021123D 10     6  -0.96857548D-07
         7  -0.26193447D-09     9   0.32659174D 07    10  -0.33592293D 09    11   0.24335757D 10
 ROW
  12     1  -0.21772783D 08     2  -0.15908390D 09     5   0.81956387D-07     6  -0.93875702D 11     7   0.34214373D 08
         8  -0.15908390D 09    11  -0.81956387D-07    12   0.11777133D 12
 ROW
  13     1  -0.10367992D 06     6   0.12441590D 08     7  -0.10367992D 06    12  -0.12441590D 08    13   0.20735983D 06
 ROW
  14     2  -0.88379945D 07     6   0.31816780D 09     8  -0.88379945D 07    12   0.31816780D 09    14   0.17675989D 08
 ROW
  15     3  -0.10367992D 06     4  -0.12441590D 08     5  -0.37324770D 07     9  -0.10367992D 06    10   0.12441590D 08
        11  -0.37324770D 07    15   0.20735983D 06
 ROW
  16     3   0.12441590D 08     4   0.99532721D 09     5   0.44789724D 09     9  -0.12441590D 08    10   0.99532721D 09
        11  -0.44789724D 09    15  -0.27939677D-08    16   0.39813088D 10
 ROW
  17     3  -0.37324770D 07     4  -0.44789724D 09     5  -0.53146342D 09     9  -0.37324770D 07    10   0.44789724D 09
        11  -0.53146342D 09    15   0.74549541D 07    16   0.59604645D-07    17   0.10629268D 10
 ROW
  18     1  -0.12441590D 08     2   0.31816780D 09     6  -0.10458714D 11     7   0.12441590D 08     8   0.31816780D 09
        12  -0.10458714D 11    13   0.27939677D-08    14  -0.63633560D 09    18   0.26869391D 11
```

```
                                                                                              PAGE -   16
CONDENSE DYNAMIC MATRICES

                                             CONDENSATION CORRESPONDENCE TABLE
                                                  INDEPENDENT COORDINATES

  JOINT ID         XT      YT      ZT      XR      YR      ZR          JOINT ID      XT      YT      ZT      XR      YR      ZR

  1                1       2       3       4       5       6           2             7       8       9       10      11      12

                                             CONDENSATION CORRESPONDENCE TABLE
                                                   DEPENDENT COORDINATES

  JOINT ID         XT      YT      ZT      XR      YR      ZR          JOINT ID      XT      YT      ZT      XR      YR      ZR

  3                1       2       3       4       5       6


**** STRUDL MESSAGE - THE ICES SYSTEM WILL TEMPORARILY BE ROLLED OUT
                      WHILE CONDENSATION IS PERFORMED.


 ROLL-OUT OF THE ICES SYSTEM COMPLETED ; CONDENSATION INITIATED
```

```
                                                                                                 PAGE -    4

                                        ****  DYNAMIC TRANSFORMATION MATRIX   ****
COL  ROW

  1     1      0.50000000D 00   0.11250000D 00   0.0              0.0              0.0              0.31250000D-02

  2     1      0.72568183D-31   0.50000000D 00   0.0              0.0              0.0             -0.53857910D-17

  3     1      0.0              0.0              0.50000000D 00  -0.31250000D-02   0.16849696D-17   0.0

  4     1      0.0              0.0              0.60000000D 02  -0.25000000D 00   0.37441595D-15   0.0

  5     1      0.0              0.0              0.56718212D-14  -0.11250000D 00   0.50000000D 00   0.0

  6     1     -0.60000000D 02  -0.27000000D 02   0.0              0.0              0.0             -0.25000000D 00

  7     1      0.50000000D 00  -0.11250000D 00   0.0              0.0              0.0             -0.31250000D-02

  8     1      0.72568183D-31   0.50000000D 00   0.0              0.0              0.0             -0.53857910D-17

  9     1      0.0              0.0              0.500000000 00   0.31250000D-02   0.42434722D-18   0.0

 10     1      0.0              0.0             -0.60000000D 02  -0.25000000D 00  -0.84571380D-16   0.0

 11     1      0.0              0.0              0.87034726D-14   0.11250000D 00   0.50000000D 00   0.0

 12     1      0.60000000D 02  -0.27000000D 02   0.0              0.0              0.0             -0.25000000D 00

$
$    LIST CONDENCED MATRICES (THE JCL ASSIGNS PUNCH TO THE
$    PRINTER FOR THIS PROBLEM).  NOTE THAT THE ENTIRE MATRIX
$    WOULD BE INPUT FOR THE SUPERELEMENT (MCAUTO 6.1.3.7.1).
$    ONLY THE 6 X 6 SUBMATRIX 2 2 WOULD BE USED FOR A ZERO
```

```
$     LENGTH MEMBER AT THE FOOTING CAP (MCAUTO 6.1.3.6.4).

$

PUNCH CONDENCED SUPERELEMENT 'PILEGP'
JNTS =
     1      2
DGOF =
     6      6
DIR =
   63     63
MAXJ =      1 NJ =      2

FINISH
```

```
 ELEMENT INCIDENCES
 'PILEGP' '1' '2'
 TYPE 'SUPER' STI MAT GLO NOD    2 NDF 6 DISPL X Y Z ROTAT X Y Z
 SUBMATRIX    1   1
 1.03679917425693E+05   6.70078044917697E-11   0.0                   -
 0.0                   -2.61934474110603E-10  -2.48831801821663E+07
 6.70078044917697E-11   3.53519779620971E+07   0.0                   -
 0.0                    0.0                   -4.28849948747326E-08
 0.0                    0.0                    1.03679917425693E+05  -
 2.43331801621663E+07  -5.02558533688273E-10   0.0
 0.0                    0.0                    2.48831801821663E+07  -
 8.52773934613996E+10  -1.07212437186831E-07   0.0
-2.61934474110603E-10   0.0                   -5.02558533688273E-10  -
-1.07212487186831E-07   2.11745558916414E+09   9.68575477600096E-08
-2.48831801821663E+07  -4.28849948747326E-08   0.0                   -
 0.0                    9.68575477600096E-08   1.11046985395768E+11
 SUBMATRIX    2   1
-1.03679917425693E+05  -8.37597556147122E-11   0.0                   -
 0.0                    2.61934474110603E-10   2.48831801821663E+07
 6.70078044917697E-11  -3.53519779620971E+07   0.0                   -
 0.0                    0.0                   -4.28849948747326E-08
 0.0                    0.0                   -1.03679917425693E+05  -
-2.48831801821663E+07  -4.18798778073561E-10   0.0
 0.0                    0.0                    2.48831801821663E+07  -
-7.33334669739598E+10   2.14424974373663E-08   0.0
 2.61934474110603E-10   0.0                   -6.70078044917697E-10  -
-8.57699897494053E-08  -2.11745558916414E+09  -9.68575477600096E-08
-2.48831801821663E+07  -2.14424974373663E-08   0.0                   -
 0.0                    8.19563865661620E-08  -9.91050589083286E+10
 SJBMATRIX    2   2
 1.03679917425693E+05  -6.70078044917697E-11   0.0                   -
 0.0                   -2.61934474110603E-10   2.48831801821663E+07
-6.70078044917697E-11   3.53519779620971E+07   0.0                   -
 0.0                    0.0                   -2.14424974373663E-08
 0.0                    0.0                    1.03679917425693E+05  -
-2.48831801821663E+07  -3.35039022458848E-10   0.0
 0.0                    0.0                   -2.48831801821663E+07  -
 8.52773934613996E+10   1.28654984624198E-07   0.0
-2.61934474110603E-10   0.0                   -3.35039022458848E-10  -
 1.28654984624198E-07   2.11745558916414E+09  -8.19563865661620E-08
 2.48831801821663E+07  -2.14424974373663E-08   0.0                   -
 0.0                   -8.19563865661620E-08   1.11043935395768E+11
 TYPE 'SUPER' MAS MAT GLO NOD    2 NDF 6 DISPL X Y Z ROTAT X Y Z
 SUBMATRIX    1   1
 2.96197420246518E+02   2.14631491493219E+00   0.0                   -
 0.0                    0.0                   -1.26060229337017E+03
 2.14631491493219E+00   2.95714499390658E+02   0.0                   -
 0.0                    0.0                   -5.15115579583727E+02
 0.0                    0.0                    2.95714499390658E+02  -
 1.14470128796383E+03   1.08209017722541E-13   0.0
 0.0                    0.0                    1.14470128796383E+03  -
 1.37364154555560E+05   1.29350321267049E-11   0.0
 0.0                    0.0                    1.08209017722541E-13  -
 1.29850821267049E-11   1.22748440728727E-27   0.0
-1.26060229337017E+03  -5.15115579583727E+02   0.0                   -
 0.0                    0.0                    1.65180395853182E+05
 SUBMATRIX    2   1
 9.05525654383892E+00  -2.14631491493219E+00   0.0                   -
 0.0                    0.0                   -1.02380028255750E+03
 2.14631491493219E+00   9.53917739969865E+00   0.0                   -
 0.0                    0.0                   -5.15115579583727E+02
 0.0                    0.0                    9.53917739969866E+00  -
 1.14470128796383E+03   1.08209017722541E-13   0.0
 0.0                    0.0                   -1.14470128796383E+03  -
```

```
-1.37364154555660E+05   -1.29850821267049E-11    0.0
 0.0                     0.0                     1.66047939149428E-13 -
 1.99257526979313E-11    1.88358845184905E-27    0.0
 1.02380028255750E+03   -5.15115579583727E+02    0.0                  -
 0.0                     0.0                    -1.09547913258139E+05
 SUBMATRIX    2    2
 2.96197420246518E+02   -2.14631491493219E+00    0.0                  -
 0.0                     0.0                     1.26060229337017E+03
-2.14631491493219E+00    2.95714499390658E+02    0.0                  -
 0.0                     0.0                    -5.15115579583727E+02
 0.0                     0.0                     2.95714499390658E+02 -
-1.14470128796383E+03    1.66047939149428E-13    0.0
 0.0                     0.0                    -1.14470128796383E+03 -
 1.37364154555660E+05   -1.99257526979313E-11    0.0
 0.0                     0.0                     1.66047939149428E-13 -
-1.99257526979313E-11    2.89038739300970E-27    0.0
 1.26060229337017E+03   -5.15115579583727E+02    0.0                  -
 0.0                     0.0                     1.65180395853182E+05
```

$$\begin{array}{c|cccccc|cccccc}
 & \multicolumn{6}{c|}{\text{Jt. 1}} & \multicolumn{6}{c}{\text{Jt. 2}} \\ \hline
 & 1.04\times10^{5} & 0 & 0 & 0 & 0 & -2.49\times10^{7} & & & & & & \\
 & 0 & 3.54\times10^{7} & 0 & 0 & 0 & 0 & & & \multicolumn{4}{c}{\text{Transpose of}} \\
\text{Jt. 1} & 0 & 0 & 1.04\times10^{5} & 2.49\times10^{7} & 0 & 0 & & & \multicolumn{4}{c}{\text{Submatrix 2-1}} \\
 & 0 & 0 & 2.49\times10^{7} & 8.53\times10^{10} & 0 & 0 & & & & & & \\
 & 0 & 0 & 0 & 0 & 2.12\times10^{9} & 0 & & & & & & \\
 & -2.49\times10^{7} & 0 & 0 & 0 & 0 & 1.11\times10^{11} & & & & & & \\ \hline
 & -1.04\times10^{5} & 0 & 0 & 0 & 0 & 2.49\times10^{7} & 1.04\times10^{5} & 0 & 0 & 0 & 0 & 2.49\times10^{7} \\
 & 0 & -3.54\times10^{7} & 0 & 0 & 0 & 0 & 0 & 3.54\times10^{7} & 0 & 0 & 0 & 0 \\
\text{Jt. 2} & 0 & 0 & -1.04\times10^{5} & -2.49\times10^{7} & 0 & 0 & 0 & 0 & 1.04\times10^{5} & -2.49\times10^{7} & 0 & 0 \\
 & 0 & 0 & 2.49\times10^{7} & -7.33\times10^{10} & 0 & 0 & 0 & 0 & -2.49\times10^{7} & 8.53\times10^{10} & 0 & 0 \\
 & 0 & 0 & 0 & 0 & -2.12\times10^{9} & 0 & 0 & 0 & 0 & 0 & 2.12\times10^{9} & 0 \\
 & -2.49\times10^{7} & 0 & 0 & 0 & 0 & -9.91\times10^{10} & 2.49\times10^{7} & 0 & 0 & 0 & 0 & 1.11\times10^{11}
\end{array}$$

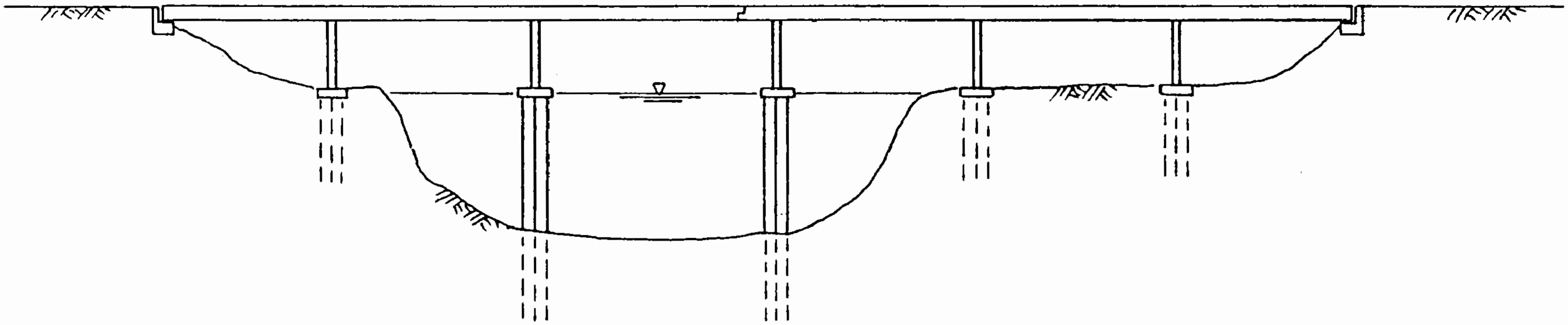
Appendix A.6

SAMPLE PROBLEM 8

Assume that Rte 80 Onramp U.C. will be placed over water and the bent footings will be modified to include a footing cap with several piles. The pile layout will be identical to that shown in problem 7 (Appendix A.5). The piles are imbedded in 25 feet of soil. Bent 3 and 4 piles pass through 40 feet of water. The stiffness matrix in kips and inches for a "zero length member" representing the stiffness properties of the pile group imbedded in soil is given below. Model bents 2 and 6 footings with uncoupled soil springs. Use a zero length member to model the piles below the soil line at bents 3 through 5. Use a superelement to model the piles passing through water. Perform a modal analysis to determine mode shapes and frequencies.

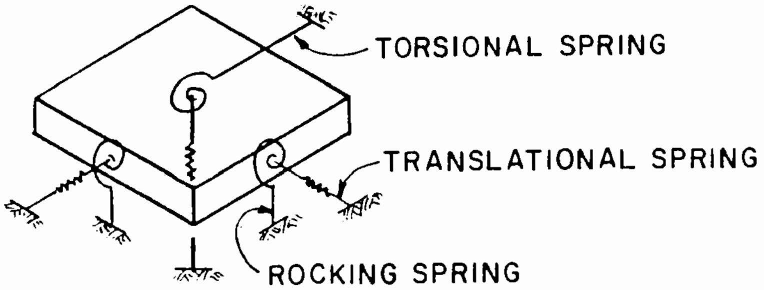
STIFFNESS MATRIX-ZERO LENGTH MEMBER

$$[K] = 10^8 \times \begin{bmatrix} 4.084 & 0 & 0 & 0 & 0 & 8.211 \\ 0 & 13.575 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4.084 & -8.211 & 0 & 0 \\ 0 & 0 & -8.211 & 222.04 & 0 & 0 \\ 0 & 0 & 0 & 0 & 129.22 & 0 \\ 8.211 & 0 & 0 & 0 & 0 & 270.40 \end{bmatrix}$$

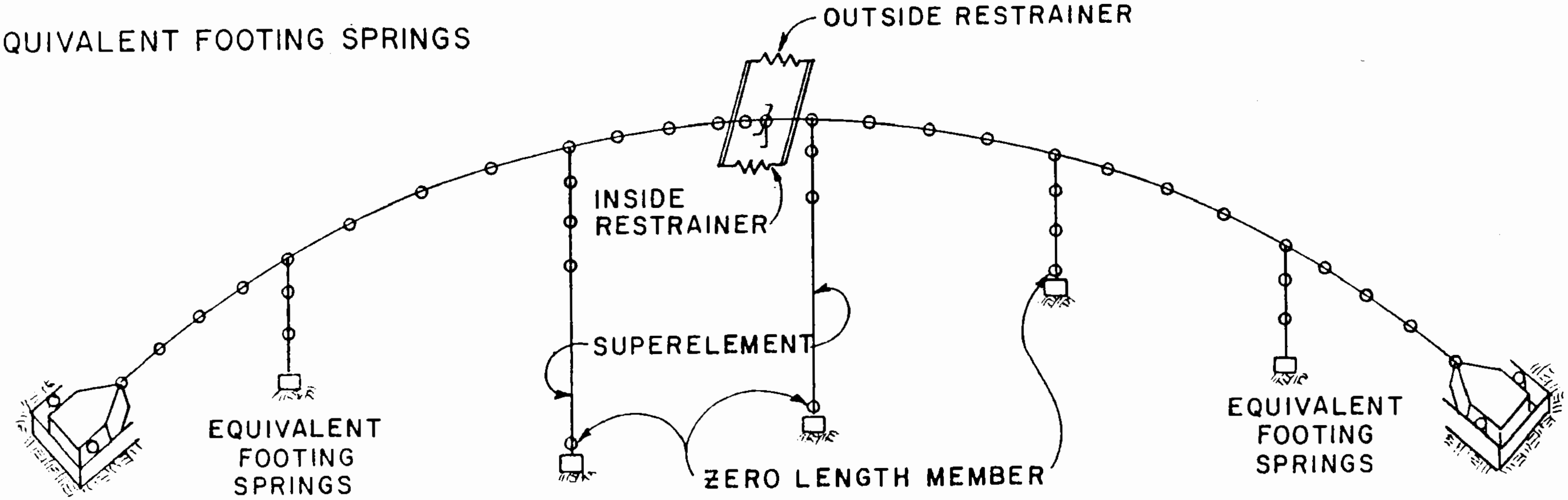


ELEVATION VIEW

MODIFIED ROUTE 80 ON-RAMP



EQUIVALENT FOOTING SPRINGS



STRUCTURE IDEALIZATION
MODIFIED ROUTE 80 ON-RAMP

80/80 INPUT CARD IMAGES

```
STRUDL 'PROB. 8' 'RTE 80 ONRAMP - MODAL ANALYSIS WITH SOIL SPRINGS '
$
$                    *****************************************
$                    ***                                   ***
$                    *** SEISMIC DESIGN OF HIGHWAY BRIDGES ***
$                    ***                                   ***
$                    *****************************************
$
$    SPONSORED BY:
$
$         FEDERAL HIGHWAY ADMINISTRATION
$
$    PRESENTED BY:
$
$         ENGINEERING COMPUTER CORPERATION
$         601 UNIVERSITY AVE.  SUITE 213
$         SACRAMENTO, CALIFORNIA
$
$    PHONE: (916) 922-9316
$
$    INSTRUCTORS:
$
$         ROY A. IMBSEN    -  PRINCIPLE INVESTIGATOR
$         RICHARD V. NUTT
$         JAMES GATES
$
$
$         PROBLEM TO ILLUSTRATE THE USE OF ZERO LENGTH MEMBERS,
$         SUPERELEMENTS, AND JOINT RELEASE SPRINGS TO MODEL THE
$         FOUNDATION CONDITIONS ON A MODIFIED VERSION OF THE
$         ROUTE 80 ONRAMP UNDERCROSSING.
$
$         ROUTE 80 IS MODIFIED BY ASSUMING THE BENTS ARE
$         SUPPORTED ON PILE GROUPS SIMILAR TO THAT USED
$         FOR SAMPLE PROBLEM 7.  ALL PILES ARE ASSUMED
$         TO BE IMBEDDED IN 25 FEET OF COHESIONLESS SOIL.
$         PILES AT BENTS 3 & 4 PASS THROUGH 40 FEET OF
$         WATER BEFORE REACHING THE SOIL.  AT BENTS 2 &
$         6, SPRINGS ARE USED TO MODEL THE PILES AND THE
$         SOIL, WHILE AT BENTS 3,4, & 5 A ZERO LENGTH
$         MEMBER IS USED.  A SUPERELEMENT IS USED TO
$         MODEL THE PILES PASSING THROUGH WATER AT BENTS
$         3 & 4.
$
$
$   ****************************************************************
$   ***                   STRUCTURE DESCRIPTION                  ***
$   ****************************************************************
$
TYPE SPACE FRAME
UNITS FEET KIPS DEGREES LBM
JOINT COORDINATES
    3011  10000.00     75.30  10000.00    SUPPORT
    1012  10000.84     75.30   9999.45
    1013  10021.35     75.30   9986.72
    1014  10042.38     75.30   9974.83
    1015  10063.87     75.30   9963.83
    1021  10085.79     75.30   9953.71
    1022  10118.29     75.30   9940.66
    1023  10151.51     75.30   9929.56
```

```
     1024  10185.32     75.30   9920.46
     1031  10219.62     75.30   9913.39
     1032  10244.01     75.30   9909.65
     1033  10268.53     75.30   9906.93
     1034  10293.14     75.30   9905.24
     2035  10317.81     75.30   9904.58
     2036  10318.81     75.30   9904.58
     1041  10333.81     75.30   9904.71
     1042  10362.45     75.30   9906.03
     1043  10391.00     75.30   9908.74
     1044  10419.37     75.30   9912.84
     1051  10447.52     75.30   9918.32
     1052  10475.36     75.30   9925.16
     1053  10502.84     75.30   9933.36
     1054  10529.88     75.30   9942.88
     1061  10556.43     75.30   9953.71
     1062  10578.36     75.30   9963.83
     1063  10599.85     75.30   9974.84
     1064  10620.87     75.30   9986.71
     1065  10641.39     75.30   9999.45
     3071  10642.23     75.30  10000.00     SUPPORT
    40211  10085.79     50.00   9953.71     SUPPORT
    40212  10085.79     58.43   9953.71
    40213  10085.79     66.87   9953.71
$
$   ****************************************************************
$   ***        A NEW NODE IS DEFINED FOR THE ZERO LENGTH         ***
$   ***                   MEMBER AT BENT 5.                      ***
$   ****************************************************************
$
     4015  10447.52     50.00   9918.32     SUPPORT
    40511  10447.52     50.00   9918.32
    40512  10447.52     58.43   9918.32
    40513  10447.52     66.87   9918.32
    40611  10556.43     50.00   9953.71     SUPPORT
    40612  10556.43     58.43   9953.71
    40613  10556.43     66.87   9953.71
$
$   ****************************************************************
$   ***        NEW NODES ARE DEFINED FOR THE SUPERELEMENT        ***
$   ***              AND ZERO LENGTH MEMBER AT BENT 3.           ***
$   ****************************************************************
$
     4003  10219.62     10.00   9913.39
     4013  10219.62     10.00   9913.39     SUPPORT
    40311  10219.62     50.00   9913.39
    40312  10219.62     58.43   9913.39
    40313  10219.62     66.87   9913.39
$
$   ****************************************************************
$   ***          NEW NODES ARE DEFINED FOR THE SUPERELEMENT      ***
$   ***              AND ZERO LENGTH MEMBER AT BENT 4.           ***
$   ****************************************************************
$
     4004  10333.81     10.00   9904.71
     4014  10333.81     10.00   9904.71     SUPPORT
    40411  10333.81     50.00   9904.71
    40412  10333.81     58.43   9904.71
    40413  10333.81     66.87   9904.71
 MEMBER INCIDENCES
```

```
    5011    3011    1012
    5012    1012    1013
    5013    1013    1014
    5014    1014    1015
    5015    1015    1021
    5021    1021    1022
    5022    1022    1023
    5023    1023    1024
    5024    1024    1031
    5031    1031    1032
    5032    1032    1033
    5033    1033    1034
    5034    1034    2035
    5035    2035    2036
    5036    2036    1041
    5041    1041    1042
    5042    1042    1043
    5043    1043    1044
    5044    1044    1051
    5051    1051    1052
    5052    1052    1053
    5053    1053    1054
    5054    1054    1061
    5061    1061    1062
    5062    1062    1063
    5063    1063    1064
    5064    1064    1065
    5065    1065    3071
     901    1034    1041
     902    1034    1041
   70211   40211   40212
   70212   40212   40213
   70213   40213    1021
$
$   *****************************************************************
$   ***               ZERO LENGTH MEMBER AT BENT 3.               ***
$   *****************************************************************
$
    7013    4003    4013
   70311   40311   40312
   70312   40312   40313
   70313   40313    1031
$
$   *****************************************************************
$   ***                ZERO LENGTH MEMBER AT BENT 4.              ***
$   *****************************************************************
$
    7014    4004    4014
   70411   40411   40412
   70412   40412   40413
   70413   40413    1041
$
$   *****************************************************************
$   ***               ZERO LENGTH MEMBER AT BENT 5.               ***
$   *****************************************************************
$
    7015   40511    4015
   70511   40511   40512
   70512   40512   40513
   70513   40513    1051
```

```
   70611   40611   40612
   70612   40612   40613
   70613   40613    1061
$
$   ****************************************************************
$   ***          DEFINE THE SUPERELEMENT INCIDENCES FOR          ***
$   ***                THE PILES AT BENTS 3 & 4.                 ***
$   ****************************************************************
$
ELEMENT INCIDENCES
   7003   40311    4003
   7004   40411    4004
 MEMBER PROPERTIES PRISMATIC
 5011 THRU 5015 AX   86.0 IX      862.0 IY   13000.0 IZ      360.0
 5021 THRU 5024 AX   86.0 IX      862.0 IY   13000.0 IZ      360.0
 5031 THRU 5036 AX   86.0 IX      862.0 IY   13000.0 IZ      360.0
 5041 THRU 5044 AX   86.0 IX      862.0 IY   13000.0 IZ      360.0
 5051 THRU 5054 AX   86.0 IX      862.0 IY   13000.0 IZ      360.0
 5061 THRU 5065 AX   86.0 IX      862.0 IY   13000.0 IZ      360.0
 MEMB PROP PRIS AX   33.0 IX      146.0 IY      143.0 IZ       73.0
 70211
 MEMB PROP PRIS AX   33.0 IX      146.0 IY      143.0 IZ       73.0
 70212
 MEMB PROP PRIS AX   33.0 IX      146.0 IY      143.0 IZ       73.0
 70213
 MEMB PROP PRIS AX   33.0 IX      146.0 IY      143.0 IZ       73.0
 70311
 MEMB PROP PRIS AX   33.0 IX      146.0 IY      143.0 IZ       73.0
 70312
 MEMB PROP PRIS AX   33.0 IX      146.0 IY      143.0 IZ       73.0
 70313
 MEMB PROP PRIS AX   33.0 IX      146.0 IY      143.0 IZ       73.0
 70411
 MEMB PROP PRIS AX   33.0 IX      146.0 IY      143.0 IZ       73.0
 70412
 MEMB PROP PRIS AX   33.0 IX      146.0 IY      143.0 IZ       73.0
 70413
 MEMB PROP PRIS AX   33.0 IX      146.0 IY      143.0 IZ       73.0
 70511
 MEMB PROP PRIS AX   33.0 IX      146.0 IY      143.0 IZ       73.0
 70512
 MEMB PROP PRIS AX   33.0 IX      146.0 IY      143.0 IZ       73.0
 70513
 MEMB PROP PRIS AX   33.0 IX      146.0 IY      143.0 IZ       73.0
 70611
 MEMB PROP PRIS AX   33.0 IX      146.0 IY      143.0 IZ       73.0
 70612
 MEMB PROP PRIS AX   33.0 IX      146.0 IY      143.0 IZ       73.0
 70613
 MEMBER PROPERTIES PRISMATIC
     901 TO 902 AX .02455 IX .000001 IY .000001 IZ .000001
$
$   ****************************************************************
$   ***      INPUT THE STIFFNESS MATRIX FOR THE ZERO LENGTH      ***
$   ***          MEMBERS.  THIS MATRIX WAS TAKEN FROM A          ***
$   ***      PROGRAM FOR CALCULATING STIFFNESS COEFFICIENTS      ***
$   ***                    FOR PILE GROUPS.                      ***
$   ****************************************************************
$
MEMBER PROPERTIES STIFFNESS
```

```
7013 MATRIX COLUMNS 1 2 3 4 5 6 CX .17261 CY 0. CZ .98499
  4.084E8        0.         0.         0.         0.  8.2108E8
       0.  1.3575E9         0.         0.         0.        0.
       0.        0.    4.084E8   -8.211E8         0.        0.
       0.        0.  -8.211E8  2.2204E10         0.        0.
       0.        0.         0.         0. 1.2922E10        0.
 8.2108E8        0.         0.         0.         0. 2.7040E10
7014 MATRIX COLUMNS 1 2 3 4 5 6 CX -.02160 CY 0. CZ .9998
  4.084E8        0.         0.         0.         0.  8.2108E8
       0.  1.3575E9         0.         0.         0.        0.
       0.        0.    4.084E8   -8.211E8         0.        0.
       0.        0.  -8.211E8  2.2204E10         0.        0.
       0.        0.         0.         0. 1.2922E10        0.
 8.2108E8        0.         0.         0.         0. 2.7040E10
7015 MATRIX COLUMNS 1 2 3 4 5 6 CX -.21497 CY 0. CZ .97662
  4.084E8        0.         0.         0.         0.  8.2108E8
       0.  1.3575E9         0.         0.         0.        0.
       0.        0.    4.084E8   -8.211E8         0.        0.
       0.        0.  -8.211E8  2.2204E10         0.        0.
       0.        0.         0.         0. 1.2922E10        0.
 8.2108E8        0.         0.         0.         0. 2.7040E10
UNITS INCHES POUNDS RADIANS
$
$   ******************************************************************
$   ***    THE SUPER ELEMENT MATRIX WHICH IS INPUT HERE IS THE     ***
$   ***    SAME MATRIX THAT WAS CALCULATED IN SAMPLE PROBLEM 7.    ***
$   ***     THIS IS FOR A PILE GROUP PASSING THROUGH WATER.        ***
$   ******************************************************************
$
ELEMENT PROPERTIES
7003 7004 TYPE 'SUPER' STIFFNESS MATRIX GLOBAL NODES 2 NDF 6 -
DISPLACEMENTS X Y Z  ROTATION X Y Z
SUBMATRIX 1 1
   1.037E5        0.         0.         0.         0. -2.4883E7
        0.  3.5352E7         0.         0.         0.        0.
        0.        0.   1.0368E5   2.4883E7         0.        0.
        0.        0.   2.4883E7 8.5277E10         0.        0.
        0.        0.         0.         0.   2.1175E9        0.
 -2.4883E7        0.         0.         0.         0. 1.1105E11
SUBMATRIX 2 1
 -1.0368E5        0.         0.         0.         0. 2.48832E7
        0. -3.5352E7         0.         0.         0.        0.
        0.        0.  -1.0368E5 -2.4883E7         0.        0.
        0.        0.  2.48832E7 -7.333E10         0.        0.
        0.        0.         0.         0. -2.1175E9        0.
 -2.4883E7        0.         0.         0.         0. -9.911E10
SUBMATRIX 2 2
 1.0368E5         0.         0.         0.         0.  2.4883E7
        0.  3.5352E7         0.         0.         0.        0.
        0.        0.   1.0368E5 -2.4883E7         0.        0.
        0.        0.  -2.4883E7 8.5277E10         0.        0.
        0.        0.         0.         0.  2.1175E9        0.
 2.4883E7         0.         0.         0.         0. 1.1104E11
UNITS FEET KIPS RADIANS
$
$   ******************************************************************
$   ***      THE EFFECT OF THE PILE GROUPS FOR BENT 2 & 6 ARE      ***
$   ***    CONSIDERED BY USING JOINT RELEASE SPRINGS.  IN THIS     ***
$   ***     CASE THE FORCES IN THE PILE GROUP ARE NOT COUPLED      ***
$   ***          AS THEY WERE FOR THE ZERO LENGTH MEMBER.          ***
```

```
$   ****************************************************************
$
JOINT RELEASES
40211 TH2 .411734 KFX 4.084E8 KFY 1.3575E9 KFZ 4.084E8 -
                KMX 2.704E10 KMY 1.292E10 KMZ 2.2204E10
40611 TH2 -.411734 KFX 4.084E8 KFY 1.3575E9 KFZ 4.084E8 -
                   KMX 2.704E10 KMY 1.292E10 KMZ 2.2204E10
UNITS DEGREES
 CONSTANTS
 DENSITY 150.0 ALL
 E  432000.  5011 THRU  5015
 E  432000.  5021 THRU  5024
 E  432000.  5031 THRU  5036
 E  432000.  5041 THRU  5044
 E  432000.  5051 THRU  5054
 E  432000.  5061 THRU  5065
 E  432000. 70211 THRU 70213
 E  432000. 70311 THRU 70313
 E  432000. 70411 THRU 70413
 E  432000. 70511 THRU 70513
 E  432000. 70611 THRU 70613
 E  432000. 7003 7004 7013 7014 7015
     E 2016000. 901 902
 BETA   23.5906 70211 THRU 70213
 BETA    9.9390 70311 THRU 70313
 BETA   -1.2375 70411 THRU 70413
 BETA  -12.4141 70511 THRU 70513
 BETA  -23.5906 70611 THRU 70613
MEMBER ECCENTRICITY
     901 LOCAL START Z  12.88 END Z  12.88
     902 LOCAL START Z -12.88 END Z -12.88
 MEMBER 5011 REL END FOR X
  JOINT 3011 RELEASE           MOM    Y  Z  -
  TH2   -33.1008
 MEMBER 5065 REL STA FOR X
  JOINT 3071 RELEASE           MOM    Y  Z  -
  TH2    33.1008
 MEMBER 5035 RELEASES END FORCE X MOMENT   Y Z
 INERTIA OF JOINTS LUMPED
 DAMPING PERCENTS  5.0   18
$
$   ****************************************************************
$   ***      THE MASS OF THE PILE GROUPS ARE LUMPED AT THE         ***
$   ***       APPROPRIATE NODES BY USING THE 'INERTIA OF           ***
$   ***      JOINTS ADD' COMMAND.  THE EFFECTS OF THE MASS         ***
$   ***     OF THE WATER IS NOT CONSIDERED  IN THIS EXAMPLE,       ***
$   ***            BUT COULD BE BY INCLUDING IT HERE.              ***
$   ****************************************************************
$
INERTIA OF JOINTS ADD
40311 40411 LINEAR ALL 114000.
4003 4004 LINEAR ALL 185000.
40211 40511 40611 LINEAR ALL 71000.
PRINT STRUCTURAL DATA
 ASSEMBLE FOR DYNAMICS REDUCE BAND
 INDEPENDENT DEGREE OF FREEDOM SELECT
 CONDENSE DYNAMIC MATRICES
 DUMP ORTHOGONALITY
$
$   ****************************************************************
```

```
$   ***                      FREE VIBRATION                       ***
$   *****************************************************************
$
 MODAL ANALYSIS HOW  18
LIST DYNAMIC NORMALIZED EIGENVECTORS
LIST DYNAMIC PARTICIPATION FACTORS
FINISH
```

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                                                                                  PAGE -          4
  ****************************
  *RESULTS OF LATEST ANALYSIS*
  ****************************

 PROBLEM - PROB. 8    TITLE - RTE 80 ONRAMP - MODAL ANALYSIS WITH SOIL SPRINGS

 ACTIVE UNITS FEET KIPS DEG. FAHR SEC  LBM


   EIGENVALUES

/--MODE--/--EIGENVALUE--/------FREQUENCY------/--------PERIOD--------/

     1      0.214627D+06        0.463278D+03          0.777071D+00
     2      0.401878D+06        0.633938D+03          0.567878D+00
     3      0.742445D+06        0.861652D+03          0.417802D+00
     4      0.862086D+06        0.928486D+03          0.387728D+00
     5      0.101474D+07        0.100734D+04          0.357376D+00
     6      0.111148D+07        0.105427D+04          0.341470D+00
     7      0.150687D+07        0.122755D+04          0.293268D+00
     8      0.173210D+07        0.131609D+04          0.273537D+00
     9      0.209874D+07        0.144870D+04          0.248498D+00
    10      0.256692D+07        0.160216D+04          0.224697D+00
    11      0.302860D+07        0.174029D+04          0.206862D+00
    12      0.492505D+07        0.221925D+04          0.162217D+00
    13      0.633005D+07        0.251596D+04          0.143087D+00
    14      0.682408D+07        0.261229D+04          0.137810D+00
    15      0.757129D+07        0.275160D+04          0.130833D+00
    16      0.784541D+07        0.280097D+04          0.128527D+00
    17      0.110543D+08        0.332481D+04          0.108277D+00
    18      0.114735D+08        0.338726D+04          0.106281D+00
    19      0.142426D+08        0.377393D+04          0.953912D-01
    20      0.153983D+08        0.392407D+04          0.917414D-01
    21      0.186498D+08        0.431853D+04          0.833616D-01
    22      0.238213D+08        0.488071D+04          0.737598D-01
    23      0.267986D+08        0.517675D+04          0.695416D-01
    24      0.269882D+08        0.519501D+04          0.692972D-01
    25      0.271868D+08        0.521409D+04          0.690436D-01
    26      0.278129D+08        0.527380D+04          0.682620D-01
    27      0.282864D+08        0.531849D+04          0.676883D-01
    28      0.288496D+08        0.537118D+04          0.670244D-01
    29      0.359578D+08        0.599648D+04          0.600352D-01
    30      0.510431D+08        0.714445D+04          0.503888D-01
    31      0.570001D+08        0.754984D+04          0.476831D-01
    32      0.645933D+08        0.803699D+04          0.447929D-01
    33      0.861642D+08        0.928246D+04          0.387828D-01
    34      0.986532D+08        0.993243D+04          0.362449D-01
    35      0.996099D+08        0.998047D+04          0.360704D-01
```

339-890 0 - 81 - 21

```
/--MODE--/--EIGENVALUE--/------FREQUENCY------/--------PERIOD--------/

     36      0.998556D+08        0.999277D+04           0.360260D-01
     37      0.104703D+09        0.102325D+05           0.351821D-01
     38      0.105738D+09        0.102829D+05           0.350096D-01
     39      0.113135D+09        0.106365D+05           0.338457D-01
     40      0.125818D+09        0.112169D+05           0.320945D-01
     41      0.174608D+09        0.132139D+05           0.272440D-01
     42      0.176499D+09        0.132853D+05           0.270976D-01
     43      0.204567D+09        0.143027D+05           0.251701D-01
     44      0.205195D+09        0.143246D+05           0.251315D-01
     45      0.211190D+09        0.145324D+05           0.247723D-01
     46      0.220874D+09        0.148618D+05           0.242231D-01
     47      0.238380D+09        0.154396D+05           0.233167D-01
     48      0.359898D+09        0.189710D+05           0.189764D-01
     49      0.360798D+09        0.189947D+05           0.189527D-01
     50      0.405853D+09        0.201458D+05           0.178697D-01
     51      0.458235D+09        0.214064D+05           0.168174D-01
     52      0.516102D+09        0.227179D+05           0.158465D-01
     53      0.549642D+09        0.234444D+05           0.153554D-01
     54      0.574447D+09        0.239676D+05           0.150203D-01
     55      0.600238D+09        0.244997D+05           0.146940D-01
     56      0.623694D+09        0.249739D+05           0.144151D-01
     57      0.642921D+09        0.253559D+05           0.141979D-01
     58      0.694968D+09        0.263622D+05           0.136559D-01
     59      0.698239D+09        0.264242D+05           0.136239D-01
     60      0.722136D+09        0.268726D+05           0.133965D-01
     61      0.771410D+09        0.277743D+05           0.129616D-01
     62      0.777920D+09        0.278912D+05           0.129073D-01
     63      0.785419D+09        0.280253D+05           0.128455D-01
     64      0.794505D+09        0.281870D+05           0.127719D-01
     65      0.813135D+09        0.285155D+05           0.126247D-01
     66      0.832594D+09        0.288547D+05           0.124763D-01
     67      0.859775D+09        0.293219D+05           0.122775D-01
     68      0.868241D+09        0.294659D+05           0.122175D-01
     69      0.921177D+09        0.303509D+05           0.118613D-01
     70      0.964256D+09        0.310525D+05           0.115933D-01
     71      0.985251D+09        0.313887D+05           0.114691D-01
     72      0.101655D+10        0.318833D+05           0.112912D-01
     73      0.113997D+10        0.337634D+05           0.106624D-01
     74      0.129446D+10        0.359786D+05           0.100059D-01
     75      0.136429D+10        0.369362D+05           0.974652D-02
     76      0.142171D+10        0.377056D+05           0.954765D-02
     77      0.147691D+10        0.384306D+05           0.936754D-02
     78      0.150098D+10        0.387424D+05           0.929213D-02
     79      0.155806D+10        0.394723D+05           0.912032D-02
     80      0.162702D+10        0.403363D+05           0.892496D-02
     81      0.165536D+10        0.406861D+05           0.884822D-02
     82      0.191929D+10        0.438097D+05           0.821735D-02
```

```
/--MODE--/--EIGENVALUE--/------FREQUENCY------/--------PERIOD--------/

     83     0.200362D+10        0.447618D+05          0.804257D-02
     84     0.201084D+10        0.448423D+05          0.802812D-02
     85     0.203832D+10        0.451477D+05          0.797382D-02
     86     0.272675D+10        0.522183D+05          0.689413D-02
     87     0.335151D+10        0.578922D+05          0.621845D-02
     88     0.360112D+10        0.600093D+05          0.599907D-02
     89     0.362468D+10        0.602053D+05          0.597954D-02
     90     0.390066D+10        0.624553D+05          0.576412D-02
     91     0.408378D+10        0.639045D+05          0.563341D-02
     92     0.410454D+10        0.640667D+05          0.561914D-02
     93     0.410487D+10        0.640692D+05          0.561892D-02
     94     0.412990D+10        0.642643D+05          0.560187D-02
     95     0.421679D+10        0.649368D+05          0.554385D-02
     96     0.450340D+10        0.671074D+05          0.536454D-02
     97     0.451034D+10        0.671590D+05          0.536041D-02
     98     0.453046D+10        0.673087D+05          0.534849D-02
     99     0.453099D+10        0.673126D+05          0.534818D-02
    100     0.522685D+10        0.722969D+05          0.497946D-02
    101     0.529682D+10        0.727792D+05          0.494647D-02
    102     0.535189D+10        0.731566D+05          0.492095D-02
    103     0.551933D+10        0.742922D+05          0.484573D-02
    104     0.679717D+10        0.824449D+05          0.436655D-02
    105     0.685836D+10        0.828152D+05          0.434703D-02
    106     0.693296D+10        0.832644D+05          0.432358D-02
    107     0.128964D+11        0.113562D+06          0.317007D-02
    108     0.128988D+11        0.113573D+06          0.316978D-02
    109     0.129055D+11        0.113602D+06          0.316895D-02
    110     0.131461D+11        0.114656D+06          0.313981D-02
    111     0.131711D+11        0.114765D+06          0.313683D-02
    112     0.152675D+11        0.123562D+06          0.291352D-02
    113     0.152929D+11        0.123664D+06          0.291110D-02
    114     0.360661D+11        0.189911D+06          0.189563D-02
    115     0.215807D+12        0.464550D+06          0.774944D-03
    116     0.215807D+12        0.464550D+06          0.774944D-03
    117     0.218906D+12        0.467874D+06          0.769438D-03
    118     0.218906D+12        0.467874D+06          0.769438D-03
    119     0.435216D+12        0.659709D+06          0.545695D-03
    120     0.441989D+12        0.664822D+06          0.541498D-03
    121     0.469936D+12        0.685519D+06          0.525150D-03
    122     0.469936D+12        0.685519D+06          0.525150D-03
    123     0.470355D+12        0.685824D+06          0.524916D-03
    124     0.470355D+12        0.685824D+06          0.524916D-03
    125     0.551565D+12        0.742674D+06          0.484735D-03
    126     0.552477D+12        0.743288D+06          0.484334D-03
    127     0.775174D+12        0.880439D+06          0.408887D-03
    128     0.775174D+12        0.880439D+06          0.408887D-03
    129     0.130379D+13        0.114183D+07          0.315282D-03
```

```
                                                                                                   PAGE -       7

/--MODE--/--EIGENVALUE--/------FREQUENCY------/---------PERIOD--------/

   130     0.156254D+13      0.125002D+07          0.287996D-03
   131     0.156254D+13      0.125002D+07          0.287996D-03
   132     0.156254D+13      0.125002D+07          0.287996D-03
```

```
                         ********** ORTHOGONALITY CHECK **********

                     MODES HAVE BEEN NORMALIZED TO UNIT GENERALIZED MASS

                             SUM=(MODE(I))TRANS*INERTIA*MODE(J))

          I     J      SUM           I    J        SUM             I    J       SUM             I    J       SUM


          1     2   1.777D-13        2    3   -1.507D-13           3    4   7.707D-13           4    5   9.749D-13
          5     6  -1.066D-13        6    7   -4.604D-15           7    8   1.656D-14           8    9   3.269D-14
          9    10   3.046D-14       10   11   -1.651D-15          11   12   7.182D-16          12   13   4.004D-15
         13    14   1.319D-14       14   15    1.134D-15          15   16  -2.047D-15          16   17  -1.666D-15
         17    18   3.600D-15

 LIST DYNAMIC NORMALIZED EIGENVECTORS
```

```
                                                                                                          PAGE -       9

                *************************
                *RESULTS OF LATEST ANALYSIS*
                *************************

               PROBLEM - PROB. 8    TITLE - RTE 80 ONRAMP - MODAL ANALYSIS WITH SOIL SPRINGS

               ACTIVE UNITS FEET KIPS DEG. FAHR SEC  LBM



                 NORMALIZED EIGENVECTORS


                  MODE    1    MAXIMUM VALUE IS  -1.280935E-02 AT JOINT 2036     IN DIRECTION DISP Z

             JOINT           /-----------------DISPLACEMENT-----------------//------------------ROTATION------------------/
                                    X DISP.        Y DISP.        Z DISP.        X ROT.         Y ROT.         Z ROT.

             3011     GLOBAL       0.0            0.0            0.0           -0.0157096     -0.0866179     -0.0240977
             1012     GLOBAL      -0.0412228     -0.0005037      0.0289577     -0.0110776     -0.0866249     -0.0270958
             1013     GLOBAL      -0.0216541     -0.0115147      0.0605045      0.1064170     -0.0909719     -0.0893002
             1014     GLOBAL      -0.0016479     -0.0176196      0.0959481      0.2350417     -0.1034358     -0.1330898
             1015     GLOBAL       0.0200281     -0.0148039      0.1383975      0.3728952     -0.1242211     -0.1596975
             1021     GLOBAL       0.0443936      0.0001647      0.1913141      0.5184488     -0.1537329     -0.1707885
             1022     GLOBAL       0.0837663      0.0216616      0.2909868      0.4652838     -0.1933880     -0.1497692
             1023     GLOBAL       0.1229939      0.0247341      0.4099313      0.4107153     -0.2133637     -0.1321497
             1024     GLOBAL       0.1570010      0.0141282      0.5378114      0.3577384     -0.2174615     -0.1088053
             1031     GLOBAL       0.1831209      0.0002459      0.6665271      0.3079351     -0.2110211     -0.0718213
             1032     GLOBAL       0.1963683     -0.0048077      0.7546621      0.3247570     -0.2023067     -0.0497367
             1033     GLOBAL       0.2054252     -0.0064666      0.8390270      0.3409509     -0.1917386     -0.0327495
             1034     GLOBAL       0.2105898     -0.0076221      0.9193097      0.3566491     -0.1824684     -0.0216901
             2035     GLOBAL       0.2127191     -0.0116348      0.9968778      0.3720513     -0.1788911     -0.0175895
             2036     GLOBAL      -0.1095571     -0.0119417      1.0000000      0.3726713      0.3788259      0.0481059
             1041     GLOBAL      -0.1087280     -0.0003154      0.9009826      0.3819858      0.3770038      0.0464477
             1042     GLOBAL      -0.1007734      0.0128970      0.7148929      0.4669592      0.3653978      0.0474455
             1043     GLOBAL      -0.0847203      0.0152672      0.5385097      0.5510438      0.3395178      0.0609474
             1044     GLOBAL      -0.0625870      0.0095106      0.3803502      0.6331793      0.2957748      0.0900575
             1051     GLOBAL      -0.0380458      0.0003264      0.2497618      0.7122900      0.2315746      0.1374904
             1052     GLOBAL      -0.0151649     -0.0016706      0.1539727      0.5465837      0.1640984      0.1575114
             1053     GLOBAL       0.0036505      0.0046404      0.0888328      0.3905609      0.1088022      0.1422123
             1054     GLOBAL       0.0174092      0.0086334      0.0480257      0.2458492      0.0652953      0.0924408
             1061     GLOBAL       0.0260588     -0.0001482      0.0253951      0.1133636      0.0334491      0.0101220
             1062     GLOBAL       0.0301708     -0.0106113      0.0163961      0.0770819      0.0143787      0.0220565
             1063     GLOBAL       0.0315067     -0.0125694      0.0137267      0.0437663      0.0006839      0.0280691
             1064     GLOBAL       0.0306776     -0.0081890      0.0151570      0.0143718     -0.0076348      0.0269050
             1065     GLOBAL       0.0285463     -0.0003538      0.0185728     -0.0101290     -0.0105629      0.0174987
             3071     GLOBAL       0.0            0.0            0.0           -0.0110310     -0.0105676      0.0169209
             40211    GLOBAL       0.0000020      0.0000001      0.0000081      0.0001421     -0.0000119     -0.0000571
```

```
JOINT           /-----------------DISPLACEMENT-----------------//-------------------ROTATION-------------------/
                        X DISP.       Y DISP.       Z DISP.        X ROT.        Y ROT.        Z ROT.
40212     GLOBAL     0.0059239     0.0000549     0.0326433     0.4048012    -0.0512319    -0.0770866
40213     GLOBAL     0.0217218     0.0001098     0.1078504     0.5775647    -0.1025128    -0.1340416
4015      GLOBAL     0.0           0.0           0.0           0.0           0.0           0.0
40511     GLOBAL    -0.0000027     0.0000001     0.0000175     0.0002069     0.0000179     0.0000323
40512     GLOBAL    -0.0053673     0.0001088     0.0414626     0.5166242     0.0771729     0.0690255
40513     GLOBAL    -0.0191971     0.0002177     0.1384815     0.7540024     0.1544196     0.1148710
40611     GLOBAL     0.0000019    -0.0000001    -0.0000005     0.0000025     0.0000026    -0.0000210
40612     GLOBAL     0.0070884    -0.0000494     0.0028750     0.0388930     0.0111470    -0.0820650
40613     GLOBAL     0.0199748    -0.0000989     0.0114016     0.0766857     0.0223047    -0.0786311
4003      GLOBAL     0.0000002     0.0000000     0.0000011    -0.0000208    -0.0000024     0.0000083
4013      GLOBAL     0.0           0.0           0.0           0.0           0.0           0.0
40311     GLOBAL     0.1625147     0.0001403     0.5796597     0.0343049    -0.1792943    -0.0023966
40312     GLOBAL     0.1656935     0.0001755     0.5944254     0.1601728    -0.1898657    -0.0384818
40313     GLOBAL     0.1731935     0.0002107     0.6250809     0.2503364    -0.2004496    -0.0612169
4004      GLOBAL    -0.0000001    -0.0000001     0.0000014    -0.0000313     0.0000044    -0.0000060
4014      GLOBAL     0.0           0.0           0.0           0.0           0.0           0.0
40411     GLOBAL    -0.0953247    -0.0001800     0.7931008     0.0360071     0.3203217    -0.0005323
40412     GLOBAL    -0.0972685    -0.0002251     0.8110330     0.1991449     0.3392082     0.0251478
40413     GLOBAL    -0.1022241    -0.0002703     0.8493349     0.3130167     0.3581172     0.0404878


   MODE    2    MAXIMUM VALUE IS    9.521995E-03 AT JOINT 40311    IN DIRECTION DISP X


JOINT           /-----------------DISPLACEMENT-----------------//-------------------ROTATION-------------------/
                        X DISP.       Y DISP.       Z DISP.        X ROT.        Y ROT.        Z ROT.

3011      GLOBAL     0.0           0.0           0.0           0.4596117    -0.1105622     0.7050212
1012      GLOBAL     0.6855015     0.0147452    -0.4490045     0.4617375    -0.1105617     0.7032641
1013      GLOBAL     0.7097610     0.3405188    -0.4093058     0.4146384    -0.1102224     0.5032417
1014      GLOBAL     0.7317078     0.5130062    -0.3685464     0.2060864    -0.1091639     0.0337758
1015      GLOBAL     0.7510818     0.4189765    -0.3272266    -0.1017805    -0.1071404    -0.6147590
1021      GLOBAL     0.7677365    -0.0025828    -0.2859047    -0.4498206    -0.1037593    -1.3401813
1022      GLOBAL     0.7966063    -0.6996462    -0.2321246    -0.3043292    -0.0929317    -0.7096878
1023      GLOBAL     0.8180480    -0.8796484    -0.1851468    -0.0408677    -0.0742786     0.2114693
1024      GLOBAL     0.8321448    -0.5276740    -0.1485741     0.1482700    -0.0533388     0.8420193
1031      GLOBAL     0.8401901    -0.0096767    -0.1230324     0.1489253    -0.0344850     0.6483163
1032      GLOBAL     0.8433456     0.1989439    -0.1108636     0.1055672    -0.0240803     0.2926807
1033      GLOBAL     0.8448749     0.2558500    -0.1023009     0.0644479    -0.0167794    -0.0366554
1034      GLOBAL     0.8451809     0.1852253    -0.0960735     0.0299231    -0.0127082    -0.2781861
2035      GLOBAL     0.8456777     0.0391990    -0.0909774     0.0015593    -0.0114095    -0.3720011
2036      GLOBAL     0.1155503     0.0327047    -0.0907783     0.0005030    -0.0261851    -0.1203791
1041      GLOBAL     0.1154736     0.0041335    -0.0838921    -0.0156214    -0.0265418    -0.0868279
1042      GLOBAL     0.1133560    -0.0144666    -0.0702465    -0.0497778    -0.0285158     0.0000743
1043      GLOBAL     0.1103553    -0.0017793    -0.0556067    -0.0795938    -0.0307460     0.0281887
1044      GLOBAL     0.1064202     0.0129151    -0.0403444    -0.1019651    -0.0315180    -0.0056529
```

```
***************************
*RESULTS OF LATEST ANALYSIS*
***************************

 PROBLEM - PROB. 8    TITLE - RTE 80 ONRAMP - MODAL ANALYSIS WITH SOIL SPRINGS
             PARTICPATION FACTORS -(OUTPUT IN INTERNAL UNITS)


MODE           /-----------------DISPLACEMENT-----------------//------------------ROTATION------------------/
                         X DISP.          Y DISP.          Z DISP.          X ROT.          Y ROT.          Z ROT.

    1               -12.9172020       -0.4295208     -116.128418          0.0             0.0             0.0
    2               103.398682        -9.0440159      -26.5034027         0.0             0.0             0.0
    3               -87.3408051      -19.3747559       -4.3783922         0.0             0.0             0.0
    4               -35.5992126       13.7899847      -31.7531891         0.0             0.0             0.0
    5               -19.9290009        4.0885954       -7.9823093         0.0             0.0             0.0
    6                43.1997528        0.3892074       17.0993195         0.0             0.0             0.0
    7                22.7614441        1.0102539      -77.8030243         0.0             0.0             0.0
    8               -21.1449585      -19.1529236      -15.1347876         0.0             0.0             0.0
    9                 8.6613655      -32.5593567       16.4158020         0.0             0.0             0.0
   10                13.3223181      -76.3327789       -2.4329376         0.0             0.0             0.0
   11                -8.3526611      -88.6617889       -2.0735464         0.0             0.0             0.0
   12                16.6310730       -2.4464178        3.8976278         0.0             0.0             0.0
   13                -5.5614777       -8.5880108      -23.5689240         0.0             0.0             0.0
   14                 3.3812809      -21.6340637        5.4007788         0.0             0.0             0.0
   15               -13.7043915       -1.5622244       11.2886524         0.0             0.0             0.0
   16                 3.8110495      -14.7450075        0.8810353         0.0             0.0             0.0
   17                -3.5858784       -1.1516628       -4.7698870         0.0             0.0             0.0
   18                -4.0281353        0.8419144      -18.6003265         0.0             0.0             0.0

FINISH
```

Appendix A.7

SAMPLE PROBLEM 9

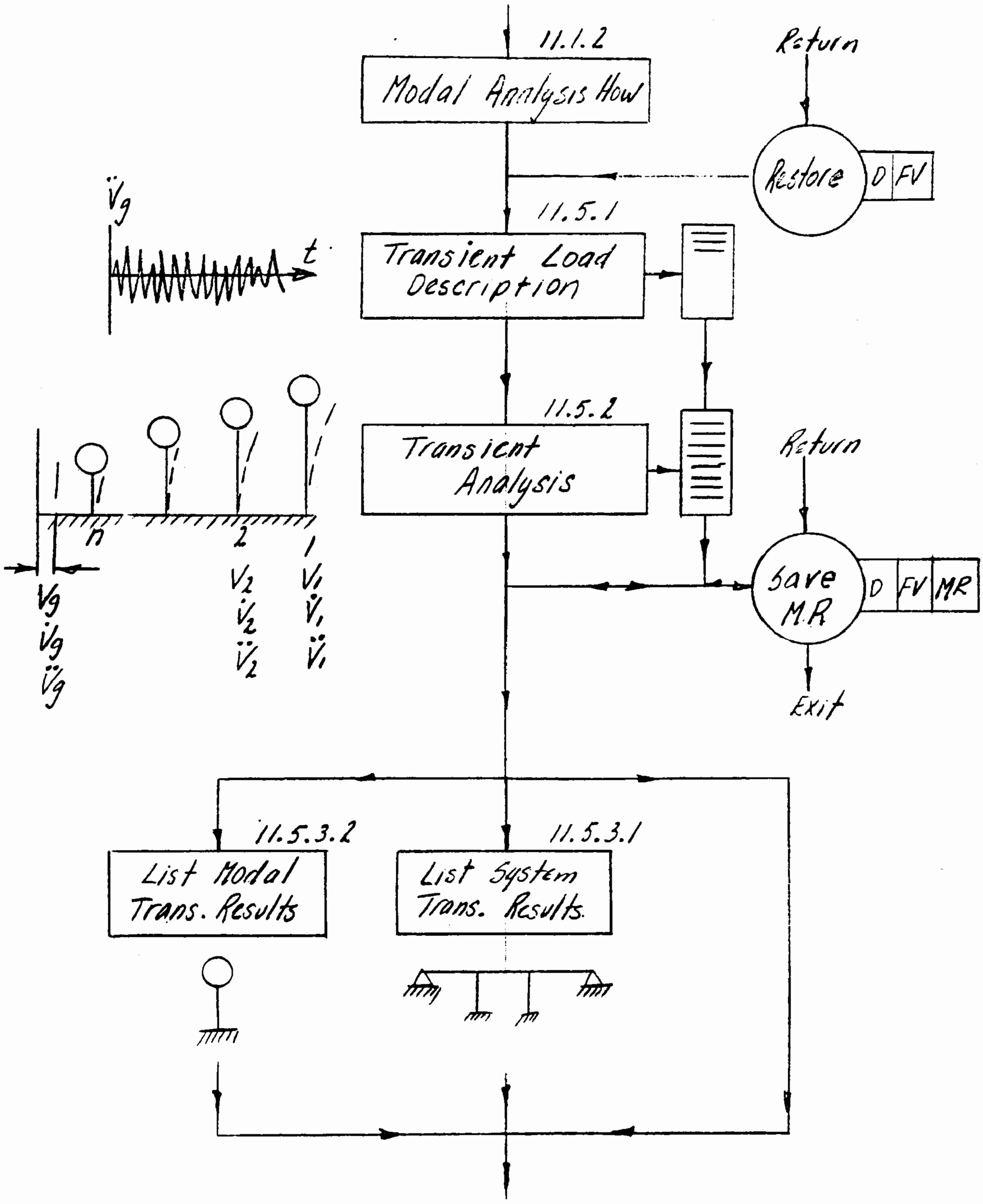
Perform a time history analysis of Route 80 Onramp U.C. using transversly applied ground motion as developed by Seed and Idress for an 8+ magnitude earthquake.

FIND:

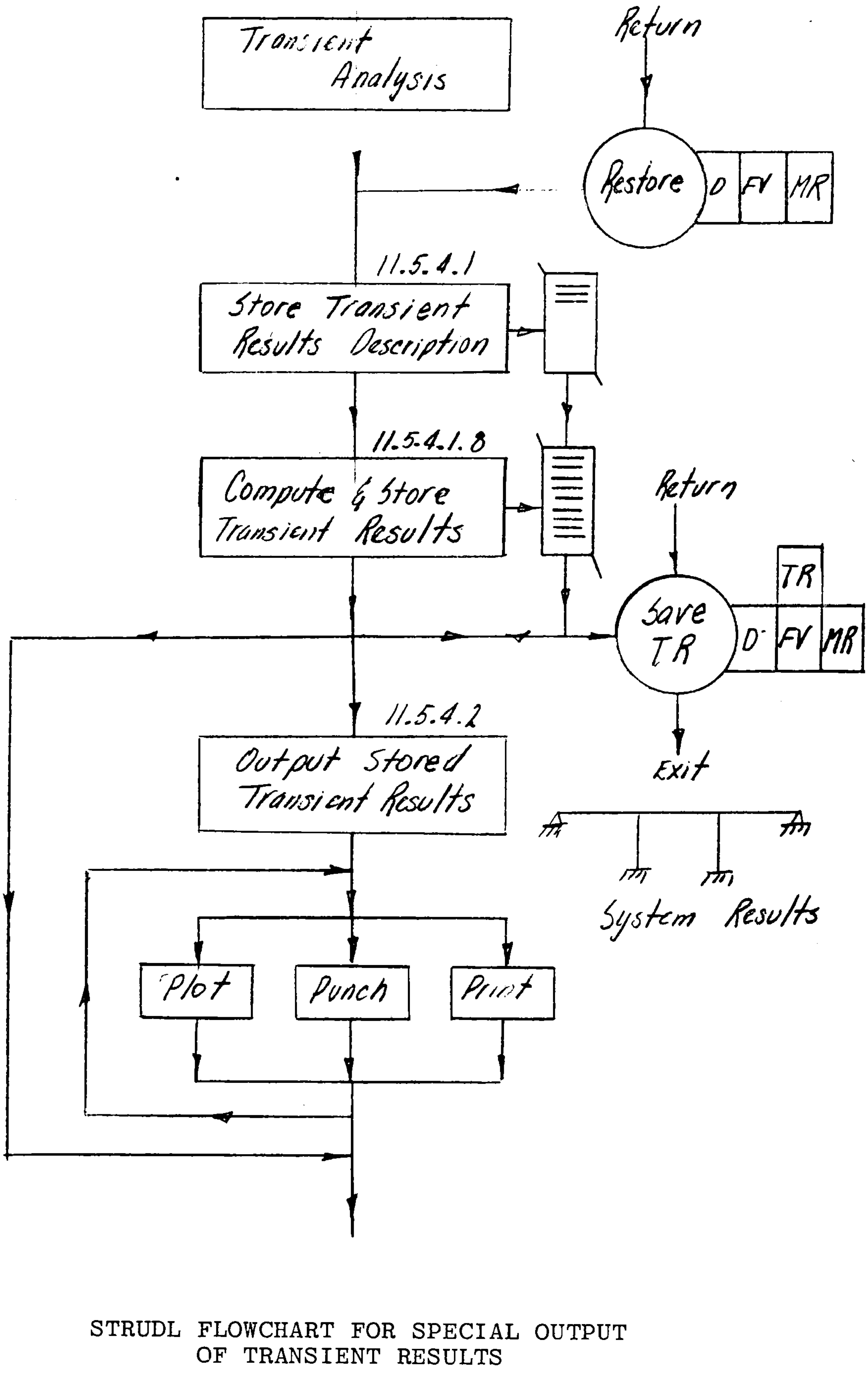
1) Maximum deck displacements at bents
2) Maximum hinge restrainer forces
3) Maximum shears and moments at column bases
4) Maximum shears at abutments and hinges

Plot the time history of bent 4 base moments in the longitudinal and transverse direction.

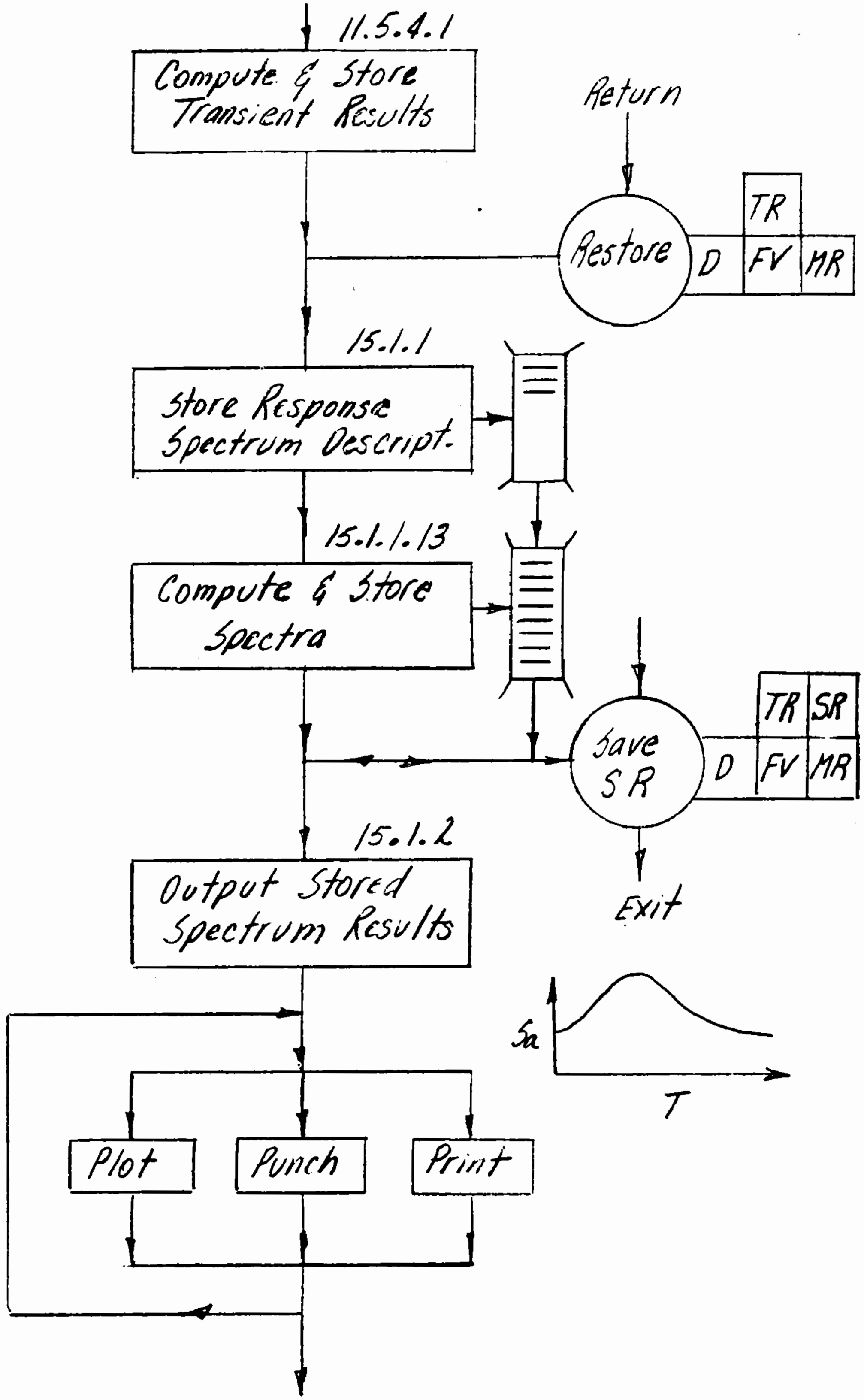
Also, assume an electrolier is to be placed on the bridge at Bent 4. It is desirable to have a simple means of evaluating its seismic response. Using the transient results for acceleration at Bent 4, develop and plot a response spectrum for base motion in the transverse direction at the electrolier.



STRUDL FLOWCHART FOR PERFORMING TRANSIENT ANALYSIS



STRUDL FLOWCHART FOR SPECIAL OUTPUT
OF TRANSIENT RESULTS



STRUDL FLOWCHART FOR COMPUTATION AND OUTPUT
OF A RESPONSE SPECTRUM

80/80 INPUT CARD IMAGES

```
 STRUDL 'PROB. 9' 'RTE 80 ONRAMP - TRANSIENT ANALYSIS '
$
$                        *****************************************
$                        ***                                   ***
$                        *** SEISMIC DESIGN OF HIGHWAY BRIDGES ***
$                        ***                                   ***
$                        *****************************************
$
$    SPONSORED BY:
$
$         FEDERAL HIGHWAY ADMINISTRATION
$
$    PRESENTED BY:
$
$         ENGINEERING COMPUTER CORPERATION
$         601 UNIVERSITY AVE.  SUITE 213
$         SACRAMENTO, CALIFORNIA
$
$    PHONE: (916) 922-9316
$
$    INSTRUCTORS:
$
$         ROY A. IMBSEN  -  PRINCIPLE INVESTIGATOR
$         RICHARD V. NUTT
$         JAMES GATES
$
$
$     THIS PROBLEM ILLUSTRATES THE USE OF A TIME HISTORY ANALYSIS
$     TO DETERMINE THE MAXIMUM FORCES IN THE ROUTE 80 ONRAMP OC.
$     THE SEED-IDRISS 8+ GROUND ACCELERATION IS APPLIED IN THE
$     TRANSVERSE DIRECTION.
$
$
$   *****************************************************************
$   ***                      STRUCTURE DESCRIPTION                ***
$   *****************************************************************
$
 JOINT COORDINATES
     3011  10000.00     75.30  10000.00     SUPPORT
     1012  10000.84     75.30   9999.45
     1013  10021.35     75.30   9986.72
     1014  10042.38     75.30   9974.83
     1015  10063.87     75.30   9963.83
     1021  10085.79     75.30   9953.71
     1022  10118.29     75.30   9940.66
     1023  10151.51     75.30   9929.56
     1024  10185.32     75.30   9920.46
     1031  10219.62     75.30   9913.39
     1032  10244.01     75.30   9909.65
     1033  10268.53     75.30   9906.93
     1034  10293.14     75.30   9905.24
     2035  10317.81     75.30   9904.58
     2036  10318.81     75.30   9904.58
     1041  10333.81     75.30   9904.71
     1042  10362.45     75.30   9906.03
     1043  10391.00     75.30   9908.74
     1044  10419.37     75.30   9912.84
     1051  10447.52     75.30   9918.32
     1052  10475.36     75.30   9925.16
     1053  10502.84     75.30   9933.36
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      1054   10529.88       75.30    9942.88
      1061   10556.43       75.30    9953.71
      1062   10578.36       75.30    9963.83
      1063   10599.85       75.30    9974.84
      1064   10620.87       75.30    9986.71
      1065   10641.39       75.30    9999.45
      3071   10642.23       75.30   10000.00      SUPPORT
     40211   10085.79       50.00    9953.71      SUPPORT
     40212   10085.79       58.43    9953.71
     40213   10085.79       66.87    9953.71
     40311   10219.62       50.00    9913.39      SUPPORT
     40312   10219.62       58.43    9913.39
     40313   10219.62       66.87    9913.39
     40411   10333.81       50.00    9904.71      SUPPORT
     40412   10333.81       58.43    9904.71
     40413   10333.81       66.87    9904.71
     40511   10447.52       50.00    9918.32      SUPPORT
     40512   10447.52       58.43    9918.32
     40513   10447.52       66.87    9918.32
     40611   10556.43       50.00    9953.71      SUPPORT
     40612   10556.43       58.43    9953.71
     40613   10556.43       66.87    9953.71
MEMBER INCIDENCES
     5011     3011     1012
     5012     1012     1013
     5013     1013     1014
     5014     1014     1015
     5015     1015     1021
     5021     1021     1022
     5022     1022     1023
     5023     1023     1024
     5024     1024     1031
     5031     1031     1032
     5032     1032     1033
     5033     1033     1034
     5034     1034     2035
     5035     2035     2036
     5036     2036     1041
     5041     1041     1042
     5042     1042     1043
     5043     1043     1044
     5044     1044     1051
     5051     1051     1052
     5052     1052     1053
     5053     1053     1054
     5054     1054     1061
     5061     1061     1062
     5062     1062     1063
     5063     1063     1064
     5064     1064     1065
     5065     1065     3071
      901    1034 1041
      902    1034 1041
    70211    40211    40212
    70212    40212    40213
    70213    40213     1021
    70311    40311    40312
    70312    40312    40313
    70313    40313     1031
    70411    40411    40412
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  70412   40412   40413
  70413   40413    1041
  70511   40511   40512
  70512   40512   40513
  70513   40513    1051
  70611   40611   40612
  70612   40612   40613
  70613   40613    1061
MEMBER PROPERTIES PRISMATIC
5011 THRU 5015 AX  86.0 IX      862.0 IY   13000.0 IZ       360.0
5021 THRU 5024 AX  86.0 IX      862.0 IY   13000.0 IZ       360.0
5031 THRU 5036 AX  86.0 IX      862.0 IY   13000.0 IZ       360.0
5041 THRU 5044 AX  86.0 IX      862.0 IY   13000.0 IZ       360.0
5051 THRU 5054 AX  86.0 IX      862.0 IY   13000.0 IZ       360.0
5061 THRU 5065 AX  86.0 IX      862.0 IY   13000.0 IZ       360.0
MEMB PROP PRIS AX  33.0 IX      146.0 IY     143.0 IZ        73.0
70211
MEMB PROP PRIS AX  33.0 IX      146.0 IY     143.0 IZ        73.0
70212
MEMB PROP PRIS AX  33.0 IX      146.0 IY     143.0 IZ        73.0
70213
MEMB PROP PRIS AX  33.0 IX      146.0 IY     143.0 IZ        73.0
70311
MEMB PROP PRIS AX  33.0 IX      146.0 IY     143.0 IZ        73.0
70312
MEMB PROP PRIS AX  33.0 IX      146.0 IY     143.0 IZ        73.0
70313
MEMB PROP PRIS AX  33.0 IX      146.0 IY     143.0 IZ        73.0
70411
MEMB PROP PRIS AX  33.0 IX      146.0 IY     143.0 IZ        73.0
70412
MEMB PROP PRIS AX  33.0 IX      146.0 IY     143.0 IZ        73.0
70413
MEMB PROP PRIS AX  33.0 IX      146.0 IY     143.0 IZ        73.0
70511
MEMB PROP PRIS AX  33.0 IX      146.0 IY     143.0 IZ        73.0
70512
MEMB PROP PRIS AX  33.0 IX      146.0 IY     143.0 IZ        73.0
70513
MEMB PROP PRIS AX  33.0 IX      146.0 IY     143.0 IZ        73.0
70611
MEMB PROP PRIS AX  33.0 IX      146.0 IY     143.0 IZ        73.0
70612
MEMB PROP PRIS AX  33.0 IX      146.0 IY     143.0 IZ        73.0
70613
MEMBER PROPERTIES PRISMATIC
    901 TO 902 AX .02455 IX .000001 IY .000001 IZ .000001
CONSTANTS
DENSITY 150.0 ALL
E  432000.  5011 THRU  5015
E  432000.  5021 THRU  5024
E  432000.  5031 THRU  5036
E  432000.  5041 THRU  5044
E  432000.  5051 THRU  5054
E  432000.  5061 THRU  5065
E  432000. 70211 THRU 70213
E  432000. 70311 THRU 70313
E  432000. 70411 THRU 70413
E  432000. 70511 THRU 70513
E  432000. 70611 THRU 70613
```

```
     E 2016000. 901 902
 BETA   23.5906 70211 THRU 70213
 BETA    9.9390 70311 THRU 70313
 BETA   -1.2375 70411 THRU 70413
 BETA  -12.4141 70511 THRU 70513
 BETA  -23.5906 70611 THRU 70613
MEMBER ECCENTRICITY
     901 LOCAL START Z  12.88 END Z  12.88
     902 LOCAL START Z -12.88 END Z -12.88
 MEMBER 5011 REL END FOR X
  JOINT 3011 RELEASE          MOM    Y  Z  -
  TH2   -33.1008
 MEMBER 5065 REL STA FOR X
  JOINT 3071 RELEASE          MOM    Y  Z  -
  TH2    33.1008
 MEMBER 5035 RELEASES END FORCE X MOMENT   Y Z
 INERTIA OF JOINTS LUMPED
 DAMPING PERCENTS  5.0   18
$
$   ******************************************************************
$   ***                 LOADING CONDITION DESCRIPTION                ***
$   ******************************************************************
$
TRANSIENT LOAD 1 'TRANSVERSE EQ'
SUPPORT ACCELERATIONS
TRANSLATION Z FILE 'RIEQSI8+' FACTOR 1.0
INTEGRATE FROM 0.0 TO 20.0 AT DELTA 0.02
END DYNAMIC LOADING
PRINT DYNAMIC LOADING DATA
ASSEMBLE FOR DYNAMICS REDUCE BAND
INDEPENDENT DEGREE OF FREEDOM SELECT
CONDENSE DYNAMIC MATRICES
DUMP ORTHOGONALITY
$
$   ******************************************************************
$   ***                         FREE VIBRATION                       ***
$   ******************************************************************
$
 MODAL ANALYSIS HOW  18
$
$   ******************************************************************
$   ***        CREATE MODAL AND RIGID BODY TRANSIENT RESULTS         ***
$                                ***
$   *** NOTE THESE RESULTS ARE CALCULATED AND TEMPORARILY STORED ***
$   ***   USING THE SAME LOADING DESCRIPTION IDENTIFIER AS THE     ***
$   ***           ORIGINAL LOADING DESCRIPTION (I.E. 1)            ***
$   ******************************************************************
$
TRANSIENT ANALYSIS LOADS 1
$
$   ******************************************************************
$   ***   CALCULATE AND LIST THE PHYSICAL SYSTEM RESULTS USING     ***
$   ***   THE MODAL RESULTS STORED AFTER THE TRANSIENT ANALYSIS    ***
$   ***                        WAS PERFORMED                       ***
$   ******************************************************************
$
LIST DYNAMIC SYSTEM TRANSIENT RESULTS MAXIMUM -
     FORCES DISTORTIONS MEMBER 901 902
LIST DYNAMIC SYSTEM TRANSIENT RESULTS MAXIMUM -
     FORCES MEMBER 70211 70311 70411 70511 70611
```

```
LIST DYNAMIC SYSTEM TRANSIENT RESULTS MAXIMUM -
     FORCES MEMBER 5011 5035 5065
LIST DYNAMIC SYSTEM TRANSIENT RESULTS MAXIMUM -
     DISPLACEMENTS JOINT 3011 1012 1021 1031 2035 2036 -
                         1041 1051 1061 1065 3071
$
$   *****************************************************************
$   ***      DESCRIBE SELECTED TRANSIENT OUTPUT DESIRED. THIS      ***
$   ***       DESCRIPTION IS STORED UNDER THE NAME SPECIFIED       ***
$   ***                     (I.E. 'PLOTTH')                        ***
$   *****************************************************************
$
STORE TRANSIENT SET IDENT 'PLOTTH'
USE MODAL RESULTS OF TRANSIENT ANALYSIS 1
JOINT TOTAL ACCELERATION
     1041 Z
MEMBER START FORCES
70411 Y Z
MEMBER START MOMENT
    70411 Y Z
END
$
$   *****************************************************************
$   ***     NOTE THAT THE ABOVE COMMANDS DESCRIBE THE RESULTS      ***
$   ***        THAT WILL BE STORED WHEN THE REFERENCED NAME        ***
$   ***    IS GIVEN IN THE 'COMPUTE AND STORE TRANSIENT' COMMAND   ***
$   *****************************************************************
$
COMPUTE AND STORE TRANSIENT 'PLOTTH'
$
$   *****************************************************************
$   ***  OUTPUT THE DESIRED TRANSIENT RESULTS STORED PREVIOUSLY    ***
$   *** UNDER THE NAME 'PLOTTH'. THE TYPE AND FORM OF THE OUTPUT   ***
$   ***                       IS SPECIFIED                         ***
$   *****************************************************************
$
OUTPUT STORED TRANSIENT RESULTS
PLOT 1 ON
ABSCISSA SCALE OF 1.67
ORDINATE SCALE ON INDIVIDUAL BASIS
USE STORE TRANSIENT SET ID 'PLOTTH'
USE TRANSIENT 1
MEMBER START MOMENT
    70411 Y Z
END SUBSET REQUEST
PLOT 1 OFF
MEMBER START FORCE
     70411 Y Z
END SUBSET REQUEST
END
$
$   *****************************************************************
$   ***      DESCRIBE SELECTED RESPONSE SPECTRUM OUTPUT TO BE      ***
$   ***      STORED. THIS DESCRIPTION IS STORED UNDER THE NAME     ***
$   ***                   SPECIFIED (I.E. 'PLOTRS')                ***
$   *****************************************************************
$
STORE SPECTRUM IDENT 'PLOTRS'
RANGE OF PERIOD LIN .01 .10 .01 .11 .51 .02 .55 1.0 .05 1.1 2.0 .05
OSCILLATOR DAMPINGS RATIO .02
```

```
USE STORE TRANSIENT SET IDENT 'PLOTTH'
USE TRANSIENT 1
JOINT TOTAL ACCELERATION
    1041 TRANS Z
END
$
$   ********************************************************************
$   ***    THE ABOVE COMMANDS ARE USED TO DESCRIBE THE RESULTS        ***
$   ***   TO BE COMPUTED AND STORED WHEN THE 'COMPUTE AND STORE       ***
$   ***                  SPECTRA' COMMAND IS GIVEN.                   ***
$   ********************************************************************
$
COMPUTE AND STORE SPECTRA FOR ID 'PLOTRS'
$
$   ********************************************************************
$   ***     OUTPUT THE RESPONSE SPECTRUM RESULTS STORED UNDER         ***
$   ***   THE NAME 'PLOTRS' IN THE ABOVE COMMANDS. THE FORMAT OF      ***
$   ***       THE PLOT AND THE TYPE OF RESULTS ARE SPECIFIED.         ***
$   ********************************************************************
$
OUTPUT STORED SPECTRUM RESULTS
PLOT 1 ON
ABSCISSA LIN OF .2
ORDINATE LIN ON INDIVIDUAL BASIS
DAMPING CURVES 1
USE STORE SPECTRUM IDENT 'PLOTRS'
USE SPECTRA FROM TRANSIENT ANALYSIS 1
JOINT TOTAL ACCELERATIONS
     1041 TRANS Z
END SUBSET
END
FINISH
```

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                                                                                                    PAGE =    3

       ******************************
       *RESULTS OF LATEST ANALYSIS*
       ******************************

      PROBLEM = ASSIGN 4   TITLE = RTE 80 ONRAMP = TRANSIENT ANALYSIS

      ACTIVE UNITS FEET KIPS DEG. FAHR SEC  LBM

    /--LOAD--/--MEMB--/----TIME----/TYPE/--NODE--/---------------------------------------------------------------------/
                                                    XT              YT              ZT              XR              YR              ZR


     1        901                      FOR 1034     0.2521016E 02 0.9980196E-05 0.1426904E-05 0.1022334E-04 0.1948794E-04 0.9766480E-04
                                           1041     0.2521016E 02 0.9980196E-05 0.1426904E-05 0.1022334E-04 0.7546674E-04 0.3497868E-03
                                       DIS 1041     0.2071883E-01 0.3248025E-01 0.1548227E-01 0.2954581E-01 0.5441039E-01 0.1697180E 00
                                                                                TIME VALUES AT MAXIMA
                                       FOR 1034     0.1526000E 02 0.1474000E 02 0.1488000E 02 0.1486000E 02 0.1474000E 02 0.1486000E 02
                                           1041     0.1526000E 02 0.1474000E 02 0.1488000E 02 0.1486000E 02 0.1488000E 02 0.1474000E 02
                                       DIS 1041     0.1526000E 02 0.1474000E 02 0.1476000E 02 0.1486000E 02 0.1476000E 02 0.1474000E 02

              902                      FOR 1034     0.4332626E 02 0.6051159E-05 0.1426904E-05 0.1022334E-04 0.1948794E-04 0.4497217E-04
                                           1041     0.4332626E 02 0.6051159E-05 0.1426904E-05 0.1022334E-04 0.7546674E-04 0.2658283E-03
                                       DIS 1041     0.3560740E-01 0.4396407E-01 0.1548227E-01 0.2954581E-01 0.5441039E-01 0.1697180E 00
                                                                                TIME VALUES AT MAXIMA
                                       FOR 1034     0.1981999E 02 0.1472000E 02 0.1488000E 02 0.1486000E 02 0.1474000E 02 0.1306000E 02
                                           1041     0.1981999E 02 0.1472000E 02 0.1488000E 02 0.1486000E 02 0.1488000E 02 0.1474000E 02
                                       DIS 1041     0.1981999E 02 0.1474000E 02 0.1476000E 02 0.1486000E 02 0.1476000E 02 0.1474000E 02

    LIST DYNAMIC SYSTEM TRANSIENT RESULTS MAXIMUM =
         FORCES MEMBER 70211 70311 70411 70511 70611
```

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     *****************************
     *RESULTS OF LATEST ANALYSIS*
     *****************************

    PROBLEM - ASSIGN 4   TITLE - RTE 80 ONRAMP - TRANSIENT ANALYSIS

    ACTIVE UNITS FEET KIPS DEG. FAHR SEC  LBM

  /--LOAD--/--MEMB--/----TIME----/TYPE/--NODE--/----------------------------------------------------------------------------------/
                                                     XT              YT              ZT              XR              YR              ZR


   1        5011                     FOR 3011      0.0            0,2283194E 03 0,3179614E 03 0,8558008E 04 0,9109724E-05 0,3876112E 02
                                         1012      0,0            0,2283194E 03 0,3179614E 03 0,8558008E 04 0,3192224E 03 0,2538834E 03
                                                                                 TIME VALUES AT MAXIMA
                                     FOR 3011      0.0            0.1478000E 02 0.1514000E 02 0.1472000E 02 0.1514000E 02 0.1472000E 02
                                         1012      0.0            0.1478000E 02 0.1514000E 02 0.1472000E 02 0.1514000E 02 0.1476000E 02

            5035                     FOR 2035      0.0            0,2020601E 03 0,1201692E 04 0.2008404E 04 0,1201701E 04 0.2020600E 03
                                         2036      0.0            0,2020601E 03 0,1201692E 04 0.2008404E 04 0.0           0.0
                                                                                 TIME VALUES AT MAXIMA
                                     FOR 2035      0.0            0.1488000E 02 0.1474000E 02 0.1488000E 02 0.1474000E 02 0.1488000E 02
                                         2036      0.0            0.1488000E 02 0.1474000E 02 0.1488000E 02 0.0           0.0

            5065                     FOR 1065      0.0            0,2187927E 03-0,3023511E 03 0.7186375E 04 0,3035498E 03 0,2456069E 03
                                         3071      0.0            0,2187927E 03 0,3023511E 03 0.7186375E 04 0.6205408E-07 0.3254871E 02
                                                                                 TIME VALUES AT MAXIMA
                                     FOR 1065      0.0            0.1474000E 02 0.1448000E 02 0.1452000E 02 0.1448000E 02 0.1474000E 02
                                         3071      0.0            0.1474000E 02 0.1448000E 02 0.1452000E 02 0.1470000E 02 0.1452000E 02

  LIST DYNAMIC SYSTEM TRANSIENT RESULTS MAXIMUM -
       DISPLACEMENTS JOINT 3011 1012 1021 1031 2035 2036 -
                           1041 1051 1061 1065 3071
```

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******************************
*RESULTS OF LATEST ANALYSIS*
******************************

PROBLEM - ASSIGN 4   TITLE - RTE 80 ONRAMP - TRANSIENT ANALYSIS

ACTIVE UNITS FEET KIPS DEG. FAHR SEC  LBM

/--LOAD--/----JOINT----/----TIME----/--TYPE--/-------------------------------------------------------------------------------/
                                                   XT             YT             ZT             XR             YR             ZR

1        3011     GLO              REL DISP      0.0            0.0            0.0            0.0472668      0.0553924      0.0725049
                                                                            TIME VALUES AT MAXIMA
                                   REL DISP      0.0            0.0            0.0           14.7799997     14.5199995     14.7799997

         1012     GLO              REL DISP      0.0672622      0.0015163      0.0452865      0.0457261      0.0553909      0.0735300
                                                                            TIME VALUES AT MAXIMA
                                   REL DISP     14.7399998     14.7799997     14.7399998     14.5999994     14.5199995     14.7799997

         1021     GLO              REL DISP      0.0294983      0.0008188      0.1208880      0.3234614      0.0526406      0.1226094
                                                                            TIME VALUES AT MAXIMA
                                   REL DISP     14.7599993     14.8799992     14.5400000     14.7199993     14.7199993     15.2399998

         1031     GLO              REL DISP      0.0212533      0.0006720      0.2137909      0.6923972      0.0293318      0.0769905
                                                                            TIME VALUES AT MAXIMA
                                   REL DISP     15.3999996     14.6199999     14.7199993     14.7199993     14.8799992     13.0400000

         2035     GLO              REL DISP      0.0216241      0.0149993      0.2452862      0.7589968      0.0229200      0.1283463
                                                                            TIME VALUES AT MAXIMA
                                   REL DISP     15.0199995     14.7799997     14.7199993     14.7199993     14.8599997     14.7199993

         2036     GLO              REL DISP      0.0178999      0.0161037      0.2454510      0.7596425      0.0404410      0.0671307
                                                                            TIME VALUES AT MAXIMA
                                   REL DISP     14.7799997     14.7799997     14.7199993     14.7199993     14.7599993     14.7599993

         1041     GLO              REL DISP      0.0178158      0.0005299      0.2380581      0.7692764      0.0389454      0.0703368
                                                                            TIME VALUES AT MAXIMA
                                   REL DISP     14.7799997     14.8799992     14.7199993     14.7199993     14.7599993     14.7599993

         1051     GLO              REL DISP      0.0135289      0.0006605      0.1749820      0.5593451      0.0421104      0.0938386
                                                                            TIME VALUES AT MAXIMA
                                   REL DISP     14.9799995     14.7399998     14.7199993     14.7199993     14.7399998     14.7199993

         1061     GLO              REL DISP      0.0227679      0.0003056      0.1022013      0.2893235      0.0427186      0.0488159
                                                                            TIME VALUES AT MAXIMA
                                   REL DISP     14.7199993     14.7399998     14.6999998     14.6999998     14.7199993     14.5199995

         1065     GLO              REL DISP      0.0577230      0.0013605      0.0389352      0.0405371      0.0478818      0.0662333
                                                                            TIME VALUES AT MAXIMA
                                   REL DISP     14.6999998     14.7399998     14.6999998     14.7399998     14.5199995     14.7399998
```

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******************************
*RESULTS OF LATEST ANALYSIS*
******************************

PROBLEM - ASSIGN 4    TITLE - RTE 80 ONRAMP - TRANSIENT ANALYSIS

ACTIVE UNITS FEET KIPS DEG. FAHR SEC  LBM

                                        TRANSIENT LOAD 1

                              TIME HISTORY FOR MEMB   70411    START   MOMENT  Y


       TIME            VALUE             TIME            VALUE             TIME            VALUE             TIME            VALUE

0.0              0.0              2.000000E-02  -1.975042E 02    4.000000E-02  -7.208811E 02    6.000000E-02  -1.450349E 03
7.999998E-02  -2.302664E 03    9.999996E-02  -3.187796E 03    1.199999E-01  -4.010485E 03    1.400000E-01  -4.606047E 03
1.600000E-01  -4.729313E 03    1.799999E-01  -4.165836E 03    2.000000E-01  -2.907096E 03    2.200000E-01  -1.094752E 03
2.399999E-01   1.061271E 03    2.600000E-01   3.317309E 03    2.800000E-01   5.393867E 03    3.000000E-01   6.978777E 03
3.200000E-01   7.783133E 03    3.400000E-01   7.621934E 03    3.600000E-01   6.460543E 03    3.800000E-01   4.414629E 03
4.000000E-01   1.737344E 03    4.200000E-01  -1.237109E 03    4.400000E-01  -4.160086E 03    4.600000E-01  -6.688844E 03
4.800000E-01  -8.454699E 03    4.999999E-01  -9.233063E 03    5.200000E-01  -9.027383E 03    5.400000E-01  -7.961590E 03
5.599999E-01  -6.215457E 03    5.800000E-01  -3.977696E 03    6.000000E-01  -1.463185E 03    6.199999E-01   1.062999E 03
6.400000E-01   3.311866E 03    6.600000E-01   5.106438E 03    6.799999E-01   6.392762E 03    7.000000E-01   7.166871E 03
7.200000E-01   7.409930E 03    7.399999E-01   7.050238E 03    7.600000E-01 . 6.058129E 03    7.800000E-01   4.503484E 03
8.000000E-01   2.543529E 03    8.200000E-01   4.114868E 02    8.400000E-01  -1.705637E 03    8.600000E-01  -3.698219E 03
8.800000E-01  -5.457438E 03    9.000000E-01  -6.794379E 03    9.200000E-01  -7.530379E 03    9.400000E-01  -7.617199E 03
9.600000E-01  -7.107992E 03    9.800000E-01  -6.046383E 03    9.999999E-01  -4.498234E 03    1.020000E 00  -2.531025E 03
1.040000E 00  -2.156615E 02    1.059999E 00   2.303736E 03    1.080000E 00   4.743016E 03    1.099999E 00   6.843410E 03
1.120000E 00   8.425820E 03    1.139999E 00   9.353902E 03    1.160000E 00   9.492785E 03    1.179999E 00   8.718285E 03
1.200000E 00   6.974938E 03    1.219999E 00   4.330406E 03    1.240000E 00   9.796118E 02    1.259999E 00  -2.797808E 03
1.280000E 00  -6.657004E 03    1.299999E 00  -1.011712E 04    1.320000E 00  -1.263601E 04    1.339999E 00  -1.380231E 04
1.360000E 00  -1.350378E 04    1.379999E 00  -1.175017E 04    1.400000E 00  -8.669789E 03    1.419999E 00  -4.598996E 03
1.440000E 00  -7.585426E 01    1.459999E 00   4.334645E 03    1.480000E 00   8.121125E 03    1.499999E 00   1.091314E 04
1.520000E 00   1.253313E 04    1.540000E 00   1.295950E 04    1.559999E 00   1.221771E 04    1.580000E 00   1.043428E 04
1.599999E 00   7.877039E 03    1.620000E 00   4.883395E 03    1.639999E 00   1.750744E 03    1.660000E 00  -1.323558E 03
1.679999E 00  -4.204117E 03    1.700000E 00  -6.749266E 03    1.719999E 00  -8.826102E 03    1.740000E 00  -1.032312E 04
1.759999E 00  -1.118139E 04    1.780000E 00  -1.139239E 04    1.799999E 00  -1.092084E 04    1.820000E 00  -9.624344E 03
1.839999E 00  -7.431781E 03    1.860000E 00  -4.428668E 03    1.879999E 00  -8.482505E 02    1.900000E 00   2.903715E 03
1.919999E 00   6.429492E 03    1.940000E 00   9.356508E 03    1.959999E 00   1.129639E 04    1.980000E 00   1.188272E 04
1.999999E 00   1.091684E 04    2.020000E 00   8.561355E 03    2.040000E 00   5.288531E 03    2.059999E 00   1.630119E 03
2.080000E 00  -2.057761E 03    2.099999E 00  -5.404008E 03    2.120000E 00  -8.003293E 03    2.139999E 00  -9.554598E 03
2.160000E 00  -9.976070E 03    2.179999E 00  -9.359684E 03    2.200000E 00  -7.960066E 03    2.219999E 00  -6.132027E 03
2.240000E 00  -4.250238E 03    2.259999E 00  -2.568784E 03    2.280000E 00  -1.156318E 03    2.299999E 00  -2.860411E 01
2.320000E 00   8.053899E 02    2.339999E 00   1.427833E 03    2.360000E 00   2.050465E 03    2.379999E 00   2.923836E 03
2.400000E 00   4.189633E 03    2.419999E 00   5.809301E 03    2.440000E 00   7.586375E 03    2.459999E 00   9.176926E 03
2.480000E 00   1.026771E 04    2.499999E 00   1.064699E 04    2.520000E 00   1.013197E 04    2.540000E 00   8.541125E 03
2.559999E 00   5.792141E 03    2.580000E 00   2.074017E 03    2.599999E 00  -2.109956E 03    2.620000E 00  -6.178180E 03
2.639999E 00  -9.625590E 03    2.660000E 00  -1.205534E 04    2.679999E 00  -1.315389E 04    2.700000E 00  -1.271659E 04
2.719999E 00  -1.076905E 04    2.740000E 00  -7.562676E 03    2.759999E 00  -3.498279E 03    2.780000E 00   9.726775E 02
```

```
      TIME            VALUE              TIME            VALUE              TIME            VALUE              TIME            VALUE

 2.799999E 00     5.349113E 03      2.820000E 00     9.114262E 03      2.839999E 00     1.186802E 04      2.860000E 00     1.339120E 04
 2.879999E 00     1.362036E 04      2.900000E 00     1.261610E 04      2.919999E 00     1.048443E 04      2.940000E 00     7.404867E 03
 2.959999E 00     3.671452E 03      2.980000E 00    -3.716541E 02      2.999999E 00    -4.414488E 03      3.020000E 00    -8.173602E 03
 3.040000E 00    -1.136594E 04      3.059999E 00    -1.375240E 04      3.080000E 00    -1.518391E 04      3.099999E 00    -1.555252E 04
 3.120000E 00    -1.481934E 04      3.139999E 00    -1.304606E 04      3.160000E 00    -1.032511E 04      3.179999E 00    -6.868766E 03
 3.200000E 00    -2.971873E 03      3.219999E 00     1.064057E 03      3.240000E 00     4.959191E 03      3.259999E 00     8.446414E 03
 3.280000E 00     1.127503E 04      3.299999E 00     1.325504E 04      3.320000E 00     1.431592E 04      3.339999E 00     1.449121E 04
 3.360000E 00     1.377279E 04      3.379999E 00     1.216739E 04      3.400000E 00     9.748422E 03      3.419999E 00     6.617664E 03
 3.440000E 00     2.905893E 03      3.459999E 00    -1.106114E 03      3.480000E 00    -5.021441E 03      3.499999E 00    -8.436789E 03
 3.520000E 00    -1.094903E 04      3.540000E 00    -1.217541E 04      3.559999E 00    -1.187951E 04      3.580000E 00    -1.004817E 04
 3.599999E 00    -6.647676E 03      3.620000E 00    -2.637220E 03      3.639999E 00     2.035779E 03      3.660000E 00     6.553719E 03
 3.679999E 00     1.044779E 04      3.700000E 00     1.331927E 04      3.719999E 00     1.487780E 04      3.740000E 00     1.498938E 04
 3.759999E 00     1.372428E 04      3.780000E 00     1.141366E 04      3.799999E 00     8.549973E 03      3.820000E 00     5.589883E 03
 3.839999E 00     2.833489E 03      3.860000E 00     4.644119E 02      3.879999E 00    -1.388773E 03      3.900000E 00    -2.685713E 03
 3.919999E 00    -3.530365E 03      3.940000E 00    -4.152410E 03      3.959999E 00    -4.762805E 03      3.980000E 00    -5.307762E 03
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1.496000E 01    2.858884E 04    1.498000E 01    1.190537E 04    1.500000E 01   -5.808508E 03    1.502000E 01   -2.198902E 04
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MAXIMUM  6.190029E 04  TIME  1.454000E 01  MINIMUM -7.243438E 04  TIME  1.472000E 01
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 *RESULTS OF LATEST ANALYSIS*
 ****************************

PROBLEM - ASSIGN 4    TITLE - RTE 80 ONRAMP - TRANSIENT ANALYSIS

ACTIVE UNITS FEET KIPS DEG. FAHR SEC  LBM

                                   SPECTRUM IDENT  PLOTRS
                         SPECTRUM FROM    JOINT  1041       ZT TOT
                                   TRANSIENT IDENT 1
                                     DAMPING = 0.020

                         TRUE               TRUE              TRUE              TRUE
    FREQUENCY        DISPLACEMENT       VELOCITY          ACCELERATION            G              PERIOD

   0.360000E 05     -0.164100E-03     -0.388641E-01      0.647840E 02      0.201193E 01       0.100000E-01
   0.180000E 05     -0.659068E-03     -0.777188E-01      0.650491E 02      0.202016E 01       0.200000E-01
   0.120000E 05     -0.146871E-02     -0.116642E 00      0.644244E 02      0.200076E 01       0.300000E-01
   0.900000E 04     -0.262100E-02     -0.156767E 00      0.646631E 02      0.200817E 01       0.400000E-01
   0.720000E 04     -0.406682E-02     -0.199527E 00      0.642227E 02      0.199449E 01       0.500000E-01
   0.599999E 04     -0.592852E-02     -0.238248E 00      0.650242E 02      0.201939E 01       0.600000E-01
   0.514286E 04     -0.809694E-02     -0.286027E 00      0.652574E 02      0.202663E 01       0.699999E-01
   0.450000E 04     -0.109878E-01     -0.287209E 00      0.677563E 02      0.210423E 01       0.799999E-01
   0.400000E 04     -0.137222E-01      0.377575E 00      0.669279E 02      0.207850E 01       0.900000E-01
   0.360000E 04     -0.199055E-01      0.594958E 00      0.784624E 02      0.243672E 01       0.100000E 00
   0.327273E 04      0.244056E-01     -0.905701E 00     -0.795806E 02     -0.247145E 01       0.110000E 00
   0.276923E 04      0.322689E-01      0.958585E 00     -0.751928E 02     -0.233518E 01       0.130000E 00
   0.240000E 04      0.496833E-01     -0.114625E 01     -0.870747E 02     -0.270418E 01       0.150000E 00
   0.211765E 04     -0.672925E-01     -0.133688E 01      0.918899E 02      0.285372E 01       0.170000E 00
   0.189474E 04      0.871134E-01     -0.196328E 01     -0.953323E 02     -0.296063E 01       0.190000E 00
   0.171428E 04     -0.125313E 00      0.225455E 01      0.111920E 03      0.347578E 01       0.210000E 00
   0.156522E 04     -0.185477E 00     -0.354686E 01      0.138390E 03      0.429784E 01       0.230000E 00
   0.144000E 04     -0.341824E 00      0.737159E 01      0.217347E 03      0.674992E 01       0.250000E 00
   0.133333E 04     -0.394416E 00      0.801414E 01      0.212239E 03      0.659128E 01       0.270000E 00
   0.124138E 04     -0.432949E 00      0.811642E 01      0.204515E 03      0.635139E 01       0.290000E 00
   0.116129E 04     -0.614151E 00      0.113071E 02      0.252982E 03      0.785657E 01       0.310000E 00
   0.109091E 04      0.857589E 00     -0.158297E 02     -0.311142E 03     -0.966279E 01       0.330000E 00
   0.102857E 04      0.944853E 00      0.159125E 02     -0.304570E 03     -0.945869E 01       0.350000E 00
   0.972972E 03      0.149890E 01      0.247733E 02     -0.430718E 03     -0.133763E 02       0.370000E 00
   0.923076E 03     -0.229738E 01     -0.370829E 02      0.596614E 03      0.185284E 02       0.390000E 00
   0.878049E 03     -0.209724E 01     -0.336249E 02      0.495086E 03      0.153753E 02       0.410000E 00
   0.837209E 03     -0.140063E 01      0.209215E 02      0.298321E 03      0.926462E 01       0.430000E 00
   0.800000E 03     -0.100046E 01      0.146190E 02      0.194560E 03      0.604224E 01       0.450000E 00
   0.765957E 03      0.123348E 01      0.170229E 02     -0.220620E 03     -0.685154E 01       0.470000E 00
   0.734694E 03     -0.116193E 01      0.159786E 02      0.191449E 03      0.594564E 01       0.490000E 00
   0.705882E 03     -0.898098E 00     -0.132640E 02      0.136740E 03      0.424658E 01       0.510000E 00
   0.654545E 03      0.858987E 00     -0.125319E 02     -0.112788E 03     -0.350273E 01       0.550000E 00
   0.600000E 03     -0.717650E 00     -0.858389E 01      0.790234E 02      0.245414E 01       0.600000E 00
   0.553846E 03     -0.569568E 00     -0.710523E 01      0.531407E 02      0.165033E 01       0.650000E 00
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                              TRUE                TRUE               TRUE                TRUE
  FREQUENCY               DISPLACEMENT          VELOCITY         ACCELERATION             G                    PERIOD

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 0.480000E 03            0.739664E 00        0.834093E 01      -0.522044E 02      -0.162125E 01          0.750000E 00
 0.450000E 03            0.947170E 00        0.955822E 01      -0.583172E 02      -0.181109E 01          0.800000E 00
 0.423529E 03           -0.106656E 01       -0.893705E 01       0.583992E 02       0.181364E 01          0.850001E 00
 0.400000E 03           -0.680974E 00       -0.582823E 01       0.332020E 02       0.103112E 01          0.900000E 00
 0.378947E 03           -0.506512E 00       -0.562349E 01       0.222538E 02       0.691112E 00          0.950000E 00
 0.360000E 03            0.459546E 00       -0.531817E 01      -0.182139E 02      -0.565648E 00          0.100000E 01
 0.327273E 03           -0.675273E 00       -0.545976E 01       0.220521E 02       0.684847E 00          0.110000E 01
 0.300000E 03           -0.885758E 00        0.584733E 01       0.242807E 02       0.754060E 00          0.120000E 01
 0.276923E 03            0.904725E 00       -0.550122E 01      -0.211513E 02      -0.656872E 00          0.130000E 01
 0.257143E 03           -0.933913E 00       -0.499840E 01       0.188349E 02       0.584934E 00          0.140000E 01
 0.240000E 03           -0.122448E 01       -0.682820E 01       0.215018E 02       0.667759E 00          0.150000E 01
 0.225000E 03            0.116538E 01       -0.775194E 01      -0.180177E 02      -0.559556E 00          0.160000E 01
 0.211765E 03           -0.143704E 01       -0.808876E 01       0.196317E 02       0.609680E 00          0.170000E 01
 0.200000E 03            0.167880E 01       -0.773394E 01      -0.205154E 02      -0.637126E 00          0.180000E 01
 0.189474E 03           -0.137245E 01       -0.658991E 01       0.150409E 02       0.467109E 00          0.190000E 01
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FINISH
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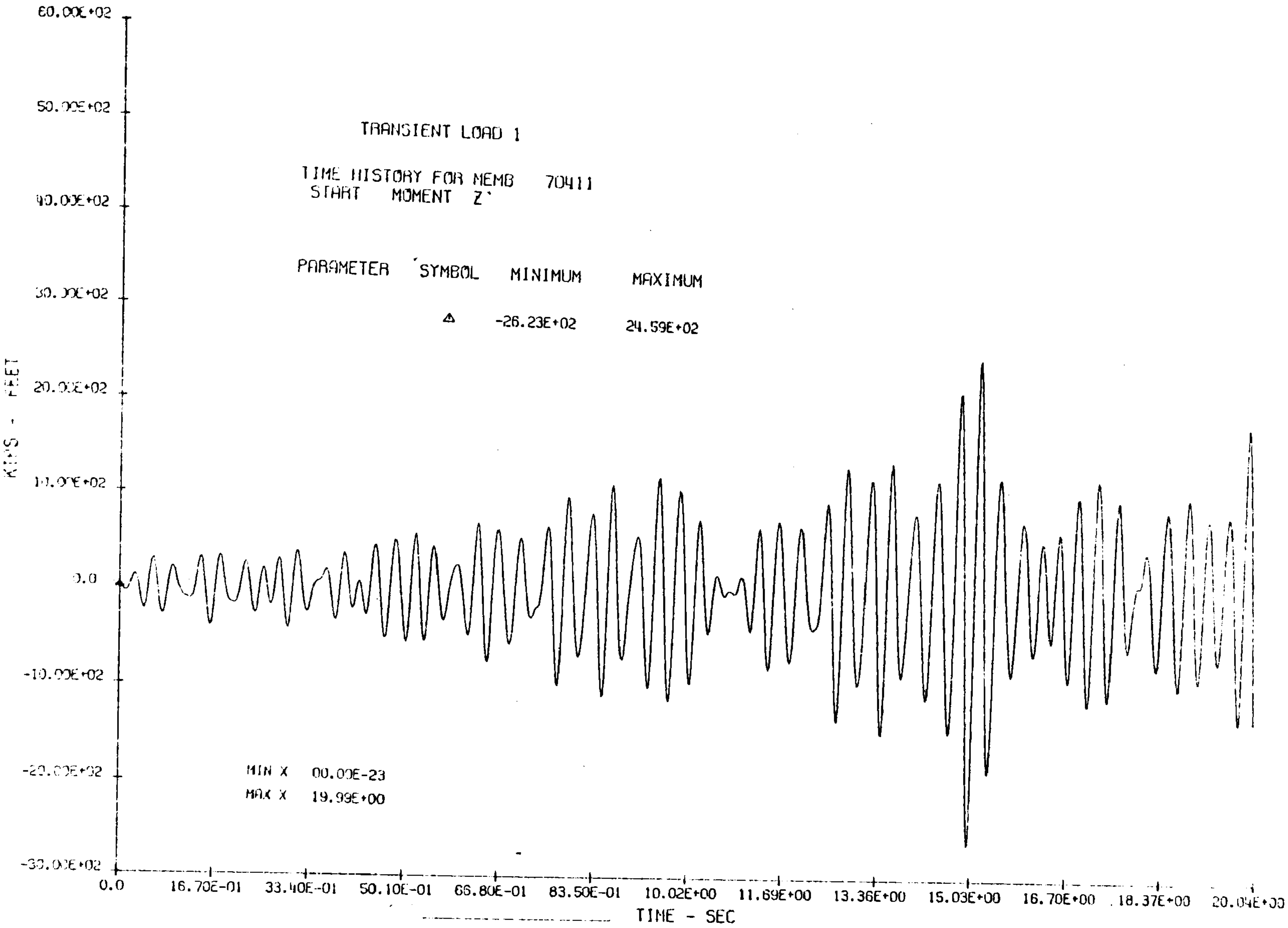
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KIPS - FEET
0.0
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33.40E-01
50.10E-01
66.80E-01
83.50E-01
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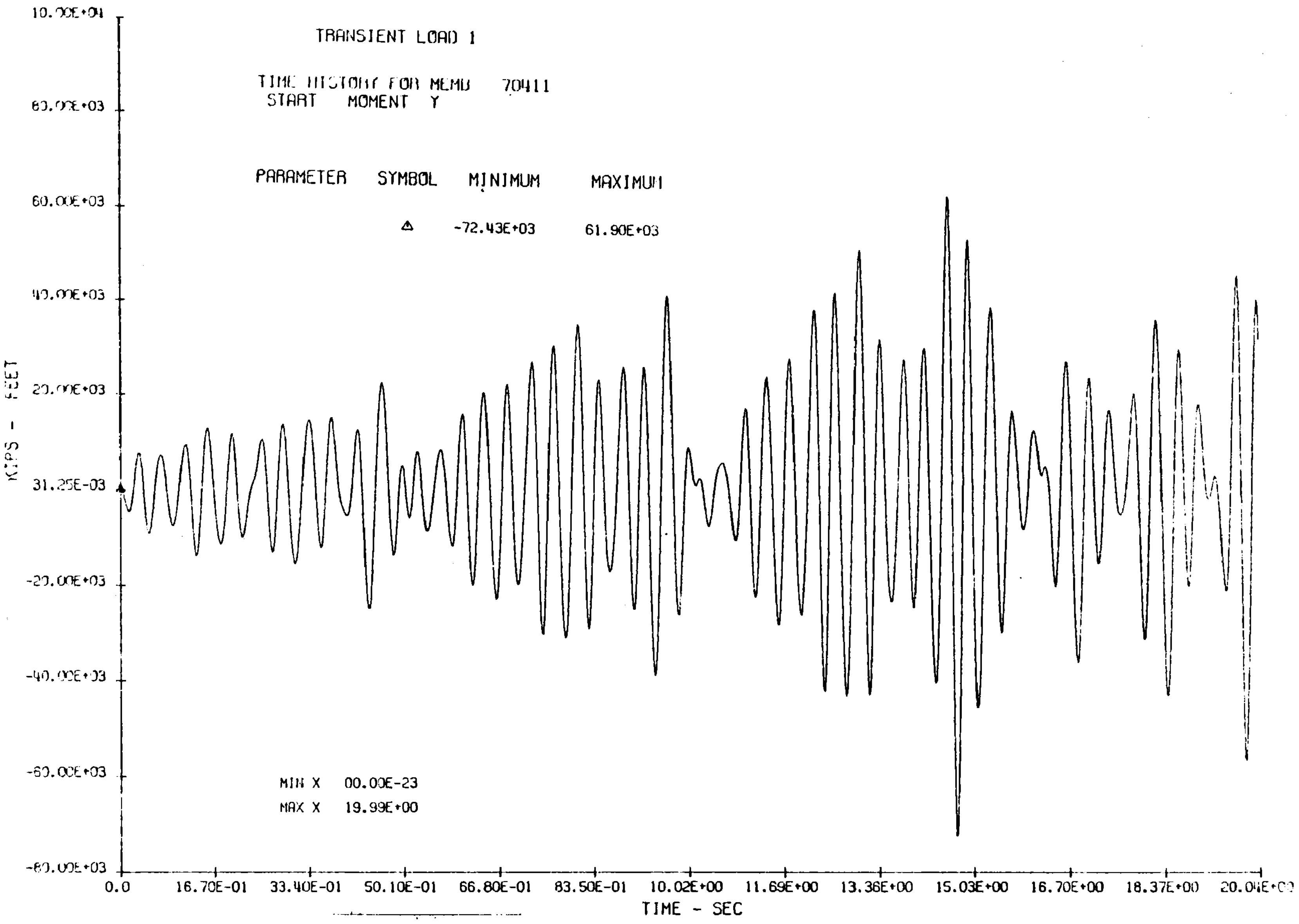

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20.00E+03
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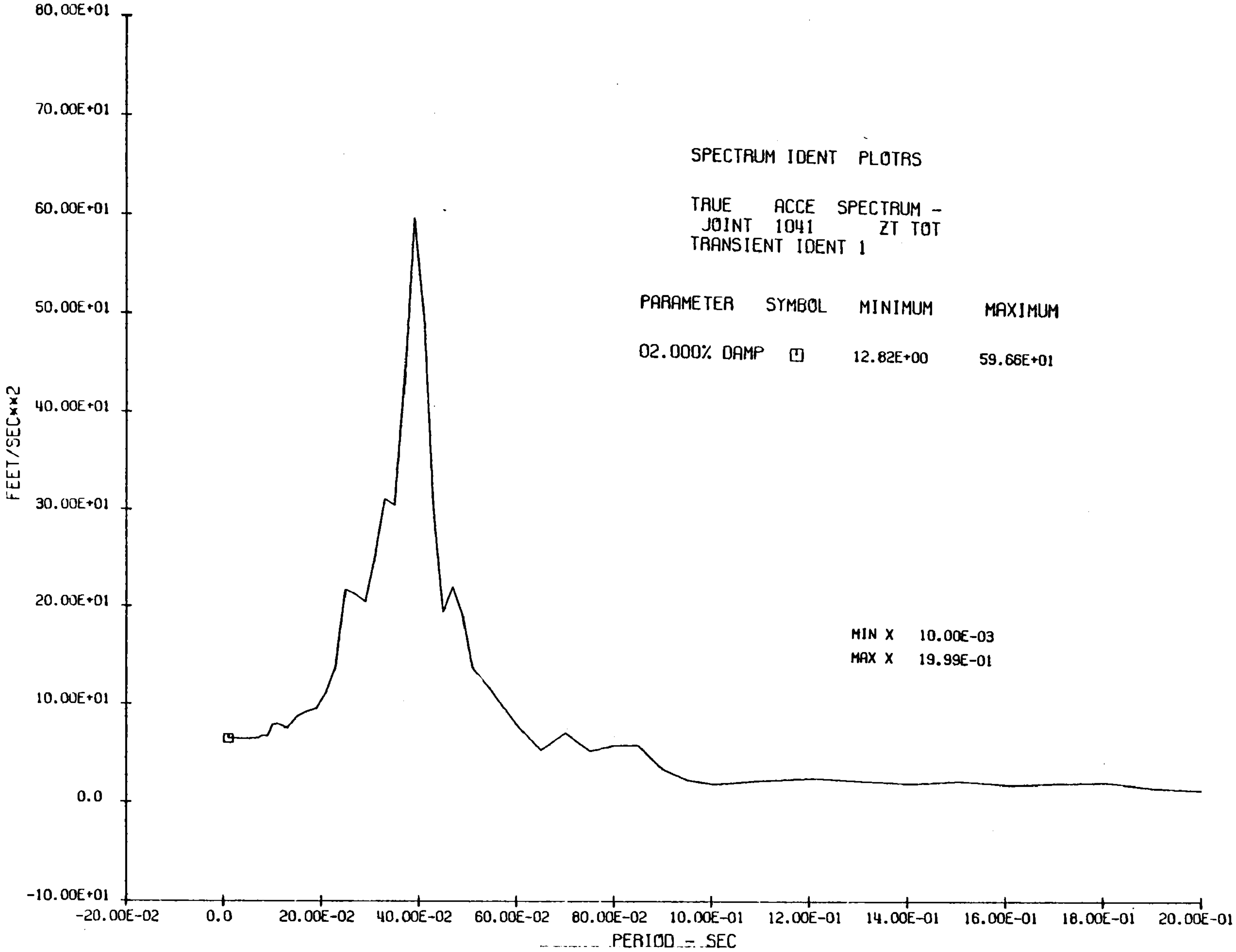
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JOINT 1041 ZT TOT
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MIN X 10.00E-03
MAX X 19.99E-01
FEET/SEC**2
80.00E+01
70.00E+01
60.00E+01
50.00E+01
40.00E+01
30.00E+01
20.00E+01
10.00E+01
0.0
-10.00E+01
-20.00E-02
0.0
20.00E-02
40.00E-02
60.00E-02
80.00E-02
10.00E-01
12.00E-01
14.00E-01
16.00E-01
18.00E-01
20.00E-01
PERIOD - SEC

Appendix B.1

SAP IV
Program Description

SAP is a structural analysis program for the static and dynamic analysis of linear systems.

SAP IV is the version of SAP originally developed at the University of California at Berkeley. This version does not have many of the user enhancements available in the later versions (SAP V and SAP VI). It does, however, have the capability of solving most of the linear dynamic analysis problems encountered by the bridge designer. It also is a relatively simple program from a computer programmer's point of view and may be easily modified to suit the needs of a particular group of users.

SAP IV is written using standard FORTRAN IV and was developed on a CDC computer. The program has also been installed with little effort on IBM and UNIVAC machines. The solution shown in this example was performed on a PRIME 400, one of a group of modern "mini-computers" which may be found in many moderate sized design offices.

The structural systems to be analyzed may be composed of combinations of a number of different structural elements. The program presently contains the following element types:

(a) three-dimensional truss element,

(b) three-dimensional beam element,

(c) plane stress and plane strain element,

(d) two-dimensional axisymmetric solid,

(e) three-dimensional solid,

(f) thick shell element,

(g) thin plate or thin shell element,

(h) boundary element,

(i) pipe element (tangent and bend).

These structural elements can be used in a static or dynamic analysis. The capacity of the program depends mainly on the total number of nodal points in the system, the number of eigenvalues needed in the dynamic analysis and the computer used. There is practically no restriction on the number of elements used, the number of load cases or the order and bandwidth of the stiffness matrix. Each nodal point in the system scan have from zero to six displacement degrees of freedom. The element stiffness and mass matrices are assembled in condensed form therefore, the program is equally efficient in the analysis of one-, two-, or three-dimensional systems.

The formulation of the structure matrices is carried out in the same way in a static or dynamic analysis. The static analysis is continued by solving the equations of equilibrium followed by the computation of element stresses. In a dynamic analysis the choice is between

1. frequency calculations only,
2. frequency calculations followed by response history analysis,
3. frequency calculations followed by response spectrum analysis,
4. response history analysis by direct integration.

To obtain the frequencies and vibration mode shapes, solution routines are used which calculate the required eigenvalues and eigenvectors directly without a transformation of the structure stiffness matrix and mass matrix to a reduced form. In the direct integration an unconditionally stable integration scheme is used, which also operates on the original structure stiffness matrix and mass matrix. This way the program operation and necessary input data for a dynamic analysis is a simple addition to what is needed for a static analysis.

Reference:

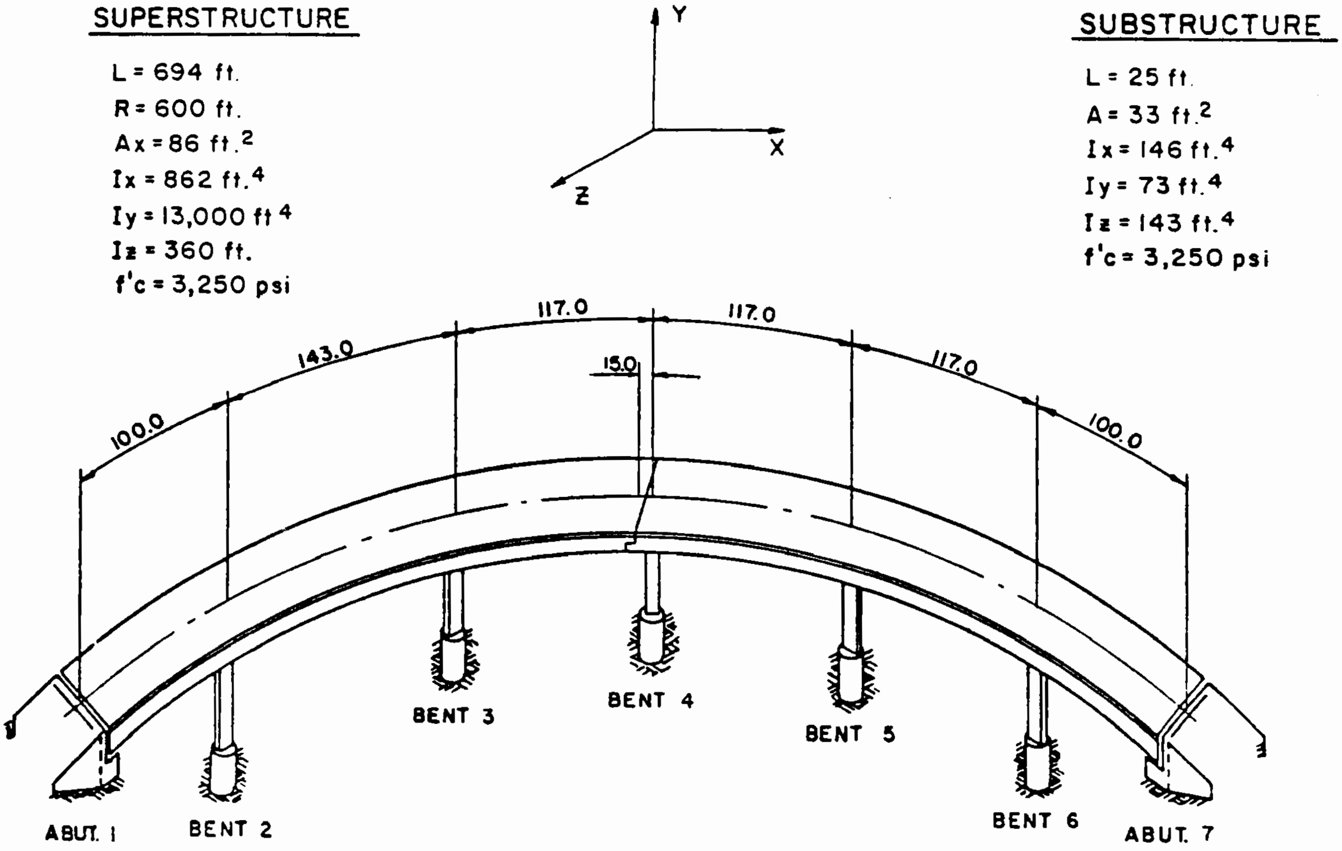
Bathe, K. J., Wilson, E. L., and Peterson, F. E., "SAP IV: A Structural Analysis Program for Static and Dynamic Response of Linear Systems," Report No. EERC-73/11, Earthquake Engineering Research Center, University of California, Berkeley, 1973 (PB 221 967)

Appendix B.2

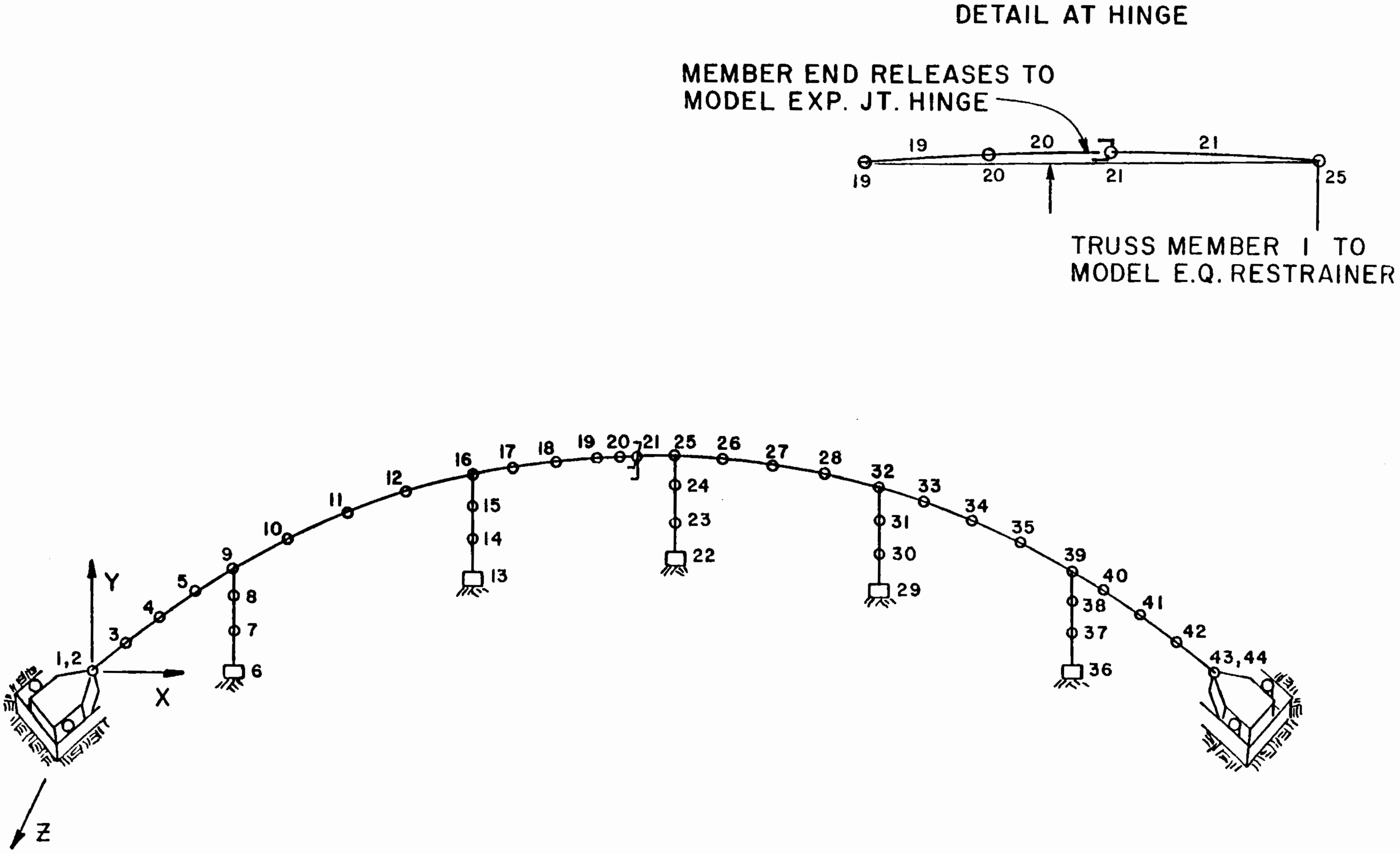
SAP IV
Example Problem

Use SAP IV to perform a frequency analysis (frequencies and mode shapes) of the Route 80 Onramp Undercrossing. Save the results on tape and use the "restart" capabilities to perform a response spectrum analysis for both longitudinal and transverse seismic excitation. Use the CALTRANS design response spectrum with the following properties:

1. Maximum rock acceleration of .5g
2. Depth to "rocklike" material between 10-80 ft.
3. Damping of 5 percent
4. No reduction for ductility or risk



ROUTE 80 ON-RAMP OVERCROSSING



STRUCTURE IDEALIZATION
ROUTE 80 ON-RAMP
(SAP IV)

SAP IV INPUT DATA ORGANIZATION
(Frequency Analysis)

I.	Heading Card	(1)*
II.	Master Control Card	(1)
III.	Nodal Point Data Cards	(45)
IV.	Element Data-Truss Element (hinge restrainer)	
	A. Control Card	(1)
	B. Material Property Card	(1)
	C. Element Load Factors Cards	(4)
	D. Element Data Card	(1)
IV.	Element Data-Beam Elements (deck and columns)	
	A. Control Card	(1)
	B. Material Property Card	(1)
	C. Element Property Cards	(2)
	D. Element Load Factor Cards	(3)
	E. Fixed-End Forces (not used)	
	F. Beam Data Cards	(43)
V.	Concentrated Load/Mass Data (one blank card)	(1)
VI.	Element Load Multipliers (one blank card)	(1)
VII.	Dynamic Analysis	
	A. Mode Shapes and Frequency Control Card	(1)

*Number of input cards used for this example. Note--Refer to reference for special job control cards needed if results are to be saved for subsequent restarts. These will vary depending on the computer used.

80/80 Listing of Input Card Images

Free Vibration Analysis

```
ROUTE 80 ONRAMP UNDERCROSSING - RESPONSE SPECTRUM ANALYSIS - SAP IV
   45    2        18    1
    1    1    1    1    1    1    1  10000.00     75.30  10000.00
    2                                10000.84     75.30   9999.45
    3                                10021.35     75.30   9986.72
    4                                10042.38     75.30   9974.83
    5                                10063.87     75.30   9963.83
    6    1    1    1    1    1    1  10085.79     50.00   9953.71
    7                                10085.79     58.43   9953.71
    8                                10085.79     66.87   9953.71
    9                                10085.79     75.30   9953.71
   10                                10118.29     75.30   9940.66
   11                                10151.51     75.30   9929.56
   12                                10185.32     75.30   9920.46
   13    1    1    1    1    1    1  10219.62     50.00   9913.39
   14                                10219.62     58.43   9913.39
   15                                10219.62     66.87   9913.39
   16                                10219.62     75.30   9913.39
   17                                10244.01     75.30   9909.65
   18                                10268.53     75.30   9906.93
   19                                10293.14     75.30   9905.24
   20                                10317.81     75.30   9904.58
   21                                10318.81     75.30   9904.58
   22    1    1    1    1    1    1  10333.81     50.00   9904.71
   23                                10333.81     58.43   9904.71
   24                                10333.81     66.87   9904.71
   25                                10333.81     75.30   9904.71
   26                                10362.45     75.30   9906.03
   27                                10391.00     75.30   9908.74
   28                                10419.37     75.30   9912.84
   29    1    1    1    1    1    1  10447.52     50.00   9918.32
   30                                10447.52     58.43   9918.32
   31                                10447.52     66.87   9918.32
   32                                10447.52     75.30   9918.32
   33                                10475.36     75.30   9925.16
   34                                10502.84     75.30   9933.36
   35                                10529.88     75.30   9942.88
   36    1    1    1    1    1    1  10556.43     50.00   9953.71
   37                                10556.43     58.43   9953.71
   38                                10556.43     66.87   9953.71
   39                                10556.43     75.30   9953.71
   40                                10578.36     75.30   9963.83
   41                                10599.85     75.30   9974.84
   42                                10620.87     75.30   9986.71
   43                                10641.39     75.30   9999.45
   44    1    1    1    1    1    1  10642.23     75.30  10000.00
   45    2    2    2    2    2    2  10320.70     75.30  10492.84
    1    1    1
    1  2016000.             .004658    .04910      .150
         0         0         0         0
         0         0         0         0
         0         0         0         0
         0         0         0         0
    1   19   25    1
    2   43    2         1
    1   432000.       .18  .0046583      .150
    1      86.0                             862.0     360.0   13000.0
    2      33.0                             146.0      73.0     143.0
       0.0       0.0       0.0       0.0
       0.0       0.0       0.0       0.0
```

80/80 Listing of Input Card Images

(continued)

```
     0.0       0.0       0.0       0.0
  1    1    2   45    1    1                     100011
  2    2    3   45    1    1
  3    3    4   45    1    1
  4    4    5   45    1    1
  5    5    9   45    1    1
  6    6    7   45    1    2
  7    7    8   45    1    2
  8    8    9   45    1    2
  9    9   10   45    1    1
 10   10   11   45    1    1
 11   11   12   45    1    1
 12   12   16   45    1    1
 13   13   14   45    1    2
 14   14   15   45    1    2
 15   15   16   45    1    2
 16   16   17   45    1    1
 17   17   18   45    1    1
 18   18   19   45    1    1
 19   19   20   45    1    1
 20   20   21   45    1    1                           100011
 21   21   25   45    1    1
 22   22   23   45    1    2
 23   23   24   45    1    2
 24   24   25   45    1    2
 25   25   26   45    1    1
 26   26   27   45    1    1
 27   27   28   45    1    1
 28   28   32   45    1    1
 29   29   30   45    1    2
 30   30   31   45    1    2
 31   31   32   45    1    2
 32   32   33   45    1    1
 33   33   34   45    1    1
 34   34   35   45    1    1
 35   35   39   45    1    1
 36   36   37   45    1    2
 37   37   38   45    1    2
 38   38   39   45    1    2
 39   39   40   45    1    1
 40   40   41   45    1    1
 41   41   42   45    1    1
 42   42   43   45    1    1
 43   43   44   45    1    1                           100011

  0    1
```

Output Results Free Vibration Analysis

```
PRINT OF FREQUENCIES

MODE          CIRCULAR
NUMBER        FREQUENCY       FREQUENCY      PERIOD
             (RAD/SEC)    (CYCLES/SEC)     (SEC)

     1        0.1555E 02    0.2475E 01    0.4041E 00

     2        0.1602E 02    0.2550E 01    0.3922E 00

     3        0.1645E 02    0.2619E 01    0.3819E 00

     4        0.1832E 02    0.2916E 01    0.3429E 00

     5        0.2008E 02    0.3196E 01    0.3129E 00

     6        0.2116E 02    0.3367E 01    0.2970E 00

     7        0.2382E 02    0.3790E 01    0.2638E 00

     8        0.2566E 02    0.4083E 01    0.2449E 00

     9        0.2655E 02    0.4226E 01    0.2367E 00

    10        0.2819E 02    0.4487E 01    0.2229E 00

    11        0.3087E 02    0.4913E 01    0.2035E 00

    12        0.4528E 02    0.7206E 01    0.1388E 00

    13        0.4977E 02    0.7922E 01    0.1262E 00

    14        0.5563E 02    0.8853E 01    0.1130E 00

    15        0.5925E 02    0.9430E 01    0.1060E 00

    16        0.7064E 02    0.1124E 02    0.8894E-01

    17        0.7903E 02    0.1258E 02    0.7950E-01

    18        0.8628E 02    0.1373E 02    0.7283E-01


PRINT OF EIGENVECTORS
```

```
N O D E   D I S P L A C E M E N T S / R O T A T I O N S

 NODE  EIGEN-            X-               Y-               Z-               X-               Y-               Z-
NUMBER VECTOR       TRANSLATION      TRANSLATION      TRANSLATION        ROTATION         ROTATION         ROTATION

   45      1     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
           2     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
           3     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
           4     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
           5     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
           6     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
           7     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
           8     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
           9     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
          10     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
          11     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
          12     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
          13     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
          14     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
          15     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
          16     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
          17     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
          18     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00

   44      1     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
           2     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
           3     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
           4     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
           5     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
           6     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
           7     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
           8     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
           9     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
          10     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
          11     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
          12     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
          13     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
          14     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
          15     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
          16     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
          17     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00
          18     0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00      0.00000E 00

   43      1    -0.26054E-03     -0.76618E-04     -0.27451E-03     -0.46241E-04     -0.86594E-04      0.60917E-04
           2     0.60886E-03      0.12729E-04      0.31463E-03      0.36648E-05     -0.70018E-04     -0.12744E-04
           3     0.20234E-01     -0.40243E-03      0.13556E-01     -0.20617E-03      0.25623E-03      0.34392E-03
           4     0.49490E-01     -0.21584E-02      0.32400E-01     -0.11772E-02     -0.37889E-05      0.17980E-02
           5    -0.22118E-01      0.45264E-04     -0.15244E-01     -0.12554E-05     -0.63497E-03     -0.54646E-04
           6    -0.42898E-01     -0.85561E-03     -0.27649E-01     -0.45693E-03      0.36619E-03      0.71904E-03
           7     0.13040E-02     -0.42171E-02      0.75779E-03     -0.23092E-02     -0.80031E-04      0.35065E-02
           8     0.11425E-01      0.11283E-02      0.85642E-02      0.64530E-03      0.90289E-03     -0.92018E-03
           9    -0.80495E-02     -0.25114E-02     -0.50213E-02     -0.13674E-02      0.20766E-03      0.20932E-02
          10     0.45754E-02      0.12743E-02      0.29387E-02      0.69558E-03     -0.47567E-04     -0.10610E-02
          11    -0.11053E-01     -0.61266E-02     -0.71811E-02     -0.33485E-02      0.46458E-04      0.50977E-02
          12     0.69645E-02     -0.28384E-04      0.65823E-02      0.15768E-04      0.16848E-02      0.44042E-04
          13     0.26208E-02     -0.73336E-04      0.16817E-02     -0.40607E-04     -0.28596E-04      0.60653E-04
          14    -0.50854E-02      0.50471E-04     -0.52117E-02      0.47919E-05     -0.15679E-02     -0.56862E-04
          15    -0.14242E-01      0.77428E-03     -0.92355E-02      0.42448E-03      0.74686E-04     -0.64303E-03
          16     0.39598E-03      0.35746E-02      0.26312E-03      0.19529E-02      0.32059E-05     -0.29724E-02
          17     0.35497E-02      0.24567E-02      0.22959E-02      0.13418E-02     -0.23553E-04     -0.20427E-02
          18    -0.26562E-02      0.82421E-03     -0.17805E-02      0.45020E-03     -0.34387E-04     -0.68522E-03

   42      1     0.84283E-03     -0.17562E-02     -0.20511E-02     -0.14551E-03     -0.86573E-04     -0.21535E-04
           2     0.14991E-02      0.28447E-03     -0.11202E-02     -0.79172E-04     -0.69715E-04     -0.57838E-04
           3     0.16961E-01     -0.91450E-02      0.18784E-01      0.18069E-03      0.25374E-03      0.44671E-03
           4     0.49503E-01     -0.48860E-01      0.32253E-01     -0.84713E-03     -0.85560E-05      0.12725E-02
           5    -0.14078E-01      0.98182E-03     -0.28116E-01     -0.64934E-03     -0.61581E-03     -0.42372E-03
           6    -0.47472E-01     -0.19141E-01     -0.20143E-01     -0.73409E-04      0.35956E-03      0.63128E-03
           7     0.23153E-02     -0.93542E-01     -0.87556E-03     -0.17753E-02     -0.78603E-04      0.21461E-02
           8     0.92022E-04      0.24860E-01      0.26735E-01      0.11700E-02      0.85589E-03     -0.13387E-03
           9    -0.10635E-01     -0.55060E-01     -0.82113E-03     -0.81860E-03      0.19765E-03      0.13322E-02
          10     0.51613E-02      0.27741E-01      0.19693E-02      0.44733E-03     -0.45617E-04     -0.62918E-03
          11    -0.11608E-01     -0.13175E 00     -0.62083E-02     -0.21644E-02      0.46154E-04      0.27915E-02
          12    -0.13764E-01     -0.52010E-03      0.39749E-01      0.79676E-03      0.14956E-02      0.48774E-03
          13     0.29542E-02     -0.14309E-02      0.10981E-02     -0.30427E-04     -0.26312E-04      0.10341E-04
          14     0.14027E-01      0.91934E-03     -0.35743E-01     -0.57617E-03     -0.13466E-02     -0.35872E-03
          15    -0.15015E-01      0.14321E-01     -0.76226E-02      0.16423E-03      0.71552E-04     -0.94664E-04
          16     0.35419E-03      0.62020E-01      0.31498E-03      0.33627E-03      0.18511E-05     -0.12346E-03
          17     0.37684E-02      0.40704E-01      0.17805E-02      0.10459E-03     -0.21679E-04      0.10527E-03
          18    -0.21924E-02      0.13131E-01     -0.23746E-02      0.40090E-05     -0.24690E-04      0.69277E-04
```

```
41     1   0.18718E-02  -0.25447E-02  -0.38719E-02  -0.23481E-03  -0.86785E-04  -0.12040E-03
       2   0.23216E-02   0.41160E-03  -0.25799E-02  -0.16850E-03  -0.69072E-04  -0.91913E-04
       3   0.13926E-01  -0.13202E-01   0.24028E-01   0.66350E-03   0.24783E-03   0.38869E-03
       4   0.49505E-01  -0.69076E-01   0.31957E-01  -0.81109E-04  -0.21242E-04   0.21689E-04
       5  -0.69414E-02   0.14138E-02  -0.40519E-01  -0.13370E-02  -0.56505E-03  -0.72540E-03
       6  -0.51458E-01  -0.26081E-01  -0.12649E-01   0.49962E-03   0.34176E-03   0.22905E-03
       7   0.32225E-02  -0.12350E 00  -0.24971E-02  -0.37236E-03  -0.75077E-04  -0.64506E-03
       8  -0.94963E-02   0.31960E-01   0.43468E-01   0.14930E-02   0.73009E-03   0.98083E-03
       9  -0.12787E-01  -0.69828E-01   0.31357E-02   0.26320E-03   0.17074E-03  -0.30196E-03
      10   0.56397E-02   0.34273E-01   0.10408E-02  -0.70688E-04  -0.40355E-04   0.24281E-03
      11  -0.12047E-01  -0.15544E 00  -0.51873E-02   0.31783E-03   0.45290E-04  -0.16186E-02
      12  -0.29105E-01  -0.44955E-03   0.66298E-01   0.16210E-02   0.10009E-02   0.85817E-03
      13   0.31715E-02  -0.10557E-02   0.56682E-03  -0.71949E-05  -0.20349E-04  -0.59474E-04
      14   0.27214E-01   0.54402E-03  -0.58395E-01  -0.11931E-02  -0.77674E-03  -0.60116E-03
      15  -0.15328E-01   0.71702E-02  -0.59206E-02  -0.21076E-03   0.63262E-04   0.61152E-03
      16   0.33119E-03   0.13838E-01   0.30928E-03  -0.15790E-02  -0.17206E-05   0.30081E-02
      17   0.37840E-02   0.12111E-02   0.12483E-02  -0.11800E-02  -0.16995E-04   0.21510E-02
      18  -0.18299E-02  -0.16929E-02  -0.25505E-02  -0.40469E-03  -0.11155E-05   0.73024E-03

40     1   0.28334E-02  -0.19454E-02  -0.57470E-02  -0.32379E-03  -0.88020E-04  -0.21553E-03
       2   0.30769E-02   0.30613E-03  -0.40606E-02  -0.26241E-03  -0.68735E-04  -0.11760E-03
       3   0.11139E-01  -0.10057E-01   0.29239E-01   0.11842E-02   0.24192E-03   0.26279E-03
       4   0.49552E-01  -0.50634E-01   0.31067E-01   0.75279E-03  -0.39394E-04  -0.13393E-02
       5  -0.93682E-03   0.11081E-02  -0.51832E-01  -0.20537E-02  -0.49340E-03  -0.97382E-03
       6  -0.54757E-01  -0.17595E-01  -0.54095E-02   0.10764E-02   0.31589E-03  -0.17164E-03
       7   0.40146E-02  -0.79062E-01  -0.40719E-02   0.78798E-03  -0.71013E-04  -0.29354E-02
       8  -0.16736E-01   0.19295E-01   0.57190E-01   0.19483E-02   0.54792E-03   0.18347E-02
       9  -0.14372E-01  -0.40887E-01   0.64128E-02   0.10988E-02   0.13131E-03  -0.14245E-02
      10   0.59838E-02   0.18852E-01   0.22312E-03  -0.41845E-03  -0.32589E-04   0.77862E-03
      11  -0.12363E-01  -0.75725E-01  -0.41379E-02   0.16666E-02   0.43904E-04  -0.38142E-02
      12  -0.36852E-01   0.38864E-04   0.80484E-01   0.24690E-02   0.31885E-03   0.11856E-02
      13   0.32457E-02   0.22942E-03   0.16074E-03  -0.24945E-04  -0.12178E-04  -0.51187E-04
      14   0.32088E-01  -0.39802E-03  -0.66890E-01  -0.18047E-02  -0.19111E-04  -0.85503E-03
      15  -0.15141E-01  -0.70625E-02  -0.42609E-02  -0.37724E-04   0.51688E-04   0.28052E-03
      16   0.34394E-03  -0.49866E-01   0.20123E-03  -0.45176E-03  -0.69466E-05   0.43432E-03
      17   0.35818E-02  -0.38973E-01   0.76127E-03  -0.18821E-03  -0.11285E-04  -0.38716E-04
      18  -0.16481E-02  -0.13652E-01  -0.21110E-02  -0.14527E-04   0.26263E-04  -0.10351E-03

39     1   0.37397E-02   0.51915E-04  -0.77107E-02  -0.42111E-03  -0.91584E-04  -0.30124E-03
       2   0.37716E-02  -0.29405E-04  -0.55777E-02  -0.35907E-03  -0.69765E-04  -0.13770E-03
       3   0.85714E-02  -0.87412E-04   0.34465E-01   0.16898E-02   0.24158E-03   0.17003E-03
       4   0.49650E-01   0.85940E-04   0.29801E-01   0.13293E-02  -0.59112E-04  -0.21588E-02
       5   0.38930E-02   0.33686E-03  -0.61710E-01  -0.27892E-02  -0.41798E-03  -0.11844E-02
       6  -0.57363E-01   0.11075E-02   0.14103E-02   0.15008E-02   0.28689E-03  -0.25329E-03
       7   0.47015E-02  -0.65233E-03  -0.56085E-02   0.80733E-03  -0.68930E-04  -0.28451E-02
       8  -0.21462E-01   0.35566E-03   0.66876E-01   0.27956E-02   0.34411E-03   0.18732E-02
       9  -0.15345E-01  -0.10906E-02   0.88589E-02   0.10540E-02   0.86085E-04  -0.75504E-03
      10   0.61833E-02   0.85725E-03  -0.43273E-03  -0.27115E-03  -0.23549E-04   0.28456E-03
      11  -0.12548E-01  -0.69156E-02  -0.30791E-02   0.20245E-03   0.42059E-04  -0.16770E-03
      12  -0.36884E-01   0.60508E-04   0.79460E-01   0.33319E-02  -0.38243E-03   0.14827E-02
      13   0.31756E-02  -0.36316E-03  -0.82761E-04  -0.97704E-04  -0.38600E-05   0.69760E-04
      14   0.28698E-01   0.32821E-04  -0.58869E-01  -0.24069E-02   0.70830E-03  -0.11274E-02
      15  -0.14453E-01   0.36888E-02  -0.27335E-02   0.65958E-03   0.39371E-04  -0.11083E-02
      16   0.39779E-03   0.11308E-01  -0.35230E-04   0.21593E-02  -0.12922E-04  -0.49856E-02
      17   0.31794E-02   0.32363E-02   0.33944E-03   0.11162E-02  -0.68931E-05  -0.26698E-02
      18  -0.15939E-02  -0.66776E-03  -0.11000E-02   0.23670E-03   0.45561E-04  -0.56741E-03

38     1   0.16422E-02   0.34622E-04  -0.41361E-02  -0.40624E-03  -0.61068E-04  -0.19707E-03
       2   0.22795E-02  -0.19610E-04  -0.27877E-02  -0.29390E-03  -0.46519E-04  -0.19864E-03
       3   0.69960E-02  -0.58296E-04   0.19212E-01   0.18154E-02   0.16108E-03  -0.45076E-03
       4   0.28724E-01   0.57317E-04   0.17113E-01   0.15690E-02  -0.39416E-04  -0.26137E-02
       5  -0.15645E-02   0.22468E-03  -0.35293E-01  -0.32497E-02  -0.27871E-03  -0.20645E-03
       6  -0.43506E-01   0.73867E-03  -0.45948E-02   0.76238E-04   0.19130E-03   0.30167E-02
       7  -0.71956E-02  -0.43513E-03  -0.71914E-02  -0.29424E-03  -0.45963E-04  -0.25011E-03
       8  -0.88820E-02   0.23726E-03   0.39113E-01   0.35202E-02   0.22745E-03   0.11312E-02
       9  -0.14227E-01  -0.72756E-03   0.26103E-02   0.46737E-03   0.57401E-04   0.80549E-03
      10   0.56609E-02   0.57190E-03   0.69927E-03  -0.23252E-04  -0.15702E-04  -0.32437E-03
      11  -0.99537E-02  -0.46141E-02  -0.30506E-02  -0.16110E-03   0.28045E-04   0.65820E-03
      12  -0.21842E-01   0.40400E-04   0.46527E-01   0.41732E-02  -0.25500E-03   0.19366E-02
      13   0.26331E-02  -0.24254E-03   0.30997E-03  -0.49208E-05  -0.25738E-05  -0.16536E-03
      14   0.17274E-01   0.21928E-04  -0.34755E-01  -0.30863E-02   0.47229E-03  -0.15139E-02
      15  -0.15022E-01   0.24652E-02  -0.45567E-02  -0.13763E-03   0.26252E-04   0.74455E-03
      16  -0.18699E-01   0.75640E-02  -0.82543E-02   0.13233E-04  -0.86166E-05  -0.55357E-04
      17  -0.78173E-02   0.21666E-02  -0.40102E-02   0.26655E-04  -0.45963E-05  -0.18632E-03
      18  -0.33894E-02  -0.44739E-03  -0.17408E-02  -0.53795E-04   0.30380E-04   0.75143E-04

37     1   0.40507E-03   0.17303E-04  -0.12102E-02  -0.26608E-03  -0.30516E-04  -0.96688E-04
       2   0.72025E-03  -0.98003E-05  -0.77330E-03  -0.17431E-03  -0.23246E-04  -0.15296E-03
       3   0.25459E-02  -0.29134E-04   0.57724E-02   0.12538E-02   0.80493E-04  -0.50923E-03
       4   0.88442E-02   0.28645E-04   0.52436E-02   0.11288E-02  -0.19696E-04  -0.18998E-02
       5  -0.12179E-02   0.11229E-03  -0.10785E-01  -0.23247E-02  -0.13927E-03   0.19078E-03
       6  -0.15383E-01   0.36918E-03  -0.24580E-02  -0.42763E-03   0.95594E-04   0.31133E-02
       7  -0.41243E-02  -0.21748E-03  -0.29739E-02  -0.56578E-03  -0.22968E-04   0.70241E-03
       8  -0.20528E-02   0.11859E-03   0.12125E-01   0.25967E-02   0.11466E-03   0.50660E-03
       9  -0.54133E-02  -0.36366E-03   0.31847E-03   0.11512E-03   0.28684E-04   0.10634E-02
      10   0.21460E-02   0.28587E-03   0.39883E-03   0.67995E-04  -0.78466E-05  -0.42216E-03
      11  -0.35887E-02  -0.23065E-02  -0.11858E-02  -0.23093E-03   0.14014E-04   0.72029E-03
      12  -0.68378E-02   0.20204E-04   0.14463E-01   0.30024E-02  -0.12743E-03   0.14574E-02
      13   0.96926E-03  -0.12131E-03   0.16591E-03   0.28793E-04  -0.12861E-05  -0.19274E-03
      14   0.54485E-02   0.10971E-04  -0.10875E-01  -0.23177E-02   0.23601E-03  -0.11570E-02
      15  -0.59564E-02   0.12335E-02  -0.17944E-02  -0.37071E-03   0.13118E-04   0.11493E-02
      16  -0.95565E-02   0.37870E-02  -0.41727E-02  -0.73727E-03  -0.43057E-05   0.16765E-02
      17  -0.43258E-02   0.10852E-02  -0.20723E-02  -0.36324E-03  -0.22468E-05   0.74172E-03
      18  -0.15468E-02  -0.22420E-03  -0.76183E-03  -0.14132E-03   0.15181E-04   0.28256E-03
```

SAP IV INPUT DATA ORGANIZATION
(Response Spectrum Analysis-Restart)

I.	Heading Card	(1)
II.	Master Control Card	(1)
III.-VI.	Omitted for restart	
VII.	Dynamic Analysis	
	A. Omitted for restart	
	B. Response History Analysis-not used	
	C. Response Spectrum Analysis	
	1. Control Card	(1)
	2. Spectrum Cards	
	a. Heading Card	(1)
	b. Control Card	(1)
	c. Spectrum Data Cards	(55)

*Note--Refer to reference for special job control cards needed if frequency results are to be restored for a restart.

80/80 Listing of Input Card Images

Response Spectrum Analysis

```
ROUTE 80 ONRAMP UNDERCROSSING - RESPONSE SPECTRUM ANALYSIS - SAP IV
   45    2         18    -3
       1.0        0.0         0.0     1
 AASHTO DESIGN SPECTRUM   - .5G PEAK ROCK ACCELERATION - 10 TO 80 FT
   55      32.2
        .000     0.7180
        .001     0.7180
        .025     0.9100
        .050     1.0560
        .075     1.1770
        .100     1.3040
        .125     1.3880
        .150     1.4740
        .175     1.5420
        .200     1.5800
        .225     1.6070
        .250     1.6220
        .275     1.6300
        .300     1.6290
        .325     1.6220
        .350     1.6090
        .375     1.5920
        .400     1.5650
        .425     1.5400
        .450     1.5120
        .475     1.4820
        .500     1.4480
        .525     1.4070
        .550     1.3670
        .575     1.3280
        .600     1.2900
        .650     1.2140
        .700     1.1520
        .750     1.0820
        .800     1.0200
        .850     0,9620
        .900     0.9040
        .950     0.8550
       1.000     0.8100
       1.100     0.7280
       1.2000    0.6570
       1.300     0.5990
       1.400     0.5550
       1.500     0.5140
       1.6000    0.4800
       1.700     0.4500
       1.800     0.4220
       1.900     0.3980
       2.000     0.3780
       2.200     0.3410
       2.400     0.3110
       2.600     0.2850
       2.800     0.2610
       3.000     0.2380
       3.500     0.1982
       4.000     0.1625
       4.500     0.1286
       5.000     0.1093
       5.500     0.1093
       6.000     0.1093
```

```
R E S P O N S E   S P E C T R U M   A N A L Y S I S


DIRECTION FACTORS

          X =     0.0000     Y =     0.0000     Z =     1.0000


INDICATOR FOR DISPLACEMENT OR ACCELERATION SPECTRUM =       1

EQ.0   DISPLACEMENT
EQ.1   ACCELERATION


MODAL PARTICIPATION FACTORS

MODE    X-DIRECTION    Y-DIRECTION    Z-DIRECTION

   1    -0.6905E 00    -0.2799E 01    -0.5445E 01

   2     0.1045E 02    -0.7579E-01    -0.6444E 01

   3     0.4489E 01    -0.1191E 01     0.1287E 02

   4     0.7326E 01    -0.5401E-01     0.1271E 01

   5     0.3116E 01    -0.4709E 00    -0.1102E 01

   6    -0.8071E 01    -0.2068E-01    -0.3667E 00

   7    -0.9590E 00    -0.1120E 01    -0.3893E 00

   8    -0.4361E 00    -0.9839E 00     0.4302E 01

   9     0.1323E 01     0.2430E 01    -0.7540E 00

  10     0.1698E 01    -0.8450E 01    -0.3708E 00

  11    -0.1936E 01    -0.9959E 01    -0.1424E 00

  12    -0.2129E 01     0.5431E-01     0.2880E 00

  13     0.1295E 01    -0.4800E 00    -0.2853E 00

  14    -0.2643E 00     0.7384E-01    -0.2877E 01

  15    -0.1478E 01     0.2252E 01    -0.2805E-01

  16    -0.4092E 00    -0.1005E 01    -0.2330E 00

  17    -0.1641E 00     0.2060E 00    -0.4847E-01

  18    -0.1735E 00    -0.3305E 00     0.1145E 00
```

```
N O D E   D I S P L A C E M E N T S / R O T A T I O N S

  NODE     MODE           X-              Y-              Z-              X-              Y-              Z-
 NUMBER   NUMBER     TRANSLATION     TRANSLATION     TRANSLATION       ROTATION        ROTATION        ROTATION

    45       1      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
             2      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
             3      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
             4      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
             5      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
             6      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
             7      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
             8      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
             9      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
            10      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
            11      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
            12      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
            13      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
            14      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
            15      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
            16      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
            17      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
            18      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
            19      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00

    44       1      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
             2      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
             3      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
             4      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
             5      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
             6      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
             7      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
             8      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
             9      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
            10      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
            11      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
            12      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
            13      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
            14      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
            15      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
            16      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
            17      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
            18      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00
            19      0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00     0.00000E 00

    43       1     -0.37406E-04    -0.11000E-04    -0.39412E-04    -0.66388E-05    -0.12432E-04     0.87459E-05
             2      0.12561E-02     0.26260E-04     0.64910E-03     0.75607E-05    -0.14445E-03    -0.26291E-04
             3      0.17120E-01    -0.34049E-03     0.11470E-01    -0.17444E-03     0.21680E-03     0.29099E-03
             4      0.56081E-01    -0.24459E-02     0.36715E-01    -0.13339E-02    -0.42936E-05     0.20375E-02
             5     -0.89464E-02     0.18309E-04    -0.61660E-02    -0.50781E-06    -0.25683E-03    -0.22104E-04
             6     -0.40581E-01    -0.80939E-03    -0.26155E-01    -0.43224E-03     0.34641E-03     0.68019E-03
             7      0.11547E-03    -0.37343E-03     0.67103E-04    -0.20448E-03    -0.70868E-05     0.31051E-03
             8      0.39457E-03     0.38966E-04     0.29577E-03     0.22286E-04     0.31182E-04    -0.31779E-04
             9     -0.78522E-03    -0.24498E-03    -0.48982E-03    -0.13339E-03     0.20257E-04     0.20419E-03
            10      0.50500E-03     0.14065E-03     0.32435E-03     0.76773E-04    -0.52500E-05    -0.11710E-03
            11     -0.11451E-02    -0.63472E-03    -0.74397E-03    -0.34691E-03     0.48131E-05     0.52813E-03
            12      0.33430E-03    -0.13625E-05     0.31596E-03     0.75688E-06     0.80872E-04     0.21140E-05
            13      0.61432E-04    -0.17190E-05     0.39419E-04    -0.95163E-06    -0.67030E-06     0.14217E-05
            14     -0.18846E-04     0.18704E-06    -0.19314E-04     0.17758E-07    -0.58107E-05    -0.21073E-06
            15     -0.25562E-03     0.13897E-04    -0.16576E-03     0.76186E-05     0.13405E-05    -0.11541E-04
            16      0.13046E-05     0.11777E-04     0.86692E-06     0.64344E-05     0.10563E-07    -0.97931E-05
            17      0.36024E-05     0.24932E-05     0.23300E-05     0.13617E-05    -0.23903E-07    -0.20731E-05
            18     -0.23260E-05     0.72176E-06    -0.15591E-05     0.39424E-06    -0.30112E-07    -0.60004E-06
            19      0.71897E-01     0.27164E-02     0.46938E-01     0.14776E-02     0.51192E-03     0.22654E-02

    42       1      0.12101E-03    -0.25214E-03    -0.29448E-03    -0.20891E-04    -0.12429E-04    -0.30918E-05
             2      0.30928E-02     0.58688E-03    -0.23112E-02    -0.16334E-03    -0.14383E-03    -0.11932E-03
             3      0.14351E-01    -0.77376E-02     0.15893E-01     0.15288E-03     0.21469E-03     0.37797E-03
             4      0.56096E-01    -0.55368E-01     0.36549E-01    -0.95996E-03    -0.96955E-05     0.14419E-02
             5     -0.56945E-02     0.39713E-03    -0.11372E-01    -0.26265E-03    -0.24998E-03    -0.17139E-03
             6     -0.44908E-01    -0.18107E-01    -0.19055E-01    -0.69443E-04     0.34014E-03     0.59718E-03
             7      0.20502E-03    -0.82832E-02    -0.77532E-04    -0.15721E-03    -0.69603E-05     0.19004E-03
             8      0.31781E-05     0.85926E-03     0.92331E-03     0.40407E-04     0.29559E-04    -0.46233E-05
             9     -0.10374E-02    -0.53710E-02    -0.80099E-04    -0.79853E-04     0.19281E-04     0.12995E-03
            10      0.56966E-03     0.30619E-02     0.21736E-03     0.49373E-04    -0.50349E-05    -0.69444E-04
            11     -0.12026E-02    -0.13650E-01    -0.64318E-03    -0.22423E-03     0.47816E-05     0.28920E-03
            12     -0.66069E-03    -0.24965E-04     0.19000E-02     0.38245E-04     0.71792E-04     0.23412E-04
            13      0.69245E-04    -0.33540E-04     0.25740E-04    -0.71321E-06    -0.61676E-06     0.24239E-06
            14      0.51985E-04     0.34070E-05    -0.13246E-03    -0.21352E-05    -0.49903E-05    -0.13294E-05
            15     -0.26948E-03     0.25703E-03    -0.13681E-03     0.29476E-05     0.12842E-05    -0.16990E-05
            16      0.11670E-05     0.20434E-03     0.10378E-05     0.11079E-05     0.60987E-08    -0.40676E-06
            17      0.38244E-05     0.41308E-04     0.18070E-05     0.10615E-06    -0.22001E-07     0.10684E-06
            18     -0.19198E-05     0.11499E-04    -0.20794E-05     0.35107E-08    -0.21621E-07     0.78179E-07
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       3   0.11783E-01  -0.11170E-01   0.20330E-01   0.56139E-03   0.20469E-03   0.32887E-03
       4   0.56098E-01  -0.78276E-01   0.36100E-01  -0.91912E-04  -0.24071E-04   0.24578E-04
       5  -0.28077E-02   0.57184E-03  -0.16389E-01  -0.54079E-03  -0.22855E-03  -0.29341E-03
       6  -0.48678E-01  -0.24672E-01  -0.11966E-01   0.47263E-03   0.32330E-03   0.21667E-03
       7   0.28536E-03  -0.10936E-01  -0.22112E-03  -0.32973E-04  -0.66481E-05  -0.57121E-04
       8  -0.32797E-03   0.11038E-02   0.15012E-02   0.51561E-04   0.25214E-04   0.33874E-04
       9  -0.12473E-02  -0.68116E-02   0.30295E-03   0.25675E-04   0.16655E-04  -0.29455E-04
      10   0.62247E-03   0.37827E-02   0.11488E-03  -0.78020E-05  -0.44540E-05   0.26799E-04
      11  -0.12481E-02  -0.16103E-01  -0.53742E-03   0.32928E-04   0.46921E-05  -0.16769E-03
      12  -0.13971E-02  -0.21579E-04   0.31824E-02   0.77810E-04   0.48042E-04   0.41193E-04
      13   0.74340E-04  -0.24745E-04   0.13286E-04  -0.16865E-06  -0.47697E-06  -0.13941E-05
      14   0.10086E-03   0.20161E-05  -0.21641E-03  -0.44215E-05  -0.28785E-05  -0.22279E-05
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      17   0.38402E-05   0.12291E-05   0.12668E-05  -0.11976E-05  -0.17247E-07   0.21829E-05
      18  -0.16024E-05  -0.14825E-05  -0.22334E-05  -0.35439E-06  -0.97685E-09   0.63947E-06
      19   0.75446E-01   0.85455E-01   0.46582E-01   0.98645E-03   0.47443E-03   0.56044E-03

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       2   0.63479E-02   0.63158E-03  -0.83774E-02  -0.54137E-03  -0.14181E-03  -0.24261E-03
       3   0.94250E-02  -0.85090E-02   0.24739E-01   0.10020E-02   0.20469E-03   0.22235E-03
       4   0.56151E-01  -0.57378E-01   0.35205E-01   0.85305E-03  -0.44641E-04  -0.15177E-02
       5  -0.37893E-03   0.44821E-03  -0.20965E-01  -0.83070E-03  -0.19957E-03  -0.39369E-03
       6  -0.51808E-01  -0.16644E-01  -0.51172E-02   0.10183E-02   0.29882E-03  -0.16255E-03
       7   0.35550E-03  -0.70010E-02  -0.36057E-03   0.69776E-04  -0.62583E-05  -0.25993E-03
       8  -0.57798E-03   0.66637E-03   0.19751E-02   0.67288E-04   0.18923E-04   0.63362E-04
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      14   0.11892E-03  -0.14750E-05  -0.24789E-03  -0.66881E-05  -0.70825E-07  -0.31687E-05
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      16   0.11332E-05  -0.16430E-03   0.66301E-06  -0.14884E-05  -0.22558E-07   0.14310E-05
      17   0.36350E-05  -0.39552E-04   0.77258E-06  -0.19101E-06  -0.11453E-07  -0.39291E-07
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39     1   0.53691E-03   0.74536E-05  -0.11070E-02  -0.60459E-04  -0.13149E-04  -0.43250E-04
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       3   0.72522E-02  -0.73960E-04   0.29161E-01   0.14298E-02   0.20440E-03   0.14386E-03
       4   0.56262E-01   0.97386E-04   0.33770E-01   0.15064E-02  -0.66985E-04  -0.24463E-02
       5   0.15747E-02   0.13626E-03  -0.24961E-01  -0.11282E-02  -0.16906E-03  -0.47905E-03
       6  -0.54264E-01   0.10476E-02   0.13341E-02   0.14197E-02   0.27140E-03  -0.23961E-03
       7   0.41632E-03  -0.57765E-04  -0.49664E-03   0.71490E-04  -0.61038E-05  -0.25194E-03
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       9  -0.14969E-02  -0.10639E-03   0.86417E-03   0.10282E-03   0.83974E-05  -0.73653E-04
      10   0.68246E-03   0.94616E-04  -0.47761E-04  -0.29927E-04  -0.25992E-05   0.31407E-04
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      12  -0.17705E-02   0.29045E-05   0.38142E-02   0.15993E-03  -0.18357E-04   0.71172E-04
      13   0.74436E-04  -0.85125E-05  -0.19399E-05  -0.22902E-05  -0.90477E-07   0.16352E-05
      14   0.10709E-03   0.12163E-06  -0.21817E-03  -0.89200E-05   0.26249E-05  -0.41783E-05
      15  -0.25941E-03   0.66207E-04  -0.49062E-04   0.11838E-04   0.70663E-06  -0.19891E-04
      16   0.13106E-05   0.37255E-04  -0.11607E-06   0.71144E-05  -0.42575E-07  -0.16426E-04
      17   0.32266E-05   0.32843E-05   0.34448E-06   0.11327E-05  -0.69955E-08  -0.27095E-05
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      19   0.78957E-01   0.12953E-02   0.52633E-01   0.28647E-02   0.41232E-03   0.25406E-02

38     1   0.23577E-03   0.49708E-05  -0.59383E-03  -0.58324E-04  -0.87676E-05  -0.28294E-04
       2   0.47028E-02  -0.40458E-04  -0.57512E-02  -0.60633E-03  -0.95973E-04  -0.40980E-03
       3   0.59193E-02  -0.49325E-04   0.16255E-01   0.15360E-02   0.13629E-03  -0.38139E-03
       4   0.32550E-01   0.64951E-04   0.19392E-01   0.17780E-02  -0.44666E-04  -0.29618E-02
       5  -0.63280E-03   0.90878E-04  -0.14275E-01  -0.13145E-02  -0.11273E-03  -0.83506E-04
       6  -0.41155E-01   0.69877E-03  -0.43466E-02   0.72119E-04   0.18097E-03   0.28537E-02
       7  -0.63718E-03  -0.38531E-04  -0.63681E-03  -0.26055E-04  -0.40700E-05  -0.22148E-04
       8  -0.30675E-03   0.81938E-05   0.13508E-02   0.12157E-03   0.79244E-05   0.39068E-04
       9  -0.13878E-02  -0.70972E-04   0.25463E-03   0.45591E-04   0.55994E-05   0.78574E-04
      10   0.62481E-03   0.63122E-04   0.77179E-04  -0.25663E-05  -0.17331E-05  -0.35801E-04
      11  -0.10312E-02  -0.47803E-03  -0.31605E-03  -0.16690E-04   0.29055E-05   0.68190E-04
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      13   0.61720E-04  -0.56851E-05   0.72657E-05  -0.11534E-06  -0.60330E-07  -0.38759E-05
      14   0.64016E-04   0.81265E-07  -0.12880E-03  -0.11438E-04   0.17503E-05  -0.56104E-05
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      16  -0.61607E-04   0.24921E-04  -0.27196E-04   0.43598E-07  -0.28389E-07  -0.18239E-06
      17  -0.79334E-05   0.21988E-05  -0.40697E-05   0.27051E-07  -0.46646E-08  -0.18909E-06
      18  -0.29681E-05  -0.39178E-06  -0.15244E-05  -0.47108E-07   0.26604E-07   0.65803E-07
      19   0.53065E-01   0.86403E-03   0.30063E-01   0.27718E-02   0.27494E-03   0.41545E-02

37     1   0.58156E-04   0.24842E-05  -0.17375E-03  -0.38202E-04  -0.43812E-05  -0.13882E-04
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       3   0.21541E-02  -0.24650E-04   0.48841E-02   0.10609E-02   0.68105E-04  -0.43086E-03
       4   0.10022E-01   0.32460E-04   0.59419E-02   0.12792E-02  -0.22320E-04  -0.21529E-02
       5  -0.49262E-03   0.45419E-04  -0.43622E-02  -0.94029E-03  -0.56332E-04   0.77167E-04
       6  -0.14552E-01   0.34924E-03  -0.23252E-02  -0.40453E-03   0.90429E-04   0.29451E-02
       7  -0.36521E-03  -0.19258E-04  -0.26334E-03  -0.50100E-04  -0.20338E-05   0.62199E-04
       8  -0.70895E-04   0.40955E-05   0.41875E-03   0.89679E-04   0.39598E-05   0.17496E-04
       9  -0.52806E-03  -0.35475E-04   0.31066E-04   0.11230E-04   0.27980E-05   0.10373E-03
      10   0.23686E-03   0.31552E-04   0.44020E-04   0.75548E-05  -0.86604E-06  -0.46594E-04
      11  -0.37180E-03  -0.23896E-03  -0.12285E-03  -0.23924E-04   0.14519E-05   0.74623E-04
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      14   0.20192E-04   0.40657E-07  -0.40301E-04  -0.85894E-05   0.87462E-06  -0.42877E-05
      15  -0.10691E-03   0.22140E-04  -0.35795E-04  -0.66534E-05   0.23345E-06   0.20629E-04
      16  -0.31221E-04   0.12477E-04  -0.13748E-04  -0.24290E-05  -0.14186E-07   0.55235E-05
      17  -0.43901E-05   0.11014E-05  -0.21031E-05  -0.76864E-06  -0.23307E-08   0.75274E-06
      18  -0.13545E-05  -0.19633E-06  -0.66713E-06  -0.12376E-06   0.13294E-07   0.24744E-06
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       19   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00

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        3   0.50862E-02   0.70861E-02   0.34576E-01   0.16366E-02   0.19884E-03   0.54652E-03
        4   0.56903E-01   0.70025E-01   0.31483E-01   0.11715E-02  -0.99758E-04  -0.14160E-02
        5   0.31654E-02   0.43017E-02  -0.28853E-01  -0.11157E-02  -0.11869E-03  -0.67405E-03
        6  -0.57252E-01   0.53279E-01   0.78554E-02   0.21767E-02   0.22265E-03  -0.15523E-02
        7   0.42749E-03   0.67088E-02  -0.67895E-03   0.41665E-04  -0.58970E-05  -0.16461E-03
        8  -0.82036E-03  -0.51054E-03   0.24972E-02   0.85945E-04   0.16864E-05   0.45673E-04
        9  -0.15737E-02   0.19016E-02   0.99857E-03   0.65309E-04   0.19117E-05  -0.30614E-05
       10   0.70628E-03  -0.49642E-04  -0.96990E-04  -0.43378E-05  -0.11160E-05  -0.24260E-04
       11  -0.13302E-02  -0.79346E-02  -0.20194E-03  -0.14420E-03   0.39353E-05   0.33934E-03
       12  -0.14009E-02  -0.24633E-04   0.28437E-02   0.94075E-04  -0.52917E-04   0.35568E-04
       13   0.70215E-04  -0.14117E-03  -0.33031E-05  -0.28030E-05   0.10699E-06   0.26402E-05
       14   0.66857E-04   0.68636E-05  -0.11523E-03  -0.47882E-05   0.48669E-05  -0.18586E-05
       15  -0.24186E-03   0.10122E-02  -0.23940E-04   0.11700E-04   0.45317E-06  -0.12242E-04
       16  -0.18273E-05   0.57690E-03  -0.30568E-05   0.27290E-05  -0.63023E-07  -0.12562E-05
       17   0.20030E-05   0.72835E-04  -0.32727E-06   0.72991E-07  -0.45938E-08   0.48957E-06
       18  -0.14271E-05   0.78229E-05   0.33813E-06  -0.24557E-07   0.40978E-07   0.17106E-06
       19   0.81534E-01   0.89067E-01   0.57741E-01   0.33262E-02   0.36987E-03   0.23180E-02

34      1   0.80679E-03   0.10238E-02  -0.18332E-02  -0.10500E-03  -0.13459E-04  -0.40323E-04
        2   0.10739E-01  -0.59898E-02  -0.19096E-01  -0.11885E-02  -0.13464E-03  -0.32356E-03
        3   0.32791E-02   0.53775E-02   0.39695E-01   0.19042E-02   0.17856E-03   0.78524E-03
        4   0.57608E-01   0.90015E-01   0.28083E-01   0.45798E-03  -0.14098E-03   0.56620E-03
        5   0.39875E-02   0.10068E-01  -0.31083E-01  -0.12355E-02  -0.44784E-04  -0.52429E-03
        6  -0.59139E-01   0.99443E-01   0.12889E-01   0.18423E-02   0.15028E-03  -0.40054E-04
        7   0.42549E-03   0.82209E-02  -0.85163E-03  -0.42324E-04  -0.53866E-05   0.57861E-04
        8  -0.78120E-03  -0.58875E-03   0.23747E-02   0.78762E-04  -0.10790E-04   0.18312E-04
        9  -0.15677E-02   0.14670E-02   0.94003E-03   0.29632E-04  -0.59771E-05   0.60102E-04
       10   0.70759E-03   0.74790E-03  -0.10332E-03   0.60952E-06   0.68291E-06  -0.25555E-04
       11  -0.13388E-02  -0.14915E-01  -0.95372E-04  -0.47563E-04   0.32555E-05   0.22243E-04
       12  -0.79207E-03  -0.66702E-05   0.10671E-02   0.25661E-04  -0.75126E-04   0.68343E-05
       13   0.62999E-04  -0.11538E-03  -0.44743E-06   0.32947E-06   0.22994E-06  -0.52402E-05
       14   0.16671E-04   0.17929E-05   0.29137E-04  -0.56774E-06   0.54646E-05   0.18177E-06
       15  -0.21396E-03   0.42361E-03  -0.37022E-05  -0.10766E-04   0.24883E-06   0.46972E-04
       16  -0.48643E-05  -0.10622E-04  -0.61349E-05  -0.99519E-05  -0.64514E-07   0.29890E-04
       17   0.64192E-06  -0.23626E-04  -0.92990E-06  -0.13768E-05  -0.35359E-08   0.37876E-05
       18  -0.12297E-05  -0.57281E-05   0.13945E-05  -0.12784E-06   0.20917E-07   0.38171E-06
       19   0.83456E-01   0.13584E 00   0.62234E-01   0.31925E-02   0.31707E-03   0.11540E-02

33      1   0.91343E-03   0.83750E-03  -0.22018E-02  -0.13517E-03  -0.13376E-04  -0.17090E-04
        2   0.11826E-01  -0.54761E-02  -0.22661E-01  -0.13629E-02  -0.12611E-03  -0.47982E-03
        3   0.18616E-02   0.43642E-03   0.44300E-01   0.22567E-02   0.15896E-03   0.75998E-03
        4   0.58307E-01   0.53093E-01   0.23455E-01  -0.88129E-04  -0.18128E-03   0.19663E-02
        5   0.40874E-02   0.91900E-02  -0.31220E-01  -0.14178E-02   0.32428E-04  -0.19131E-03
        6  -0.59969E-01   0.78881E-01   0.16017E-01   0.11835E-02   0.77125E-04   0.23705E-02
        7   0.40991E-03   0.38752E-02  -0.10091E-02  -0.95000E-04  -0.48497E-05   0.17343E-03
        8  -0.64600E-03  -0.22610E-03   0.19092E-02   0.67830E-04  -0.22684E-04   0.33404E-05
        9  -0.14931E-02  -0.79321E-04   0.66845E-03   0.17875E-04  -0.13447E-04   0.47293E-04
       10   0.68969E-03   0.95674E-03  -0.62243E-04  -0.76121E-05   0.23671E-05   0.12317E-04
       11  -0.13259E-02  -0.93929E-02  -0.48240E-05   0.77838E-04   0.25522E-05  -0.35080E-03
       12  -0.15485E-03   0.20573E-04  -0.10784E-02  -0.45637E-04  -0.77245E-04  -0.13082E-04
       13   0.54110E-04   0.57403E-04   0.41479E-05   0.66067E-06   0.23245E-06  -0.38440E-05
       14  -0.24493E-04  -0.57938E-05   0.16509E-03   0.39923E-05   0.41294E-05   0.11700E-05
       15  -0.17774E-03  -0.81202E-03   0.12925E-04  -0.43140E-05   0.16290E-06   0.13747E-04
       16  -0.78647E-05  -0.57887E-03  -0.86439E-05  -0.16122E-05  -0.38985E-07  -0.35191E-05
       17  -0.77592E-06  -0.70947E-04  -0.14316E-05   0.86281E-07  -0.13328E-08  -0.15324E-05
       18  -0.71946E-06  -0.61984E-05   0.17250E-05   0.90653E-07  -0.93569E-08  -0.29808E-06
       19   0.84629E-01   0.96237E-01   0.65334E-01   0.32263E-02   0.29652E-03   0.32383E-02

32      1   0.10031E-02  -0.26315E-04  -0.25837E-02  -0.16488E-03  -0.14223E-04   0.40017E-05
        2   0.12692E-01   0.53504E-04  -0.26161E-01  -0.15404E-02  -0.12811E-03  -0.62069E-03
        3   0.72615E-03   0.14166E-03   0.48633E-01   0.26940E-02   0.15832E-03   0.42107E-03
        4   0.58844E-01  -0.25127E-03   0.17769E-01  -0.10498E-03  -0.21001E-03   0.13713E-02
        5   0.37124E-02  -0.20201E-03  -0.29413E-01  -0.15913E-02   0.91382E-04   0.10020E-03
        6  -0.59994E-01   0.16148E-03   0.17447E-01   0.88626E-03   0.25511E-04   0.33573E-02
        7   0.38328E-03   0.26563E-03  -0.11550E-02  -0.65402E-04  -0.46564E-05  -0.19831E-04
        8  -0.46036E-03  -0.22325E-04   0.11451E-02   0.49342E-04  -0.31461E-04   0.16601E-04
        9  -0.13785E-02   0.17386E-03   0.21138E-03   0.34800E-04  -0.19006E-04  -0.71011E-04
       10   0.65911E-03  -0.50698E-04   0.19896E-04  -0.14395E-04   0.36131E-05   0.50231E-04
       11  -0.12937E-02  -0.48030E-03   0.69812E-04   0.41345E-04   0.20056E-05  -0.10355E-03
       12   0.33115E-03   0.26306E-05  -0.30133E-02  -0.11915E-03  -0.58797E-04  -0.24862E-04
       13   0.44693E-04  -0.27337E-04   0.72599E-05  -0.27870E-05   0.11829E-06   0.11079E-04
       14  -0.45267E-04   0.78127E-06   0.24345E-03   0.87546E-05   0.13567E-05   0.14570E-05
       15  -0.13549E-03   0.12846E-03   0.28315E-04   0.18665E-04   0.19204E-06  -0.72453E-04
       16  -0.10796E-04  -0.45072E-04  -0.99225E-05   0.31449E-05   0.19547E-08  -0.15731E-04
       17  -0.21799E-07  -0.22040E-04  -0.17482E-05  -0.45381E-06   0.38563E-08   0.17521E-07
       18   0.17421E-07  -0.46044E-05   0.12443E-05  -0.11609E-06  -0.32081E-07   0.71754E-06
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       3   0.21202E-02   0.94473E-04   0.25947E-01   0.25621E-02   0.10557E-03  -0.37744E-04
       4   0.48788E-01  -0.16758E-03   0.13571E-01   0.93492E-03  -0.14004E-03  -0.30949E-02
       5   0.31280E-02  -0.13473E-03  -0.15838E-01  -0.15492E-02   0.60933E-04  -0.19539E-03
       6  -0.31906E-01   0.10771E-03   0.96073E-02   0.91927E-03   0.17011E-04   0.31580E-02
       7   0.20988E-03   0.17718E-03  -0.61113E-03  -0.60824E-04  -0.31315E-05  -0.20176E-04
       8  -0.27927E-03  -0.14893E-04   0.66414E-03   0.60296E-04  -0.20978E-04   0.24220E-04
       9  -0.12902E-02   0.11598E-03   0.25905E-04   0.11176E-04  -0.12673E-04   0.72329E-04
      10   0.67826E-03  -0.33822E-04   0.76471E-04   0.10149E-05   0.24092E-05  -0.34543E-04
      11  -0.13506E-02  -0.32046E-03  -0.10383E-03   0.37519E-05   0.13373E-05   0.67727E-04
      12   0.15185E-03   0.17564E-05  -0.17918E-02  -0.15822E-03  -0.39206E-04  -0.17482E-04
      13   0.75229E-04  -0.18257E-04   0.15953E-04   0.36822E-06   0.78818E-07  -0.22896E-05
      14  -0.28191E-04   0.52198E-06   0.14824E-03   0.12757E-04   0.90464E-06   0.23739E-05
      15  -0.37615E-03   0.85846E-04  -0.49731E-04   0.15779E-05   0.12805E-06   0.66715E-05
      16  -0.68070E-04  -0.30150E-04  -0.19418E-04  -0.49232E-06   0.12901E-08   0.44548E-06
      17   0.50579E-05  -0.14755E-04   0.42234E-06  -0.94106E-07   0.25714E-08   0.12677E-06
      18   0.27719E-05  -0.30851E-05   0.13930E-05   0.62985E-07  -0.21925E-07   0.67633E-08
      19   0.58886E-01   0.47549E-03   0.37305E-01   0.35545E-02   0.21048E-03   0.44781E-02

30     1   0.26775E-03  -0.87703E-05  -0.36092E-03  -0.81225E-04  -0.47392E-05  -0.54237E-04
       2   0.21291E-02   0.17832E-04  -0.38967E-02  -0.86553E-03  -0.42686E-04  -0.46232E-03
       3   0.98007E-03   0.47213E-04   0.75634E-02   0.16658E-02   0.52754E-04  -0.17907E-03
       4   0.17856E-01  -0.83752E-04   0.48107E-02   0.97267E-03  -0.69977E-04  -0.35634E-02
       5   0.11511E-02  -0.67326E-04  -0.46477E-02  -0.10204E-02   0.30449E-04  -0.22922E-03
       6  -0.92876E-02   0.53831E-04   0.28623E-02   0.62414E-03   0.85003E-05   0.20465E-02
       7   0.62351E-04   0.88558E-04  -0.17716E-03  -0.39114E-04  -0.15548E-05  -0.13612E-04
       8  -0.88509E-04  -0.74439E-05   0.20479E-03   0.43962E-04  -0.10483E-04   0.18769E-04
       9  -0.49245E-03   0.57971E-04  -0.10653E-04  -0.49388E-06  -0.63327E-05   0.96616E-04
      10   0.26625E-03  -0.16906E-04   0.36101E-04   0.65454E-05   0.12039E-05  -0.51662E-04
      11  -0.53246E-03  -0.16019E-03  -0.59862E-04  -0.10171E-04   0.66827E-06   0.10313E-03
      12   0.38970E-04   0.87838E-06  -0.56225E-03  -0.11971E-03  -0.19591E-04  -0.91258E-05
      13   0.32839E-04  -0.91318E-05   0.72119E-05   0.13263E-05   0.39386E-07  -0.61160E-05
      14  -0.90928E-05   0.26115E-06   0.47247E-04   0.99880E-05   0.45205E-06   0.19125E-05
      15  -0.17448E-03   0.42956E-04  -0.28288E-04  -0.48168E-05   0.63989E-07   0.31776E-04
      16  -0.33237E-04  -0.15095E-04  -0.87040E-05  -0.16033E-05   0.64465E-09   0.59410E-05
      17   0.28118E-05  -0.73909E-05   0.41258E-06   0.61108E-07   0.12849E-08  -0.48146E-06
      18   0.14100E-05  -0.15461E-05   0.56154E-06   0.10720E-06  -0.10956E-07  -0.24816E-06
      19   0.20313E-01   0.23766E-03   0.11218E-01   0.24343E-02   0.10518E-03   0.41487E-02

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       5   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
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      19   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00

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       3  -0.88429E-04   0.45143E-02   0.53150E-01   0.29238E-02   0.15606E-03   0.37722E-03
       4   0.60416E-01  -0.61953E-01   0.11662E-01  -0.62077E-03  -0.22780E-03   0.23024E-02
       5   0.31993E-02  -0.15453E-01  -0.26110E-01  -0.13584E-02   0.14420E-03   0.28160E-03
       6  -0.59755E-01  -0.74055E-01   0.17666E-01   0.80041E-03  -0.15642E-04   0.18933E-02
       7   0.34941E-03   0.41187E-02  -0.12966E-02  -0.22158E-04  -0.43775E-05  -0.20395E-03
       8  -0.27331E-03  -0.44990E-03   0.17225E-03  -0.37946E-05  -0.37298E-04   0.17253E-04
       9  -0.12920E-02   0.49478E-02  -0.38386E-03   0.37144E-04  -0.22637E-04  -0.18590E-03
      10   0.64498E-03  -0.23224E-02   0.13536E-03  -0.20390E-04   0.44197E-05   0.74935E-04
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      13   0.38447E-04  -0.50567E-03   0.68162E-05  -0.40334E-05  -0.74866E-07   0.10182E-04
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      15  -0.10660E-03   0.25563E-02   0.40481E-04   0.16034E-04   0.24910E-06  -0.28222E-04
      16  -0.16956E-04   0.81180E-04  -0.10424E-04  -0.72615E-06   0.46062E-07   0.55589E-05
      17  -0.31570E-05  -0.13995E-03  -0.17086E-05  -0.70380E-06   0.12376E-07   0.62280E-06
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      19   0.86125E-01   0.98496E-01   0.69799E-01   0.37926E-02   0.34123E-03   0.30783E-02

27     1   0.11646E-02  -0.32829E-02  -0.34021E-02  -0.21160E-03  -0.13915E-04  -0.13988E-05
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       3  -0.64557E-03   0.61660E-02   0.57261E-01   0.31436E-02   0.13167E-03   0.38591E-03
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       5   0.26179E-02  -0.23957E-01  -0.21154E-01  -0.10276E-02   0.20326E-03  -0.89643E-04
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       7   0.30536E-03   0.84822E-02  -0.14165E-02  -0.34685E-04  -0.34171E-05  -0.69356E-04
       8  -0.11234E-03  -0.77968E-03  -0.94403E-03  -0.52869E-04  -0.41021E-04  -0.45111E-05
       9  -0.12143E-02   0.82244E-02  -0.10622E-02  -0.12429E-04  -0.24719E-04  -0.95194E-05
      10   0.62924E-03  -0.33832E-02   0.26838E-03  -0.38485E-05   0.48606E-05  -0.13603E-04
      11  -0.12705E-02  -0.71892E-02   0.15813E-03  -0.21639E-04   0.86896E-06   0.69058E-04
      12   0.59893E-03   0.14166E-03  -0.41964E-02  -0.75234E-04   0.22981E-04  -0.73363E-05
      13   0.32200E-04  -0.46352E-03   0.56362E-06   0.46566E-06  -0.27393E-06  -0.19209E-04
      14  -0.30958E-04   0.33993E-05   0.12301E-03  -0.14233E-05  -0.54715E-05   0.38628E-06
      15  -0.74320E-04   0.82152E-03   0.52912E-04  -0.11458E-04   0.28773E-06   0.12394E-03
      16  -0.22500E-04  -0.15433E-03  -0.92792E-05  -0.11809E-05   0.87050E-07   0.49705E-05
      17  -0.39826E-05   0.73697E-05  -0.11107E-05   0.92905E-06   0.21790E-07  -0.77682E-05
      18   0.17472E-05   0.10563E-04  -0.48649E-06   0.22585E-06  -0.23611E-07  -0.17803E-05
      19   0.86559E-01   0.14481E 00   0.71897E-01   0.39861E-02   0.36849E-03   0.95677E-03
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26      1   0.12100E-02  -0.28024E-02  -0.37845E-02  -0.22391E-03  -0.13064E-04  -0.75559E-04
        2   0.14289E-01   0.71889E-02  -0.36333E-01  -0.20995E-02  -0.10158E-03   0.70496E-04
        3  -0.94676E-03   0.41305E-02   0.60672E-01   0.33664E-02   0.10967E-03   0.36035E-03
        4   0.62241E-01  -0.88862E-01  -0.19000E-02  -0.57512E-03  -0.24698E-03  -0.21402E-02
        5   0.21274E-02  -0.16790E-01  -0.14608E-01  -0.68042E-03   0.25127E-03  -0.53869E-03
        6  -0.59174E-01  -0.44036E-01   0.14503E-01   0.91769E-03  -0.92614E-04  -0.19624E-02
        7   0.25215E-03   0.69080E-02  -0.15066E-02  -0.63245E-04  -0.25771E-05   0.17074E-03
        8   0.32119E-05  -0.53416E-03  -0.21465E-02  -0.10156E-03  -0.43023E-04  -0.24462E-04
        9  -0.11573E-02   0.52897E-02  -0.17845E-02  -0.67769E-04  -0.25702E-04   0.18036E-03
       10   0.61455E-03  -0.17933E-02   0.41003E-03   0.11919E-04   0.50494E-05  -0.79555E-04
       11  -0.12376E-02  -0.56414E-02   0.18025E-03   0.11036E-05   0.50749E-06  -0.16483E-03
       12   0.49655E-03   0.25464E-04  -0.29293E-02  -0.53739E-04   0.63573E-04   0.90256E-06
       13   0.25519E-04   0.24341E-03  -0.11349E-04   0.12991E-05  -0.47929E-06  -0.12480E-04
       14  -0.13755E-04  -0.10129E-04  -0.68260E-04  -0.65313E-05  -0.76782E-05  -0.34954E-06
       15  -0.39487E-04  -0.21880E-02   0.65292E-04  -0.11408E-04   0.34936E-06   0.28273E-04
       16  -0.27051E-04  -0.10389E-03  -0.67566E-05   0.23286E-06   0.11504E-06  -0.65305E-05
       17  -0.45855E-05   0.13938E-03  -0.65518E-06   0.41299E-06   0.28433E-07   0.13723E-05
       18   0.23451E-05   0.29410E-04  -0.75838E-06  -0.84122E-08   0.86636E-09   0.79119E-06
       19   0.86445E-01   0.10153E 00   0.73903E-01   0.41771E-02   0.40234E-03   0.29932E-02

25      1   0.12341E-02   0.62674E-04  -0.41632E-02  -0.23640E-03  -0.13813E-04  -0.14081E-03
        2   0.14403E-01  -0.64628E-03  -0.39232E-01  -0.22904E-02  -0.10486E-03   0.24496E-03
        3  -0.10695E-02   0.10571E-03   0.63811E-01   0.35966E-02   0.11659E-03   0.21609E-03
        4   0.62399E-01  -0.19516E-03  -0.90676E-02  -0.53722E-03  -0.25297E-03  -0.37740E-02
        5   0.18968E-02  -0.12353E-03  -0.69979E-02  -0.35815E-03   0.27479E-03  -0.56877E-03
        6  -0.57086E-01  -0.11767E-02   0.11665E-01   0.72776E-03  -0.10474E-03  -0.36081E-03
        7   0.19310E-03  -0.30832E-04  -0.15818E-02  -0.83475E-04  -0.27025E-05   0.25438E-03
        8   0.63449E-04  -0.24014E-04  -0.34042E-02  -0.15253E-03  -0.45010E-04  -0.16422E-04
        9  -0.11263E-02   0.29524E-03  -0.25366E-02  -0.10642E-03  -0.27024E-04   0.95596E-04
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       11  -0.11930E-02  -0.15585E-03   0.19609E-03   0.10068E-04   0.51996E-06  -0.14872E-03
       12   0.40302E-03   0.56824E-05  -0.72583E-03  -0.31179E-04   0.86468E-04  -0.62979E-05
       13   0.17948E-04  -0.78154E-04  -0.27298E-04  -0.14709E-05  -0.59436E-06   0.39437E-04
       14  -0.30009E-05   0.49079E-06  -0.30400E-03  -0.11617E-04  -0.85980E-05  -0.10661E-05
       15  -0.31256E-05   0.15129E-03   0.78705E-04   0.14035E-05   0.48660E-06  -0.17049E-03
       16  -0.30287E-04  -0.48460E-04  -0.34514E-05   0.52060E-07   0.12154E-06   0.99800E-05
       17  -0.49063E-05   0.27753E-04   0.19386E-06  -0.64895E-08   0.31120E-07   0.33732E-06
       18   0.27459E-05   0.11151E-04  -0.40571E-06  -0.64471E-07   0.20232E-07  -0.14278E-05
       19   0.85845E-01   0.14341E-02   0.76922E-01   0.43848E-02   0.43081E-03   0.38666E-02

24      1   0.38683E-03   0.41797E-04  -0.21993E-02  -0.21935E-03  -0.92103E-05  -0.65042E-04
        2   0.11598E-01  -0.43101E-03  -0.20490E-01  -0.20670E-02  -0.69918E-04  -0.75789E-03
        3   0.17887E-04   0.70498E-04   0.33812E-01   0.33618E-02   0.77740E-04   0.56450E-04
        4   0.32118E-01  -0.13016E-03  -0.47067E-02  -0.47771E-03  -0.16868E-03  -0.32857E-02
        5  -0.72758E-03  -0.82392E-04  -0.38448E-02  -0.36858E-03   0.18323E-03  -0.10023E-03
        6  -0.43707E-01  -0.78487E-03   0.59180E-02   0.61448E-03  -0.69841E-04   0.30017E-02
        7   0.10985E-02  -0.20566E-04  -0.85979E-03  -0.83295E-04  -0.18020E-05  -0.98930E-05
        8  -0.14621E-04  -0.16019E-04  -0.19525E-02  -0.17921E-03  -0.30012E-04  -0.33563E-05
        9  -0.47715E-03   0.19695E-03  -0.14821E-02  -0.13352E-03  -0.18020E-04   0.59271E-04
       10   0.45320E-03  -0.13458E-03   0.34507E-03   0.29364E-04   0.35804E-05  -0.31606E-04
       11  -0.14457E-02  -0.10398E-03   0.10765E-03   0.10324E-04   0.34671E-06   0.62372E-04
       12   0.27667E-03   0.39275E-05  -0.42220E-03  -0.38121E-04   0.57657E-04  -0.21065E-04
       13   0.16242E-03  -0.52196E-04  -0.14752E-04  -0.14347E-05  -0.39632E-06  -0.79485E-06
       14  -0.62999E-05   0.32791E-06  -0.18255E-03  -0.15934E-04  -0.57331E-05   0.14937E-06
       15  -0.64881E-03   0.10110E-03   0.53258E-04   0.41257E-05   0.32447E-06  -0.68349E-06
       16   0.15244E-04  -0.32417E-04  -0.27708E-05  -0.17996E-06   0.81045E-07   0.16299E-05
       17  -0.24145E-05   0.18580E-04   0.17570E-06   0.10060E-07   0.20751E-07   0.25420E-06
       18  -0.34041E-05   0.74707E-05  -0.60365E-07  -0.21254E-07   0.13490E-07  -0.15467E-06
       19   0.55510E-01   0.95658E-03   0.40583E-01   0.40526E-02   0.28726E-03   0.45174E-02

23      1   0.56179E-04   0.20888E-04  -0.63650E-03  -0.14065E-03  -0.46024E-05  -0.18115E-04
        2   0.41977E-02  -0.21540E-03  -0.58808E-02  -0.13046E-02  -0.34939E-04  -0.84155E-03
        3   0.12797E-03   0.35232E-04   0.98073E-02   0.21649E-02   0.38847E-04  -0.15720E-04
        4   0.91235E-02  -0.65049E-04  -0.13447E-02  -0.29893E-03  -0.84288E-04  -0.20334E-02
        5  -0.57494E-03  -0.41178E-04  -0.11441E-02  -0.24961E-03   0.91562E-04   0.89779E-04
        6  -0.15515E-01  -0.39227E-03   0.16617E-02   0.37240E-03  -0.34900E-04   0.31345E-02
        7   0.52824E-03  -0.10279E-04  -0.25404E-03  -0.55600E-04  -0.90048E-06  -0.95200E-04
        8  -0.14392E-04  -0.80068E-05  -0.59788E-03  -0.12875E-03  -0.14997E-04   0.21402E-05
        9  -0.11354E-03   0.98446E-04  -0.45917E-03  -0.98362E-04  -0.90045E-05   0.27591E-04
       10   0.15992E-03  -0.67270E-04   0.11055E-03   0.23333E-04   0.17891E-05  -0.32386E-04
       11  -0.59133E-03  -0.51980E-04   0.32031E-04   0.69881E-05   0.17325E-06   0.11293E-03
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       13   0.79667E-04  -0.26107E-04  -0.43495E-05  -0.95261E-06  -0.19804E-06  -0.14243E-04
       14  -0.28434E-05   0.16405E-06  -0.57709E-04  -0.12244E-04  -0.28649E-05   0.52277E-06
       15  -0.32674E-03   0.50591E-04   0.17928E-04   0.37025E-05   0.16214E-06   0.57860E-04
       16   0.11107E-04  -0.16230E-04  -0.10069E-05  -0.20132E-06   0.40498E-07  -0.17634E-05
       17  -0.67480E-06   0.93067E-05   0.67203E-07   0.13169E-07   0.10369E-07   0.15107E-06
       18  -0.20421E-05   0.37438E-05   0.14524E-07  -0.47520E-10   0.67412E-08   0.34139E-06
       19   0.18513E-01   0.47809E-03   0.11735E-01   0.25942E-02   0.14355E-03   0.38350E-02

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        3   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
        4   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
        5   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
        6   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
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        8   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
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       10   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
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         3  -0.10844E-02  -0.24518E-02   0.65635E-01   0.35517E-02   0.12413E-03   0.19417E-03
         4   0.62459E-01   0.58251E-01  -0.12881E-01  -0.54419E-03  -0.25487E-03  -0.39647E-02
         5   0.18623E-02   0.87306E-02  -0.28459E-02  -0.15004E-03   0.27740E-03  -0.60434E-03
         6  -0.57105E-01   0.44343E-02   0.10091E-01   0.58364E-03  -0.10496E-03  -0.37217E-03
         7   0.19357E-03  -0.43563E-02  -0.16245E-02  -0.78359E-04  -0.29229E-05   0.30430E-03
         8   0.69294E-04   0.15998E-03  -0.40867E-02  -0.13451E-03  -0.45747E-04  -0.12055E-04
         9  -0.11238E-02  -0.49418E-03  -0.29475E-02  -0.82066E-04  -0.27580E-04   0.29919E-04
        10   0.60193E-03  -0.77396E-03   0.64001E-03   0.68091E-05   0.55121E-05   0.56576E-04
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        12   0.39309E-03   0.97520E-04   0.60244E-03  -0.94552E-05   0.89591E-04  -0.62731E-05
        13   0.18082E-04  -0.93738E-03  -0.36417E-04  -0.24703E-05  -0.61480E-06   0.66176E-04
        14  -0.19002E-05   0.18388E-04  -0.43404E-03  -0.86591E-05  -0.87050E-05  -0.13886E-05
        15  -0.31989E-05   0.34638E-02   0.86511E-04   0.12090E-05   0.53738E-06  -0.24599E-03
        16  -0.30500E-04  -0.31805E-03  -0.16437E-05  -0.14223E-06   0.12017E-06   0.21968E-04
        17  -0.49503E-05   0.73064E-04   0.66275E-06   0.14508E-06   0.31364E-07  -0.46988E-05
        18   0.27703E-05   0.56280E-04  -0.68244E-07  -0.29263E-07   0.23607E-07  -0.37996E-05
        19   0.85904E-01   0.59950E-01   0.79375E-01   0.43423E-02   0.43752E-03   0.40932E-02

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         3   0.29906E-01  -0.19071E-02   0.65546E-01   0.35486E-02  -0.89502E-04  -0.54467E-03
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         6   0.31524E-02   0.43486E-02   0.98851E-02   0.57403E-03  -0.20598E-03   0.85727E-04
         7  -0.11929E-02  -0.45572E-02  -0.16044E-02  -0.77990E-04   0.20094E-04   0.20081E-03
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         9   0.20756E-02   0.20010E-04  -0.29236E-02  -0.80480E-04   0.23939E-04  -0.51399E-03
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        14   0.37981E-05   0.17770E-04  -0.42452E-03  -0.84621E-05   0.95167E-05   0.61883E-06
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        17   0.44733E-05   0.77177E-04   0.62823E-06   0.15241E-06  -0.34519E-07  -0.41027E-05
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         3   0.29813E-01   0.85838E-02   0.63220E-01   0.34763E-02  -0.10323E-03  -0.47085E-03
         4  -0.89638E-03   0.47626E-01  -0.13288E-01  -0.55366E-03  -0.13543E-04   0.57891E-03
         5   0.78292E-02   0.86006E-02   0.46357E-02   0.20673E-03   0.28617E-03   0.33591E-04
         6   0.30132E-02   0.19263E-02   0.48268E-02   0.33673E-03  -0.20321E-03   0.85860E-04
         7  -0.11772E-02   0.85735E-02  -0.11069E-02  -0.71368E-04   0.20307E-04   0.97575E-04
         8  -0.17836E-03  -0.81617E-03  -0.30719E-02  -0.10395E-03   0.40132E-04   0.31557E-04
         9   0.20867E-02   0.11081E-01  -0.23270E-02  -0.36531E-04   0.24612E-04  -0.31965E-03
        10   0.88024E-03  -0.84095E-02   0.40812E-03  -0.15786E-04  -0.91527E-05   0.19911E-03
        11  -0.10039E-03  -0.16590E-02   0.16643E-03  -0.13158E-05  -0.15291E-05   0.11093E-03
        12   0.42309E-03  -0.28662E-04   0.21452E-02   0.30359E-04   0.54409E-04   0.23742E-05
        13   0.16482E-03  -0.13219E-02  -0.35790E-04  -0.46878E-05   0.36542E-07  -0.10273E-04
        14   0.98708E-05   0.62681E-05  -0.19223E-03  -0.36018E-05   0.92115E-05   0.62332E-06
        15   0.11731E-04   0.11306E-02   0.48289E-04  -0.12700E-07  -0.14934E-05   0.10205E-03
        16  -0.29163E-05  -0.20727E-03  -0.62059E-06  -0.47327E-06   0.40133E-07  -0.97172E-05
        17   0.43558E-05   0.11880E-03  -0.20913E-06   0.49172E-06  -0.33102E-07   0.26817E-05
        18  -0.18540E-04   0.77166E-05   0.26356E-05  -0.11842E-07   0.98493E-07   0.18545E-05
        19   0.15368E 00   0.91747E-01   0.80892E-01   0.43164E-02   0.45867E-03   0.23392E-02

18       1  -0.21443E-02   0.95470E-02  -0.46255E-02  -0.22248E-03  -0.27765E-05   0.33004E-04
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         3   0.29581E-01   0.12409E-01   0.60337E-01   0.34075E-02  -0.13226E-03  -0.30479E-03
         4  -0.10111E-02   0.29986E-01  -0.13523E-01  -0.56468E-03  -0.56812E-05   0.89404E-03
         5   0.82688E-02   0.63407E-02   0.11435E-01   0.54865E-03   0.26414E-03   0.88673E-04
         6   0.27627E-02  -0.39209E-03  -0.85475E-04   0.98266E-04  -0.19437E-03   0.66688E-04
         7  -0.11383E-02  -0.84421E-02  -0.60408E-03  -0.67329E-04   0.20516E-04  -0.98539E-04
         8  -0.10870E-03  -0.10805E-02  -0.20660E-02  -0.75175E-04   0.41520E-04   0.12796E-05
         9   0.21196E-02   0.14132E-01  -0.17073E-02   0.14385E-04   0.25613E-04   0.82745E-04
        10   0.85801E-03  -0.97211E-02   0.17969E-03  -0.41883E-04  -0.94301E-05  -0.90732E-04
        11  -0.10036E-03  -0.31037E-02   0.12697E-03  -0.10981E-04  -0.16603E-05   0.22795E-05
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        14   0.23819E-04  -0.32929E-05   0.21814E-04   0.12271E-05   0.79709E-05   0.17832E-06
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        16  -0.26606E-05   0.10637E-03   0.34137E-06  -0.61057E-07   0.38300E-07  -0.11240E-04
        17   0.40460E-05  -0.35123E-04  -0.95522E-06   0.41785E-06  -0.28313E-07   0.69425E-05
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        19   0.15306E 00   0.10995E 00   0.83209E-01   0.43676E-02   0.43774E-03   0.10069E-02

17       1  -0.21456E-02   0.68095E-02  -0.46622E-02  -0.21295E-03  -0.28215E-06   0.23496E-03
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         3   0.29109E-01   0.87186E-02   0.56761E-01   0.33359E-02  -0.15797E-03  -0.15612E-03
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         5   0.88965E-02   0.25234E-02   0.17539E-01   0.68011E-03   0.23206E-03   0.38055E-04
         6   0.23333E-02  -0.13926E-02  -0.46838E-02  -0.14311E-03  -0.17892E-03   0.15012E-04
         7  -0.10747E-02  -0.42171E-02  -0.10500E-03  -0.53093E-04   0.20127E-04  -0.20147E-03
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         9   0.21692E-02   0.80589E-02  -0.10741E-02   0.49420E-04   0.25593E-04   0.35050E-03
        10   0.82146E-03  -0.48883E-02  -0.51537E-04  -0.52847E-04  -0.94638E-05  -0.24502E-03
        11  -0.10157E-03  -0.19448E-02   0.84832E-04  -0.17111E-04  -0.17267E-05  -0.79567E-04
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        13   0.15473E-03   0.64107E-03  -0.31936E-04   0.65212E-06   0.59714E-07  -0.16281E-04
        14   0.40940E-04  -0.52571E-05   0.18978E-03   0.60042E-05   0.55221E-05  -0.89318E-06
        15   0.49860E-05  -0.13743E-02  -0.19253E-04  -0.14849E-04  -0.11195E-05  -0.37669E-04
        16  -0.22908E-05   0.20822E-03   0.11746E-05   0.16317E-05  -0.31167E-07   0.38277E-05
        17   0.35556E-05  -0.11992E-03  -0.15102E-05  -0.70116E-06  -0.20400E-07  -0.14057E-05
        18  -0.14706E-04  -0.49957E-05   0.57395E-05  -0.74251E-07   0.37315E-07  -0.99126E-06
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        3   0.28394E-01  -0.40494E-03   0.52812E-01   0.32466E-02  -0.16204E-03  -0.13444E-03
        4  -0.11941E-02   0.77265E-03  -0.13407E-01  -0.75947E-03   0.67806E-05   0.21748E-04
        5   0.96320E-02   0.62797E-04   0.22778E-01   0.11882E-02   0.19688E-03  -0.18638E-03
        6   0.17735E-02   0.59951E-04  -0.88162E-02  -0.38658E-03  -0.15840E-03  -0.53739E-04
        7  -0.98987E-03  -0.17186E-03   0.37002E-03  -0.74415E-05   0.18797E-04  -0.73918E-04
        8   0.15529E-03  -0.24319E-04  -0.52141E-04  -0.11549E-04   0.39198E-04  -0.13609E-04
        9   0.22311E-02   0.27168E-03  -0.46266E-03   0.24897E-04   0.23827E-04   0.18422E-03
       10   0.77185E-03  -0.36826E-03  -0.27605E-03  -0.15002E-04  -0.90469E-05  -0.41657E-04
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       14   0.53592E-04   0.49718E-07   0.28573E-03   0.10773E-04   0.22345E-05  -0.20818E-05
       15   0.11037E-05  -0.15051E-03  -0.40824E-04  -0.73505E-05  -0.62668E-06  -0.24375E-04
       16  -0.18452E-05   0.21626E-04   0.17450E-05   0.11233E-05   0.19450E-07   0.53461E-05
       17   0.29239E-05  -0.11795E-04  -0.18141E-05  -0.73328E-06  -0.11907E-07  -0.37728E-05
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        9   0.23487E-02   0.18124E-03  -0.43703E-03  -0.24275E-04   0.15888E-04  -0.11698E-03
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14      1   0.16529E-03  -0.40483E-04  -0.81039E-03  -0.17480E-03   0.34968E-06  -0.15330E-04
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        3   0.71171E-02  -0.13496E-03   0.76099E-02   0.17016E-02  -0.53992E-04  -0.14530E-02
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        5   0.21518E-02   0.20933E-04   0.36817E-02   0.80491E-03   0.65599E-04  -0.44613E-03
        6   0.36049E-03   0.19985E-04  -0.15631E-02  -0.33608E-03  -0.52780E-04  -0.75763E-04
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        9   0.92746E-03   0.90588E-04  -0.16728E-03  -0.32783E-04   0.79392E-05  -0.17956E-03
       10   0.12276E-03  -0.12280E-03  -0.43637E-04  -0.95747E-05  -0.30145E-05  -0.26896E-04
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       15  -0.46288E-04  -0.50330E-04   0.32797E-05   0.31034E-06  -0.20881E-06   0.81873E-05
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       12   0.14615E-03  -0.70190E-04   0.20149E-02   0.68749E-04  -0.55218E-04  -0.16366E-04
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       14   0.48747E-04   0.10706E-05   0.27751E-03   0.49148E-05  -0.27551E-05  -0.11058E-05
       15  -0.79243E-05  -0.56349E-03  -0.46719E-04  -0.43143E-06   0.19342E-06   0.16606E-04
       16   0.19269E-06   0.16206E-04   0.17191E-05  -0.34232E-06   0.36137E-08  -0.23648E-05
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         7  -0.79310E-03  -0.28563E-02   0.13600E-02   0.26468E-04   0.85212E-05   0.29877E-04
         8   0.61610E-03  -0.37489E-03   0.19953E-02   0.45161E-04   0.17270E-04  -0.55465E-05
         9   0.25512E-02   0.13188E-02   0.70988E-03   0.35061E-05   0.84659E-05  -0.11242E-03
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        14   0.37234E-05  -0.23771E-05   0.11805E-03  -0.85329E-06  -0.62637E-05   0.26028E-06
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        17  -0.10723E-05  -0.43632E-04  -0.11016E-05  -0.44024E-06   0.30286E-08  -0.59582E-06
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         5   0.10663E-01  -0.13236E-02   0.28681E-01   0.11423E-02  -0.11291E-03  -0.47347E-03
         6   0.33103E-03   0.12911E-02  -0.16465E-01  -0.48453E-03   0.28253E-04   0.30177E-03
         7  -0.74007E-03  -0.24004E-02   0.15300E-02   0.77799E-05   0.17223E-05  -0.64279E-04
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        16   0.38115E-05  -0.58238E-04   0.17525E-06   0.21684E-06  -0.13458E-07   0.10242E-05
        17  -0.29373E-05   0.39702E-04  -0.28608E-06  -0.30595E-06   0.73303E-08  -0.11710E-05
        18   0.61899E-05  -0.65985E-04   0.89551E-06   0.11486E-05   0.56903E-08   0.35621E-05
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        10   0.48673E-03  -0.57096E-03  -0.82988E-03   0.64206E-04   0.12879E-05   0.21369E-03
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        13   0.14506E-03   0.21903E-03  -0.53264E-04   0.31430E-04  -0.42078E-06   0.71360E-04
        14  -0.13524E-03  -0.36005E-06  -0.27127E-03  -0.11627E-04  -0.33403E-05   0.52279E-05
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        17  -0.44887E-05   0.14697E-05   0.70038E-06   0.60950E-06   0.12177E-07   0.15403E-05
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         3   0.87465E-02  -0.21691E-03   0.12593E-01   0.11765E-02  -0.19728E-03  -0.10387E-02
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         5   0.44612E-02  -0.96317E-05   0.13172E-01   0.12434E-02  -0.12687E-03  -0.45244E-03
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        14  -0.81030E-04  -0.24056E-06  -0.15813E-03  -0.14224E-04  -0.22273E-05   0.70836E-05
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        17   0.17771E-05   0.49295E-06  -0.98955E-06  -0.17003E-06   0.40573E-08  -0.28303E-06
        18  -0.73143E-05   0.30305E-05   0.39013E-05   0.67894E-06  -0.25172E-08   0.12243E-05
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      6   0.20852E-02   0.23534E-02  -0.12728E-01  -0.45278E-03   0.10208E-03   0.13591E-03
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      5   0.53926E-03   0.44445E-04   0.82303E-02   0.27871E-03  -0.28001E-03  -0.16499E-03
      6   0.49936E-02   0.24576E-02  -0.75602E-02  -0.93654E-04   0.13901E-03   0.14310E-03
      7  -0.10405E-02   0.15264E-02   0.10202E-02   0.38479E-04  -0.10830E-04   0.29349E-04
      8  -0.42398E-04   0.36506E-03   0.88528E-03   0.28975E-04  -0.27508E-04  -0.50835E-05
      9   0.19277E-02  -0.76556E-02  -0.56535E-03  -0.11899E-03  -0.22528E-04  -0.18238E-03
     10   0.64307E-03  -0.16747E-01  -0.60472E-03  -0.29411E-03   0.58130E-05  -0.36607E-03
     11  -0.11110E-03  -0.81713E-03   0.26184E-04  -0.14587E-04   0.14935E-05  -0.16652E-04
     12  -0.57337E-03   0.39064E-04  -0.15785E-02  -0.31817E-04   0.59959E-04   0.19895E-04
     13   0.15003E-03   0.11276E-02  -0.88089E-04   0.13365E-04  -0.34969E-06   0.14459E-04
     14  -0.66935E-04   0.20194E-05  -0.16487E-03  -0.27871E-05   0.62573E-05   0.16968E-05
     15  -0.12265E-04   0.18227E-03   0.32665E-04   0.18955E-05  -0.72329E-06   0.13210E-05
     16   0.57897E-05  -0.20923E-04  -0.30229E-05  -0.99295E-07  -0.25147E-07  -0.50728E-07
     17  -0.53882E-05   0.23639E-04   0.27160E-05   0.54013E-07   0.26085E-07  -0.56211E-07
     18   0.15045E-04  -0.12802E-03  -0.69079E-05  -0.23866E-07  -0.91297E-07   0.85615E-06
     19   0.14217E 00   0.67781E-01   0.92738E-01   0.12573E-02   0.45454E-03   0.18452E-02

 2    1   0.26471E-03   0.21018E-03  -0.21291E-03   0.11350E-03   0.32987E-04   0.17582E-03
      2   0.14145E 00   0.26975E-02  -0.92586E-01   0.14640E-02  -0.27435E-04   0.22518E-02
      3   0.58927E-02   0.76419E-03  -0.34843E-02   0.43153E-03  -0.31163E-03   0.62695E-03
      4   0.32427E-02  -0.51802E-03  -0.22178E-02  -0.28691E-03   0.78770E-04  -0.42864E-03
      5  -0.31023E-02   0.12422E-05   0.23775E-02   0.11509E-04  -0.28849E-03  -0.60370E-05
      6   0.68217E-02   0.11001E-03  -0.46404E-02   0.54730E-04   0.14467E-03   0.95081E-04
      7  -0.11864E-02   0.68932E-04   0.79060E-03   0.30061E-04  -0.11502E-04   0.56992E-04
      8  -0.40672E-03   0.16533E-04   0.30118E-03   0.99164E-05  -0.29053E-04   0.13183E-04
      9   0.16362E-02  -0.34932E-03  -0.10431E-02  -0.19039E-03  -0.23531E-04  -0.29102E-03
     10   0.72281E-03  -0.76949E-03  -0.48078E-03  -0.42094E-03   0.62588E-05  -0.64006E-03
     11  -0.91646E-04  -0.30006E-04   0.58114E-04  -0.20817E-04   0.15780E-05  -0.31594E-04
     12   0.25690E-03   0.20467E-05  -0.24929E-03  -0.14052E-06   0.67551E-04   0.25249E-05
     13   0.14692E-03   0.57873E-04  -0.95778E-04   0.31652E-04  -0.33361E-06   0.48135E-04
     14   0.21809E-04   0.11503E-06  -0.23028E-04  -0.46491E-07   0.72579E-05   0.16709E-06
     15  -0.22981E-04   0.98614E-05   0.16028E-04   0.53987E-05  -0.86748E-06   0.61947E-05
     16   0.55549E-05  -0.12063E-05  -0.36083E-05  -0.65850E-06  -0.25920E-07  -0.10035E-05
     17  -0.51491E-05   0.14272E-05   0.33842E-05   0.77922E-06   0.27670E-07   0.11869E-05
     18   0.14141E-04  -0.80375E-05  -0.91348E-05  -0.43896E-05  -0.10185E-06  -0.66825E-05
     19   0.14183E 00   0.29848E-02   0.92836E-01   0.16261E-02   0.46423E-03   0.24877E-02
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1          1   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
           2   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
           3   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
           4   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
           5   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
           6   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
           7   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
           8   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
           9   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
          10   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
          11   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
          12   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
          13   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
          14   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
          15   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
          16   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
          17   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
          18   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
          19   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
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R E S P O N S E   S P E C T R U M   S T R E S S   C O M P O N E N T S

SQUARE ROOT OF THE SUM OF THE SQUARES OF THE MODAL STRESSES
(FOR ALL ELEMENTS)



ELEMENT TYPE (T R U S S                ) / / /  ELEMENT NUMBER (    1)

      P/A         P
0.8171E 04 0.4012E 03


ELEMENT TYPE (B E A M                  ) / / /  ELEMENT NUMBER (    1)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.2425E-06 0.2599E 03 0.4170E 03 0.3936E 04 0.0000E 00 0.0000E 00 0.8083E-07 0.2599E 03 0.4170E 03 0.3936E 04 0.4187E 03 0.2609E 03


ELEMENT TYPE (B E A M                  ) / / /  ELEMENT NUMBER (    2)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.2230E 03 0.2564E 03 0.4122E 03 0.3936E 04 0.4190E 03 0.2609E 03 0.2230E 03 0.2564E 03 0.4122E 03 0.3936E 04 0.1036E 05 0.6450E 04


ELEMENT TYPE (B E A M                  ) / / /  ELEMENT NUMBER (    3)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.6490E 03 0.1957E 03 0.2163E 03 0.3999E 04 0.1033E 05 0.6450E 04 0.6490E 03 0.1957E 03 0.2163E 03 0.3999E 04 0.1538E 05 0.1117E 05


ELEMENT TYPE (B E A M                  ) / / /  ELEMENT NUMBER (    4)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.1070E 04 0.9378E 02 0.1578E 03 0.4183E 04 0.1533E 05 0.1117E 05 0.1070E 04 0.9378E 02 0.1578E 03 0.4183E 04 0.1412E 05 0.1320E 05


ELEMENT TYPE (B E A M                  ) / / /  ELEMENT NUMBER (    5)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.1485E 04 0.1235E 03 0.3238E 03 0.4435E 04 0.1404E 05 0.1320E 05 0.1485E 04 0.1235E 03 0.3238E 03 0.4435E 04 0.1050E 05 0.1190E 05


ELEMENT TYPE (B E A M                  ) / / /  ELEMENT NUMBER (    6)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.6169E 03 0.7132E 03 0.2831E 04 0.3979E 03 0.4005E 05 0.1512E 05 0.6169E 03 0.7132E 03 0.2831E 04 0.3979E 03 0.1625E 05 0.9129E 04


ELEMENT TYPE (B E A M                  ) / / /  ELEMENT NUMBER (    7)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.6167E 03 0.7098E 03 0.2819E 04 0.3979E 03 0.1625E 05 0.9129E 04 0.6167E 03 0.7098E 03 0.2819E 04 0.3979E 03 0.7911E 04 0.3251E 04
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ELEMENT TYPE (B E A M                    )  / / /   ELEMENT NUMBER (    8)

    P1(1)        V2(I)        V3(I)        T1(I)        M2(I)        M3(I)        P1(J)        V2(J)        V3(J)        T1(J)        M2(J)        M3(J)
0.6162E 03 0.6984E 03 0.2783E 04 0.3979E 03 0.7911E 04 0.3251E 04 0.6162E 03 0.6984E 03 0.2783E 04 0.3979E 03 0.3119E 05 0.2989E 04


ELEMENT TYPE (B E A M                    )  / / /   ELEMENT NUMBER (    9)

    P1(I)        V2(I)        V3(I)        T1(I)        M2(I)        M3(I)        P1(J)        V2(J)        V3(J)        T1(J)        M2(J)        M3(J)
0.8124E 03 0.4102E 03 0.4445E 03 0.2012E 04 0.2328E 05 0.1176E 05 0.8124E 03 0.4102E 03 0.4445E 03 0.2012E 04 0.1471E 05 0.2327E 05


ELEMENT TYPE (B E A M                    )  / / /  .ELEMENT NUMBER (   10)

    P1(I)        V2(I)        V3(I)        T1(I)        M2(I)        M3(I)        P1(J)        V2(J)        V3(J)        T1(J)        M2(J)        M3(J)
0.3508E 03 0.1898E 03 0.4423E 03 0.1833E 04 0.1473E 05 0.2327E 05 0.3508E 03 0.1898E 03 0.4423E 03 0.1833E 04 0.1009E 05 0.2775E 05


ELEMENT TYPE (B E A M                    )  / / /   ELEMENT NUMBER (   11)

    P1(I)        V2(I)        V3(I)        T1(I)        M2(I)        M3(I)        P1(J)        V2(J)        V3(J)        T1(J)        M2(J)        M3(J)
0.5489E 03 0.1980E 03 0.4494E 03 0.2302E 04 0.9990E 04 0.2775E 05 0.5489E 03 0.1980E 03 0.4494E 03 0.2302E 04 0.1538E 05 0.2281E 05


ELEMENT TYPE (B E A M                    )  / / /   ELEMENT NUMBER (   12)

    P1(I)        V2(I)        V3(I)        T1(I)        M2(I)        M3(I)        P1(J)        V2(J)        V3(J)        T1(J)        M2(J)        M3(J)
0.1070E 04 0.4815E 03 0.4479E 03 0.2803E 04 0.1529E 05 0.2281E 05 0.1070E 04 0.4815E 03 0.4479E 03 0.2803E 04 0.2453E 05 0.1131E 05


ELEMENT TYPE (B E A M                    )  / / /   ELEMENT NUMBER (   13)

    P1(I)        V2(I)        V3(I)        T1(I)        M2(I)        M3(I)        P1(J)        V2(J)        V3(J)        T1(J)        M2(J)        M3(J)
0.7544E 03 0.1031E 04 0.2644E 04 0.4064E 03 0.3773E 05 0.2316E 05 0.7544E 03 0.1031E 04 0.2644E 04 0.4064E 03 0.1550E 05 0.1450E 05


ELEMENT TYPE (B E A M                    )  / / /   ELEMENT NUMBER (   14)

    P1(I)        V2(I)        V3(I)        T1(I)        M2(I)        M3(I)        P1(J)        V2(J)        V3(J)        T1(J)        M2(J)        M3(J)
0.7541E 03 0.1027E 04 0.2632E 04 0.4064E 03 0.1550E 05 0.1450E 05 0.7541E 03 0.1027E 04 0.2632E 04 0.4064E 03 0.7021E 04 0.5957E 04


ELEMENT TYPE (B E A M                    )  / / /   ELEMENT NUMBER (   15)

    P1(I)        V2(I)        V3(I)        T1(I)        M2(I)        M3(I)        P1(J)        V2(J)        V3(J)        T1(J)        M2(J)        M3(J)
0.7536E 03 0.1011E 04 0.2598E 04 0.4064E 03 0.7021E 04 0.5957E 04 0.7536E 03 0.1011E 04 0.2598E 04 0.4064E 03 0.2876E 05 0.3157E 04


ELEMENT TYPE (B E A M                    )  / / /   ELEMENT NUMBER (   16)

    P1(I)        V2(I)        V3(I)        T1(I)        M2(I)        M3(I)        P1(J)        V2(J)        V3(J)        T1(J)        M2(J)        M3(J)
0.1075E 04 0.3233E 03 0.3922E 03 0.2889E 04 0.1047E 05 0.1153E 05 0.1075E 04 0.3233E 03 0.3922E 03 0.2889E 04 0.1277E 05 0.1271E 05
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ELEMENT TYPE (B E A M                    )  / / /  ELEMENT NUMBER (  17)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.6839E 03 0.1663E 03 0.2195E 03 0.2753E 04 0.1280E 05 0.1271E 05 0.6839E 03 0.1663E 03 0.2195E 03 0.2753E 04 0.1488E 05 0.1268E 05


ELEMENT TYPE (B E A M                    )  / / /  ELEMENT NUMBER (  18)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.3254E 03 0.1795E 03 0.1999E 03 0.2721E 04 0.1489E 05 0.1268E 05 0.3254E 03 0.1795E 03 0.1999E 03 0.2721E 04 0.1069E 05 0.8766E 04


ELEMENT TYPE (B E A M                    )  / / /  ELEMENT NUMBER (  19)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.2030E 03 0.3381E 03 0.4133E 03 0.2786E 04 0.1067E 05 0.8766E 04 0.2030E 03 0.3381E 03 0.4133E 03 0.2786E 04 0.4894E 03 0.4279E 03


ELEMENT TYPE (B E A M                    )  / / /  ELEMENT NUMBER (  20)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.0000E 00 0.4279E 03 0.4685E 03 0.2789E 04 0.4685E 03 0.4279E 03 0.0000E 00 0.4279E 03 0.4685E 03 0.2789E 04 0.0000E 00 0.0000E 00


ELEMENT TYPE (B E A M                    )  / / /  ELEMENT NUMBER (  21)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.1063E 03 0.4889E 03 0.5127E 03 0.2789E 04 0.2417E 02 0.1344E-05 0.1063E 03 0.4889E 03 0.5127E 03 0.2789E 04 0.7695E 04 0.7334E 04


ELEMENT TYPE (B E A M                    )  / / /  ELEMENT NUMBER (  22)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.8085E 03 0.1010E 04 0.1558E 04 0.4551E 03 0.2059E 05 0.2345E 05 0.8085E 03 0.1010E 04 0.1558E 04 0.4551E 03 0.8383E 04 0.1494E 05


ELEMENT TYPE (B E A M                    )  / / /  ELEMENT NUMBER (  23)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.8082E 03 0.1005E 04 0.1548E 04 0.4551E 03 0.8383E 04 0.1494E 05 0.8082E 03 0.1005E 04 0.1548E 04 0.4551E 03 0.7805E 04 0.6481E 04


ELEMENT TYPE (B E A M                    )  / / /  ELEMENT NUMBER (  24)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.8076E 03 0.9906E 03 0.1520E 04 0.4551E 03 0.7805E 04 0.6481E 04 0.8076E 03 0.9906E 03 0.1520E 04 0.4551E 03 0.1960E 05 0.2053E 04


ELEMENT TYPE (B E A M                    )  / / /  ELEMENT NUMBER (  25)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.1275E 04 0.4535E 03 0.7905E 03 0.2500E 04 0.1897E 05 0.7033E 04 0.1275E 04 0.4535E 03 0.7905E 03 0.2500E 04 0.1463E 05 0.1170E 05
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ELEMENT TYPE (B E A M                    )   / / /   ELEMENT NUMBER (  26)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.9614E 03 0.2302E 03 0.4569E 03 0.2555E 04 0.1462E 05 0.1170E 05 0.9614E 03 0.2302E 03 0.4569E 03 0.2555E 04 0.2087E 05 0.1805E 05


ELEMENT TYPE (B E A M                    )   / / /   ELEMENT NUMBER (  27)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.7237E 03 0.1053E 03 0.4121E 03 0.3010E 04 0.2081E 05 0.1805E 05 0.7237E 03 0.1053E 03 0.4121E 03 0.3010E 04 0.1408E 05 0.1816E 05


ELEMENT TYPE (B E A M                    )   / / /   ELEMENT NUMBER (  28)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.6436E 03 0.3290E 03 0.7762E 03 0.3396E 04 0.1399E 05 0.1816E 05 0.6436E 03 0.3290E 03 0.7762E 03 0.3396E 04 0.1556E 05 0.1369E 05


ELEMENT TYPE (B E A M                    )   / / /   ELEMENT NUMBER (  29)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.4019E 03 0.9593E 03 0.1902E 04 0.3334E 03 0.2346E 05 0.2148E 05 0.4019E 03 0.9593E 03 0.1902E 04 0.3334E 03 0.8185E 04 0.1343E 05


ELEMENT TYPE (B E A M                    )   / / /   ELEMENT NUMBER (  30)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.4017E 03 0.9545E 03 0.1893E 04 0.3334E 03 0.8185E 04 0.1343E 05 0.4017E 03 0.9545E 03 0.1893E 04 0.3334E 03 0.9839E 04 0.5514E 04


ELEMENT TYPE (B E A M                    )   / / /   ELEMENT NUMBER (  31)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.4012E 03 0.9382E 03 0.1867E 04 0.3334E 03 0.9839E 04 0.5514E 04 0.4012E 03 0.9382E 03 0.1867E 04 0.3334E 03 0.2503E 05 0.3058E 04


ELEMENT TYPE (B E A M                    )   / / /   ELEMENT NUMBER (  32)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.1171E 04 0.4129E 03 0.7176E 03 0.2496E 04 0.1175E 05 0.1355E 05 0.1171E 04 0.4129E 03 0.7176E 03 0.2496E 04 0.1343E 05 0.2137E 05


ELEMENT TYPE (B E A M                    )   / / /   ELEMENT NUMBER (  33)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.8678E 03 0.1959E 03 0.3180E 03 0.2372E 04 0.1346E 05 0.2137E 05 0.8678E 03 0.1959E 03 0.3180E 03 0.2372E 04 0.1948E 05 0.2454E 05


ELEMENT TYPE (B E A M                    )   / / /   ELEMENT NUMBER (  34)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.6289E 03 0.1893E 03 0.4572E 03 0.2475E 04 0.1947E 05 0.2454E 05 0.6289E 03 0.1893E 03 0.4572E 03 0.2475E 04 0.1099E 05 0.2136E 05
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ELEMENT TYPE (B E A M                    )  / / /  ELEMENT NUMBER (  35)

    P1(I)       V2(I)       V3(I)       T1(I)       M2(I)       M3(I)       P1(J)       V2(J)       V3(J)       T1(J)       M2(J)       M3(J)
0.5403E 03 0.3850E 03 0.8104E 03 0.2595E 04 0.1096E 05 0.2136E 05 0.5403E 03 0.3850E 03 0.8104E 03 0.2595E 04 0.1922E 05 0.1303E 05


ELEMENT TYPE (B E A M                    )  / / /  ELEMENT NUMBER (  36)

    P1(I)       V2(I)       V3(I)       T1(I)       M2(I)       M3(I)       P1(J)       V2(J)       V3(J)       T1(J)       M2(J)       M3(J)
0.7303E 03 0.6462E 03 0.1601E 04 0.4356E 03 0.2122E 05 0.1389E 05 0.7303E 03 0.6462E 03 0.1601E 04 0.4356E 03 0.8260E 04 0.8473E 04


ELEMENT TYPE (B E A M                    )  / / /  ELEMENT NUMBER (  37)

    P1(I)       V2(I)       V3(I)       T1(I)       M2(I)       M3(I)       P1(J)       V2(J)       V3(J)       T1(J)       M2(J)       M3(J)
0.7300E 03 0.6426E 03 0.1591E 04 0.4356E 03 0.8260E 84 0.8473E 04 0.7300E 03 0.6426E 03 0.1591E 04 0.4356E 03 0.7043E 04 0.3166E 04


ELEMENT TYPE (B E A M                    )  / / /  ELEMENT NUMBER (  38)

    P1(I)       V2(I)       V3(I)       T1(I)       M2(I)       M3(I)       P1(J)       V2(J)       V3(J)       T1(J)       M2(J)       M3(J)
0.7293E 03 0.6306E 03 0.1563E 04 0.4356E 03 0.7043E 04 0.3166E 04 0.7293E 03 0.6306E 03 0.1563E 04 0.4356E 03 0.1955E 05 0.2554E 04


ELEMENT TYPE (B E A M                    )  / / /  ELEMENT NUMBER (  39)

    P1(I)       V2(I)       V3(I)       T1(I)       M2(I)       M3(I)       P1(J)       V2(J)       V3(J)       T1(J)       M2(J)       M3(J)
0.1089E 04 0.9330E 02 0.3880E 03 0.4211E 04 0.6907E 04 0.1319E 05 0.1089E 04 0.9330E 02 0.3880E 03 0.4211E 04 0.1115E 05 0.1400E 05


ELEMENT TYPE (B E A M                    )  / / /  ELEMENT NUMBER (  40)

    P1(I)       V2(I)       V3(I)       T1(I)       M2(I)       M3(I)       P1(J)       V2(J)       V3(J)       T1(J)       M2(J)       M3(J)
0.7823E 03 0.1058E 03 0.1877E 03 0.3973E 04 0.1124E 05 0.1400E 05 0.7823E 03 0.1058E 03 0.1877E 03 0.3973E 04 0.1421E 05 0.1159E 05


ELEMENT TYPE (B E A M                    )  / / /  ELEMENT NUMBER (  41)

    P1(I)       V2(I)       V3(I)       T1(I)       M2(I)       M3(I)       P1(J)       V2(J)       V3(J)       T1(J)       M2(J)       M3(J)
0.4731E 03 0.2056E 03 0.1789E 03 0.3740E 04 0.1428E 05 0.1159E 05 0.4731E 03 0.2056E 03 0.1789E 03 0.3740E 04 0.1013E 05 0.6634E 04


ELEMENT TYPE (B E A M                    )  / / /  ELEMENT NUMBER (  42)

    P1(I)       V2(I)       V3(I)       T1(I)       M2(I)       M3(I)       P1(J)       V2(J)       V3(J)       T1(J)       M2(J)       M3(J)
0.1623E 03 0.2636E 03 0.4048E 03 0.3624E 04 0.1017E 05 0.6634E 04 0.1623E 03 0.2636E 03 0.4048E 03 0.3624E 04 0.4006E 03 0.2663E 03


ELEMENT TYPE (B E A M                    )  / / /  ELEMENT NUMBER (  43)

    P1(I)       V2(I)       V3(I)       T1(I)       M2(I)       M3(I)       P1(J)       V2(J)       V3(J)       T1(J)       M2(J)       M3(J)
0.4094E-07 0.2652E 03 0.4103E 03 0.3622E 04 0.4120E 03 0.2663E 03 0.1228E-06 0.2652E 03 0.4103E 03 0.3622E 04 0.0000E 00 0.0000E 00
```

80/80 Listing of Input Card Images

Response Spectrum Analysis

```
ROUTE 80 ONRAMP UNDERCROSSING - RESPONSE SPECTRUM ANALYSIS - SAP IV
   45    2         18   -3
       0.0       0.0       1.0    1
 AASHTO DESIGN SPECTRUM  - .5G PEAK ROCK ACCELERATION - 10 TO 80 FT
   55       32.2
      .000    0.7180
      .001    0.7180
      .025    0.9100
      .050    1.0560
      .075    1.1770
      .100    1.3040
      .125    1.3880
      .150    1.4740
      .175    1.5420
      .200    1.5800
      .225    1.6070
      .250    1.6220
      .275    1.6300
      .300    1.6290
      .325    1.6220
      .350    1.6090
      .375    1.5920
      .400    1.5650
      .425    1.5400
      .450    1.5120
      .475    1.4820
      .500    1.4480
      .525    1.4070
      .550    1.3670
      .575    1.3280
      .600    1.2900
      .650    1.2140
      .700    1.1520
      .750    1.0820
      .800    1.0200
      .850    0.9620
      .900    0.9040
      .950    0.8550
     1.000    0.8100
     1.100    0.7280
     1.2000   0.6570
     1.300    0.5990
     1.400    0.5550
     1.500    0.5140
     1.6000   0.4800
     1.700    0.4500
     1.800    0.4220
     1.900    0.3980
     2.000    0.3780
     2.200    0.3410
     2.400    0.3110
     2.600    0.2850
     2.800    0.2610
     3.000    0.2380
     3.500    0.1982
     4.000    0.1625
     4.500    0.1286
     5.000    0.1093
     5.500    0.1093
     6.000    0.1093
```

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N O D E   D I S P L A C E M E N T S / R O T A T I O N S

  NODE   MODE              X-            Y-            Z-             X-            Y-            Z-
NUMBER  NUMBER    TRANSLATION   TRANSLATION   TRANSLATION       ROTATION      ROTATION      ROTATION

   45       1     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
            2     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
            3     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
            4     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
            5     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
            6     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
            7     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
            8     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
            9     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
           10     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
           11     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
           12     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
           13     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
           14     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
           15     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
           16     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
           17     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
           18     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
           19     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00

   44       1     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
            2     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
            3     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
            4     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
            5     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
            6     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
            7     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
            8     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
            9     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
           10     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
           11     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
           12     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
           13     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
           14     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
           15     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
           16     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
           17     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
           18     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00
           19     0.00000E 00   0.00000E 00   0.00000E 00    0.00000E 00   0.00000E 00   0.00000E 00

   43       1    -0.29496E-03  -0.86740E-04  -0.31078E-03   -0.52350E-04  -0.98034E-04   0.68965E-04
            2     0.77453E-03   0.16192E-04   0.40024E-03    0.46620E-05  -0.89070E-04  -0.16211E-04
            3     0.49071E-01  -0.97596E-03   0.32875E-01   -0.50000E-03   0.62141E-03   0.83408E-03
            4     0.97335E-02  -0.42451E-03   0.63722E-02   -0.23152E-03  -0.74519E-06   0.35362E-03
            5    -0.31635E-02   0.64740E-05  -0.21803E-02   -0.17956E-06  -0.90817E-04  -0.78162E-05
            6    -0.18439E-02  -0.36777E-04  -0.11884E-02   -0.19640E-04   0.15740E-04   0.30907E-04
            7     0.46873E-04  -0.15158E-03   0.27238E-04   -0.83003E-04  -0.28767E-05   0.12604E-03
            8     0.38924E-02   0.38440E-03   0.29179E-02    0.21985E-03   0.30762E-03  -0.31350E-03
            9    -0.44745E-03  -0.13960E-03  -0.27912E-03   -0.76009E-04   0.11543E-04   0.11635E-03
           10     0.11030E-03   0.30721E-04   0.70845E-04    0.16768E-04  -0.11467E-05  -0.25577E-04
           11    -0.84241E-04  -0.46695E-04  -0.54733E-04   -0.25521E-04   0.35409E-06   0.38854E-04
           12     0.45215E-04  -0.18428E-06   0.42734E-04    0.10237E-06   0.10938E-04   0.28593E-06
           13     0.13530E-04  -0.37860E-06   0.86218E-05   -0.20964E-06  -0.14763E-06   0.31312E-06
           14    -0.20514E-03   0.20359E-05  -0.21023E-03    0.19330E-06  -0.63248E-04  -0.22937E-05
           15    -0.48522E-05   0.26380E-06  -0.31465E-05    0.14462E-06   0.25445E-07  -0.21908E-06
           16     0.74289E-06   0.67062E-05   0.49364E-06    0.36639E-05   0.60145E-08  -0.55764E-05
           17     0.10644E-05   0.73663E-06   0.68843E-06    0.40234E-06  -0.70624E-08  -0.61251E-06
           18    -0.15345E-05   0.47614E-06  -0.10286E-05    0.26008E-06  -0.19865E-07  -0.39585E-06
           19     0.50321E-01   0.11555E-02   0.33712E-01    0.60723E-03   0.71490E-03   0.97810E-03

   42       1     0.95417E-03  -0.19882E-02  -0.23221E-02   -0.16474E-03  -0.98011E-04  -0.24380E-04
            2     0.19071E-02   0.36188E-03  -0.14251E-02   -0.10071E-03  -0.88684E-04  -0.73576E-04
            3     0.41135E-01  -0.22178E-01   0.45559E-01    0.43820E-03   0.61537E-03   0.10834E-02
            4     0.97361E-02  -0.96097E-02   0.63434E-02   -0.16661E-03  -0.16828E-05   0.25026E-03
            5    -0.20136E-02   0.14043E-03  -0.40213E-02   -0.92873E-04  -0.88077E-04  -0.60603E-04
            6    -0.20405E-02  -0.82276E-03  -0.86581E-03   -0.31554E-05   0.15455E-04   0.27135E-04
            7     0.83223E-04  -0.33623E-02  -0.31472E-04   -0.63813E-04  -0.28254E-05   0.77141E-04
            8     0.31352E-04   0.84766E-02   0.91084E-02    0.39862E-03   0.29160E-03  -0.45609E-04
            9    -0.59115E-03  -0.30606E-02  -0.45644E-04   -0.45503E-04   0.10987E-04   0.74053E-04
           10     0.12442E-03   0.66877E-03   0.47475E-04    0.10784E-04  -0.10997E-05  -0.15168E-04
           11    -0.88476E-04  -0.10042E-02  -0.47318E-04   -0.16496E-04   0.35178E-06   0.21276E-04
           12    -0.89359E-04  -0.33766E-05   0.25606E-03    0.51724E-05   0.97100E-05   0.31665E-05
           13     0.15251E-04  -0.73870E-05   0.56691E-05   -0.15708E-06  -0.13584E-06   0.53385E-07
           14     0.56584E-03   0.37085E-04  -0.14419E-02   -0.23242E-04  -0.54318E-04  -0.14470E-04
           15    -0.51154E-05   0.48790E-05  -0.25970E-05    0.55975E-07   0.24378E-07  -0.32252E-07
           16     0.66449E-06   0.11636E-03   0.57027E-06    0.63988E-06   0.34729E-08  -0.23162E-06
           17     0.11300E-05   0.12205E-04   0.53399E-06    0.31362E-07  -0.60005E-08   0.31566E-07
           18    -0.12665E-05   0.75859E-05  -0.13712E-05    0.23149E-08  -0.14263E-07   0.51575E-07
           19     0.42430E-01   0.26134E-01   0.47170E-01    0.65705E-03   0.77168E-03   0.11230E-02
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41     1   0.21191E-02  -0.28808E-02  -0.43834E-02  -0.26593E-03  -0.98250E-04  -0.13631E-03
       2   0.29533E-02   0.52360E-03  -0.32819E-02  -0.21435E-02  -0.87866E-04  -0.11692E-03
       3   0.33774E-01  -0.32017E-01   0.58273E-01   0.16091E-02   0.60104E-03   0.94265E-03
       4   0.97364E-02  -0.13586E-01   0.62656E-02  -0.15952E-04  -0.41777E-05   0.42657E-05
       5  -0.99281E-03   0.20221E-03  -0.57952E-02  -0.19123E-03  -0.80817E-04  -0.10375E-03
       6  -0.22119E-02  -0.11210E-02  -0.54370E-03   0.21475E-04   0.14690E-04   0.98451E-05
       7   0.11583E-03  -0.44391E-02  -0.89757E-04  -0.13384E-04  -0.26986E-05  -0.23187E-04
       8  -0.32354E-02   0.10889E-01   0.14810E-01   0.50865E-03   0.24874E-03   0.33417E-03
       9  -0.71077E-03  -0.38815E-02   0.17264E-03   0.14631E-04   0.94408E-05  -0.16785E-04
      10   0.13596E-03   0.82621E-03   0.25091E-04  -0.17041E-05  -0.97284E-06   0.58533E-05
      11  -0.91822E-04  -0.11847E-02  -0.39537E-04   0.24225E-05   0.34519E-06  -0.12336E-04
      12  -0.18896E-03  -0.29186E-05   0.43042E-03   0.10524E-04   0.64979E-05   0.55715E-05
      13   0.16373E-04  -0.54500E-05   0.29262E-05  -0.37144E-07  -0.10505E-06  -0.30704E-06
      14   0.10978E-02   0.21945E-04  -0.23556E-02  -0.48127E-04  -0.31332E-04  -0.24250E-04
      15  -0.52223E-05   0.24429E-05  -0.20171E-05  -0.71804E-07   0.21553E-07   0.20834E-06
      16   0.62134E-06   0.25962E-04   0.57836E-06  -0.29624E-05  -0.32280E-08   0.56435E-05
      17   0.11346E-05   0.36316E-06   0.37429E-06  -0.35383E-06  -0.50959E-08   0.64497E-06
      18  -0.10571E-05  -0.97798E-06  -0.14734E-05  -0.23379E-06  -0.64442E-09   0.42186E-06
      19   0.35593E-01   0.37080E-01   0.61024E-01   0.17334E-02   0.66961E-03   0.10223E-02

40     1   0.32077E-02  -0.22024E-02  -0.65063E-02  -0.36657E-03  -0.99648E-04  -0.24853E-03
       2   0.39141E-02   0.38944E-03  -0.51656E-02  -0.33381E-03  -0.87439E-04  -0.14960E-03
       3   0.27015E-01  -0.24389E-01   0.70909E-01   0.28720E-02   0.58670E-03   0.63732E-03
       4   0.97456E-02  -0.99585E-02   0.61102E-02   0.14806E-03  -0.77480E-05  -0.26341E-03
       5  -0.13399E-03   0.15849E-03  -0.74133E-02  -0.29374E-03  -0.70569E-04  -0.13928E-03
       6  -0.23541E-02  -0.75628E-03  -0.23252E-03   0.46269E-04   0.13578E-04  -0.73862E-05
       7   0.14430E-03  -0.28419E-02  -0.14636E-03   0.29324E-04  -0.25526E-05  -0.10551E-03
       8  -0.57018E-02   0.65737E-02   0.19485E-01   0.66380E-03   0.18668E-03   0.62507E-03
       9  -0.79887E-03  -0.22728E-02   0.35647E-03   0.60525E-04   0.72988E-05  -0.79181E-04
      10   0.14425E-03   0.45448E-03   0.53788E-05  -0.10088E-04  -0.78563E-06   0.18770E-04
      11  -0.94225E-04  -0.57716E-03  -0.31538E-04   0.12703E-04   0.33462E-06  -0.29071E-04
      12  -0.23925E-03   0.25232E-06   0.52252E-03   0.16029E-04   0.20700E-05   0.76974E-05
      13   0.16756E-04   0.11844E-05   0.82983E-06  -0.12878E-06  -0.62871E-07  -0.26425E-06
      14   0.12944E-02  -0.16055E-04  -0.26982E-02  -0.72799E-04  -0.77092E-06  -0.34490E-04
      15  -0.51587E-05  -0.24062E-05  -0.14517E-05  -0.12852E-07   0.17610E-07   0.95574E-07
      16   0.64526E-06  -0.93553E-04   0.37753E-06  -0.84753E-06  -0.12845E-07   0.81482E-06
      17   0.10740E-05  -0.11686E-04   0.22827E-06  -0.56434E-07  -0.33838E-08  -0.11609E-07
      18  -0.95209E-06  -0.78864E-05  -0.12195E-05  -0.83919E-08   0.15172E-07  -0.59799E-07
      19   0.29848E-01   0.27507E-01   0.74678E-01   0.30092E-02   0.63398E-03   0.99484E-03

39     1   0.42337E-02   0.58774E-04  -0.87293E-02  -0.47674E-03  -0.10368E-03  -0.34104E-03
       2   0.47978E-02  -0.37406E-04  -0.70954E-02  -0.45678E-03  -0.88749E-04  -0.17517E-03
       3   0.20787E-01  -0.21199E-03   0.83583E-01   0.40982E-02   0.58586E-03   0.41235E-03
       4   0.97649E-02   0.16902E-04   0.58611E-02   0.26145E-03  -0.11626E-04  -0.42458E-03
       5   0.55680E-03   0.48180E-04  -0.88262E-02  -0.39893E-03  -0.59782E-04  -0.16939E-03
       6  -0.24656E-02   0.47603E-04   0.60621E-04   0.64509E-04   0.12332E-04  -0.10887E-04
       7   0.16899E-03  -0.23448E-04  -0.20160E-03   0.29019E-04  -0.24777E-05  -0.10227E-03
       8  -0.73121E-02   0.12117E-03   0.22785E-01   0.95245E-03   0.11724E-03   0.63821E-03
       9  -0.85300E-03  -0.60625E-04   0.49244E-03   0.58591E-04   0.47852E-05  -0.41970E-04
      10   0.14906E-03   0.20666E-04  -0.10432E-04  -0.65365E-05  -0.56770E-06   0.68599E-05
      11  -0.95638E-04  -0.52709E-04  -0.23468E-04   0.15430E-05   0.32056E-06  -0.12782E-05
      12  -0.23946E-03   0.39284E-06   0.51587E-03   0.21631E-04  -0.24828E-05   0.96262E-05
      13   0.16394E-04  -0.18748E-05  -0.42726E-06  -0.50440E-06  -0.19927E-07   0.36014E-06
      14   0.11657E-02   0.13239E-05  -0.23747E-02  -0.97092E-04   0.28571E-04  -0.45479E-04
      15  -0.49242E-05   0.12568E-05  -0.93131E-06   0.22472E-06   0.13413E-07  -0.37758E-06
      16   0.74629E-06   0.21214E-04  -0.66094E-07   0.40511E-05  -0.24243E-07  -0.93534E-05
      17   0.95334E-06   0.97039E-06   0.10178E-06   0.33468E-06  -0.20669E-08  -0.80055E-06
      18  -0.92078E-06  -0.38576E-06  -0.63544E-06   0.13674E-06   0.26321E-07  -0.32779E-06
      19   0.25109E-01   0.27792E-03   0.88035E-01   0.42877E-02   0.61670E-03   0.97362E-03

38     1   0.18591E-02   0.39197E-04  -0.46825E-02  -0.45990E-03  -0.69136E-04  -0.22311E-03
       2   0.28998E-02  -0.24946E-04  -0.35462E-02  -0.37387E-03  -0.59177E-04  -0.25269E-03
       3   0.16967E-01  -0.14138E-03   0.46592E-01   0.44026E-02   0.39065E-03  -0.10932E-02
       4   0.56494E-02   0.11273E-04   0.33657E-02   0.30859E-03  -0.77522E-05  -0.51405E-03
       5  -0.22376E-03   0.32135E-04  -0.50478E-02  -0.46480E-03  -0.39362E-04  -0.29528E-04
       6  -0.18700E-02   0.31751E-04  -0.19750E-03   0.32770E-05   0.82227E-05   0.12967E-03
       7  -0.25865E-03  -0.15641E-04  -0.25849E-03  -0.10576E-04  -0.16521E-05  -0.89902E-05
       8  -0.30261E-02   0.80833E-04   0.13326E-01   0.11993E-02   0.78174E-04   0.38541E-03
       9  -0.79084E-03  -0.40443E-04   0.14510E-03   0.25979E-04   0.31908E-05   0.44774E-04
      10   0.13647E-03   0.13787E-04   0.16857E-04  -0.56053E-06  -0.37854E-06  -0.78195E-05
      11  -0.75865E-04  -0.35168E-04  -0.23251E-04  -0.12279E-05   0.21375E-06   0.50166E-05
      12  -0.14180E-03   0.26229E-06   0.30207E-03   0.27093E-04  -0.16555E-05   0.12573E-04
      13   0.13594E-04  -0.12521E-05   0.16002E-05  -0.25404E-07  -0.13287E-07  -0.85365E-06
      14   0.69680E-03   0.88455E-06  -0.14019E-02  -0.12450E-03   0.19051E-04  -0.61068E-04
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      16  -0.35080E-04   0.14191E-04  -0.15486E-04   0.24826E-07  -0.16165E-07  -0.10385E-06
      17  -0.23440E-05   0.64965E-06  -0.12024E-05   0.79925E-08  -0.13782E-08  -0.55868E-07
      18  -0.19580E-05  -0.25846E-06  -0.10057E-05  -0.31077E-07   0.17550E-07   0.43410E-07
      19   0.18590E-01   0.18537E-03   0.49212E-01   0.46369E-02   0.41122E-03   0.13210E-02

37     1   0.45858E-03   0.19588E-04  -0.13701E-02  -0.30124E-03  -0.34547E-04  -0.10946E-03
       2   0.91624E-03  -0.12467E-04  -0.98372E-03  -0.22173E-03  -0.29571E-04  -0.19459E-03
       3   0.61742E-02  -0.70655E-04   0.13999E-01   0.30407E-02   0.19521E-03  -0.12350E-02
       4   0.17394E-02   0.56338E-05   0.10313E-02   0.22202E-03  -0.38738E-05  -0.37365E-03
       5  -0.17419E-03   0.16060E-04  -0.15425E-02  -0.33249E-03  -0.19919E-04   0.27286E-04
       6  -0.66120E-03   0.15869E-04  -0.10565E-03  -0.18381E-04   0.41089E-05   0.13382E-03
       7  -0.14825E-03  -0.78174E-05  -0.10689E-03  -0.20337E-04  -0.82557E-06   0.25248E-04
       8  -0.69938E-03   0.40403E-04   0.41310E-02   0.88469E-03   0.39064E-04   0.17260E-03
       9  -0.30091E-03  -0.20215E-04   0.17702E-04   0.63991E-05   0.15944E-05   0.59111E-04
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      11  -0.27352E-04  -0.17580E-04  -0.90376E-05  -0.17603E-05   0.10681E-06   0.54899E-05
      12  -0.44393E-04  -0.13117E-06   0.93899E-04   0.20077E-04  -0.82728E-06   0.94617E-05
      13   0.50038E-05  -0.62623E-06   0.85652E-06   0.14864E-06  -0.66378E-08  -0.99502E-06
      14   0.21978E-03   0.44254E-06  -0.47867E-03  -0.93497E-04   0.95201E-05  -0.46671E-04
      15  -0.20293E-05   0.42024E-06  -0.67948E-06  -0.126[illegible]E-06  0.44694E-08   0.39158E-06
      16  -0.17835E-04   0.71047E-05  -0.78284E-05  -0.13[illegible]E-05  -0.40770E-08   0.31452E-05
      17  -0.12471E-05   0.32541E-06  -0.62138E-06  -0.10892E-06  -0.68860E-09   0.22240E-06
      18  -0.89356E-06  -0.12902E-06  -0.49011E-05  -0.81647E-07   0.87703E-08   0.16323E-06
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         19   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00

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          3   0.14579E-01   0.20311E-01   0.99106E-01   0.46909E-02   0.56992E-03   0.15665E-02
          4   0.98761E-02   0.12154E-01   0.54641E-02   0.20333E-03  -0.17314E-04  -0.24577E-03
          5   0.11193E-02   0.15211E-02  -0.10203E-01  -0.39452E-03  -0.41969E-04  -0.23835E-03
          6  -0.26014E-02   0.24209E-02   0.35694E-03   0.98905E-04   0.10117E-04  -0.70533E-04
          7   0.17353E-03   0.27232E-02  -0.27560E-03   0.16913E-04  -0.23937E-05  -0.66819E-04
          8  -0.80929E-02  -0.50365E-02   0.24635E-01   0.84785E-03   0.16636E-04   0.45056E-03
          9  -0.89678E-03   0.10836E-02   0.56903E-03   0.37216E-04   0.10893E-05  -0.17445E-05
         10   0.15426E-03  -0.10843E-04  -0.21184E-04  -0.94746E-06  -0.24376E-06  -0.52989E-05
         11  -0.97862E-04  -0.58373E-03  -0.14856E-04  -0.10608E-04   0.29952E-06   0.24965E-04
         12  -0.18948E-03  -0.33317E-05   0.38462E-03   0.12724E-04  -0.71572E-05   0.48106E-05
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         14   0.72772E-03   0.74709E-04  -0.12543E-02  -0.52119E-04   0.52975E-04  -0.20230E-04
         15  -0.45911E-05   0.19214E-04  -0.45444E-06   0.22210E-06   0.86022E-08  -0.23238E-06
         16  -0.10405E-05   0.32850E-03  -0.17406E-05   0.15539E-05  -0.35887E-07  -0.71529E-06
         17   0.59181E-06   0.21520E-04  -0.96695E-07   0.21566E-07  -0.13573E-08   0.14465E-06
         18  -0.94146E-06   0.51607E-05   0.22307E-06  -0.16200E-07   0.27033E-07   0.11285E-06
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          3   0.93988E-02   0.15414E-01   0.11378E 00   0.54580E-02   0.51186E-03   0.22507E-02
          4   0.99984E-02   0.15623E-01   0.48741E-02   0.79487E-04  -0.24468E-04   0.98271E-04
          5   0.14100E-02   0.35601E-02  -0.10991E-01  -0.43688E-03  -0.15836E-04  -0.18539E-03
          6  -0.26872E-02   0.45185E-02   0.58566E-03   0.83712E-04   0.68286E-05  -0.18200E-05
          7   0.17272E-03   0.33370E-02  -0.34569E-03  -0.17180E-04  -0.21865E-05   0.23487E-04
          8  -0.77066E-02  -0.58080E-02   0.23426E-01   0.77699E-03  -0.10645E-03   0.18064E-03
          9  -0.89335E-03   0.83596E-03   0.53567E-03   0.16886E-04  -0.34060E-05   0.34249E-04
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          3   0.53359E-02   0.12509E-02   0.12698E 00   0.64682E-02   0.45562E-03   0.21783E-02
          4   0.10120E-01   0.92148E-02   0.40708E-02  -0.15296E-04  -0.31463E-04   0.34128E-03
          5   0.14453E-02   0.32496E-02  -0.11039E-01  -0.50133E-03   0.11467E-04  -0.67649E-04
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          8  -0.45415E-02  -0.22024E-03   0.11297E-01   0.48675E-03  -0.31037E-03   0.16377E-03
          9  -0.78550E-03   0.99070E-04   0.12045E-03   0.19831E-04  -0.10830E-04  -0.40465E-04
         10   0.14396E-03  -0.11073E-04   0.43457E-05  -0.35811E-05   0.78915E-06   0.10971E-04
         11  -0.95174E-04  -0.35335E-04   0.51360E-05   0.30417E-05   0.14755E-06  -0.76179E-05
         12   0.44789E-04   0.35580E-06  -0.40756E-03  -0.16115E-04  -0.79524E-05  -0.33653E-05
         13   0.98435E-05  -0.60208E-05   0.15980E-05  -0.61383E-06   0.26034E-07   0.24401E-05
         14  -0.49272E-03   0.85039E-05   0.26496E-02   0.75292E-04   0.14767E-04   0.15859E-04
         15  -0.25719E-05   0.24384E-05   0.53748E-06   0.35430E-06   0.36454E-08  -0.13753E-05
         16  -0.61472E-05  -0.25665E-04  -0.56501E-05   0.17908E-05   0.11017E-08  -0.89573E-05
         17  -0.64396E-06  -0.65119E-05  -0.51152E-06  -0.13400E-06   0.11394E-08   0.51773E-06
         18   0.11492E-07  -0.30378E-05   0.81045E-06  -0.76915E-07  -0.21672E-07   0.47336E-04
         19   0.16221E-01   0.53685E-03   0.14269E 00   0.79236E-02   0.56692E-03   0.13093E-02
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         3   0.60770E-02   0.27079E-03   0.74371E-01   0.73437E-02   0.30260E-03  -0.10819E-03
         4   0.84677E-02  -0.29086E-04   0.23554E-02   0.16227E-03  -0.24305E-04  -0.53716E-03
         5   0.11061E-02  -0.47642E-04  -0.56002E-02  -0.54781E-03   0.21546E-04  -0.69090E-04
         6  -0.14498E-02   0.48940E-05   0.43654E-03   0.41770E-04   0.77293E-06   0.14349E-03
         7   0.85195E-04   0.71922E-04  -0.24807E-03  -0.24690E-04  -0.12712E-05  -0.81899E-05
         8  -0.27550E-02  -0.14692E-03   0.65517E-02   0.59473E-03  -0.20695E-03   0.23893E-03
         9  -0.73522E-03   0.66089E-04   0.14762E-04   0.63684E-05  -0.72216E-05   0.41216E-04
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        12   0.20538E-04   0.23756E-06  -0.24234E-03  -0.21400E-04  -0.53027E-05  -0.23645E-05
        13   0.16569E-04  -0.40210E-05   0.35136E-05   0.81099E-07   0.17359E-07  -0.50428E-06
        14  -0.30686E-03   0.56816E-05   0.16136E-02   0.13886E-03   0.98468E-05   0.25840E-04
        15  -0.71403E-05   0.16296E-05  -0.94402E-06   0.29953E-07   0.24308E-08   0.12664E-06
        16  -0.38761E-04  -0.17168E-04  -0.11057E-04  -0.28033E-06   0.73459E-09   0.25367E-06
        17   0.14944E-05  -0.43595E-05   0.12478E-06  -0.27804E-07   0.75973E-09   0.37454E-07
        18   0.18286E-05  -0.20353E-05   0.91235E-06   0.41551E-07  -0.14464E-07   0.44617E-08
        19   0.13232E-01   0.35806E-03   0.76085E-01   0.75171E-02   0.37947E-03   0.85351E-03

30       1   0.21113E-02  -0.69157E-04  -0.28460E-02  -0.64049E-03  -0.37370E-04  -0.42768E-03
         2   0.13128E-02   0.10996E-04  -0.24028E-02  -0.53369E-03  -0.26321E-04  -0.28507E-03
         3   0.28092E-02   0.13533E-03   0.21679E-01   0.47746E-02   0.15121E-03  -0.51326E-03
         4   0.30991E-02  -0.14536E-04   0.83495E-03   0.16882E-03  -0.12145E-04  -0.61846E-03
         5   0.40703E-03  -0.23810E-04  -0.16435E-02  -0.36083E-03   0.10767E-04  -0.81054E-04
         6  -0.42201E-03   0.24460E-05   0.13006E-03   0.28360E-04   0.38624E-06   0.92990E-04
         7   0.25310E-04   0.35948E-04  -0.71912E-04  -0.15877E-04  -0.63520E-06  -0.55253E-05
         8  -0.87314E-03  -0.73434E-04   0.20202E-02   0.43369E-03  -0.10342E-03   0.18515E-03
         9  -0.28062E-03   0.33034E-04  -0.60703E-05  -0.28143E-06  -0.36086E-05   0.55056E-04
        10   0.58153E-04  -0.36926E-05   0.78851E-05   0.14296E-05   0.26295E-06  -0.11284E-04
        11  -0.39172E-04  -0.11785E-04  -0.44039E-05  -0.74828E-06   0.49164E-07   0.75873E-05
        12   0.52708E-05   0.11880E-06  -0.76045E-04  -0.16191E-04  -0.26498E-05  -0.12343E-05
        13   0.72327E-05  -0.20112E-05   0.15884E-05   0.29212E-06   0.86745E-08  -0.13470E-05
        14  -0.98973E-04   0.28425E-05   0.51427E-03   0.10872E-03   0.49205E-05   0.20817E-04
        15  -0.33119E-05   0.81540E-06  -0.53698E-06  -0.91435E-07   0.12147E-08   0.60318E-06
        16  -0.18926E-04  -0.85953E-05  -0.49563E-05  -0.91296E-06   0.36708E-09   0.33829E-05
        17   0.83077E-06  -0.21837E-05   0.12190E-06   0.18055E-07   0.37964E-09  -0.14225E-06
        18   0.93015E-06  -0.10199E-05   0.37045E-06   0.70722E-07  -0.72277E-08  -0.16371E-06
        19   0.49878E-02   0.17895E-03   0.22173E-01   0.48839E-02   0.18962E-03   0.98148E-03

29       1   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
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         5   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
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         2   0.82948E-02   0.45959E-02  -0.18391E-01  -0.10525E-02  -0.79123E-04  -0.30855E-03
         3  -0.25346E-03   0.12939E-01   0.15234E 00   0.83803E-02   0.44731E-03   0.10812E-02
         4   0.10486E-01  -0.10753E-01   0.20240E-02  -0.10774E-03  -0.39538E-04   0.39960E-03
         5   0.11313E-02  -0.54643E-02  -0.92325E-02  -0.48035E-03   0.50988E-04   0.99574E-04
         6  -0.27152E-02  -0.33649E-02   0.80270E-03   0.36369E-04  -0.71076E-06   0.86029E-04
         7   0.14183E-03   0.16719E-02  -0.52631E-03  -0.89945E-05  -0.17769E-05  -0.82786E-04
         8  -0.26962E-02  -0.44382E-02   0.16992E-02  -0.37434E-04  -0.36794E-03   0.17020E-03
         9  -0.73625E-03   0.28195E-02  -0.21874E-03   0.21166E-04  -0.12899E-04  -0.10594E-03
        10   0.14087E-03  -0.50726E-03   0.29564E-04  -0.44535E-05   0.96533E-06   0.16367E-04
        11  -0.94890E-04  -0.20048E-03   0.89637E-05  -0.18521E-05   0.10779E-06   0.13502E-04
        12   0.78179E-04   0.11877E-04  -0.56774E-03  -0.13033E-04  -0.30735E-05  -0.27908E-05
        13   0.84677E-05  -0.11137E-03   0.15012E-05  -0.88835E-06  -0.16489E-07   0.22424E-05
        14  -0.49167E-03   0.13124E-03   0.25373E-02   0.40793E-04  -0.23304E-04   0.55615E-05
        15  -0.20235E-05   0.48525E-04   0.76842E-06   0.30436E-06   0.47285E-08  -0.53572E-06
        16  -0.96549E-05   0.46225E-04  -0.59359E-05  -0.41349E-06   0.26229E-07   0.31653E-05
        17  -0.93276E-06  -0.41348E-04  -0.50482E-06  -0.20794E-06   0.36566E-08   0.18401E-06
        18   0.62357E-06  -0.21698E-04   0.22240E-06  -0.66115E-07  -0.25051E-07  -0.98487E-07
        19   0.16435E-01   0.24026E-01   0.15558E 00   0.85987E-02   0.60017E-03   0.12551E-02

27       1   0.91830E-02  -0.25887E-01  -0.26826E-01  -0.16685E-02  -0.10972E-03  -0.11030E-04
         2   0.86241E-02   0.65189E-02  -0.20513E-01  -0.11723E-02  -0.70668E-04  -0.13809E-03
         3  -0.18504E-02   0.17674E-01   0.16413E 00   0.90105E-02   0.37739E-03   0.11061E-02
         4   0.10686E-01  -0.19117E-01   0.87640E-03  -0.11590E-03  -0.41557E-04   0.96834E-04
         5   0.92571E-03  -0.84712E-02  -0.74802E-02  -0.36335E-03   0.71874E-04  -0.31698E-04
         6  -0.26857E-02  -0.40385E-02   0.75568E-03   0.41525E-04  -0.27199E-05  -0.27686E-04
         7   0.12395E-03   0.34431E-02  -0.57499E-03  -0.14080E-04  -0.13871E-05  -0.28153E-04
         8  -0.11082E-02  -0.76915E-02  -0.93129E-02  -0.52176E-03  -0.40467E-03  -0.44502E-04
         9  -0.69197E-03   0.46866E-02  -0.60527E-03  -0.70828E-05  -0.14086E-04  -0.54245E-05
        10   0.13744E-03  -0.73894E-03   0.58618E-04  -0.84059E-06   0.10616E-05  -0.29712E-05
        11  -0.93471E-04  -0.52889E-03   0.11633E-04  -0.15917E-05   0.63928E-07   0.50805E-05
        12   0.81006E-04   0.19160E-04  -0.56757E-03  -0.10179E-04   0.31063E-05  -0.99225E-06
        13   0.70918E-05  -0.80064E-04   0.12413E-06   0.10256E-06  -0.64671E-07  -0.42306E-05
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        15  -0.14108E-05   0.15594E-04   0.10044E-05  -0.21751E-06   0.54617E-08   0.23527E-05
        16  -0.12812E-04  -0.87878E-04  -0.52838E-05  -0.67242E-06   0.49568E-07   0.28303E-05
        17  -0.11767E-05   0.21774E-05  -0.39317E-06   0.27184E-06   0.64063E-08  -0.22052E-05
        18   0.11526E-05   0.69686E-05  -0.31498E-06   0.14899E-06  -0.15076E-07  -0.11745E-05
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       3  -0.27137E-02   0.11839E-01   0.17390E 00   0.96490E-02   0.31433E-03   0.10329E-02
       4   0.10803E-01  -0.15423E-01  -0.32477E-03  -0.99818E-04  -0.42866E-04  -0.37146E-03
       5   0.75227E-03  -0.59370E-02  -0.51656E-02  -0.24060E-03   0.88852E-04  -0.19048E-03
       6  -0.26433E-02  -0.20009E-02   0.65900E-03   0.41698E-04  -0.42082E-05  -0.89168E-04
       7   0.10235E-03   0.27635E-02  -0.61158E-03  -0.25672E-04  -0.10461E-05   0.69308E-04
       8   0.31685E-04  -0.52695E-02  -0.21175E-01  -0.10049E-02  -0.42442E-03  -0.24132E-03
       9  -0.65949E-03   0.30143E-02  -0.10169E-02  -0.38617E-04  -0.14646E-04   0.10278E-03
      10   0.13423E-03  -0.39170E-03   0.89558E-04   0.26032E-05   0.11029E-05  -0.17376E-04
      11  -0.91050E-04  -0.41503E-03   0.13261E-04   0.81189E-07   0.37335E-07  -0.12126E-04
      12   0.67159E-04   0.34441E-05  -0.39619E-03  -0.72682E-05   0.85984E-05   0.12207E-06
      13   0.56205E-05   0.53609E-04  -0.24997E-05   0.28612E-06  -0.10556E-06  -0.27486E-05
      14  -0.14972E-03  -0.11024E-03  -0.74299E-03  -0.71092E-04  -0.83576E-04  -0.38047E-05
      15  -0.74955E-06  -0.41533E-04   0.12394E-05  -0.21655E-06   0.66316E-08   0.53668E-06
      16  -0.15403E-04  -0.59158E-04  -0.38474E-05   0.13259E-06   0.65508E-07  -0.37186E-05
      17  -0.13548E-05   0.41180E-04  -0.19358E-06   0.12202E-06   0.84007E-08   0.40545E-06
      18   0.15471E-05   0.19402E-04  -0.50030E-06  -0.55495E-08   0.57153E-09   0.52195E-06
      19   0.17343E-01   0.31146E-01   0.17920E 00   0.99490E-02   0.55723E-03   0.12961E-02

25     1   0.97316E-02   0.49421E-03  -0.32829E-01  -0.18641E-02  -0.10892E-03  -0.11104E-02
       2   0.88807E-02  -0.39850E-03  -0.24191E-01  -0.14123E-02  -0.64655E-04   0.15104E-03
       3  -0.30656E-02   0.30299E-03   0.18290E 00   0.10309E-01   0.33417E-03   0.61939E-03
       4   0.10830E-01  -0.33871E-04  -0.15738E-02  -0.93240E-04  -0.43905E-04  -0.65502E-03
       5   0.67073E-03  -0.43681E-04  -0.24745E-02  -0.12664E-03   0.97168E-04  -0.20112E-03
       6  -0.25939E-02  -0.53469E-04   0.53002E-03   0.33068E-04  -0.47592E-05  -0.16395E-04
       7   0.78383E-04  -0.12515E-04  -0.64207E-03  -0.33884E-04  -0.10970E-05   0.10326E-03
       8   0.62593E-03  -0.23690E-03  -0.33583E-01  -0.15047E-02  -0.44402E-03  -0.16200E-03
       9  -0.64180E-03   0.16824E-03  -0.14455E-02  -0.60643E-04  -0.15399E-04   0.54475E-04
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      12   0.54509E-04   0.79561E-06  -0.98170E-04  -0.42170E-05   0.11695E-04  -0.85180E-06
      13   0.39530E-05  -0.17213E-04  -0.60122E-05  -0.32396E-06  -0.13090E-06   0.86859E-05
      14  -0.32664E-04   0.53422E-05  -0.33090E-02  -0.12644E-03  -0.93587E-04  -0.11605E-04
      15  -0.59331E-07   0.28718E-05   0.14940E-05   0.26641E-07   0.92369E-08  -0.32364E-05
      16  -0.17246E-04  -0.27594E-04  -0.19653E-05   0.29644E-07   0.69209E-07   0.56828E-05
      17  -0.14496E-05   0.81999E-05   0.57276E-07  -0.25083E-08   0.91949E-08   0.99664E-07
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      19   0.17558E-01   0.76722E-03   0.19044E 00   0.10680E-01   0.58770E-03   0.14661E-02

24     1   0.30503E-02   0.32959E-03  -0.17342E-01  -0.17296E-02  -0.72627E-04  -0.51288E-03
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       3   0.51269E-04   0.20207E-03   0.96915E-01   0.96359E-02   0.22283E-03   0.16180E-03
       4   0.55745E-02  -0.22590E-04  -0.81690E-03  -0.82912E-04  -0.29276E-04  -0.57026E-03
       5  -0.25728E-03  -0.29134E-04  -0.13595E-02  -0.13033E-03   0.64792E-04  -0.35441E-04
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       7   0.44592E-03  -0.83482E-05  -0.34901E-03  -0.33811E-04  -0.73149E-06  -0.40158E-05
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       12   0.62052E-04  -0.28380E-06   0.46318E-03   0.19178E-04  -0.33702E-05  -0.31423E-05
       13   0.32133E-04  -0.41372E-04  -0.64864E-05   0.34040E-05   0.62071E-08   0.19415E-04
       14   0.58334E-03   0.54117E-06   0.31101E-02   0.11726E-03   0.24322E-04  -0.22560E-04
       15   0.20951E-07  -0.28570E-05  -0.77455E-06  -0.13953E-06  -0.11896E-07  -0.46269E-06
       16  -0.10507E-05   0.12315E-04   0.99362E-06   0.63962E-06   0.11075E-07   0.30442E-05
       17   0.86389E-06  -0.34850E-05  -0.53601E-06  -0.21665E-06  -0.35181E-08  -0.11147E-05
       18  -0.77191E-05  -0.75614E-05   0.38542E-05   0.36983E-06   0.65525E-08   0.15397E-05
       19   0.12316E 00   0.16262E-02   0.16118E 00   0.96242E-02   0.62598E-03   0.36641E-02

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        3   0.58899E-01  -0.77408E-03   0.77305E-01   0.79757E-02  -0.39970E-03  -0.42840E-02
        4  -0.13944E-03   0.89437E-04  -0.12306E-02  -0.12258E-03   0.78472E-06   0.10917E-04
        5   0.22787E-02   0.14810E-04   0.43953E-02   0.42424E-03   0.46420E-04  -0.17924E-03
        6   0.50645E-04   0.18169E-05  -0.23113E-03  -0.21095E-04  -0.47993E-05  -0.42372E-05
        7  -0.41090E-03  -0.46533E-04   0.12278E-03   0.78955E-05   0.50878E-05   0.21068E-04
        8   0.63296E-03  -0.16004E-03   0.45925E-04  -0.27134E-04   0.25784E-03  -0.80639E-04
        9   0.13384E-02   0.10327E-03  -0.24904E-03  -0.13833E-04   0.90535E-05  -0.66657E-04
       10   0.91020E-04  -0.53661E-04  -0.32449E-04  -0.31721E-05  -0.13176E-05  -0.88635E-05
       11  -0.17422E-04  -0.44732E-05   0.42097E-05   0.16393E-06  -0.81220E-07   0.39592E-06
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       14   0.34900E-03   0.36156E-06   0.18735E-02   0.16301E-03   0.16218E-04  -0.30566E-04
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       16   0.10809E-04   0.82376E-05  -0.16910E-05   0.55763E-07   0.73850E-08   0.75384E-07
       17  -0.36128E-05  -0.23331E-05   0.42606E-06  -0.29524E-07  -0.23459E-08  -0.53935E-07
       18   0.61212E-07  -0.50661E-05   0.14846E-05   0.20174E-06   0.43692E-08   0.40876E-06
       19   0.83954E-01   0.10845E-02   0.83606E-01   0.84918E-02   0.41740E-03   0.64833E-02

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        3   0.20400E-01  -0.38685E-03   0.21812E-01   0.48773E-02  -0.15476E-03  -0.41647E-02
        4  -0.46658E-04   0.44698E-04  -0.35649E-03  -0.78739E-04   0.39213E-06   0.96663E-05
        5   0.76088E-03   0.74019E-05   0.13018E-02   0.28462E-03   0.23196E-04  -0.15775E-03
        6   0.16380E-04   0.90807E-06  -0.71022E-04  -0.15271E-04  -0.23982E-05  -0.34425E-05
        7  -0.16092E-03  -0.23258E-04   0.44716E-04   0.89406E-05   0.25424E-05   0.31257E-04
        8   0.14634E-03  -0.79993E-04   0.80147E-04   0.10875E-04   0.12885E-03  -0.36096E-04
        9   0.52850E-03   0.51621E-04  -0.95323E-04  -0.18681E-04   0.45240E-05  -0.10232E-03
       10   0.26812E-04  -0.26823E-04  -0.95310E-05  -0.20913E-05  -0.65841E-06  -0.58745E-05
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       13   0.45367E-04  -0.13820E-04  -0.81766E-05  -0.14924E-05   0.20682E-08  -0.82545E-05
       14   0.11012E-03   0.18089E-06   0.59336E-03   0.12579E-03   0.81043E-05  -0.23381E-04
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       16   0.55874E-05   0.41243E-05  -0.96723E-06  -0.16431E-06   0.36903E-08  -0.97998E-06
       17  -0.19308E-05  -0.11687E-05   0.27702E-06   0.45273E-07  -0.11722E-08   0.33473E-06
       18   0.89861E-06  -0.25388E-05   0.31718E-06   0.81510E-07   0.21833E-08  -0.10775E-06
       19   0.28462E-01   0.54199E-03   0.23898E-01   0.53108E-02   0.20858E-03   0.58603E-02

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        6   0.42090E-04   0.20614E-03  -0.61634E-03  -0.19460E-04  -0.50704E-05   0.17316E-06
        7  -0.36071E-03  -0.24454E-03   0.38910E-03   0.11830E-04   0.60028E-05   0.25152E-04
        8   0.39616E-02   0.33190E-04   0.11530E-01   0.20012E-03   0.30431E-03   0.50118E-04
        9   0.13768E-02  -0.11369E-02   0.14746E-03  -0.30736E-05   0.10224E-04  -0.23916E-04
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       13   0.34230E-04  -0.91835E-03  -0.69435E-05  -0.25477E-05  -0.83106E-08   0.49445E-05
       14   0.53060E-03   0.11653E-04   0.30206E-02   0.53496E-04  -0.29771E-04  -0.12080E-04
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       16   0.10972E-06   0.92279E-05   0.97899E-06  -0.17493E-06   0.20960E-08  -0.13466E-05
       17   0.27704E-06   0.42033E-05  -0.48905E-06   0.12540E-06  -0.81745E-09   0.52198E-06
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       3   0.74428E-01  -0.17323E 00   0.11554E 00   0.60999E-02  -0.64893E-03  -0.13652E-02
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       6   0.14677E-04   0.23352E-03  -0.73754E-03  -0.20014E-04  -0.19260E-05   0.90575E-05
       7  -0.32193E-03  -0.11594E-02   0.55206E-03   0.10744E-04   0.34589E-05   0.12128E-04
       8   0.60779E-02  -0.36983E-02   0.19684E-01   0.44551E-03   0.17037E-03  -0.54716E-04
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      16   0.12077E-05   0.26479E-04   0.64150E-06   0.49186E-06  -0.39333E-08   0.99870E-06
      17  -0.31683E-06  -0.12891E-04  -0.32549E-06  -0.13007E-06   0.89482E-09  -0.17604E-06
      18   0.61345E-08   0.70025E-04   0.18759E-05   0.23943E-06  -0.82122E-09  -0.63113E-06
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       9   0.12858E-02  -0.53619E-04  -0.19589E-04   0.17848E-04  -0.42534E-05  -0.73543E-04
      10   0.25439E-03  -0.83197E-04  -0.18725E-03  -0.95064E-05   0.18757E-06  -0.55173E-05
      11  -0.69982E-05  -0.23139E-05  -0.14945E-05  -0.17681E-06   0.27993E-07   0.55356E-06
      12  -0.11952E-03   0.39332E-08  -0.25011E-03  -0.22480E-04  -0.14085E-05   0.10617E-04
      13   0.83238E-04   0.32217E-04  -0.34920E-04  -0.58623E-06  -0.61796E-07  -0.16090E-05
      14  -0.88200E-03  -0.26184E-05  -0.17212E-02  -0.15482E-03  -0.24244E-04   0.77104E-04
      15   0.95556E-06   0.49103E-06  -0.13915E-07   0.36877E-07   0.74616E-08   0.51472E-08
      16  -0.14445E-05  -0.12783E-05   0.10147E-05  -0.30453E-07  -0.66915E-08  -0.15816E-06
      17   0.74057E-06   0.29071E-06  -0.53120E-06   0.12004E-07   0.23989E-08   0.71960E-07
      18  -0.78696E-05   0.39893E-05   0.48786E-05  -0.56133E-07  -0.33232E-08  -0.43121E-06
      19   0.64917E-01   0.10316E-02   0.52046E-01   0.47216E-02   0.59196E-03   0.55142E-02

 7     1  -0.53767E-02  -0.37625E-03  -0.18431E-02  -0.47586E-03   0.74198E-04   0.10172E-02
       2   0.19475E-01   0.15673E-03  -0.10959E-01  -0.23039E-02  -0.16400E-04  -0.40394E-02
       3   0.62504E-02  -0.31071E-03   0.10926E-01   0.23654E-02  -0.28257E-03  -0.14839E-02
       4   0.22990E-03   0.16656E-04  -0.33152E-03  -0.68584E-04   0.38133E-05  -0.40675E-04
       5   0.45094E-03  -0.17022E-05   0.14004E-02   0.30408E-03  -0.22418E-04  -0.10022E-03
       6   0.26919E-04   0.14093E-05  -0.12030E-03  -0.25784E-04   0.11924E-05  -0.51129E-05
       7  -0.12843E-03  -0.17414E-05   0.13749E-03   0.28433E-04  -0.48699E-06   0.24836E-04
       8   0.74488E-03   0.99368E-05   0.38725E-02   0.82914E-03  -0.34803E-04  -0.17450E-03
       9   0.48768E-03  -0.26801E-04  -0.47484E-04  -0.62091E-05  -0.21254E-05  -0.95923E-04
      10   0.11555E-03  -0.41587E-04  -0.73558E-04  -0.14270E-04   0.93728E-07  -0.21238E-04
      11  -0.23314E-05  -0.11567E-05  -0.37377E-06  -0.88630E-07   0.13928E-07   0.48369E-06
      12  -0.37376E-04   0.19670E-08  -0.77648E-04  -0.16612E-04  -0.70384E-06   0.79700E-05
      13   0.38288E-04   0.16115E-04  -0.16251E-04  -0.29582E-05  -0.30880E-07  -0.69984E-05
      14  -0.27862E-03  -0.13100E-05  -0.53439E-03  -0.11429E-03  -0.12115E-04   0.59124E-04
      15   0.48989E-06   0.24570E-06  -0.85168E-07  -0.10468E-07   0.37286E-08  -0.86252E-07
      16  -0.10605E-05  -0.64000E-06   0.57376E-06   0.97841E-07  -0.33438E-08   0.16800E-06
      17   0.52506E-06   0.14562E-06  -0.29237E-06  -0.50237E-07   0.11988E-08  -0.83860E-07
      18  -0.48789E-05   0.19992E-05   0.25737E-05   0.44783E-06  -0.16606E-08   0.80767E-06
      19   0.21175E-01   0.51552E-03   0.16133E-01   0.34539E-02   0.29580E-03   0.44283E-02
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6      1   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
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       3   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
       4   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
       5   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
       6   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
       7   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
       8   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
       9   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
      10   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
      11   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
      12   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
      13   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
      14   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
      15   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
      16   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
      17   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
      18   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
      19   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00

5      1  -0.68819E-02   0.41963E-01  -0.17465E-01  -0.12733E-02   0.23469E-03  -0.70552E-03
       2   0.87455E-01   0.42196E-01  -0.55072E-01  -0.91342E-03  -0.40909E-04  -0.85636E-03
       3   0.48030E-01   0.54263E-01   0.45351E-01   0.25669E-02  -0.85303E-03  -0.29999E-02
       4   0.90342E-04  -0.20907E-02  -0.12047E-02  -0.28188E-04   0.12062E-04   0.85776E-04
       5   0.22816E-02   0.97479E-05   0.67452E-02   0.30097E-03  -0.79605E-04  -0.14749E-03
       6   0.94749E-04   0.10693E-03  -0.57832E-03  -0.20574E-04   0.46382E-05   0.61756E-05
       7  -0.33156E-03   0.52016E-03   0.56346E-03   0.65155E-05  -0.26172E-05  -0.25365E-04
       8   0.49031E-02   0.27807E-02   0.18341E-01   0.63888E-03  -0.17116E-03  -0.44018E-03
       9   0.13384E-02  -0.32387E-02   0.16122E-03   0.77540E-04  -0.90165E-05   0.11732E-03
      10   0.11404E-03  -0.24910E-02  -0.17248E-03   0.39195E-04   0.65349E-06   0.11030E-03
      11  -0.10125E-04  -0.34680E-04  -0.20654E-05   0.49936E-06   0.69085E-07   0.18628E-05
      12  -0.20248E-03  -0.17575E-06  -0.43414E-03  -0.13472E-04   0.17087E-05   0.63943E-05
      13   0.32862E-04  -0.44172E-04  -0.14649E-04  -0.29325E-05  -0.92952E-07  -0.55971E-05
      14  -0.16387E-02  -0.10125E-04  -0.33663E-02  -0.93962E-04   0.69001E-06   0.45155E-04
      15  -0.39955E-07  -0.17480E-05   0.86446E-06  -0.21473E-07   0.25119E-08  -0.64308E-07
      16   0.31848E-05   0.96364E-05  -0.94569E-06   0.11310E-06  -0.11584E-07   0.74569E-07
      17  -0.14940E-05  -0.67103E-05   0.39732E-06  -0.39004E-07   0.49705E-08   0.81579E-08
      18   0.91157E-05   0.88087E-04  -0.18140E-05   0.13062E-06  -0.21792E-07  -0.66044E-06
      19   0.10018E 00   0.80715E-01   0.76094E-01   0.30921E-02   0.90571E-03   0.32377E-02

4      1  -0.42184E-02   0.54876E-01  -0.12289E-01  -0.35259E-03   0.24728E-03   0.33247E-03
       2   0.87446E-01   0.54955E-01  -0.56024E-01  -0.12399E-03  -0.29591E-04   0.20646E-03
       3   0.38629E-01   0.71880E-01   0.26803E-01   0.23058E-02  -0.87038E-03  -0.10967E-02
       4   0.23063E-03  -0.28732E-02  -0.93880E-03  -0.42546E-04   0.12833E-04   0.20684E-04
       5   0.13312E-02   0.20239E-04   0.49126E-02   0.19763E-03  -0.91015E-04  -0.10695E-03
       6   0.15396E-03   0.15325E-03  -0.46892E-03  -0.12314E-04   0.56282E-05   0.65063E-05
       7  -0.37114E-03   0.81746E-03   0.49799E-03   0.10189E-04  -0.36662E-05  -0.85309E-05
       8   0.26243E-02   0.46272E-02   0.14030E-01   0.45031E-03  -0.23048E-03  -0.26887E-03
       9   0.12367E-02  -0.55341E-02  -0.63411E-04   0.15091E-04  -0.11292E-04   0.27122E-04
      10   0.12544E-03  -0.45218E-02  -0.15558E-03  -0.11572E-05   0.10125E-05   0.37620E-04
      11  -0.93301E-05  -0.70977E-04  -0.25924E-06  -0.25901E-07   0.93204E-07   0.82906E-06
      12  -0.16076E-03   0.44371E-05  -0.35747E-03  -0.88533E-05   0.54184E-05   0.46070E-05
      13   0.33210E-04   0.18230E-03  -0.17251E-04  -0.34806E-05  -0.85217E-07  -0.78550E-05
      14  -0.13955E-02   0.12597E-04  -0.29402E-02  -0.61852E-04   0.39174E-04   0.32376E-04
      15  -0.90956E-07   0.17231E-05   0.83158E-06  -0.58654E-07  -0.67098E-08  -0.14438E-06
      16   0.33058E-05  -0.26437E-05  -0.13437E-05   0.32000E-06  -0.13161E-07   0.56981E-06
      17  -0.15842E-05   0.20098E-06   0.59945E-06  -0.20664E-06   0.65104E-08  -0.36715E-06
      18   0.98715E-05   0.10978E-04  -0.30776E-05   0.26241E-05  -0.42899E-07   0.46880E-05
      19   0.95756E-01   0.10621E 00   0.65109E-01   0.23884E-02   0.93962E-03   0.12015E-02

3      1  -0.12109E-02   0.37800E-01  -0.69869E-02   0.44227E-03   0.25656E-03   0.11450E-02
       2   0.87377E-01   0.37915E-01  -0.56670E-01   0.55734E-03  -0.20504E-04   0.10777E-02
       3   0.28219E-01   0.49915E-01   0.82938E-02   0.19651E-02  -0.88656E-03   0.67370E-03
       4   0.38912E-03  -0.20348E-02  -0.66321E-03  -0.54844E-04   0.13436E-04  -0.41287E-04
       5   0.19068E-03   0.15716E-04   0.29103E-02   0.98553E-04  -0.99014E-04  -0.58340E-04
       6   0.22690E-03   0.11167E-03  -0.34352E-03  -0.42555E-05   0.63164E-05   0.65021E-05
       7  -0.42237E-03   0.61959E-03   0.41414E-03   0.15619E-04  -0.43960E-05   0.11913E-04
       8  -0.41826E-03   0.36013E-02   0.87333E-02   0.28584E-03  -0.27137E-03  -0.50149E-04
       9   0.10985E-02  -0.43624E-02  -0.32216E-03  -0.67806E-04  -0.12837E-04  -0.10393E-03
      10   0.14046E-03  -0.36577E-02  -0.13208E-03  -0.64239E-04   0.12697E-05  -0.79957E-04
      11  -0.81731E-05  -0.60115E-04   0.19263E-05  -0.10731E-05   0.10987E-06  -0.12250E-05
      12  -0.77550E-04   0.52834E-05  -0.21349E-03  -0.43034E-05   0.81096E-05   0.26908E-05
      13   0.33043E-04   0.24835E-03  -0.19401E-04   0.29436E-05  -0.77018E-07   0.31845E-05
      14  -0.72857E-03   0.21981E-04  -0.17945E-02  -0.30337E-04   0.68109E-04   0.18469E-04
      15  -0.23283E-06   0.34599E-05   0.62006E-06   0.35980E-07  -0.13730E-07   0.25075E-07
      16   0.32968E-05  -0.11914E-04  -0.17213E-05  -0.56541E-07  -0.14319E-07  -0.28885E-07
      17  -0.15920E-05   0.69845E-05   0.80247E-06   0.15959E-07   0.77071E-08  -0.16608E-07
      18   0.99253E-05  -0.84455E-04  -0.45571E-05  -0.15745E-07  -0.60229E-07   0.56480E-06
      19   0.91842E-01   0.73538E-01   0.58463E-01   0.21148E-02   0.96994E-03   0.17181E-02

2      1   0.20873E-02   0.16573E-02  -0.16789E-02   0.89502E-03   0.26011E-03   0.13864E-02
       2   0.87221E-01   0.16633E-02  -0.57089E-01   0.90272E-03  -0.16917E-04   0.13885E-02
       3   0.16890E-01   0.21904E-02  -0.99871E-02   0.12369E-02  -0.89323E-03   0.17970E-02
       4   0.56281E-03  -0.89907E-04  -0.38491E-03  -0.49795E-04   0.13671E-04  -0.74396E-04
       5  -0.10970E-02   0.43924E-06   0.84069E-03   0.40682E-05  -0.10201E-03  -0.21347E-05
       6   0.30997E-03   0.49988E-05  -0.21085E-03   0.24858E-05   0.65737E-05   0.43203E-05
       7  -0.48158E-03   0.27940E-04   0.32092E-03   0.15450E-04  -0.46689E-05   0.23134E-04
       8  -0.40123E-02   0.16310E-03   0.29711E-02   0.97825E-04  -0.28660E-03   0.13005E-03
       9   0.93236E-03  -0.19905E-03  -0.59438E-03  -0.10849E-03  -0.13409E-04  -0.16584E-03
      10   0.15787E-03  -0.16807E-03  -0.10501E-03  -0.71949E-04   0.13670E-05  -0.13980E-03
      11  -0.67424E-05  -0.27960E-05   0.42753E-05  -0.15315E-05   0.11609E-06  -0.23243E-05
      12   0.34747E-04   0.27682E-06  -0.33717E-04  -0.19005E-07   0.91364E-05   0.34150E-06
      13   0.32359E-04   0.12746E-04  -0.21099E-04   0.69667E-05  -0.73475E-07   0.10601E-04
      14   0.23739E-03   0.12521E-05  -0.25065E-03  -0.50635E-06   0.79427E-04   0.18188E-05
      15  -0.43623E-06   0.18719E-06   0.30539E-06   0.10248E-06  -0.16457E-07   0.15556E-06
      16   0.31631E-05  -0.68692E-06  -0.20533E-05  -0.37497E-06  -0.14759E-07  -0.57140E-06
      17  -0.15214E-05   0.42167E-06   0.98631E-06   0.23023E-06   0.81754E-08   0.35067E-06
      18   0.93288E-05  -0.53023E-05  -0.60275E-05  -0.28958E-05  -0.67189E-07  -0.44085E-05
      19   0.88973E-01   0.32272E-02   0.58069E-01   0.17828E-02   0.98243E-03   0.26739E-02
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          7  0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
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         14  0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
         15  0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
         16  0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
         17  0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
         18  0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
         19  0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00   0.00000E 00
```

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R E S P O N S E   S P E C T R U M   S T R E S S   C O M P O N E N T S

SQUARE ROOT OF THE SUM OF THE SQUARES OF THE MODAL STRESSES
(FOR ALL ELEMENTS)


ELEMENT TYPE (T R U S S              )  / / /  ELEMENT NUMBER (   1)

      P/A        P
0.6218E 04 0.3053E 03


ELEMENT TYPE (B E A M                )  / / /  ELEMENT NUMBER (   1)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.1520E-06 0.3751E 03 0.4191E 03 0.8349E 04 0.0000E 00 0.0000E 00 0.5066E-07 0.3751E 03 0.4191E 03 0.8349E 04 0.4208E 03 0.3766E 03


ELEMENT TYPE (B E A M                )  / / /  ELEMENT NUMBER (   2)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.1409E 03 0.3724E 03 0.4148E 03 0.8342E 04 0.5443E 03 0.3766E 03 0.1409E 03 0.3724E 03 0.4148E 03 0.8342E 04 0.1053E 05 0.9367E 04


ELEMENT TYPE (B E A M                )  / / /  ELEMENT NUMBER (   3)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.4099E 03 0.2827E 03 0.2289E 03 0.8140E 04 0.1069E 05 0.9367E 04 0.4099E 03 0.2827E 03 0.2289E 03 0.8140E 04 0.1617E 05 0.1617E 05


ELEMENT TYPE (B E A M                )  / / /  ELEMENT NUMBER (   4)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.6764E 03 0.1485E 03 0.7568E 02 0.7874E 04 0.1630E 05 0.1617E 05 0.6764E 03 0.1485E 03 0.7568E 02 0.7874E 04 0.1535E 05 0.1914E 05


ELEMENT TYPE (B E A M                )  / / /  ELEMENT NUMBER (   5)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.9394E 03 0.2192E 03 0.2596E 03 0.7701E 04 0.1544E 05 0.1914E 05 0.9394E 03 0.2192E 03 0.2596E 03 0.7701E 04 0.9975E 04 0.1797E 05


ELEMENT TYPE (B E A M                )  / / /  ELEMENT NUMBER (   6)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.8718E 03 0.1247E 04 0.1878E 04 0.9378E 03 0.2527E 05 0.2770E 05 0.8718E 03 0.1247E 04 0.1878E 04 0.9378E 03 0.1032E 05 0.1723E 05


ELEMENT TYPE (B E A M                )  / / /  ELEMENT NUMBER (   7)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.8716E 03 0.1242E 04 0.1871E 04 0.9378E 03 0.1032E 05 0.1723E 05 0.8716E 03 0.1242E 04 0.1871E 04 0.9378E 03 0.8694E 04 0.6916E 04
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ELEMENT TYPE (B E A M                 )   / / /   ELEMENT NUMBER (   8)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.8713E 03 0.1222E 04 0.1850E 04 0.9378E 03 0.8694E 04 0.6916E 04 0.8713E 03 0.1222E 04 0.1850E 04 0.9378E 03 0.2313E 05 0.4121E 04


ELEMENT TYPE (B E A M                 )   / / /   ELEMENT NUMBER (   9)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.9897E 03 0.7205E 03 0.1033E 04 0.4706E 04 0.2530E 05 0.1793E 05 0.9897E 03 0.7205E 03 0.1033E 04 0.4706E 04 0.1734E 05 0.3346E 05


ELEMENT TYPE (B E A M                 )   / / /   ELEMENT NUMBER (  10)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.9399E 03 0.3170E 03 0.4895E 03 0.5137E 04 0.1722E 05 0.3346E 05 0.9399E 03 0.3170E 03 0.4895E 03 0.5137E 04 0.2918E 05 0.4030E 05


ELEMENT TYPE (B E A M                 )   / / /   ELEMENT NUMBER (  11)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.1057E 04 0.3388E 03 0.5591E 03 0.6143E 04 0.2898E 05 0.4030E 05 0.1057E 04 0.3388E 03 0.5591E 03 0.6143E 04 0.1490E 05 0.3065E 05


ELEMENT TYPE (B E A M                 )   / / /   ELEMENT NUMBER (  12)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.1311E 04 0.8629E 03 0.1061E 04 0.6601E 04 0.1470E 05 0.3065E 05 0.1311E 04 0.8629E 03 0.1061E 04 0.6601E 04 0.2909E 05 0.1407E 05


ELEMENT TYPE (B E A M                 )   / / /   ELEMENT NUMBER (  13)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.9166E 03 0.2231E 04 0.2347E 04 0.6612E 03 0.3003E 05 0.5153E 05 0.9166E 03 0.2231E 04 0.2347E 04 0.6612E 03 0.1092E 05 0.3273E 05


ELEMENT TYPE (B E A M                 )   / / /   ELEMENT NUMBER (  14)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.9164E 03 0.2222E 04 0.2338E 04 0.6612E 03 0.1092E 05 0.3273E 05 0.9164E 03 0.2222E 04 0.2338E 04 0.6612E 03 0.1090E 05 0.1401E 05


ELEMENT TYPE (B E A M                 )   / / /   ELEMENT NUMBER (  15)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.9160E 03 0.2190E 04 0.2313E 04 0.6612E 03 0.1090E 05 0.1401E 05 0.9160E 03 0.2190E 04 0.2313E 04 0.6612E 03 0.2972E 05 0.4612E 04


ELEMENT TYPE (B E A M                 )   / / /   ELEMENT NUMBER (  16)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.7967E 03 0.8652E 03 0.3021E 03 0.3024E 04 0.7140E 04 0.1457E 05 0.7967E 03 0.8652E 03 0.3021E 03 0.3024E 04 0.1229E 05 0.1578E 05
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ELEMENT TYPE (B E A M                  )  / / /   ELEMENT NUMBER (  17)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.5148E 03 0.4048E 03 0.1210E 03 0.2805E 04 0.1234E 05 0.1578E 05 0.5148E 03 0.4048E 03 0.1210E 03 0.2805E 04 0.1435E 05 0.2235E 05


ELEMENT TYPE (B E A M                  )  / / /   ELEMENT NUMBER (  18)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.2132E 03 0.2207E 03 0.1822E 03 0.2585E 04 0.1439E 05 0.2235E 05 0.2132E 03 0.2207E 03 0.1822E 03 0.2585E 04 0.9953E 04 0.1802E 05


ELEMENT TYPE (B E A M                  )  / / /   ELEMENT NUMBER (  19)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.1847E 03 0.6908E 03 0.3880E 03 0.2495E 04 0.9976E 04 0.1802E 05 0.1847E 03 0.6908E 03 0.3880E 03 0.2495E 04 0.4093E 03 0.9828E 03


ELEMENT TYPE (B E A M                  )  / / /   ELEMENT NUMBER (  20)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.0000E 00 0.9828E 03 0.4114E 03 0.2494E 04 0.4114E 03 0.9828E 03 0.0000E 00 0.9828E 03 0.4114E 03 0.2494E 04 0.0000E 00 0.0000E 00


ELEMENT TYPE (B E A M                  )  / / /   ELEMENT NUMBER (  21)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.1644E 02 0.1168E 04 0.4245E 03 0.2494E 04 0.2162E 02 0.2913E-05 0.1644E 02 0.1168E 04 0.4245E 03 0.2494E 04 0.6365E 04 0.1753E 05


ELEMENT TYPE (B E A M                  )  / / /   ELEMENT NUMBER (  22)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.4325E 03 0.2561E 04 0.3252E 03 0.6208E 03 0.3691E 04 0.5827E 05 0.4325E 03 0.2561E 04 0.3252E 03 0.6208E 03 0.1633E 04 0.3674E 05


ELEMENT TYPE (B E A M                  )  / / /   ELEMENT NUMBER (  23)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.4323E 03 0.2549E 04 0.3242E 03 0.6208E 03 0.1633E 04 0.3674E 05 0.4323E 03 0.2549E 04 0.3242E 03 0.6208E 03 0.2595E 04 0.1543E 05


ELEMENT TYPE (B E A M                  )  / / /   ELEMENT NUMBER (  24)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.4321E 03 0.2511E 04 0.3216E 03 0.6208E 03 0.2595E 04 0.1543E 05 0.4321E 03 0.2511E 04 0.3216E 03 0.6208E 03 0.5059E 04 0.6894E 04


ELEMENT TYPE (B E A M                  )  / / /   ELEMENT NUMBER (  25)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.1455E 03 0.8617E 03 0.1539E 03 0.4604E 04 0.4719E 04 0.1702E 05 0.1455E 03 0.8617E 03 0.1539E 03 0.4604E 04 0.4579E 04 0.9597E 04
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ELEMENT TYPE (B E A M                    )   / / /   ELEMENT NUMBER (  26)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.1508E 03 0.3354E 03 0.6739E 02 0.4586E 04 0.4597E 04 0.9597E 04 0.1508E 03 0.3354E 03 0.6739E 02 0.4586E 04 0.4550E 04 0.1914E 05


ELEMENT TYPE (B E A M                    )   / / /   ELEMENT NUMBER (  27)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.1990E 03 0.2654E 03 0.1225E 03 0.4636E 04 0.4499E 04 0.1914E 05 0.1990E 03 0.2654E 03 0.1225E 03 0.4636E 04 0.1491E 04 0.1698E 05


ELEMENT TYPE (B E A M                    )   / / /   ELEMENT NUMBER (  28)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.2915E 03 0.6932E 03 0.2024E 03 0.4623E 04 0.1530E 04 0.1698E 05 0.2915E 03 0.6932E 03 0.2024E 03 0.4623E 04 0.5246E 04 0.1759E 05


ELEMENT TYPE (B E A M                    )   / / /   ELEMENT NUMBER (  29)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.3026E 03 0.1767E 04 0.7134E 03 0.6012E 03 0.9334E 04 0.4176E 05 0.3026E 03 0.1767E 04 0.7134E 03 0.6012E 03 0.3380E 04 0.2689E 05


ELEMENT TYPE (B E A M                    )   / / /   ELEMENT NUMBER (  30)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.3025E 03 0.1759E 04 0.7104E 03 0.6012E 03 0.3380E 04 0.2689E 05 0.3025E 03 0.1759E 04 0.7104E 03 0.6012E 03 0.2823E 04 0.1212E 05


ELEMENT TYPE (B E A M                    )   / / /   ELEMENT NUMBER (  31)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.3024E 03 0.1733E 04 0.7016E 03 0.6012E 03 0.2823E 04 0.1212E 05 0.3024E 03 0.1733E 04 0.7016E 03 0.6012E 03 0.8661E 04 0.3151E 04


ELEMENT TYPE (B E A M                    )   / / /   ELEMENT NUMBER (  32)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.3254E 03 0.6502E 03 0.2126E 03 0.5636E 04 0.9762E 04 0.1709E 05 0.3254E 03 0.6502E 03 0.2126E 03 0.5636E 04 0.4582E 04 0.2382E 05


ELEMENT TYPE (B E A M                    )   / / /   ELEMENT NUMBER (  33)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.2249E 03 0.2666E 03 0.1891E 03 0.5421E 04 0.4836E 04 0.2382E 05 0.2249E 03 0.2666E 03 0.1891E 03 0.5421E 04 0.3046E 04 0.2902E 05


ELEMENT TYPE (B E A M                    )   / / /   ELEMENT NUMBER (  34)

    P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.1280E 03 0.2248E 03 0.1346E 03 0.5439E 04 0.3013E 04 0.2902E 05 0.1280E 03 0.2248E 03 0.1346E 03 0.5439E 04 0.4596E 04 0.2580E 05
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ELEMENT TYPE (B E A M                    )  / / /   ELEMENT NUMBER (  35)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.1742E 03 0.5135E 03 0.1149E 03 0.5624E 04 0.4369E 04 0.2580E 05 0.1742E 03 0.5135E 03 0.1149E 03 0.5624E 04 0.5718E 04 0.1776E 05


ELEMENT TYPE (B E A M                    )  / / /   ELEMENT NUMBER (  36)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.1567E 03 0.9905E 03 0.8838E 03 0.6514E 03 0.1273E 05 0.2256E 05 0.1567E 03 0.9905E 03 0.8838E 03 0.6514E 03 0.5304E 04 0.1426E 05


ELEMENT TYPE (B E A M                    )  / / /   ELEMENT NUMBER (  37)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.1566E 03 0.9855E 03 0.8797E 03 0.6514E 03 0.5304E 04 0.1426E 05 0.1566E 03 0.9855E 03 0.8797E 03 0.6514E 03 0.2286E 04 0.6139E 04


ELEMENT TYPE (B E A M                    )  / / /   ELEMENT NUMBER (  38)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.1565E 03 0.9684E 03 0.8671E 03 0.6514E 03 0.2286E 04 0.6139E 04 0.1565E 03 0.9684E 03 0.8671E 03 0.6514E 03 0.9507E 04 0.3060E 04


ELEMENT TYPE (B E A M                    )  / / /   ELEMENT NUMBER (  39)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.5645E 03 0.2196E 03 0.1375E 03 0.7032E 04 0.4558E 04 0.1777E 05 0.5645E 03 0.2196E 03 0.1375E 03 0.7032E 04 0.5658E 04 0.1823E 05


ELEMENT TYPE (B E A M                    )  / / /   ELEMENT NUMBER (  40)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.4175E 03 0.1533E 03 0.7091E 02 0.6825E 04 0.5906E 04 0.1823E 05 0.4175E 03 0.1533E 03 0.7091E 02 0.6825E 04 0.6234E 04 0.1518E 05


ELEMENT TYPE (B E A M                    )  / / /   ELEMENT NUMBER (  41)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.2577E 03 0.2677E 03 0.9767E 02 0.6635E 04 0.6435E 04 0.1518E 05 0.2577E 03 0.2677E 03 0.9767E 02 0.6635E 04 0.4193E 04 0.8750E 04


ELEMENT TYPE (B E A M                    )  / / /   ELEMENT NUMBER (  42)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.8960E 02 0.3477E 03 0.1779E 03 0.6519E 04 0.4371E 04 0.8750E 04 0.8960E 02 0.3477E 03 0.1779E 03 0.6519E 04 0.1368E 03 0.3518E 03


ELEMENT TYPE (B E A M                    )  / / /   ELEMENT NUMBER (  43)

   P1(I)      V2(I)      V3(I)      T1(I)      M2(I)      M3(I)      P1(J)      V2(J)      V3(J)      T1(J)      M2(J)      M3(J)
0.2888E-07 0.3504E 03 0.1799E 03 0.6518E 04 0.1806E 03 0.3518E 03 0.8664E-07 0.3504E 03 0.1799E 03 0.6518E 04 0.0000E 00 0.0000E 00
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Appendix C.1

EAC/EASE2 - Program Description

EAC/EASE2 is a static or dynamic, finite element, linear analysis program. Emphasis is on ease of input, utility of output and cost effectiveness. The program is particularly well-suited for the efficient analysis of very large structural models. Operational on CDC CYBER 70, 170 and 6000-Series computer systems, the program is available in batch mode through Control Data's CYBERNET Services, and McDonnell Douglas Automation Company.

ANALYSIS OPTIONS

The analysis options available in EASE2 are as follows:

Static--Loading conditions include temperature, thermally induced bending in beams and shells, normal pressures and edge tractions on membranes and shells, distributed beam and pipe loads, and face pressures on solid elements. Load cases may be assembled automatically into linear combinations. Data generation options are provided in all sections of the input, and output production is user-controlled. Print formats are user-selected, displacements/reactions can be referenced to special coordinate systems, a choice is given between stress and/or stress resultant element output, and solution results can be selectively diverted to external tape.

Eigenvalue Extraction--Two eigenvalue analysis algorithms are available in EASE2:

1. Determinant search, a polynomial iteration method for small bandwidth models.

2. Subspace iteration, a vector iteration method which is very effective for the low mode response of large models.

The eigenvalue problem is solved directly without transformation to standard form; that is, the system mass matrix can have zero diagonal elements. No computational advantage is realized by lumping masses at "dynamic" degrees of freedom only; to the contrary, more realistic eigenvalues are calculated when a refined mass matrix is used to model continuous systems. With EASE2 the transition from statics to dynamics is nominal. A dynamic analysis is executed by appending new data to a static analysis deck. Frequencies and mode shapes can require as few as two additional data cards.

Transient Response Analysis--Two types of dynamic response analysis are available in EASE2:

1. Mode superposition, effective for long duration input, low mode response (for example, seismic excitation of a building).

2. Direct integration, effective for short duration input, higher mode response (for example, pressure transients in a piping network).

Forcing functions are either external, time dependent loads or base (ground) acceleration histories. Static load cases define spatial distributions, and arbitrary function tables assign time dependencies to the applied forces. Any static load distribution (such as element pressure or span loading) is allowed to act dynamically that is, time of dependent loads are not limited to being concentrated nodal forces only.

Printed output is organized into user-requested component versus time tables printer plots and/or max/min summaries only. The components that can be requested include element stresses or stress resultants, node displacements, velocities or accelerations, and inertial, damping, elastic or applied forces at selected degrees of freedom.

Response Spectrum Analysis--Modal response maxima due to independent X, Y, or Z-direction ground shock spectra are combined using the absolute sum, the square root of the sum of the squares or the closely spaced modes method. Several "response cases" can be analyzed in the same run. The applied shock spectra and/or the method of modal combination can be varied from response case to response case, and the maxima of all response cases will be printed for each stress and/or displacement component.

EASE 2 SYSTEM

The EASE2 system includes the following programs:

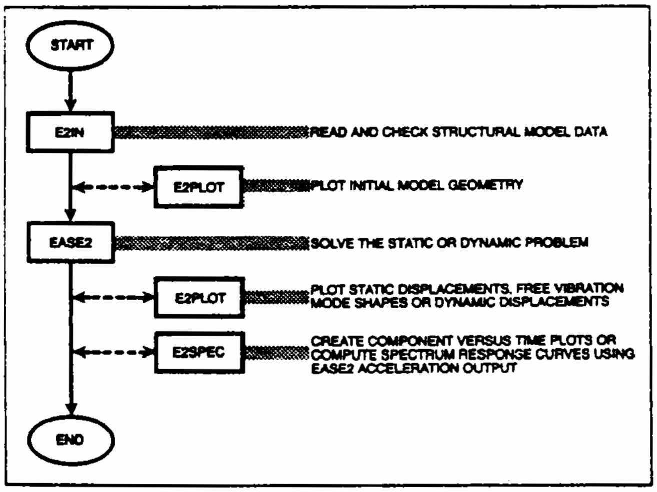
E2IN - A pre-processor which reads and checks the structural model data.

EASE2 - A linear static or dynamic structural analysis program.

E2PLOT - A post-processor which creates initial or deformed geometry plots.

E2SPEC - A post-processor which plots structural output versus time or wich will create spectral response curves from EASE2 acceleration output.

The following figure outlines the EASE2 analysis process:



PROGRAM FEATURES

Input Pre-Processor--E2IN is a multi-pass pre-processor that reads and checks input at low core. During the initial pass, card input is checked for admissibility, and the actual number of cards in each data block is noted. The second pass checks all cards for valid formats and notes any difference between actual and specified lengths of the data sets. Finally, the model data are tested in detail:

1. All integer references are checked for range.

2. All elements are tested to ensure finite length, area or volume.

3. Core requirements are determined and an optimum core allocation procedure is executed.

4. Omitted or extraneous data entries are noted.

Plotting Capabilities--The E2PLOT post-processor generates undeformed geometry plots, allowing:

1. Node/element number labels.

2. Special symbols for boundary nodes.

3. "Blow-ups" of specific areas of the mesh.

4. User-selection of which elements are to be plotted.

E2PLOT also plots static displacement vectors, free vibration mode shapes or dynamic displacement, velocity or acceleration vectors. Deformed geometry plot options include:

1. For reference, the undeformed structure can be drawn with dashed line segments.

2. Rotations can be shown for deformed beam, shell or pipe element types.

In an EASE2 transient response analysis, any output component can be plotted versus time on the line printer. As an alternative, the E2SPEC post-processor will create component versus time plots on external plotters. E2SPEC will also read acceleration histories from an EASE2 analysis and generate spectral response curves for up to six damping ratios.

Restart Provisions--The decomposed stiffness matrix can be reinstated to:

1. Solve additional static load cases.
2. Solve the eigenvalue problem.
3. Continue a direct integration analysis.

Eigenvalues/vectors can be reinstated to:

1. Solve for additional eigenpairs.
2. Solve for the transient response of the system.
3. Perform a response spectrum analysis.

To continue a transient response calculation, dynamic system vectors can be reinstated as initial conditions. A static system vector can be read as initial displacements for a transient response analysis.

Appendix C.2

EAC/EASE2
Example Problem

Use EAC/EASE2 to perform a response spectrum analysis of Route 80 Onramp U.C. for both transverse and longitudinal excitation (see Global coordinate system). Use the CALTRANS design spectrum with the following properties:

1) Maximum rock acceleration of .5g

2) Depth to "rocklike" material between 10-80 ft.

3) Damping of 5 percent.

4) No reduction for ductility or risk.

FIND:

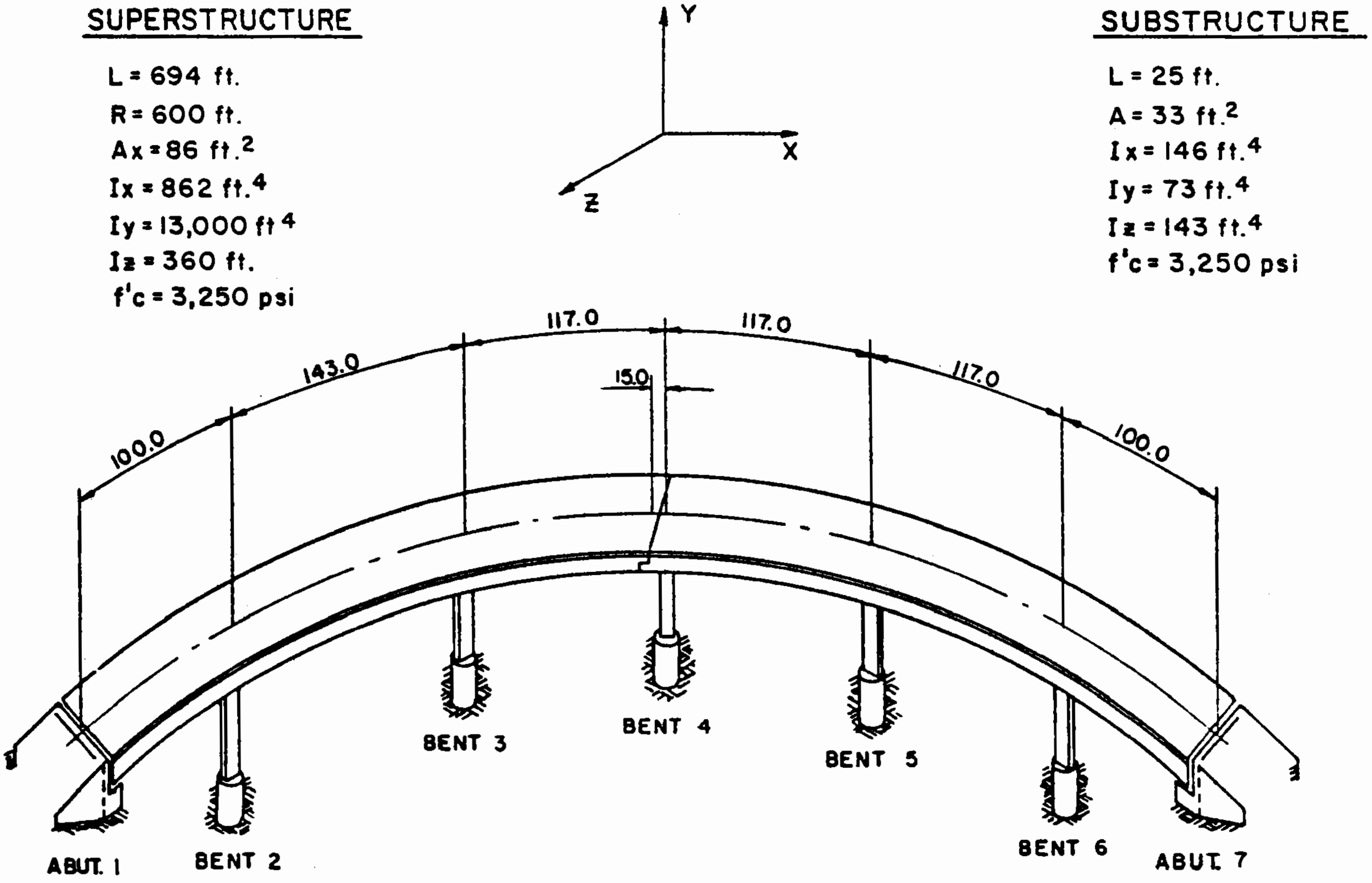
1) RMS displacements.

2) RMS hinge restrainer forces.

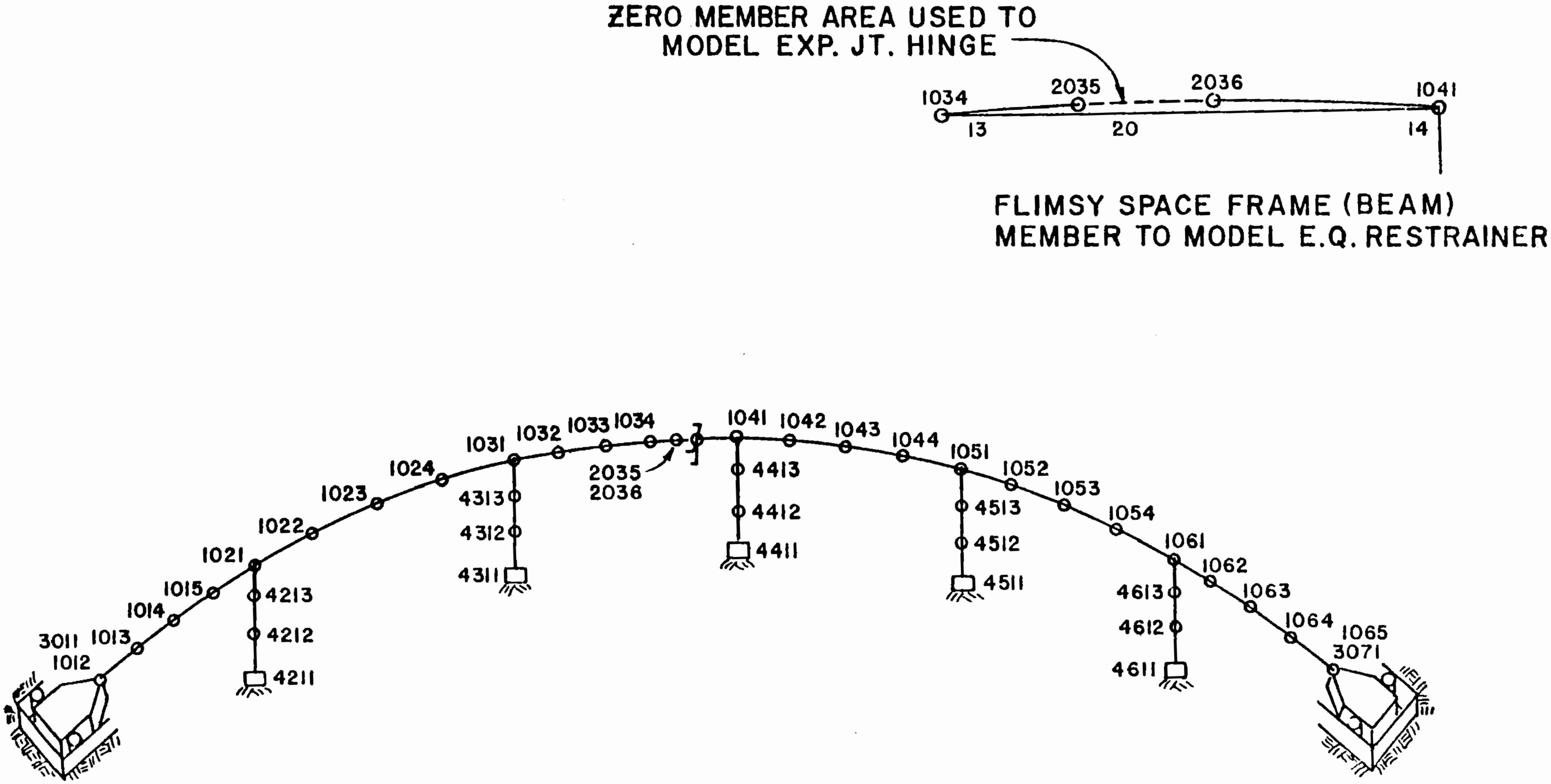
3) RMS shears and moments at column bases.

4) RMS shears at abutments and hinges.

Use the EASE2 plotting post-processor, E2PLOT, to plot the first 10 mode shapes.



ROUTE 80 ON-RAMP OVERCROSSING



STRUCTURE IDEALIZATION
ROUTE 80 ON-RAMP
(EASE 2)

INPUT DATA ORGANIZATION FOR EASE2
(Input via E2IN, an EASE2 pre-processor)

1. EASE2
2. NODES
3. VECTORS
4. SKEW NODES
5. RESTRAINTS
6. MATERIALS
7. SECTIONS
8. BEAMS
9. DYNAMIC JOB
10. EIGENVALUES
11. SPECTRUM
12. SHOCK TABLE
13. RESPONSE CASE
14. RSA OUTPUT

*NOTE: A echo print of the input card images is included in the attached partial computer output listing.

```
/- - - - -    E A S E 2    C A R D    I M A G E    L I S T I N G    - - - - -/
....+...10....+...20....+...30....+...40....+...50....+...60....+...70....+...80

EASE2              5         2                                                    EASE2         1
 RESPONSE SPECTRUM ANALYSIS OF ROUTE 80 ONRAMP UNDERCROSSING                      EASE2         2
          FHWA  'SEISMIC DESIGN OF HIGHWAY BRIDGES'                               EASE2         3
NODES             44 4613                                                         NODES         1
 3011       10000.00     75.30  10000.00                                          NODES         2
 1012       10000.84     75.30   9999.45                                          NODES         3
 1013       10021.35     75.30   9986.72                                          NODES         4
 1014       10042.38     75.30   9974.83                                          NODES         5
 1015       10063.87     75.30   9963.83                                          NODES         6
 1021       10085.79     75.30   9953.71                                          NODES         7
 1022       10118.29     75.30   9940.66                                          NODES         8
 1023       10151.51     75.30   9929.56                                          NODES         9
 1024       10185.32     75.30   9920.46                                          NODES        10
 1031       10219.62     75.30   9913.39                                          NODES        11
 1032       10244.01     75.30   9909.65                                          NODES        12
 1033       10268.53     75.30   9906.93                                          NODES        13
 1034       10293.14     75.30   9905.24                                          NODES        14
 2035       10317.81     75.30   9904.58                                          NODES        15
 2036       10318.81     75.30   9904.58                                          NODES        16
 1041       10333.81     75.30   9904.71                                          NODES        17
 1042       10362.45     75.30   9906.03                                          NODES        18
 1043       10391.00     75.30   9908.74                                          NODES        19
 1044       10419.37     75.30   9912.84                                          NODES        20
 1051       10447.52     75.30   9918.32                                          NODES        21
 1052       10475.36     75.30   9925.16                                          NODES        22
 1053       10502.84     75.30   9933.36                                          NODES        23
 1054       10529.88     75.30   9942.88                                          NODES        24
 1061       10556.43     75.30   9953.71                                          NODES        25
 1062       10578.36     75.30   9963.83                                          NODES        26
 1063       10599.85     75.30   9974.84                                          NODES        27
 1064       10620.87     75.30   9986.71                                          NODES        28
 1065       10641.39     75.30   9999.45                                          NODES        29
 3071       10642.23     75.30  10000.00                                          NODES        30
 4211       10085.79     50.00   9953.71                                          NODES        31
 4212       10085.79     58.43   9953.71                                          NODES        32
 4213       10085.79     66.87   9953.71                                          NODES        33
 4311       10219.62     50.00   9913.39                                          NODES        34
 4312       10219.62     58.43   9913.39                                          NODES        35
 4313       10219.62     66.87   9913.39                                          NODES        36
 4411       10333.81     50.00   9904.71                                          NODES        37
 4412       10333.81     58.43   9904.71                                          NODES        38
 4413       10333.81     66.87   9904.71                                          NODES        39
 4511       10447.52     50.00   9918.32                                          NODES        40
 4512       10447.52     58.43   9918.32                                          NODES        41
 4513       10447.52     66.87   9918.32                                          NODES        42
 4611       10556.43     50.00   9953.71                                          NODES        43
 4612       10556.43     58.43   9953.71                                          NODES        44
 4613       10556.43     66.87   9953.71                                          NODES        45
VECTORS           14                                                              VECTORS       1
    4    2 3011 1012                                                              VECTORS       2

....+...10....+...20....+...30....+...40....+...50....+...60....+...70....+...80
```

```
/- - - - -    E A S E 2   C A R D   I M A G E   L I S T I N G   - - - - -/
....+...10....+...20....+...30....+...40....+...50....+...60....+...70....+...80

    5    2 4211 4213                                                              VECTORS      3
    6    3    4    5                                                              VECTORS      4
    7    2 1065 3071                                                              VECTORS      5
    8    2 4611 4613                                                              VECTORS      6
    9    3    7    8                                                              VECTORS      7
   10    2 1015 1022                                                              VECTORS      8
   11    2 1024 1032                                                              VECTORS      9
   12    2 2036 1042                                                              VECTORS     10
   13    2 1044 1052                                                              VECTORS     11
   14    2 1054 1062                                                              VECTORS     12
SKEW NODES         2                                                              SKEW NODES   1
 3011    4    5                                                                   SKEW NODES   2
 3071    7    8                                                                   SKEW NODES   3
RESTRAINTS         7                                                              RESTRAINTS   1
 30110RRR00                                                                       RESTRAINTS   2
 4211RRRRRR                                                                       RESTRAINTS   3
 4311RRRRRR                                                                       RESTRAINTS   4
 4411RRRRRR                                                                       RESTRAINTS   5
 4511RRRRRR                                                                       RESTRAINTS   6
 4611RRRRRR                                                                       RESTRAINTS   7
 30710RRR00                                                                       RESTRAINTS   8
MATERIALS          2                                                              MATERIALS    1
    1CONCR                .150   432000.       .18                                MATERIALS    2
    2STEEL                .480  2016000.       .26                                MATERIALS    3
SECTIONS           4                                                              SECTIONS     1
    1           86.0    13000.     360.0     862.0                                SECTIONS     2
    2           33.0     143.0      73.0     146.0                                SECTIONS     3
    3         .04910                                                              SECTIONS     4
    4                   13000.     360.0      862.                                SECTIONS     5
BEAMS             44                                                              BEAMS        1
    1 3011 1012    1    1    2                                                    BEAMS        2
    2 1012 1013                                                                   BEAMS        3
    3 1013 1014                                                                   BEAMS        4
    4 1014 1015                                                                   BEAMS        5
    5 1015 1021                                                                   BEAMS        6
    6 1021 1022                                                                   BEAMS        7
    7 1022 1023                                                                   BEAMS        8
    8 1023 1024                                                                   BEAMS        9
    9 1024 1031                                                                   BEAMS       10
   10 1031 1032                                                                   BEAMS       11
   11 1032 1033                                                                   BEAMS       12
   12 1033 1034                                                                   BEAMS       13
   13 1034 2035                                                                   BEAMS       14
   14 2036 1041                                                                   BEAMS       15
   15 1041 1042                                                                   BEAMS       16
   16 1042 1043                                                                   BEAMS       17
   17 1043 1044                                                                   BEAMS       18
   18 1044 1051                                                                   BEAMS       19
   19 1051 1052                                                                   BEAMS       20
   20 1052 1053                                                                   BEAMS       21

....+...10....+...20....+...30....+...40....+...50....+...60....+...70....+...80
```

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/- - - - -    E A S E 2   C A R D   I M A G E   L I S T I N G   - - - - -/
....+...10....+...20....+...30....+...40....+...50....+...60....+...70....+...80

   21 1053 1054                                                                   BEAMS       22
   22 1054 1061                                                                   BEAMS       23
   23 1061 1062                                                                   BEAMS       24
   24 1062 1063                                                                   BEAMS       25
   25 1063 1064                                                                   BEAMS       26
   26 1064 1065                                                                   BEAMS       27
   27 1065 3071                                                                   BEAMS       28
   28 1034 1041    2    3                                                         BEAMS       29
   29 4211 4212    1    2   10                                                    BEAMS       30
   30 4212 4213                                                                   BEAMS       31
   31 4213 1021                                                                   BEAMS       32
   32 4311 4312             11                                                    BEAMS       33
   33 4312 4313                                                                   BEAMS       34
   34 4313 1031                                                                   BEAMS       35
   35 4411 4412             12                                                    BEAMS       36
   36 4412 4413                                                                   BEAMS       37
   37 4413 1041                                                                   BEAMS       38
   38 4511 4512             13                                                    BEAMS       39
   39 4512 4513                                                                   BEAMS       40
   40 4513 1051                                                                   BEAMS       41
   41 4611 4612             14                                                    BEAMS       42
   42 4612 4613                                                                   BEAMS       43
   43 4613 1061                                                                   BEAMS       44
   44 2035 2036    1    4    2       011                                          BEAMS       45
DYNAMIC JOB        1      32.2                                                    DYNAMC JOB   1
EIGENVALUES       18   18 1161  1                                                 EIGENVALUE   1
SPECTRUM           1    2    1                                                    SPECTRUM     1
SHOCK TABLE        1   54    2      32.0                                          SHOCK TABL   1
 ACCELEATION SPECTRUM FOR AASHTO .7G SHOCK -  10 - 80 FT ALLUVIUM - DAMPING .05   SHOCK TABL   2
     .1667     .1093     .1818     .1093     .2000     .1093     .2222     .1286  SHOCK TABL   3
     .2500     .1625     .2857     .1982     .3333     .2390     .3571     .2610  SHOCK TABL   4
     .3846     .2850     .4167     .3110     .4545     .3410     .5000     .3780  SHOCK TABL   5
     .5263     .3980     .5556     .4220     .5882     .4500     .6250     .4800  SHOCK TABL   6
     .6667     .5140     .7143     .5550     .7692     .5990     .8333     .6570  SHOCK TABL   7
     .9091     .7280    1.0000    0.8100    1.0526    0.8550    1.1111    0.9040  SHOCK TABL   8
    1.1765    0.9620    1.2500    1.0200    1.3333    1.0820    1.4286    1.1520  SHOCK TABL   9
    1.5385    1.2140    1.6667    1.2900    1.7391    1.3280    1.8182    1.3670  SHOCK TABL  10
    1.9048    1.4070    2.0000    1.4480    2.1053    1.4820    2.2222    1.5120  SHOCK TABL  11
    2.3529    1.5400    2.5000    1.5650    2.6667    1.5920    2.8571    1.6090  SHOCK TABL  12
    3.0769    1.6220    3.3333    1.6290    3.6364    1.6300    4.0000    1.6220  SHOCK TABL  13
    4.4444    1.6070    5.0000    1.5800    5.7143    1.5420    6.6667    1.4740  SHOCK TABL  14
    8.000    1.3880   10.0000    1.3040   13.3333    1.1770   20.0000    1.0560   SHOCK TABL  15
   40.0000    0.9100 1000.0000    0.7181                                          SHOCK TABL  16
RESPONSE CASE      1                    LONGITUDINAL DESIGN EARTHQUAKE            RESPONSE C   1
    1                      1.0                                                    RESPONSE C   2
RESPONSE CASE      2                    TRANSVERSE DESIGN EARTHQUAKE              RESPONSE C   1
              1                                1.0                                RESPONSE C   2
RSA OUTPUT                   0                                                    RSA OUTPUT   1
 NODE                                                                             RSA OUTPUT   2
 BEAM                                                                             RSA OUTPUT   3

....+...10....+...20....+...30....+...40....+...50....+...60....+...70....+...80
```

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E I G E N V A L U E   T A B L E

 RESPONSE SPECTRUM ANALYSIS OF ROUTE 80 ONRAMP UNDERCROSSING                                                            PAGE     5
          FHWA  'SEISMIC DESIGN OF HIGHWAY BRIDGES'                                                                     22  NOV 78

  ROOT    E I G E N V A L U E S    CIRCULAR FREQUENCY            FREQUENCY             PERIOD
NUMBER   (RADIANS/UNIT TIME)**2   (RADIANS/UNIT TIME)   (CYCLES/UNIT TIME)   (TIME/ONE CYCLE)

     1      .24186826119937E+03           .155521E+02               2.4752            .404008
     2      .25656949474062E+03           .160178E+02               2.5493            .392263
     3      .27155818199573E+03           .164790E+02               2.6227            .381284
     4      .33611874067592E+03           .183335E+02               2.9179            .342715
     5      .40345093186865E+03           .200861E+02               3.1968            .312813
     6      .44767958142647E+03           .211584E+02               3.3675            .296959
     7      .56782800908658E+03           .238291E+02               3.7925            .263677
     8      .66182696742511E+03           .257260E+02               4.0944            .244235
     9      .70498675390941E+03           .265516E+02               4.2258            .236641
    10      .79473760882097E+03           .281911E+02               4.4868            .222878
    11      .95312134083148E+03           .308727E+02               4.9135            .203519
    12      .20501301083788E+04           .452784E+02               7.2063            .138768
    13      .24787344197206E+04           .497869E+02               7.9238            .126202
    14      .31105797247042E+04           .557726E+02               8.8765            .112657
    15      .35615583056347E+04           .596788E+02               9.4982            .105283
    16      .50095657082390E+04           .707783E+02              11.2647            .088773
    17      .62592409162787E+04           .791154E+02              12.5916            .079418
    18      .75448383424468E+04           .868610E+02              13.8244            .072336
```

```
RESPONSE SPECTRUM ANALYSIS OF ROUTE 80 ONRAMP UNDERCROSSING                                                      PAGE     8
         FHWA  'SEISMIC DESIGN OF HIGHWAY BRIDGES'                                                               22  NOV  78

           COORDINATE             X (OR X*)      Y (OR Y*)      Z (OR Z*)      X (OR X*)      Y (OR Y*)      Z (OR Z*)
  NODE         SYSTEM EIGEN-  TRANSLATIONAL  TRANSLATIONAL  TRANSLATIONAL     ROTATIONAL     ROTATIONAL     ROTATIONAL
 NUMBER   NUMBER/TYPE VECTOR      COMPONENT      COMPONENT      COMPONENT      COMPONENT      COMPONENT      COMPONENT


  1012            0/0      1       -.013052       -.009034        .010209       -.004880       -.001385       -.007556
                           2        .836709        .015874       -.547646        .008614       -.000166        .013253
                           3        .091554        .011447       -.054177        .006470       -.004807        .009388
                           4       -.029659        .004723        .020358        .002619       -.000782        .003906
                           5        .097929       -.000074       -.075217       -.000387        .009246        .000165
                           6        .067905        .001116       -.046174        .000556        .001426        .000963
                           7        .108059       -.006323       -.071943       -.003495        .000991       -.005237
                           8        .102750       -.004494       -.075777       -.002671        .007082       -.003599
                           9        .114744       -.024782       -.072979       -.013504       -.001792       -.020649
                          10       -.035075        .037169        .023337        .020333       -.000310        .030917
                          11        .005578        .002347       -.003524        .001286       -.000107        .001951
                          12       -.061446       -.000435        .059568        .000063       -.016110       -.000558
                          13        .035076        .013871       -.022853        .007581       -.000094        .011537
                          14       -.048746       -.000263        .051716        .000104       -.016496       -.000381
                          15       -.007044        .002767        .004934        .001515       -.000268        .002300
                          16       -.010299        .002067        .006720        .001128        .000020        .001719
                          17        .040808       -.009354       -.026600       -.005107       -.000100       -.007779
                          18        .151638       -.043746       -.098786       -.023886       -.000417       -.036376

  1013            0/0      1        .004513       -.206038        .038473       -.002456       -.001366       -.006214
                           2        .838223        .361863       -.543532        .005283       -.000200        .010308
                           3        .152503        .260853        .044189        .010418       -.004770        .003430
                           4       -.019729        .106887        .036273        .002952       -.000768        .002126
                           5       -.018785       -.002193       -.262797       -.008947        .008974        .005269
                           6        .049873        .024922       -.074959       -.000914        .001371        .001430
                           7        .095469       -.140207       -.091720       -.003488        .000933       -.002723
                           8        .013931       -.099174       -.218134       -.007241        .006703        .001006
                           9        .136959       -.543090       -.036608       -.008359       -.001715       -.012984
                          10       -.031128        .808919        .029467        .014221       -.000288        .017674
                          11        .006896        .050455       -.001363        .000907       -.000101        .001024
                          12        .136589       -.008245        .376596        .007618       -.014305       -.004737
                          13        .035984        .270260       -.020727        .003193       -.000095        .003472
                          14        .152129       -.004654        .372780        .006308       -.014166       -.003845
                          15       -.003730        .051067        .010048        .000529       -.000223        .000361
                          16       -.010389        .035986        .006166        .000178        .000025        .000101
                          17        .041251       -.156319       -.023867       -.000427       -.000117        .000220
                          18        .153116       -.707197       -.087335       -.000640       -.000469        .003524

  1014            0/0      1        .020525       -.299106        .066690        .001831       -.001315       -.001763
                           2        .838874        .524512       -.537286       -.001256       -.000284        .002011
                           3        .208491        .375561        .143743        .012348       -.004680       -.005898
                           4       -.010677        .150911        .052028        .002377       -.000733       -.001164
                           5       -.122157       -.002915       -.444291       -.017917        .008250        .009696
                           6        .034038        .034195       -.102169       -.002677        .001221        .001413
                           7        .084567       -.184944       -.109471       -.002212        .000776        .001882
                           8       -.061233       -.127330       -.348907       -.011130        .005687        .006724
                           9        .155450       -.688848       -.002115        .002044       -.001504        .003292
                          10       -.027728       1.000000        .034792        .000285       -.000229       -.008336
                          11        .007963        .059569        .000639        .000034       -.000085       -.000703
```

```
MASS  MODAL  PARTICIPATION  FACTORS

 RESPONSE SPECTRUM ANALYSIS OF ROUTE 80 ONRAMP UNDERCROSSING                                              PAGE    25
          FHWA  'SEISMIC DESIGN OF HIGHWAY BRIDGES'                                                       22  NOV 78

  MODE        GLOBAL X        GLOBAL Y        GLOBAL Z
 NUMBER      DIRECTION       DIRECTION       DIRECTION


      1   -.633298E+00    -.281826E+01    -.531738E+01
      2   -.104291E+02     .623861E-01     .646334E+01
      3   -.458982E+01     .115823E+01    -.129230E+02
      4   -.729015E+01     .738300E-01    -.116860E+01
      5   -.312564E+01     .485193E+00     .110005E+01
      6   -.807786E+01    -.229628E-01    -.306409E+00
      7   -.969248E+00    -.110079E+01    -.428563E+00
      8   -.423779E+00    -.106244E+01     .428218E+01
      9   -.132246E+01    -.241094E+01     .680192E+00
     10   -.169453E+01     .844565E+01     .378006E+00
     11   -.193089E+01    -.996586E+01    -.144936E+00
     12   -.212102E+01     .538410E-01     .285239E+00
     13   -.127823E+01     .439674E+00     .280307E+00
     14   -.252421E+00     .748770E-01    -.287439E+01
     15    .147875E+01    -.213960E+01     .289862E-01
     16    .385264E+00     .998528E+00     .234423E+00
     17    .168383E+00    -.222494E+00     .379816E-01
     18   -.157175E+00    -.929350E+00     .677468E-01
```

```
  NODE   RESPONSE X-TRANSLATION Y-TRANSLATION Z-TRANSLATION  X-ROTATION    Y-ROTATION    Z-ROTATION
 NUMBER      CASE  DISPLACEMENT  DISPLACEMENT  DISPLACEMENT DISPLACEMENT  DISPLACEMENT  DISPLACEMENT

  1012          1   .140655E+00*  .294754E-02   .920653E-01*  .160587E-02   .468459E-03   .245659E-02
                2   .887473E-01   .317016E-02*  .579163E-01   .175197E-02*  .983896E-03*  .262621E-02*
      (MAXIMUM)     .140655E+00   .317016E-02   .920653E-01   .175197E-02   .983896E-03   .262621E-02

  1013          1   .141018E+00*  .669320E-01   .919502E-01*  .124822E-02   .458855E-03   .182284E-02*
                2   .916678E-01   .722377E-01*  .582484E-01   .209858E-02*  .971391E-03*  .168195E-02
      (MAXIMUM)     .141018E+00   .722377E-01   .919502E-01   .209858E-02   .971391E-03   .182284E-02

  1014          1   .141442E+00*  .959069E-01   .924253E-01*  .108361E-02   .434839E-03   .655927E-03
                2   .956291E-01   .104318E+00*  .648844E-01   .238931E-02*  .940957E-03*  .120391E-02*
      (MAXIMUM)     .141442E+00   .104318E+00   .924253E-01   .238931E-02   .940957E-03   .120391E-02

  1015          1   .141890E+00*  .724466E-01   .932297E-01*  .201864E-02   .405230E-03   .192872E-02
                2   .100092E+00   .792735E-01*  .758985E-01   .309400E-02*  .906692E-03*  .321208E-02*
      (MAXIMUM)     .141890E+00   .792735E-01   .932297E-01   .309400E-02   .906692E-03   .321208E-02

  1021          1   .142255E+00*  .108690E-02   .942242E-01*  .312142E-02   .381073E-03   .329230E-02
                2   .104658E+00   .150266E-02*  .894928E-01   .414671E-02*  .888077E-03*  .502829E-02*
      (MAXIMUM)     .142255E+00   .150266E-02   .942242E-01   .414671E-02   .888077E-03   .502829E-02

  1022          1   .144743E+00*  .713118E-01   .966134E-01   .236029E-02   .332449E-03   .225260E-02
                2   .111919E+00   .166784E+00*  .111773E+00*  .500157E-02*  .821206E-03*  .526521E-02*
      (MAXIMUM)     .144743E+00   .166784E+00   .111773E+00   .500157E-02   .821206E-03   .526521E-02

  1023          1   .146804E+00*  .778955E-01   .970307E-01   .278043E-02   .291983E-03   .247894E-02*
                2   .117535E+00   .245741E+00*  .131972E+00*  .658722E-02*  .686927E-03*  .210053E-02
      (MAXIMUM)     .146804E+00   .245741E+00   .131972E+00   .658722E-02   .686927E-03   .247894E-02

  1024          1   .148332E+00*  .702491E-01   .942511E-01   .368038E-02   .326143E-03   .120137E-02
                2   .121107E+00   .155459E+00*  .147661E+00*  .843396E-02*  .599508E-03*  .388015E-02*
      (MAXIMUM)     .148332E+00   .155459E+00   .147661E+00   .843396E-02   .599508E-03   .388015E-02

  1031          1   .149096E+00*  .131954E-02   .894157E-01   .458788E-02   .381094E-03   .329843E-02
                2   .123038E+00   .159008E-02*  .160808E+00*  .960912E-02*  .616643E-03*  .359635E-02*
      (MAXIMUM)     .149096E+00   .159008E-02   .160808E+00   .960912E-02   .616643E-03   .359635E-02

  1032          1   .150707E+00*  .774627E-01*  .864811E-01   .450796E-02   .408824E-03   .206330E-02
                2   .125077E+00   .742213E-01   .170743E+00*  .984946E-02*  .626302E-03*  .220028E-02*
      (MAXIMUM)     .150707E+00   .774627E-01   .170743E+00   .984946E-02   .626302E-03   .220028E-02

  1033          1   .151822E+00*  .109355E+00*  .838481E-01   .440653E-02   .434760E-03   .997044E-03*
                2   .126455E+00   .103388E+00   .180285E+00*  .100514E-01*  .583028E-03*  .930794E-03
      (MAXIMUM)     .151822E+00   .109355E+00   .180285E+00   .100514E-01   .583028E-03   .997044E-03

  1034          1   .152432E+00*  .910962E-01*  .815183E-01   .435700E-02   .455993E-03   .232409E-02*
                2   .127158E+00   .793169E-01   .188437E+00*  .102543E-01*  .531472E-03*  .221038E-02
      (MAXIMUM)     .152432E+00   .910962E-01   .188437E+00   .102543E-01   .531472E-03   .232409E-02

  1041          1   .850494E-01*  .142654E-02*  .775173E-01   .442480E-02   .429341E-03   .381133E-02*
                2   .164172E-01   .751224E-03   .189334E+00*  .106248E-01*  .571117E-03*  .139927E-02
      (MAXIMUM)     .850494E-01   .142654E-02   .189334E+00   .106248E-01   .571117E-03   .381133E-02

  1042          1   .856638E-01*  .100249E+00*  .745542E-01   .421657E-02   .400803E-03   .296303E-02*
                2   .162169E-01   .297529E-01   .178541E+00*  .990643E-02*  .542280E-03*  .127272E-02
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RESPONSE SPECTRUM ANALYSIS OF ROUTE 80 ONRAMP UNDERCROSSING                                                      PAGE    28
         FHWA  'SEISMIC DESIGN OF HIGHWAY BRIDGES'                                                               22  NOV  78

  NODE   RESPONSE X-TRANSLATION Y-TRANSLATION Z-TRANSLATION   X-ROTATION    Y-ROTATION    Z-ROTATION
 NUMBER      CASE  DISPLACEMENT  DISPLACEMENT  DISPLACEMENT DISPLACEMENT  DISPLACEMENT  DISPLACEMENT

  1043          1   .858052E-01*  .143203E+00*  .725677E-01   .402534E-02   .366722E-03   .950795E-03
                2   .158377E-01   .380669E-01   .167713E+00*  .923044E-02*  .564350E-03*  .112480E-02*
     (MAXIMUM)      .858052E-01   .143203E+00   .167713E+00   .923044E-02   .564350E-03   .112480E-02

  1044          1   .854085E-01*  .975557E-01*  .704451E-01   .383150E-02   .339272E-03   .304539E-02*
                2   .154478E-01   .232316E-01   .155574E+00*  .858096E-02*  .588751E-03*  .124369E-02
     (MAXIMUM)      .854085E-01   .975557E-01   .155574E+00   .858096E-02   .588751E-03   .304539E-02

  1051          1   .844326E-01*  .709203E-03*  .680597E-01   .364484E-02   .313586E-03   .368571E-02*
                2   .153894E-01   .537128E-03   .142896E+00*  .792037E-02*  .560084E-03*  .129556E-02
     (MAXIMUM)      .844326E-01   .709203E-03   .142896E+00   .792037E-02   .560084E-03   .368571E-02

  1052          1   .839894E-01*  .955376E-01*  .658147E-01   .325743E-02   .294756E-03   .322226E-02*
                2   .163165E-01   .123496E-01   .131179E+00*  .665851E-02*  .521033E-03*  .224070E-02
     (MAXIMUM)      .839894E-01   .955376E-01   .131179E+00   .665851E-02   .521033E-03   .322226E-02

  1053          1   .828445E-01*  .134816E+00*  .625784E-01   .321476E-02   .316731E-03   .115980E-02
                2   .181330E-01   .245575E-01   .118697E+00*  .564164E-02*  .540328E-03*  .230922E-02*
     (MAXIMUM)      .828445E-01   .134816E+00   .118697E+00   .564164E-02   .540328E-03   .230922E-02

  1054          1   .809381E-01*  .883446E-01*  .579144E-01   .333814E-02   .371294E-03   .230224E-02*
                2   .210748E-01   .249083E-01   .104339E+00*  .487602E-02*  .589103E-03*  .171639E-02
     (MAXIMUM)      .809381E-01   .883446E-01   .104339E+00   .487602E-02   .589103E-03   .230224E-02

  1061          1   .783642E-01*  .128675E-02*  .526132E-01   .287242E-02   .414932E-03   .251653E-02*
                2   .253084E-01   .276036E-03   .885342E-01*  .430559E-02*  .615265E-03*  .936478E-03
     (MAXIMUM)      .783642E-01   .128675E-02   .885342E-01   .430559E-02   .615265E-03   .251653E-02

  1062          1   .766877E-01*  .608765E-01*  .488854E-01   .196282E-02   .443855E-03   .167573E-02*
                2   .302453E-01   .276913E-01   .751973E-01*  .301959E-02*  .632595E-03*  .974251E-03
     (MAXIMUM)      .766877E-01   .608765E-01   .751973E-01   .301959E-02   .632595E-03   .167573E-02

  1063          1   .748224E-01*  .847092E-01*  .462626E-01   .994982E-03   .478218E-03   .564757E-03
                2   .361433E-01   .372919E-01   .615568E-01*  .173301E-02*  .668127E-03*  .102322E-02*
     (MAXIMUM)      .748224E-01   .847092E-01   .615568E-01   .173301E-02   .668127E-03   .102322E-02

  1064          1   .729439E-01*  .606949E-01*  .453238E-01   .105625E-02*  .505385E-03   .165071E-02*
                2   .430934E-01   .262692E-01   .477013E-01*  .643782E-03   .700051E-03*  .113352E-02
     (MAXIMUM)      .729439E-01   .606949E-01   .477013E-01   .105625E-02   .700051E-03   .165071E-02

  1065          1   .712442E-01*  .269317E-02*  .465101E-01*  .146485E-02*  .516129E-03   .224610E-02*
                2   .510602E-01   .116137E-02   .342007E-01   .609613E-03   .713200E-03*  .983435E-03
     (MAXIMUM)      .712442E-01   .269317E-02   .465101E-01   .146485E-02   .713200E-03   .224610E-02

  2035          1   .152741E+00*  .591474E-01*  .799673E-01   .438054E-02   .465140E-03   .310345E-02*
                2   .127471E+00   .239843E-01   .195524E+00*  .104747E-01*  .508073E-03*  .296543E-02
     (MAXIMUM)      .152741E+00   .591474E-01   .195524E+00   .104747E-01   .508073E-03   .310345E-02

  2036          1   .851043E-01*  .590327E-01*  .799324E-01   .438286E-02   .436048E-03   .403029E-02*
                2   .164449E-01   .225675E-01   .195809E+00*  .104840E-01*  .589960E-03*  .159225E-02
     (MAXIMUM)      .851043E-01   .590327E-01   .195809E+00   .104840E-01   .589960E-03   .403029E-02

  3011          1   .168106E+00*  .411929E-17*  .256348E-17   .398709E-16   .468474E-03   .293612E-02
                2   .105969E+00   .411444E-17   .371573E-17*  .835809E-16*  .983916E-03*  .315784E-02*
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 NODE   RESPONSE X-TRANSLATION Y-TRANSLATION Z-TRANSLATION   X-ROTATION    Y-ROTATION    Z-ROTATION
NUMBER      CASE  DISPLACEMENT  DISPLACEMENT  DISPLACEMENT DISPLACEMENT  DISPLACEMENT  DISPLACEMENT

 3071          1    .850805E-01*  .407076E-17*  .265694E-17   .365443E-16   .516147E-03   .268276E-02*
               2    .614519E-01   .180891E-17   .346762E-17*  .652136E-16*  .713221E-03*  .115689E-02
    (MAXIMUM)       .850805E-01   .407076E-17   .346762E-17   .652136E-16   .713221E-03   .268276E-02

 4211          1    .244127E-16*  .612765E-17   .157333E-16*  .260395E-15   .402543E-17   .337404E-15*
               2    .169012E-16   .846944E-17*  .147678E-16   .282107E-15*  .938112E-17*  .246308E-15
    (MAXIMUM)       .244127E-16   .846944E-17   .157333E-16   .282107E-15   .938112E-17   .337404E-15

 4212          1    .315263E-01*  .362346E-03   .187573E-01*  .395603E-02*  .126974E-03   .654140E-02*
               2    .211457E-01   .500823E-03*  .160702E-01   .344192E-02   .295909E-03*  .442368E-02
    (MAXIMUM)       .315263E-01   .500823E-03   .187573E-01   .395603E-02   .295909E-03   .654140E-02

 4213          1    .946028E-01*  .724980E-03   .584436E-01*  .496154E-02*  .254099E-03   .748867E-02*
               2    .648716E-01   .100214E-02*  .518929E-01   .471366E-02   .592168E-03*  .551073E-02
    (MAXIMUM)       .946028E-01   .100214E-02   .584436E-01   .496154E-02   .592168E-03   .748867E-02

 4311          1    .252964E-16*  .743879E-17   .123367E-16   .259071E-15   .402565E-17   .356134E-15*
               2    .252067E-16   .896232E-17*  .201829E-16*  .484579E-15*  .651385E-17*  .347244E-15
    (MAXIMUM)       .252964E-16   .896232E-17   .201829E-16   .484579E-15   .651385E-17   .356134E-15

 4312          1    .326477E-01*  .439878E-03   .147940E-01   .321817E-02   .126981E-03   .678849E-02*
               2    .283592E-01   .529969E-03*  .238330E-01*  .529673E-02*  .205466E-03*  .584212E-02
    (MAXIMUM)       .326477E-01   .529969E-03   .238330E-01   .529673E-02   .205466E-03   .678849E-02

 4313          1    .984345E-01*  .880124E-03   .494199E-01   .471003E-02   .254113E-03   .784896E-02*
               2    .837447E-01   .106045E-02*  .833921E-01*  .847238E-02*  .411177E-03*  .647712E-02
    (MAXIMUM)       .984345E-01   .106045E-02   .833921E-01   .847238E-02   .411177E-03   .784896E-02

 4411          1    .155428E-16*  .804205E-17*  .100786E-16   .235072E-15   .453531E-17   .206327E-15*
               2    .303681E-17   .423439E-17   .254137E-16*  .578801E-15*  .603294E-17*  .351754E-16
    (MAXIMUM)       .155428E-16   .804205E-17   .254137E-16   .578801E-15   .603294E-17   .206327E-15

 4412          1    .183681E-01*  .475550E-03*  .118158E-01   .261243E-02   .143057E-03   .380443E-02*
               2    .310091E-02   .250392E-03   .292107E-01*  .644304E-02*  .190297E-03*  .640418E-03
    (MAXIMUM)       .183681E-01   .475550E-03   .292107E-01   .644304E-02   .190297E-03   .380443E-02

 4413          1    .550548E-01*  .951495E-03*  .408761E-01   .408392E-02   .286284E-03   .447547E-02*
               2    .935751E-02   .501019E-03   .100537E+00*  .997458E-02*  .380820E-03*  .864598E-03
    (MAXIMUM)       .550548E-01   .951495E-03   .100537E+00   .997458E-02   .380820E-03   .447547E-02

 4511          1    .183828E-16*  .399954E-17*  .104420E-16   .215698E-15   .331254E-17   .232989E-15*
               2    .452597E-17   .302770E-17   .185384E-16*  .425272E-15*  .591640E-17*  .507854E-16
    (MAXIMUM)       .183828E-16   .399954E-17   .185384E-16   .425272E-15   .591640E-17   .232989E-15

 4512          1    .201107E-01*  .236505E-03*  .112834E-01   .244977E-02   .104487E-03   .410835E-02*
               2    .488671E-02   .179037E-03   .222308E-01*  .489561E-02*  .186621E-03*  .958463E-03
    (MAXIMUM)       .201107E-01   .236505E-03   .222308E-01   .489561E-02   .186621E-03   .410835E-02

 4513          1    .583353E-01*  .473142E-03*  .375676E-01   .358516E-02   .209099E-03   .444275E-02*
               2    .128550E-01   .358237E-03   .762477E-01*  .752802E-02*  .373463E-03*  .809667E-03
    (MAXIMUM)       .583353E-01   .473142E-03   .762477E-01   .752802E-02   .373463E-03   .444275E-02

 4611          1    .153837E-16*  .725544E-17*  .763907E-17   .143448E-15   .438310E-17   .208230E-15*
               2    .482643E-17   .155517E-17   .124981E-16*  .256393E-15*  .649930E-17*  .471426E-16
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RESPONSE SPECTRUM ANALYSIS OF ROUTE 80 ONRAMP UNDERCROSSING                                                              PAGE    30
         FHWA  'SEISMIC DESIGN OF HIGHWAY BRIDGES'                                                                       22  NOV 78

   NODE   RESPONSE X-TRANSLATION Y-TRANSLATION Z-TRANSLATION    X-ROTATION    Y-ROTATION    Z-ROTATION
  NUMBER      CASE  DISPLACEMENT  DISPLACEMENT  DISPLACEMENT  DISPLACEMENT  DISPLACEMENT  DISPLACEMENT

   4612          1   .177743E-01*  .429036E-03*  .928233E-02   .198621E-02   .138256E-03   .366733E-02*
                 2   .669346E-02   .919621E-04   .149124E-01*  .323555E-02*  .205007E-03*  .135197E-02
       (MAXIMUM)     .177743E-01   .429036E-03   .149124E-01   .323555E-02   .205007E-03   .366733E-02

   4613          1   .527059E-01*  .858362E-03*  .299809E-01   .277077E-02   .276676E-03   .412326E-02*
                 2   .188714E-01   .184044E-03   .495136E-01*  .466320E-02*  .410258E-03*  .133151E-02
       (MAXIMUM)     .527059E-01   .858362E-03   .495136E-01   .466320E-02   .410258E-03   .412326E-02
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RESPONSE SPECTRUM ANALYSIS OF ROUTE 80 ONRAMP UNDERCROSSING                                                      PAGE    32
         FHWA  'SEISMIC DESIGN OF HIGHWAY BRIDGES'                                                               22  NOV 78

    BEAM  RESPONSE         END 1        END 1        END 1        END 1        END 1        END 1        END 2        END 2
NUMBER        CASE      X-THRUST      Y-SHEAR      Z-SHEAR     X-TORQUE     Y-MOMENT     Z-MOMENT     Y-MOMENT     Z-MOMENT

     1           1   .870837E+01* .411929E+03* .256348E+03  .398709E+04  .148379E-03* .699210E-07  .257384E+03  .413594E+03*
                 2   .552060E+01  .411444E+03  .371573E+03* .835809E+04* .919114E-04  .819658E-07* .373075E+03* .413107E+03
         (MAXIMUM)   .870837E+01  .411929E+03  .371573E+03  .835809E+04  .148379E-03  .819658E-07  .373075E+03  .413594E+03

     2           1   .229654E+03* .407191E+03  .252843E+03  .398699E+04  .257384E+03  .414538E+03  .636085E+04  .102330E+05
                 2   .145955E+03  .407219E+03* .368896E+03* .835098E+04* .373075E+03* .537939E+03* .927801E+04* .103386E+05*
         (MAXIMUM)   .229654E+03  .407219E+03  .368896E+03  .835098E+04  .373075E+03  .537939E+03  .927801E+04  .103386E+05

     3           1   .651769E+03* .213485E+03  .192744E+03  .404688E+04  .636085E+04  .102095E+05  .110070E+05  .151853E+05
                 2   .414164E+03  .224334E+03* .279757E+03* .815010E+04* .927801E+04* .104977E+05* .160099E+05* .158716E+05*
         (MAXIMUM)   .651769E+03  .224334E+03  .279757E+03  .815010E+04  .927801E+04  .104977E+05  .160099E+05  .158716E+05

     4           1   .106945E+04* .156576E+03* .927618E+02  .422441E+04  .110070E+05  .151369E+05  .129910E+05  .139365E+05
                 2   .679879E+03  .755608E+02  .146184E+03* .788553E+04* .160099E+05* .160047E+05* .189272E+05* .150499E+05*
         (MAXIMUM)   .106945E+04  .156576E+03  .146184E+03  .788553E+04  .160099E+05  .160047E+05  .189272E+05  .150499E+05

     5           1   .147992E+04* .320681E+03* .126341E+03  .446797E+04  .129910E+05  .138604E+05  .117178E+05  .103722E+05*
                 2   .942101E+03  .256413E+03  .217869E+03* .771158E+04* .189272E+05* .151398E+05* .177142E+05* .976409E+04
         (MAXIMUM)   .147992E+04  .320681E+03  .217869E+03  .771158E+04  .189272E+05  .151398E+05  .177142E+05  .103722E+05

     6           1   .803265E+03  .440871E+03  .413959E+03  .200342E+04  .115730E+05  .231589E+05  .231445E+05  .145285E+05
                 2   .964606E+03* .100346E+04* .721411E+03* .468361E+04* .176688E+05* .246811E+05* .333828E+05* .170701E+05*
         (MAXIMUM)   .964606E+03  .100346E+04  .721411E+03  .468361E+04  .176688E+05  .246811E+05  .333828E+05  .170701E+05

     7           1   .344708E+03  .440038E+03  .191815E+03  .185140E+04  .231445E+05  .145487E+05  .277239E+05  .995149E+04
                 2   .914593E+03* .477724E+03* .316567E+03* .511916E+04* .333828E+05* .169446E+05* .402989E+05* .283927E+05*
         (MAXIMUM)   .914593E+03  .477724E+03  .316567E+03  .511916E+04  .333828E+05  .169446E+05  .402989E+05  .283927E+05

     8           1   .544744E+03  .443216E+03  .196512E+03  .232210E+04  .277239E+05  .985230E+04  .228168E+05  .152929E+05*
                 2   .103525E+04* .548385E+03* .338419E+03* .610860E+04* .402989E+05* .281963E+05* .306360E+05* .145926E+05
         (MAXIMUM)   .103525E+04  .548385E+03  .338419E+03  .610860E+04  .402989E+05  .281963E+05  .306360E+05  .152929E+05

     9           1   .106320E+04  .439223E+03  .481440E+03  .280891E+04  .228168E+05  .152110E+05* .111458E+05  .241945E+05
                 2   .129426E+04* .103246E+04* .863617E+03* .655157E+04* .306360E+05* .143992E+05  .137754E+05* .285240E+05*
         (MAXIMUM)   .129426E+04  .103246E+04  .863617E+03  .655157E+04  .306360E+05  .152110E+05  .137754E+05  .285240E+05

    10           1   .105700E+04* .389262E+03* .325856E+03  .287553E+04  .113728E+05  .103497E+05* .127817E+05  .126906E+05*
                 2   .791791E+03  .298390E+03  .861899E+03* .294815E+04* .142682E+05* .707571E+04  .158856E+05* .120288E+05
         (MAXIMUM)   .105700E+04  .389262E+03  .861899E+03  .294815E+04  .142682E+05  .103497E+05  .158856E+05  .126906E+05

    11           1   .669742E+03* .216848E+03* .164272E+03  .274008E+04* .127817E+05  .127205E+05* .128508E+05  .148030E+05*
                 2   .510308E+03  .120888E+03  .400713E+03* .273674E+04  .158856E+05* .120787E+05  .224648E+05* .140545E+05
         (MAXIMUM)   .669742E+03  .216848E+03  .400713E+03  .274008E+04  .158856E+05  .127205E+05  .224648E+05  .148030E+05

    12           1   .315918E+03* .198228E+03* .179953E+03  .270736E+04* .128508E+05  .148091E+05* .890968E+04  .106337E+05*
                 2   .208753E+03  .178252E+03  .221705E+03* .252004E+04  .224648E+05* .140950E+05  .180751E+05* .975780E+04
         (MAXIMUM)   .315918E+03  .198228E+03  .221705E+03  .270736E+04  .224648E+05  .148091E+05  .180751E+05  .106337E+05

    13           1   .193373E+03* .411298E+03* .343760E+03  .277142E+04* .890968E+04  .106172E+05* .431622E+03  .483838E+03*
                 2   .178100E+03  .380453E+03  .693289E+03* .242881E+04  .180751E+05* .978090E+04  .972861E+03* .399676E+03
         (MAXIMUM)   .193373E+03  .411298E+03  .693289E+03  .277142E+04  .180751E+05  .106172E+05  .972861E+03  .483838E+03

    14           1   .988617E+02* .503207E+03* .489520E+03  .277484E+04* .182582E-06  .240487E+02* .734307E+04  .755319E+04*
                 2   .149720E+02  .413874E+03  .114604E+04* .242830E+04  .536366E-06* .210453E+02  .171913E+05* .620636E+04
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RESPONSE SPECTRUM ANALYSIS OF ROUTE 80 ONRAMP UNDERCROSSING                                                   PAGE    33
         FHWA  'SEISMIC DESIGN OF HIGHWAY BRIDGES'                                                            22  NOV 78

  BEAM  RESPONSE          END 1         END 1         END 1         END 1         END 1         END 1         END 2         END 2
 NUMBER    CASE        X-THRUST       Y-SHEAR       Z-SHEAR      X-TORQUE      Y-MOMENT      Z-MOMENT      Y-MOMENT      Z-MOMENT

    15          1   .127161E+04*  .784261E+03*  .456697E+03   .250610E+04   .704355E+04   .188378E+05*  .117070E+05*  .144209E+05*
                2   .136964E+03   .150436E+03   .862702E+03*  .451971E+04*  .167038E+05*  .470762E+04   .977803E+04   .438385E+04
      (MAXIMUM)     .127161E+04   .784261E+03   .862702E+03   .451971E+04   .167038E+05   .188378E+05   .117070E+05   .144209E+05

    16          1   .958429E+03*  .454145E+03*  .231560E+03   .255809E+04   .117070E+05*  .144118E+05*  .181000E+05   .206446E+05*
                2   .145395E+03   .669702E+02   .335995E+03*  .449796E+04*  .977803E+04   .440617E+04   .193511E+05*  .433683E+04
      (MAXIMUM)     .958429E+03   .454145E+03   .335995E+03   .449796E+04   .117070E+05   .144118E+05   .193511E+05   .206446E+05

    17          1   .718447E+03*  .406689E+03*  .106374E+03   .300845E+04   .181000E+05   .205838E+05*  .182154E+05*  .139919E+05*
                2   .195933E+03   .116510E+03   .263817E+03*  .454467E+04*  .193511E+05*  .428785E+04   .170189E+05   .140598E+04
      (MAXIMUM)     .718447E+03   .406689E+03   .263817E+03   .454467E+04   .193511E+05   .205838E+05   .182154E+05   .139919E+05

    18          1   .633320E+03*  .767146E+03*  .330893E+03   .339365E+04   .182154E+05*  .139035E+05*  .137274E+05   .153693E+05*
                2   .290825E+03   .193393E+03   .693083E+03*  .453268E+04*  .170189E+05   .144419E+04   .171510E+05*  .501511E+04
      (MAXIMUM)     .633320E+03   .767146E+03   .693083E+03   .453268E+04   .182154E+05   .139035E+05   .171510E+05   .153693E+05

    19          1   .116578E+04*  .712717E+03*  .414834E+03   .251290E+04   .135792E+05   .116768E+05*  .215145E+05   .133410E+05*
                2   .326952E+03   .212735E+03   .649609E+03*  .561031E+04*  .166719E+05*  .988125E+04   .235728E+05*  .459226E+04
      (MAXIMUM)     .116578E+04   .712717E+03   .649609E+03   .561031E+04   .166719E+05   .116768E+05   .235728E+05   .133410E+05

    20          1   .864498E+03*  .315695E+03*  .194544E+03   .238906E+04   .215145E+05   .133637E+05*  .247091E+05   .193391E+05*
                2   .224456E+03   .191184E+03   .265569E+03*  .539292E+04*  .235728E+05*  .484571E+04   .287961E+05*  .289273E+04
      (MAXIMUM)     .864498E+03   .315695E+03   .265569E+03   .539292E+04   .235728E+05   .133637E+05   .287961E+05   .193391E+05

    21          1   .627134E+03*  .454324E+03*  .189256E+03   .249144E+04   .247091E+05   .193262E+05*  .214753E+05   .108841E+05*
                2   .123433E+03   .135746E+03   .225656E+03*  .541482E+04*  .287961E+05*  .285151E+04   .255459E+05*  .462303E+04
      (MAXIMUM)     .627134E+03   .454324E+03   .225656E+03   .541482E+04   .287961E+05   .193262E+05   .255459E+05   .108841E+05

    22          1   .537557E+03*  .805217E+03*  .387764E+03   .261083E+04   .214753E+05   .108561E+05*  .130316E+05   .191041E+05*
                2   .171748E+03   .111040E+03   .514516E+03*  .560455E+04*  .255459E+05*  .439109E+04   .175002E+05*  .580986E+04
      (MAXIMUM)     .537557E+03   .805217E+03   .514516E+03   .560455E+04   .255459E+05   .108561E+05   .175002E+05   .191041E+05

    23          1   .108567E+04*  .385245E+03*  .945735E+02   .423741E+04   .132023E+05   .684149E+04*  .140114E+05   .110480E+05*
                2   .576689E+03   .137009E+03   .219508E+03*  .704987E+04*  .175083E+05*  .459464E+04   .179995E+05*  .570927E+04
      (MAXIMUM)     .108567E+04   .385245E+03   .219508E+03   .704987E+04   .175083E+05   .684149E+04   .179995E+05   .110480E+05

    24          1   .781361E+03*  .186359E+03*  .105809E+03   .400254E+04   .140114E+05   .111353E+05*  .116055E+05   .140946E+05*
                2   .427319E+03   .698821E+02   .151127E+03*  .683776E+04*  .179995E+05*  .596167E+04   .150120E+05*  .626912E+04
      (MAXIMUM)     .781361E+03   .186359E+03   .151127E+03   .683776E+04   .179995E+05   .111353E+05   .150120E+05   .140946E+05

    25          1   .475006E+03*  .177356E+03*  .205758E+03   .377149E+04   .116055E+05   .141582E+05*  .664598E+04   .100460E+05*
                2   .265063E+03   .985526E+02   .264597E+03*  .664283E+04*  .150120E+05*  .647532E+04   .865667E+04*  .421000E+04
      (MAXIMUM)     .475006E+03   .177356E+03   .264597E+03   .664283E+04   .150120E+05   .141582E+05   .865667E+04   .100460E+05

    26          1   .167137E+03*  .401625E+03*  .264115E+03   .365571E+04   .664598E+04   .100887E+05*  .266768E+03   .397139E+03*
                2   .943957E+02   .178907E+03   .343992E+03*  .652252E+04*  .865667E+04*  .439408E+04   .348163E+03*  .133535E+03
      (MAXIMUM)     .167137E+03   .401625E+03   .343992E+03   .652252E+04   .865667E+04   .100887E+05   .348163E+03   .397139E+03

    27          1   .639129E+01*  .407076E+03*  .265694E+03   .365443E+04   .266768E+03   .408722E+03*  .129470E-04*  .212792E-07*
                2   .345076E+01   .180891E+03   .346762E+03*  .652136E+04*  .348163E+03*  .181622E+03   .764920E-05   .123131E-07
      (MAXIMUM)     .639129E+01   .407076E+03   .346762E+03   .652136E+04   .348163E+03   .408722E+03   .129470E-04   .212792E-07

    28          1   .398495E+03* 0.            0.            0.            0.            0.            0.            0.
                2   .302831E+03  0.            0.            0.            0.            0.            0.            0.
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  BEAM  RESPONSE        END 1         END 1         END 1         END 1         END 1         END 1         END 2         END 2
NUMBER      CASE     X-THRUST       Y-SHEAR       Z-SHEAR      X-TORQUE      Y-MOMENT      Z-MOMENT      Y-MOMENT      Z-MOMENT

  29           1  .612765E+03   .281028E+04*  .733148E+03   .402543E+03   .154635E+05   .397159E+05*  .930492E+04   .160951E+05*
               2  .846944E+03*  .186313E+04   .125145E+04*  .938112E+03*  .277392E+05*  .251606E+05   .172298E+05*  .103055E+05
   (MAXIMUM)      .846944E+03   .281028E+04   .125145E+04   .938112E+03   .277392E+05   .397159E+05   .172298E+05   .160951E+05

  30           1  .612525E+03   .279830E+04*  .729732E+03   .402543E+03   .930492E+04   .160951E+05*  .324931E+04   .788285E+04
               2  .846774E+03*  .185586E+04   .124580E+04*  .938112E+03*  .172298E+05*  .103055E+05   .687539E+04*  .850384E+04*
   (MAXIMUM)      .846774E+03   .279830E+04   .124580E+04   .938112E+03   .172298E+05   .160951E+05   .687539E+04   .850384E+04

  31           1  .612046E+03   .276228E+04*  .718233E+03   .402543E+03   .324931E+04   .788285E+04   .311922E+04   .309880E+05*
               2  .846435E+03*  .183463E+04   .122660E+04*  .938112E+03*  .687539E+04*  .850384E+04*  .415806E+04*  .228252E+05
   (MAXIMUM)      .846435E+03   .276228E+04   .122660E+04   .938112E+03   .687539E+04   .850384E+04   .415806E+04   .309880E+05

  32           1  .743879E+03   .261860E+04*  .103150E+04   .402565E+03   .232219E+05   .374197E+05*  .145588E+05   .154014E+05*
               2  .896232E+03*  .232305E+04   .224293E+04*  .651385E+03*  .516463E+05*  .297760E+05   .327441E+05*  .108501E+05
   (MAXIMUM)      .896232E+03   .261860E+04   .224293E+04   .651385E+03   .516463E+05   .374197E+05   .327441E+05   .154014E+05

  33           1  .743620E+03   .260727E+04*  .102698E+04   .402565E+03   .145588E+05   .154014E+05*  .601548E+04   .691624E+04
               2  .896042E+03*  .231425E+04   .223371E+04*  .651385E+03*  .327441E+05*  .108501E+05   .139119E+05*  .107232E+05*
   (MAXIMUM)      .896042E+03   .260727E+04   .223371E+04   .651385E+03   .327441E+05   .154014E+05   .139119E+05   .107232E+05

  34           1  .743101E+03   .257306E+04*  .101155E+04   .402565E+03   .601548E+04   .691624E+04   .313547E+04   .284435E+05
               2  .895661E+03*  .228910E+04   .220197E+04*  .651385E+03*  .139119E+05*  .107232E+05*  .478427E+04*  .293613E+05*
   (MAXIMUM)      .895661E+03   .257306E+04   .220197E+04   .651385E+03   .139119E+05   .107232E+05   .478427E+04   .293613E+05

  35           1  .804205E+03*  .154503E+04*  .102197E+04   .453531E+03   .237115E+05   .203976E+05*  .151034E+05   .828187E+04*
               2  .423439E+03   .334895E+03   .253744E+04*  .603294E+03*  .578627E+05*  .379335E+04   .365266E+05*  .154905E+04
   (MAXIMUM)      .804205E+03   .154503E+04   .253744E+04   .603294E+03   .578627E+05   .203976E+05   .365266E+05   .828187E+04

  36           1  .803919E+03*  .153537E+04*  .101764E+04   .453531E+03   .151034E+05   .828187E+04*  .653972E+04   .772853E+04*
               2  .423334E+03   .333821E+03   .252638E+04*  .603294E+03*  .365266E+05*  .154905E+04   .154051E+05*  .251793E+04
   (MAXIMUM)      .803919E+03   .153537E+04   .252638E+04   .603294E+03   .365266E+05   .828187E+04   .154051E+05   .772853E+04

  37           1  .803347E+03*  .150773E+04*  .100270E+04   .453531E+03   .653972E+04   .772853E+04*  .209647E+04   .194404E+05*
               2  .423122E+03   .331072E+03   .248850E+04*  .603294E+03*  .154051E+05*  .251793E+04   .671857E+04*  .509416E+04
   (MAXIMUM)      .803347E+03   .150773E+04   .248850E+04   .603294E+03   .154051E+05   .772853E+04   .671857E+04   .194404E+05

  38           1  .399954E+03*  .187992E+04*  .967217E+03   .331254E+03   .216676E+05   .232080E+05*  .135498E+05   .811318E+04*
               2  .302770E+03   .716040E+03   .176885E+04*  .591640E+03*  .417828E+05*  .941024E+04   .268919E+05*  .342750E+04
   (MAXIMUM)      .399954E+03   .187992E+04   .176885E+04   .591640E+03   .417828E+05   .232080E+05   .268919E+05   .811318E+04

  39           1  .399704E+03*  .187106E+04*  .962397E+03   .331254E+03   .135498E+05   .811318E+04*  .556608E+04   .972468E+04*
               2  .302687E+03   .713015E+03   .176117E+04*  .591640E+03*  .268919E+05*  .342750E+04   .120971E+05*  .277825E+04
   (MAXIMUM)      .399704E+03   .187106E+04   .176117E+04   .591640E+03   .268919E+05   .811318E+04   .120971E+05   .972468E+04

  40           1  .399203E+03*  .184616E+04*  .945951E+03   .331254E+03   .556608E+04   .972468E+04*  .307314E+04   .247327E+05*
               2  .302522E+03   .704226E+03   .173466E+04*  .591640E+03*  .120971E+05*  .277825E+04   .315462E+04*  .864438E+04
   (MAXIMUM)      .399203E+03   .184616E+04   .173466E+04   .591640E+03   .120971E+05   .972468E+04   .315462E+04   .247327E+05

  41           1  .725544E+03*  .159012E+04*  .649354E+03   .438310E+03   .139937E+05   .210605E+05*  .854554E+04   .818485E+04*
               2  .155517E+03   .895857E+03   .996201E+03*  .649930E+03*  .226647E+05*  .128806E+05   .143174E+05*  .535541E+04
   (MAXIMUM)      .725544E+03   .159012E+04   .996201E+03   .649930E+03   .226647E+05   .210605E+05   .143174E+05   .818485E+04

  42           1  .725175E+03*  .158036E+04*  .645775E+03   .438310E+03   .854554E+04   .818485E+04*  .320743E+04   .700415E+04*
               2  .155536E+03   .891640E+03   .991181E+03*  .649930E+03*  .143174E+05*  .535541E+04   .613655E+04*  .232358E+04
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RESPONSE SPECTRUM ANALYSIS OF ROUTE 80 ONRAMP UNDERCROSSING                                                    PAGE    35
        FHWA  'SEISMIC DESIGN OF HIGHWAY BRIDGES'                                                              22  NOV 78

   BEAM RESPONSE         END 1        END 1        END 1        END 1        END 1        END 1        END 2        END 2
 NUMBER     CASE      X-THRUST      Y-SHEAR      Z-SHEAR     X-TORQUE     Y-MOMENT     Z-MOMENT     Y-MOMENT     Z-MOMENT

    43         1   .724440E+03* .155226E+04* .633693E+03  .438310E+03  .320743E+04  .700415E+04* .253005E+04  .194294E+05*
               2   .155569E+03  .878904E+03  .974107E+03* .649930E+03* .613655E+04* .232358E+04  .305933E+04* .965056E+04
       (MAXIMUM)   .724440E+03  .155226E+04  .974107E+03  .649930E+03  .613655E+04  .700415E+04  .305933E+04  .194294E+05

    44         1  0.            .463175E+03* .431622E+03  .277495E+04* .431622E+03  .463175E+03* 0.          0.
               2  0.            .402199E+03  .972861E+03* .242840E+04  .972861E+03* .402199E+03  0.          0.
       (MAXIMUM)  0.            .463175E+03  .972861E+03  .277495E+04  .972861E+03  .463175E+03  0.          0.
```

INPUT DATA SECTIONS USED FOR E2PLOT
(An EASE2 plotting post-processor)

1. E2PLOT
2. VIEW
3. FRAME

*NOTE: The input card images for E2PLOT and the resulting mode plots are included at the end of this section. Special job control cards are needed to use the plotting post processor. Refer to the user instructions for more information.

```
E2PLOT             1    2
 ROUTE 80 ONRAMP UNDERCROSSING  -  MODE SHAPES
VIEW               1               100.0     100.0    Y    3
 3 DIMENSIONAL VIEW
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    0    RRRRRR
    2    0RRR00
    5    000000
FRAME              1    1         1
 MODE 1
 BEAM    1   42    1    1
FRAME              2    1         2
 MODE 2
 BEAM    1   42    1    1
FRAME              3    1         3
 MODE 3
 BEAM    1   42    1    1
FRAME              4    1         4
 MODE 4
 BEAM    1   42    1    1
FRAME              5    1         5
 MODE 5
 BEAM    1   42    1    1
FRAME              6    1         6
 MODE 6
 BEAM    1   42    1    1
FRAME              7    1         7
 MODE 7
 BEAM    1   42    1    1
FRAME              8    1         8
 MODE 8
 BEAM    1   42    1    1
FRAME              9    1         9
 MODE 9
 BEAM    1   42    1    1
FRAME             10    1        10
 MODE 10
 BEAM    1   42    1    1
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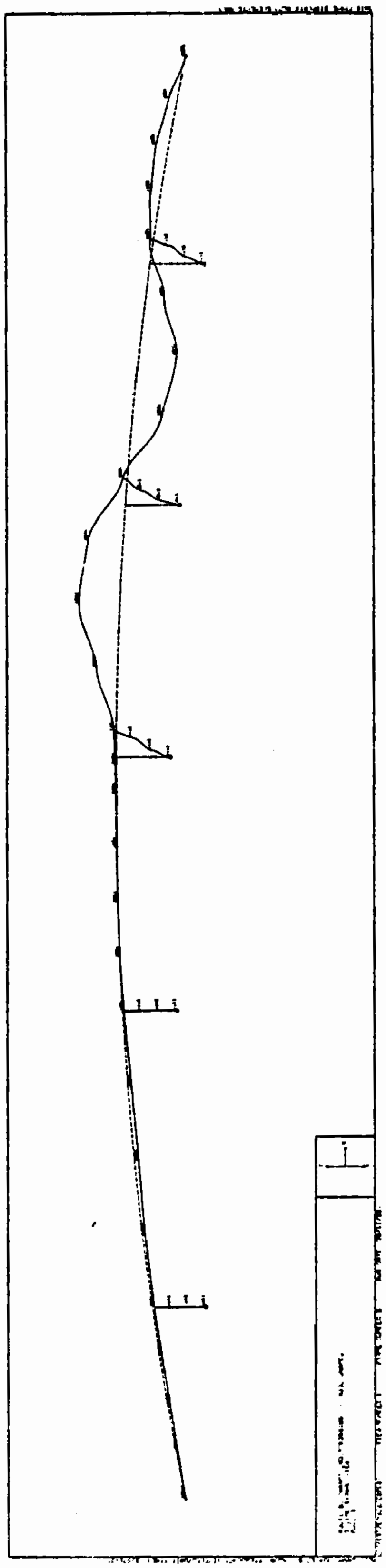

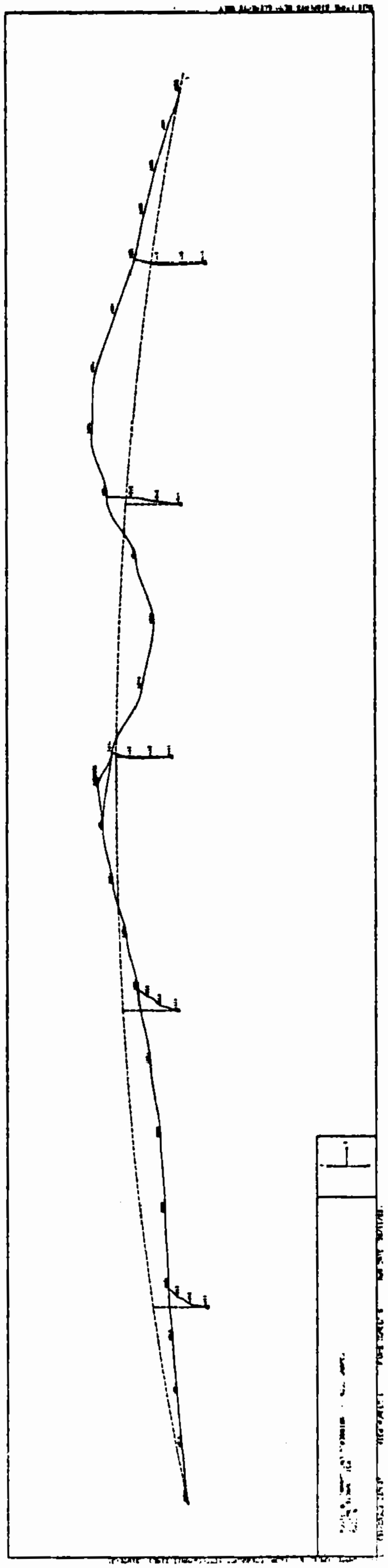

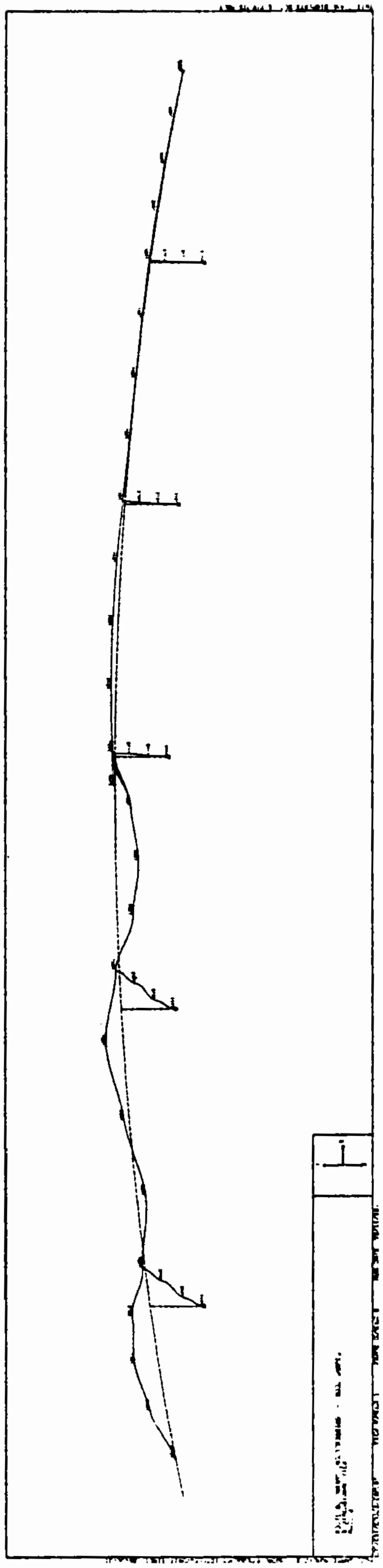

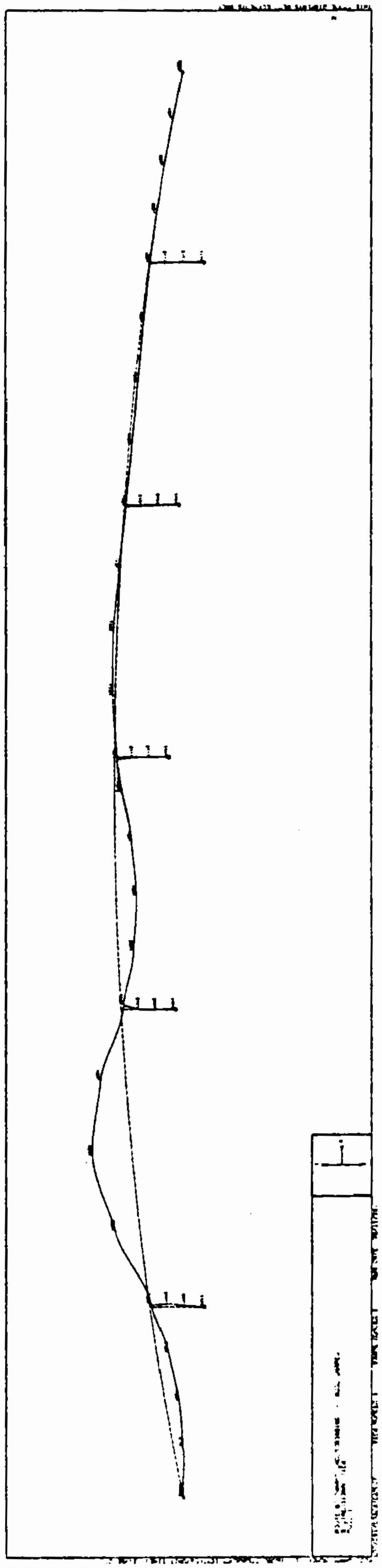

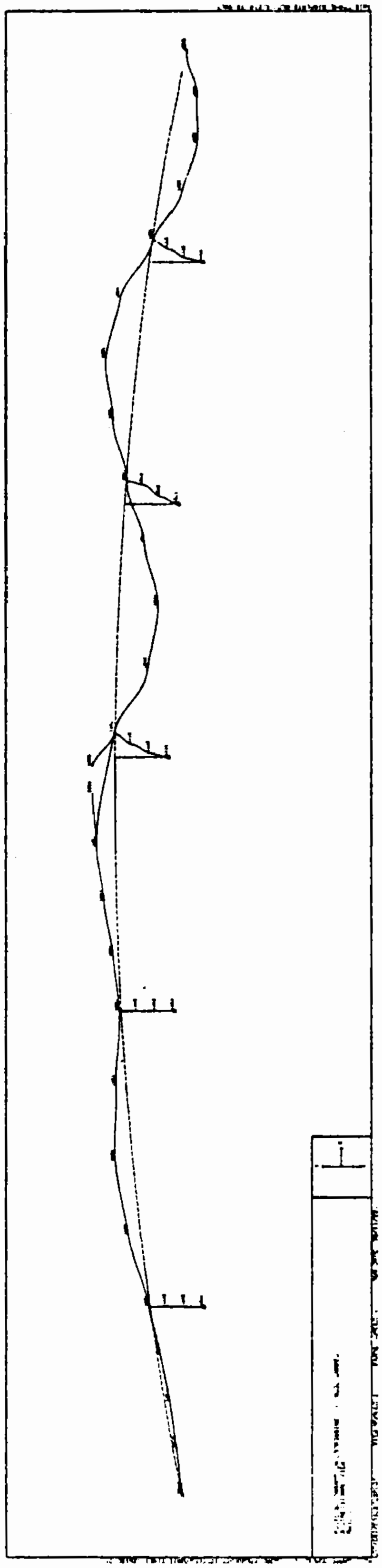

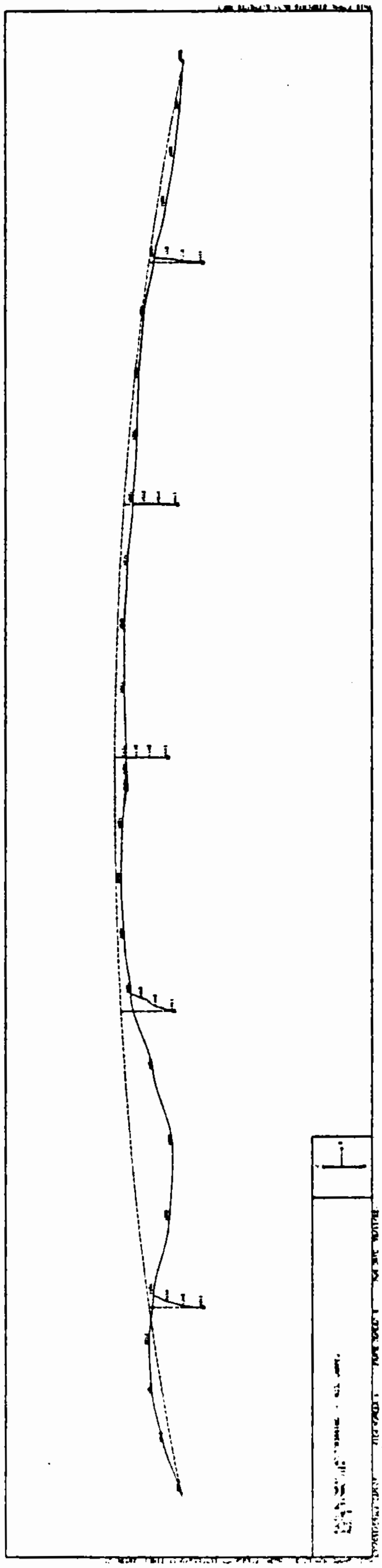

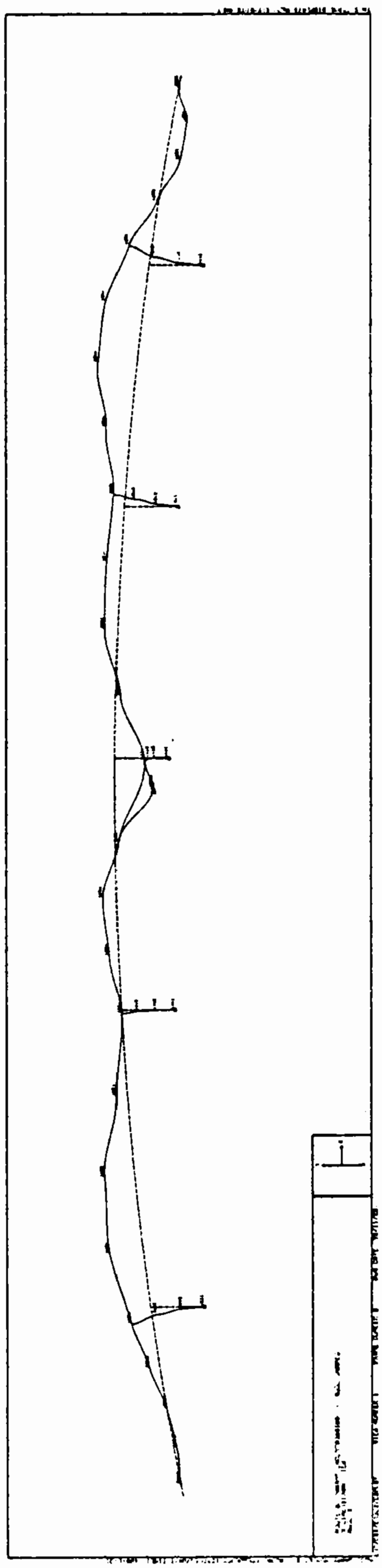

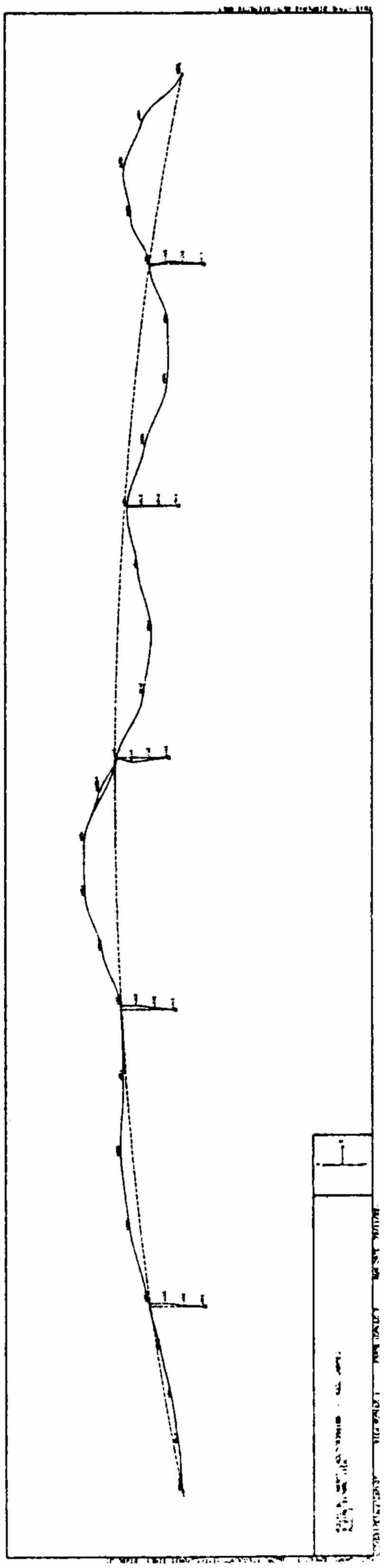

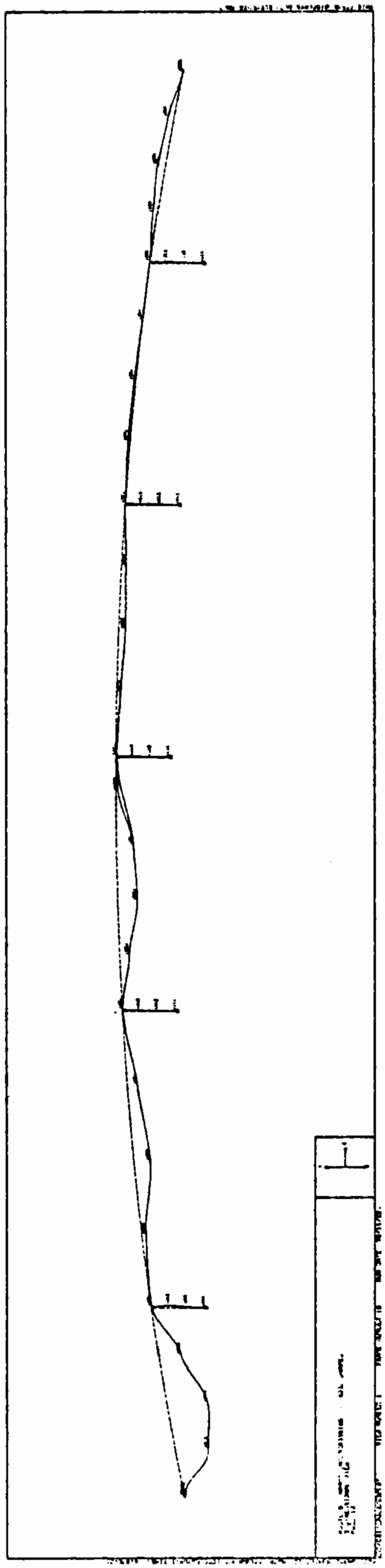

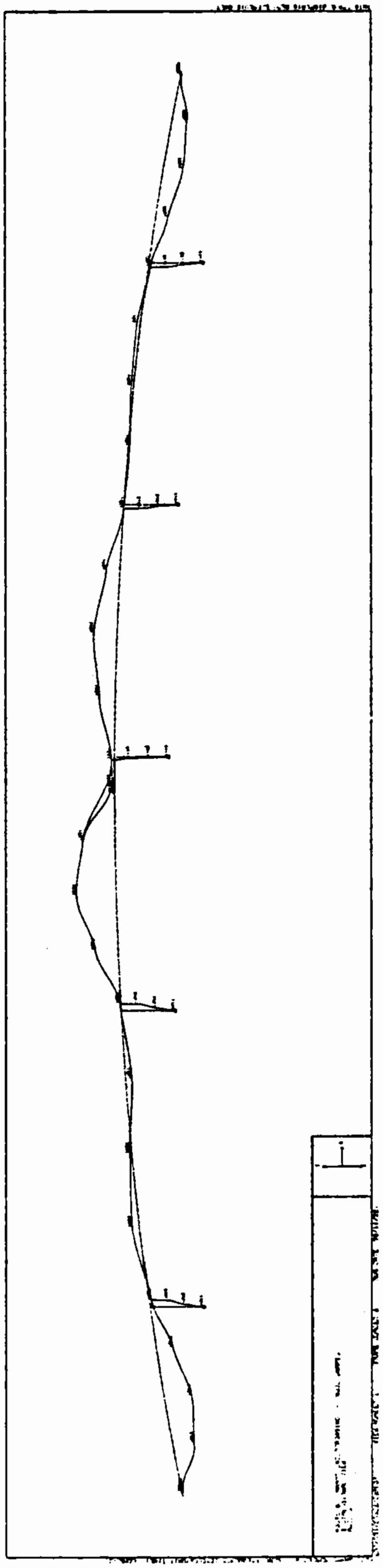

APPENDIX D.1

EARTHQUAKE ENGINEERING RESEARCH CENTER

SHAKE

A COMPUTER PROGRAM FOR EARTHQUAKE RESPONSE ANALYSIS OF HORIZONTALLY LAYERED SITES

by

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John Lysmer

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A computer program distributed by
NISEE/Computer Applications

Report No. EERC 72-12

December 1972

College of Engineering
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Berkeley, California

TABLE OF CONTENTS

1. INTRODUCTION

Several methods for evaluating the effect of local soil conditions on ground response during earthquakes are presently available. Most of these methods are based on the assumption that the main responses in a soil deposit are caused by the upward propagation of shear waves from the underlying rock formation. Analytical procedures based on this concept incorporating nonlinear soil behavior, have been shown to give results in good agreement with field observations in a number of cases. Accordingly they are finding increasing use in earthquake engineering for predicting responses within soil deposits and the characteristics of ground surface motions.

The analytical procedure generally involves the following steps:

1. Determine the characteristics of the motions likely to develop in the rock formation underlying the site, and select an accelerogram with these characteristics for use in the analysis.
   The maximum acceleration, predominant period, and effective duration are the most important parameters of an earthquake motion. Empirical relationships between these parameters and the distance from the causative fault to the site have been established for different magnitude earthquakes (Gutenberg and Richter, 1956, Seed et al., 1969, Schnabel and Seed, 1972). A design motion with the desired characteristics can be selected from the strong motion accelerograms that have been recorded during previous earthquakes (Seed and Idriss, 1969) or from artificially generated accelerograms (Housner and Jennings, 1964).

2. Determine the dynamic properties of the soil deposit.

   Average relationships between the dynamic shear moduli and damping ratios of soils, as functions of shear strain and static properties, have been established for various soil types (Hardin and Drnevich, 1970, Seed and Idriss, 1970). Thus a relatively simple testing program to obtain the static properties for use in these relationships will often serve to establish the dynamic properties with a sufficient degree of accuracy. However more elaborate dynamic testing procedures are required for special problems and for cases involving soil types for which empirical relationships with static properties have not been established.

3. Compute the response of the soil deposit to the base-rock motions.

   A one-dimensional method of analysis can be used if the soil structure is essentially horizontal. Programs developed for performing this analysis are in general based on either the solution to the wave equation (Kanai, 1951; Matthiesen et al., 1964; Roesset and Whitman, 1969; Lysmer et al., 1971) or on a lumped mass simulation (Idriss and Seed, 1968). More irregular soil deposits may require a finite element analysis.

In the following sections the theory and use of a computer program based on the one-dimensional wave propagation method are described. The program can compute the responses for a design motion given anywhere in the system. Thus accelerograms obtained from instruments on soil deposits can be used to generate new rock motions which, in turn, can be used as design motion for other soil deposits, see Fig. 1 (Schnabel et al., 1971). The program also incorporates nonlinear soil behavior, the effect of the elasticity of the base rock and systems with variable damping.

2. THEORY

The theory considers the responses associated with vertical propagation of shear waves through the linear viscoelastic system shown in Fig. 2. The system consists of N horizontal layers which extend to infinity in the horizontal direction and has a halfspace as the bottom layer. Each layer is homogeneous and isotropic and is characterized by the thickness, h, mass density, ρ, shear modulus, G, and damping factor, β.

2.1 Propagation of harmonic shear waves in a one-dimensional system.

Vertical propagation of shear waves through the system shown in Fig. 2 will cause only horizontal displacements:

$$u = u(x,t) \tag{1}$$

which must satisfy the wave equation:

$$\rho \frac{\partial^2 u}{\partial t^2} = G \frac{\partial^2 u}{\partial x^2} + \eta \frac{\partial^3 u}{\partial x^2 \partial t} \tag{2}$$

Harmonic displacements with frequency ω can be written in the form:

$$u(x,t) = U(x) \cdot e^{i\omega t} \tag{3}$$

Substituting Eq. 3 into Eq. 2 results in an ordinary differential equation:

$$(G + i\omega\eta) \frac{d^2 U}{dx^2} = \rho\omega^2 U \tag{4}$$

which has the general solution

$$U(x) = Ee^{ikx} + Fe^{-ikx} \tag{5}$$

in which

$$k^2 = \frac{\rho\omega^2}{G + i\omega\eta} = \frac{\rho\omega^2}{G^*} \tag{6}$$

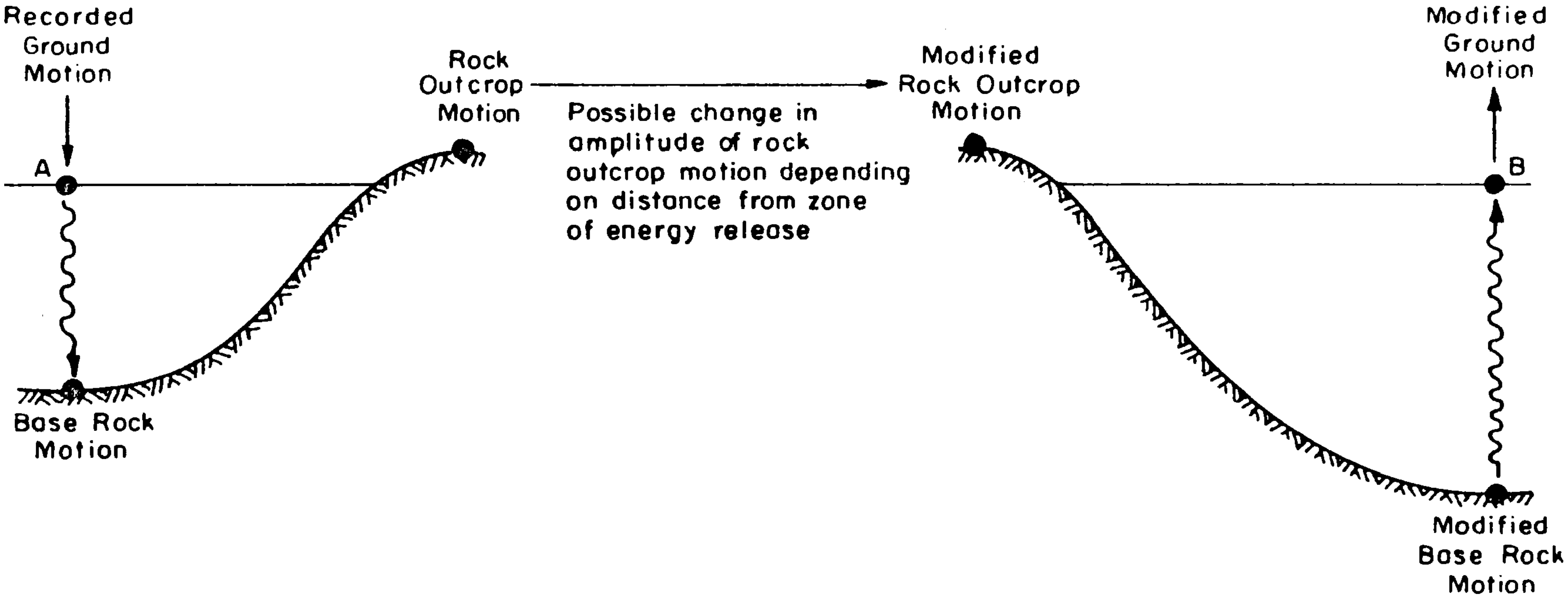


Fig. 1 SCHEMATIC REPRESENTATION OF PROCEDURE FOR COMPUTING EFFECTS OF LOCAL SOIL CONDITIONS ON GROUND MOTIONS

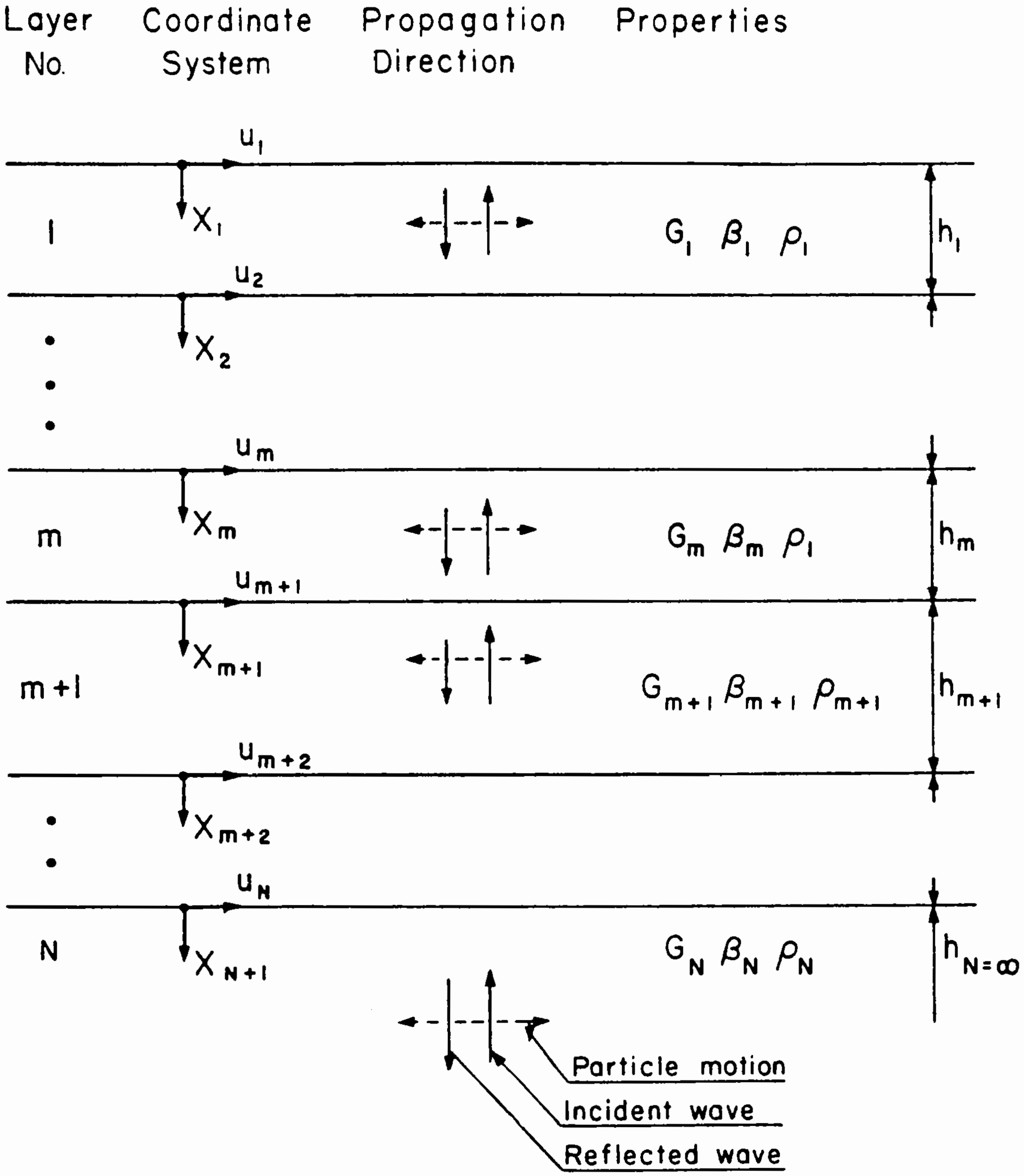


Fig. 2 ONE-DIMENSIONAL SYSTEM

where k is the complex wave number and G* is the complex shear modulus. The critical damping ratio, β, is related to the viscosity η by

$$\omega\eta = 2G\beta$$

Experiments on many soil materials indicate that G and β are nearly constant over the frequency range which is of main interest in the analysis. It is therefore convenient to express the complex shear modulus in terms of the critical damping ratio instead of the viscosity:

$$G^* = G + i\omega\eta = G(1+2i\beta) \qquad (7)$$

where G* can be assumed to be independent of frequency.

Equations 3 and 5 give the solution to the wave equation for a harmonic motion of frequency ω:

$$u(x,t) = Ee^{i(kx+\omega t)} + Fe^{-i(kx-\omega t)} \qquad (8)$$

where the first term represents the incident wave travelling in the negative x-direction (upwards) and the second term represents the reflected wave travelling in the positive x-direction (downwards).

Equation 8 is valid for each of the layers in Fig. 2. Introducing a local coordinate system X for each layer, the displacements at the top and bottom of layer m are:

$$u_m(X=0) = (E_m + F_m)e^{i\omega t} \qquad (9)$$

$$u_m(X=h_m) = (E_m \cdot e^{ik_m h_m} + F_m e^{-ik_m h_m}) \cdot e^{i\omega t} \qquad (10)$$

The shear stress on a horizontal plane is:

$$\tau(x,t) = G \cdot \frac{\partial u}{\partial x} + \eta \frac{\partial^2 u}{\partial x \partial t} = G^* \frac{\partial u}{\partial x} \qquad (11)$$

or by Eq. 8:

$$\tau(x,t) = ikG^*(Ee^{ikx} - Fe^{-ikx})e^{i\omega t} \tag{12}$$

and the shear stresses at the top and bottom of layer m are respectively:

$$\tau_m(X=0) = ik_m G_m^* (E_m - F_m)e^{i\omega t} \tag{13}$$

$$\tau_m(X=h_m) = ik_m G_m^* (Ee^{ik_m h_m} - F_m e^{-ik_m h_m})e^{i\omega t} \tag{14}$$

Stresses and displacements must be continuous at all interfaces. Hence, by Eq. 9, 10, 13 and 14:

$$E_{m+1} + F_{m+1} = E_m e^{ik_m h_m} + F_m e^{-ik_m h_m} \tag{15}$$

$$E_{m+1} - F_{m+1} = \frac{k_m G_m^*}{k_{m+1} G_{m+1}^*}(E_m e^{ik_m h_m} - F_m e^{-ik_m h_m}) \tag{16}$$

Subtraction and addition of Eqs. 15 and 16 yield the following recursion formulas for the amplitudes, E_{m+1} and F_{m+1}, of the incident and reflected wave in layer m+1, expressed in terms of the amplitudes in layer m:

$$E_{m+1} = \frac{1}{2}E_m(1+\alpha_m)e^{ik_m h_m} + \frac{1}{2}F_m(1-\alpha_m)e^{-ik_m h_m} \tag{17}$$

$$F_{m+1} = \frac{1}{2}E_m(1-\alpha_m)e^{ik_m h_m} + \frac{1}{2}F_m(1+\alpha_m)e^{-ik_m h_m} \tag{18}$$

where α_m is the complex impedance ratio

$$\alpha_m = \frac{k_m G_m^*}{k_{m+1} G_{m+1}^*} = \left(\frac{\rho_m G_m^*}{\rho_{m+1} G_{m+1}^*}\right)^{1/2} \tag{19}$$

which again is independent of frequency.

At the free surface, the shear stresses must be zero. In addition, Eq. 12 with τ_1 and X_1 equal to zero gives $E_1 = F_1$ --i.e., the amplitudes of the incident and reflected waves are always equal at a free surface. Beginning with the surface layer, repeated use of the recursion formulas Eqs. 17 and 18 leads to the following relationships between the amplitudes in layer m and those in the surface layer:

$$E_m = e_m(\omega)\ E_1 \tag{20}$$

$$F_m = f_m(\omega)\ E_1 \tag{21}$$

The transfer functions e_m and f_m are simply the amplitudes for the case $E_1 = F_1 = 1$, and can be determined by substituting this condition into the above recursion formulas.

Other transfer functions are easily obtained from the e_m and f_m functions. The transfer function $A_{n,m}$ between the displacements at level n and m is defined by

$$A_{n,m}(\omega) = u_m/u_n$$

and by substituting Eqs. 9, 20 and 21:

$$A_{n,m}(\omega) = \frac{e_m(\omega) + f_m(\omega)}{e_n(\omega) + f_n(\omega)} \tag{22}$$

Based on these equations the transfer function $A(\omega)$ can be found between any two layers in the system. Hence, if the motion is known in any one layer in the system, the motion can be computed in any other layer.

The amplitudes, E and F can thus be computed for all layers in the system, and the strains and accelerations can be derived from the displacement function. Accelerations are expressed by the equation:

$$\ddot{u}(x,t) = \frac{\partial^2 u}{\partial t^2} = -\,\omega^2(Ee^{i(kx+\omega t)} + Fe^{-i(kx-\omega t)}) \tag{23}$$

and strains by:

$$\gamma = \frac{\partial u}{\partial x} = ik(Ee^{i(kx+\omega t)} - Fe^{-i(kx-\omega t)}) \tag{24}$$

2.2 Ratio between rock outcrop motions and base rock motions.

If the amplitudes of the incident and reflected wave components, E_N and F_N, in the elastic halfspace, Fig. 3a, are known, the motions in the halfspace with the soil system removed, Fig. 3c, are easily computed. The shear stresses are zero at any free surface; thus $F_N = E_N$, and the incident wave is completely reflected with a resulting amplitude $2E_N$ at the free surface of the halfspace. The amplitude of the incident wave in the halfspace is independent of the properties of the system above it since the reflected wave is completely absorbed in the halfspace and does not contribute to the incident wave. The incident wave component, E_N, is therefore equal in all systems shown in Fig. 3.

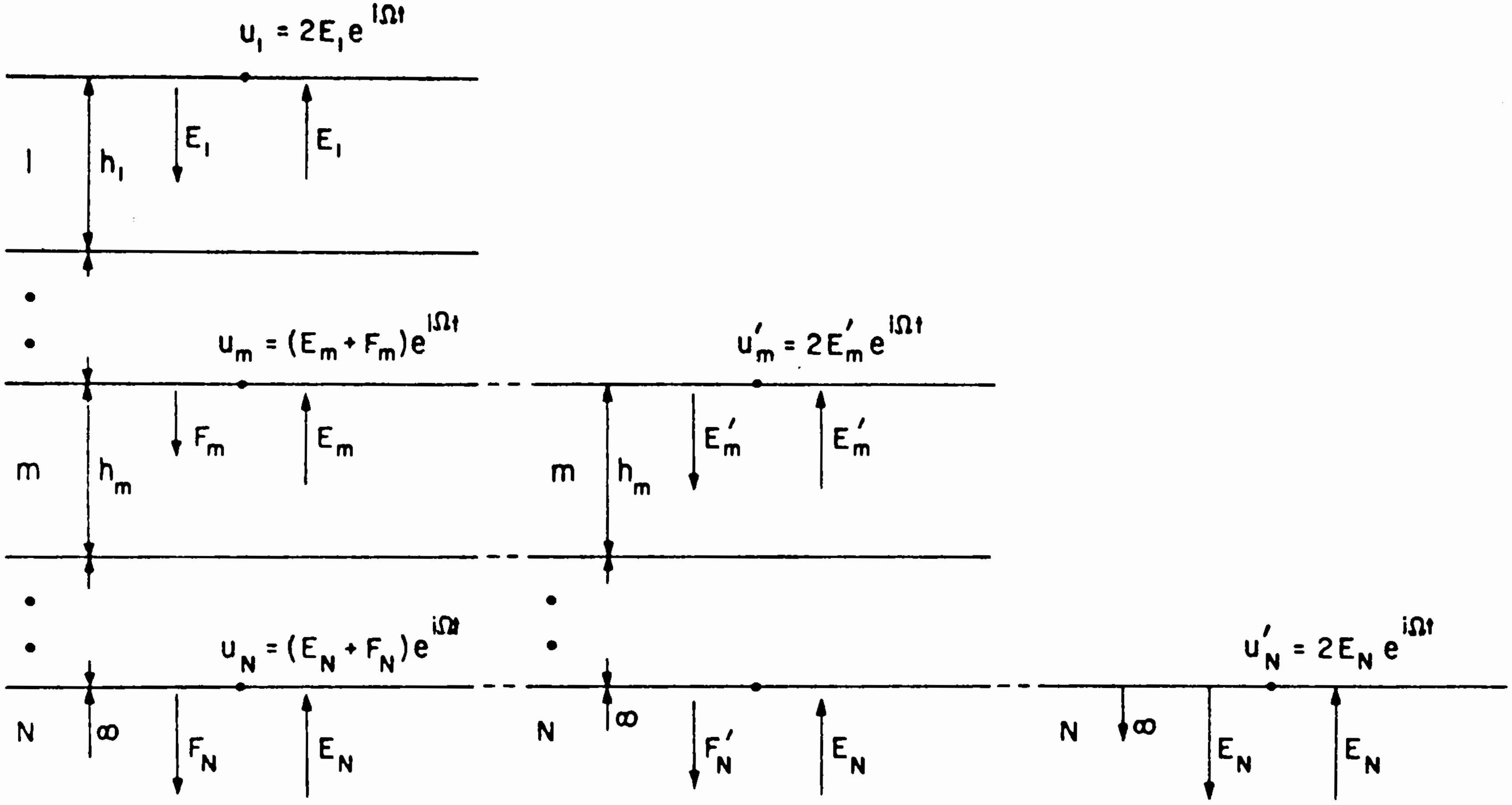
The ratio between the base motion, u_N, and the motion, u'_N, at the free surface may be computed from the transfer function:

$$A'_N(\omega) = \frac{u_N}{u'_N} = \frac{e_N(\omega) + f_N(\omega)}{2e_N(\omega)} \tag{25}$$

The transfer function between the motion at the surface of the deposit, u_1, and the motion at the free surface of the halfspace is:

$$A'_{N,1}(\omega) = \frac{1}{e_N(\omega)} \tag{26}$$

If the halfspace is the rock formation underlying a soil deposit, Eq. 25 shows the ratio between the motion in the base rock and in the outcropping rock. The ratio between the amplitudes of the base rock motion



a) System analyzed

b) Layer m outcropping

c) Halfspace outcropping

Fig. 3 ONE-DIMENSIONAL SYSTEM WITH OUTCROPPING LAYERS and the outcropping rock motion is always less than 1, with minimum values at the resonance frequencies of the deposit. Transfer functions for the deposit used in the example,(Sect. 6), are shown in Fig. 4. The amplitude of the base rock motion is only 65% of the amplitude of the rock outcrop motion at the fundamental frequency of the deposit. This difference is a function of the impedance ratio between the deposit and the rock and of the damping in the deposit.

The differences in the computed responses resulting from the use of a rigid base, relative to the use of an elastic base, depend also on which frequencies are dominant in the rock motion. Rock motions with frequency dominance near the resonant frequencies of the deposit will be considerably more affected than motions with frequency dominance between the resonance frequencies, see Fig. 4. The effect of the elasticity of the base rock is, therefore, not only a function of the impedance ratio between deposit and rock and of the damping in the deposit, but also of the frequency distribution of the energy in the rock motion relative to the resonance frequencies of the deposit.

An approximation for the free surface motion for one of the layers in the system, Fig. 3b, may be obtained in the same way as for the halfspace, provided the incident wave component in the outcropping layer and in the layer within the system are equal--i.e. $E_m = E'_m$. This is approximately the case when the properties of layer m and all layers below are equal in the two systems and when the impedance, $\rho_m V_m$, is of the same order of magnitude as for the halfspace. This is the case for example, in sedimentary rock layers overlying a crystalline rock base. For a more accuate solution, the motion in outcropping layers must be computed in a separate system from the motion in the halfspace.

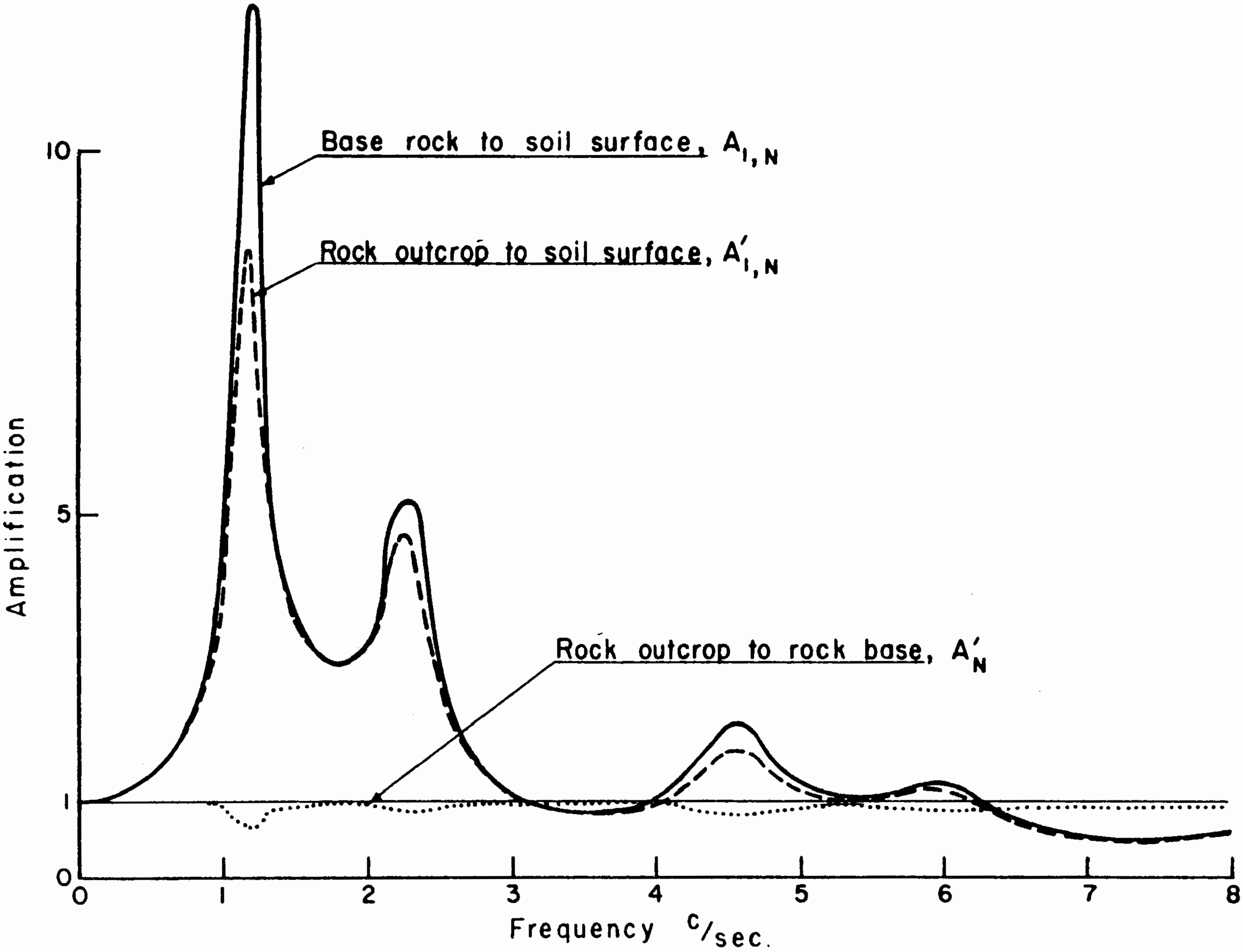


Fig. 4 TRANSFER FUNCTIONS

2.3 Transient motions

The expressions developed above are valid for steady state harmonic motions. The theory can be extended to transient motions through the use of Fourier transformation.

A digitized seismogram with n equidistant acceleration values, $\ddot{u}_j(j \cdot \Delta t)$, $j = 0,\ldots,n-1$, can be represented by a finite sum of harmonic motions:

$$\ddot{u}(t) = \sum_{s=0}^{n/2} (a_s e^{i\omega_s t} + b_s e^{-i\omega_s t}) \tag{27}$$

where ω_s, $s=0,\ldots,n/2$ are the equidistant frequencies:

$$\omega_s = \frac{2\pi}{n \cdot \Delta t} \cdot s \tag{28}$$

a_s and b_s designates the complex Fourier coefficients:

$$a_s = \frac{1}{n} \sum_{j=0}^{n-1} \ddot{u}(t) e^{-i\omega_s t}, \quad b_s = \frac{1}{n} \sum_{j=0}^{n-1} \ddot{u}(t) e^{i\omega_s t} \tag{29}$$

and each term in Eq. 27 is a harmonic motion oscillating with frequency ω_s.

If the series in Eq. 27 represent the motion in a layer m, a new series representing the motion in any other layer n, is obtained by applying the appropriate amplification factor from Eq. 22 to each term in the series:

$$\ddot{u}_n(t) = \sum_{s=0}^{n/2} A_{m,n}(\omega_s) \cdot (a_{m,s} e^{i\omega_s t} + b_{m,s} e^{-i\omega_s t}) \tag{30}$$

The representation of a discrete motion with its Fourier transform gives an exact representation of the motion at the discrete points $t = j \cdot \Delta t$, $j = 0,\ldots,n-1$. Cyclic repetition of the motion with the period $T = n \cdot \Delta t$

is implied in the solution. The solution applies, therefore, to an infinite train of identical accelerograms rather than the given single accelerogram. For systems with damping this is not of any significant consequence since the individual accelerograms can be separated by a quiet zone of zeros causing the responses from one cycle to damp out before the beginning of the next cycle.

The Fourier Transformation can be performed in several ways. The SHAKE program utilizes the Fast Fourier Transform algorithm developed by Cooley and Tukey (1965), which is faster by a factor n/logn over the conventional method. This technique computes all values in the series simultaneously. The method requires that the number of terms in the series be some power of 2. A typical analysis using an acceleration record of 800 terms with time-step Δt = .02 sec. will use 1024 values in the Fast Fourier Transform, with all values between 800 and 1024 set equal to 0. This will satisfy both the requirements of a quiet zone after the acceleration record and that the total number of terms must be a power of two.

3. DESCRIPTION OF PROGRAM SHAKE

Program SHAKE computes the responses in a system of homogeneous, visco-elastic layers of infinite horizontal extent subjected to vertically travelling shear waves. The system is shown in Fig. 2. The program is based on the continuous solution to the wave-equation (Kanai, 1951) adapted for use with transient motions through the Fast Fourier Transform algorithm (Cooley and Tukey, 1965). The nonlinearity of the shear modulus and damping is accounted for by the use of equivalent linear soil properties (Idriss and Seed, 1968, Seed and Idriss, 1970) using an iterative procedure to obtain values for modulus and damping compatible with the effective strains in each layer.

The following assumptions are implied in the analysis:

1. The soil system extends infinitely in the horizontal direction.
2. Each layer in the system is completely defined by its value of shear modulus, critical damping ratio, density, and thickness. These values are independent of frequency.
3. The responses in the system are caused by the upward propagation of shear waves from the underlying rock formation.
4. The shear waves are given as acceleration values of equally spaced time intervals. Cyclic repetition of the acceleration time history is implied in the solution.
5. The strain dependence of modulus and damping is accounted for by an equivalent linear procedure based on an average effective strain level computed for each layer.

The program is able to handle systems with variation in both moduli and damping and takes into account the effect of the elastic base. The motion used as a basis for the analysis, the object motion, can be given in any one layer in the system and new motions can be computed in any other layer.

The following set of operations can be performed by the program:

1. Read the input motion, find the maximum acceleration, scale the values up or down, and compute the predominant period.
2. Read data for the soil deposit and compute the fundamental period of the deposit.
3. Compute the maximum stresses and strains in the middle of each sublayer and obtain new values for modulus and damping compatible with a specified percentage of the maximum strain.

4. Compute new motions at the top of any sublayer inside the system or outcropping from the system.
5. Print, plot and punch the motions developed at the top of any sublayer.
6. Plot Fourier Spectra for the motions.
7. Compute, print and plot response spectra for motions.
8. Compute print and plot the amplification function between any two sublayers.
9. Increase or decrease the time interval without changing the predominant period or duration of the record.
10. Set a computed motion as a new object motion. Change the acceleration level and predominant period of the object motion.
11. Compute, print and plot the stress or strain time-history in the middle of any sublayer.

These operations are performed by exercising the various available options in the program. A list of these options is given in Section 5, Required Input Data.

4. SYSTEM AND OPERATION DOCUMENTATION

4.1 Computer equipment

The program has been developed on a CDC 6400 computer using FORTRAN IV language. The CDC 6400 has a 131 k core memory and uses a 60 bit words. The program has been run without modifications on CDC 6600, 7600 and UNIVAC 1108 computers, and with minor modifications on IBM 360 and 370 computers.

4.2 Storage requirements

The program requires approximately 50,000 octal words of storage excluding the blank common X. The additional storage is a function of the maximum number of terms used in the Fourier Transform as shown in Table 1.

Table 1. Storage Requirements

Number of terms	Length of array X	Field length octal
0	0	50,000
512	3220	57,000
1024	6420	65,000
2048	12820	102,000
4096	25620	134,000
8192	51220	220,000

4.3 Runtime

The runtime is a function of the number of terms, n, used in the Fourier Transformation and of the number of sublayers in the deposit. The time involved in the Fast Fourier Transformation is proportional to n·logn; all other operations are approximately proportional to n. In the computation of strain compatible soil properties, the time will also increase in proportion to the number of sublayers.

Table 2. Runtimes.

Number of terms	Time interval, sec.	Run time sec.
512	.04	45
1024	.02	80
2048	.01	170

5. REQUIRED INPUT DATA

5.1 Organization of input data.

Following is a description of the operations performed by the different options, the required format for the input data, and explanations of some of the input parameters.

The various options can be executed and repeated in any logical sequence. The operations in an option will be performed on the data given or computed in the program when the option is called, and the data may be changed at any time during the execution by repeating the option with new data.

For example, in order to compute new motions in a soil deposit, (Option 5) object motion (Option 1), soil profile data (Option 2), specification of location of object motion (Option 3), dynamic soil property-strain relation (Option 8), and strain iterations (Option 4--if strain compatible properties are desired), must precede Option 5. Soil responses for a new (additional) soil deposit may be obtained by repeating Options 2, 3, 4, and 5. The last-read soil deposit may be subjected to a new earthquake by repeating Options 1, 4, and 5.

5.2 Initialization card (I5,F10.0)

Cols. 1-5	MAMAX	Maximum number of terms to be used in the Fourier Transformation in any of the problems to be run. Must be a power of 2 such as 512, 1024, 2048, etc.
6-15	SKO	Coefficient of earth pressure at rest for sand layers. If blank the value is set equal to 0.45. May be left blank if all layers are clay.

After the initialization card follows one run option card.

5.3 Run option card (I5)

Cols. 1-5	KK	Run of option

0 - stop, no more data
1 - read input motion, and set as object motion
2 - read soil profile data
3 - assign the object motion to a specified sublayer
4 - iterate to obtain strain—compatible soil properties
5 - compute new motions at the top of specified sublayers, print maximum accelerations and punch acceleration time history
6 - print or punch acceleration time history of object motion or any specified computed motion
7 - modify object motion or set the motion in any specified sublayer as new object motion
8 - read relations between dynamic soil properties and strain
9 - compute response spectra for any specified motion
10 - increase time interval in motions
11 - decrease time interval in motions
12 - plot Fourier Spectrum of object motion
13 - compute and plot Fourier Spectrum of motion in any specified sublayer
14 - plot acceleration time history of object motion or any specified computed motion
15 - compute and plot amplification function between any two specified sublayers
16 - compute and plot stress or strain history in the middle of any specified sublayer.

After the run option card follows the data set for the selected option:

5.4 Data cards and explanatory notes for the various options

Option 1. Read Input Motion.

Operations performed

(1) Acceleration values are read from cards.

(2) The sequence of the cards is checked.

(3) The maximum acceleration value in the record is found.

(4) The acceleration values may be scaled either by a specified factor or to a specified maximum acceleration.

(5) Trailing zeros are added to the record to obtain sufficient length on the quiet zone (a) and a total number of values which are a power of 2.

(6) The higher frequencies in the record are removed and the maximum acceleration in the modified record is found--optional.

(7) The motion is set as the new object motion.

Data Cards

1st Card (2I5,F10.0,5A6)

Cols.	1-5	NV	Number of acceleration values to be read from cards
	6-10	MA(a)	Number of values to be used in Fourier transform Must be a power of 2.
	11-20	DT(b)	Time interval between acceleration values (sec.)
	22-50	TITLE(I)	Identification for earthquake.

2nd Card (3F10.0)

Cols.	1-10	XF	Multiplication factor for acceleration values. Used only if XMAX is 0, left blank otherwise.
	11-20	XMAX	Maximum acceleration value to be used. The acceleration values in the record will be scaled to give maximum acceleration = XMAX , unless XF is left blank.
	21-30	FMAX(c)	Maximum frequency to be used in the calculations. Acceleration amplitudes at all frequencies greater than FMAX are set equal to 0.

3rd and consecutive cards. Acceleration record. (8F9.6,I7)

Cols.	1-72	X(I)	8 acceleration values. (g's)
	73-79	K	Card number. Warning will be given for cards not in sequence.

Explanatory notes for Option 1.

(a) The acceleration values between NV and MA are set equal to 0. in the program. Cyclic repetition of the motion is implied in the Fourier transform and a quiet zone of 0.'s or low values are necessary to avoid interference between the cycles. For most problems a quiet zone of 2-4 seconds is adequate with longer time required for profiles deeper than about 250 ft and/or damping values less than about 5 percent.

(b) The predominant period of the earthquake record can be changed by altering the time interval Δt from that originally assigned to the acceleration record. If the original record has time interval Δt, and corresponding predominant period T_1, a new predominant period T_2 is obtained by changing the time interval to

$$\Delta t_2 = \frac{T_2}{T_1} \Delta t_1$$

(c) Frequencies above 10-15 c/sec carry a relatively small amount of the energy in earthquake motions, and the amplitudes of these frequencies can often be set equal to 0 without causing any significant change in the responses within a soil system. Table 3 shows the maximum accelerations and strains in the soil system used in the example run, sect. 6, computed for the Pasadena motion with time interval 0.02 sec and a maximum frequency of 25 c/sec. Results are also shown for the same motion with all amplitudes above 5 c/sec set equal to 0. The difference in maximum accelerations was less than 6.5% and in maximum strains less than 0.7% in the two cases. The difference in response spectral values was less than 1% for periods above 0.2 sec and less than 10% for periods from .0 to 0.2 sec.

Table 3. Effect of the Higher Frequencies on the Maximum Accelerations and Strains.

Depth	Maximum acceleration, g's		Difference %	Maximum strain, %		Difference %
	f_{max} = 25 c/sec	5 c/sec		f_{max} = 25 c/sec	5 c/sec	
0	.0971	.0962	.9			
				.00725	.00724	.1
7	.0958	.0949	.3			
				.1292	.1283	.7
20	.0600	.0599	.1			
				.0391	.0390	.3
30	.0553	.0556	.6			
				.0287	.0287	-
42	.0508	.0507	.2			
				.00982	.00989	.7
62	.0470	.0469	.2			
				.0505	.0504	.2
80	.0319	.0299	6.3			
				.0349	.0348	.3
100	.0239	.0235	1.7			
				.0320	.0319	.3
120	.0178	.0189	6.2			

In the computation of responses in deep soil systems from a motion given near the surface of the deposit, errors in the higher frequencies will be amplified and may cause erroneous results. To avoid this source of error, the amplitudes of all frequencies above 10-20 c/sec. may be set equal to 0., since these frequencies generally are of little interest and do not affect the response. Several runs should be performed with different amounts of the higher frequencies removed to investigate the effect on the response and to ensure a stable solution.

Removal of the higher frequencies in a motion has a smoothening effect on the acceleration time history as shown in Fig. 5 for a segment of the Pasadena motion. In this case the maximum acceleration for the modified and original motion were approximately equal, but the maximum accelerations may decrease or increase with the removal of the higher frequencies depending on the shape of the acceleration curve near the maximum value.

Option 2. Read Data for Soil Deposit.

Operations performed

(1) The properties of the soil deposit are read from cards.
(2) The sequence of the layer cards is checked.
(3) The layers are subdivided into sublayers--optional.
(4) Effective pressures in the middle of each sublayer are computed.
(5) The fundamental period of the deposit is computed.

Data Cards

1st Card (3I5,6A6)

Cols.	1-5	MSOIL	Soil deposit number. Can be left blank.
	6-10	ML(a)	Number of layer cards to be read including card for halfspace. There is one card for each layer whose properties are individually specified.(b)

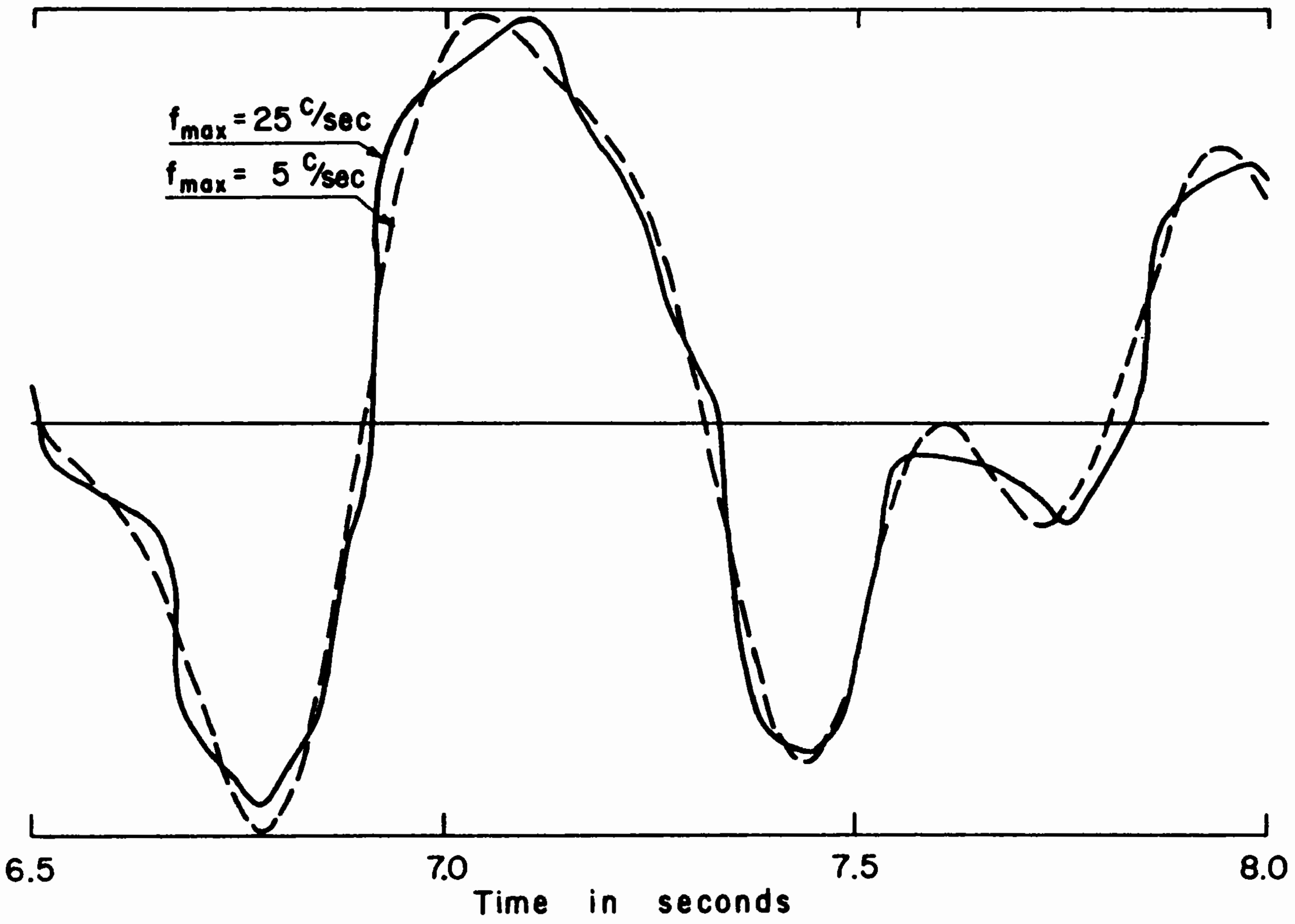


Fig. 5 EFFECT OF THE HIGHER FREQUENCIES ON THE ACCELERATION TIME-HISTORY

Cols. 11-15	MWL	Number of first submerged sublayer (b). [If no ground water table present, put groundwater table at top of halfspace.]
17-51	IDNT(I)	Identification for soil profile.

2nd and consecutive cards. One card for each layer including halfspace. (3I5,6F10.0,F5.0)

Cols. 1-5	K	Layer number. The layer cards must be in sequence with the surface layer as layer 1. Note that the number of layers may be ≤ the number of sublayers(b).
10	TYPE(c)	Soil type 1 - clay 2 - sand 3 - rock
11-15	NLN(a,b)	Number of sublayers in layer K. The Kth layer will be divided into NLN sublayers of thickness = HL/NLN.*
16-25	HL(a)	Layer thickness (ft.)
26-35	GMOD(d)	Initial estimate of shear modulus (kips/sq.ft.) Not necessary if VS is given.
36-45	B(d)	Initial estimate of critical damping ratio (decimal).
46-55	W	Unit weight (kips/cu. ft.).
56-65	VS(d)	Initial estimate of shear wave velocity (ft/sec). Not necessary if GMOD is given.
66-75	FACTOR(c)	Factor for shear modulus Clay - F_c = undrained shear strength (kips/sq. ft.) Sand - F_s = factor modifying the average curve read in under Option 8. Set F_s = 1. for no change. Rock - F_R = Shear wave velocity for low strain values—in thousands of ft./sec.
76-80	BFAC	Factor modifying the standard damping curve read in under Option 8. For example, a factor of 1.2 increases each and every value by 20 percent.

For the elastic half space, soil layer card number ML, it is sufficient to give values for K, GMOD or VS and W.

---

*Maximum total number of sublayers including the base is 20.

Explanatory notes for Option 2.

(a) With the wave propagation method the responses can be computed in a homogeneous layer of any thickness. A soil deposit will, however, have varying properties not only due to the variation in the soil itself but also due to the differences in the strain-level induced during shaking. Since the soil deposit must be represented by a set of homogeneous layers, each with a constant value of modulus and damping, the thickness of each layer must be limited based on the variation in the soil properties. For a fairly uniform deposit, a sublayer thickness increasing from about 5' at the surface to 50-200' below 100' depth should give sufficient accuracy. Accuracy may be checked by making a trial run and comparing results with a subsequent run where more layers and/or sublayers are used.

(b) The division of a layer into sublayers is for convenience to avoid punching of several cards with the same properties, and all sublayers are treated as separate layers in the following computations. The sublayers are numbered consecutively starting at the top of the soil deposit and the halfspace is counted as the last layer and the last sublayer in the deposit.

(c) Computations of shear moduli for the different soil types are based on the following expressions:

Clay $G_c = K_c \cdot F_c$

Sand $G_s = K_s \cdot 1000 \cdot (\sigma'_m)^{1/2} \cdot F_s$

Rock $G_R = K_R \cdot \rho \cdot (1000 \cdot F_R)^2/2000.$

where

K = strain function given in Option 8.

F = factor given as input (FACTOR)

ρ = mass density in kips/cu. ft.

σ'_m = mean effective pressure (psf).

The strain function for clays, K_c, gives the average relationship between G/S_u and strain for saturated clays. While the undrained shear strength of the clay, S_u, is normally used in this modulus-strain relation, the factor for clay, F_c, should be given a value which gives the correct modulus-strain relation; thus F_c is not necessarily equal to S_u. If the modulus of the clay is found from seismic investigations, the value of F_c should be set to G_c/K_c where K_c is the value for 10^{-4} percent strain in the curve given in Option 8.

(d) The modulus and damping are in general used as initial values on the first iteration for the computation of strain-compatible properties, but they can also be used directly to compute the responses for the values given, by omitting Option 4. Typical values of the modulus for strong shaking are of the order of 500 kips/sq. ft. near the surface increasing to 3000 kips/sq. ft. at 100-200' depth for sand, 500-2000 kips/sq.ft. for clay with values as low as 50-100 kips/sq. ft. for soft clay. Usually 3-5 iterations are sufficient to obtain strain compatible values within a 5-10% error limit.

The results are not highly sensitive to errors in the damping ratio and values selected between 0.05 to 0.15 will usually give strain-compatible values with 2 to 3 iterations.

Option 3. Assign Object Motion to a Specified Sublayer.

Operations performed

The object motion is assigned to the top of one sublayer in the soil deposit.

Data Cards

1st Card (2I5)

Cols.	1-5	IN	Number of sublayer where object motion is assigned.
	6-10	INT	Type of sublayer 0 - Outcropping[(a)] sublayer 1 - sublayer within profile

Explanatory notes to Option 3.

(a) See Section 2.2.

Option 4. Obtain Strain Compatible Soil Properties.

Operations performed

(1) Parameters for the iterations are read from card.

(2) Maximum strains, stresses and times for the maxima are computed in the middle of each sublayer.

(3) Effective strains are obtained from the maximum strains and used to compute new soil properties.

(4) The operation is repeated until strain-compatible soil properties are obtained within a given error limit or until a specified maximum number of iterations is reached.

(5) The fundamental period of the deposit is computed after the final iteration.

(6) A set of soil data cards with the new strain compatible properties is punched--optional.

Data Cards

1st Card (2I5,2F10.0)

Cols.	1-5	KS(a)	Set equal to 1 for punched set of soil data cards with the soil properties after final iteration. Leave blank if punched cards are not wanted.
	6-10	ITMAX(b)	Maximum number of iterations.
	11-20	ERR(b)	Maximum acceptable difference between the last-used modulus and damping values and the strain-compatible values (percent).
	21-30	PRMUL(c)	Ratio between effective strain and maximum strain (decimal).

Explanatory notes for Option 4.

(a) The most time consuming part of the computations is to obtain strain compatible soil properties. A set of soil data cards with strain-compatible properties may save computer or punching time if additional computer runs are to be made subsequently.

(b) The iterations stop when the specified maximum number of iterations (ITMAX) is reached or when the difference between the modulus and damping used and the strain-compatible modulus and damping values is less than the acceptable difference (ERR). Usually 3-5 iterations are sufficient to obtain an error of less than 5-10%. The values given as "new values" in the final iteration are used in all computations following Option 4, and the actual error is less than the error values given in the final iteration.

(c) The effective strain is used to compute new soil properties. The ratio between the effective and the maximum strain has been empirically found to be between 0.5 and 0.7. The responses, however, are not highly sensitive to this value and an estimate between 0.55 to 0.65 is usually adequate, with the higher value appropriate for giving more uniform strain histories.

Option 5. Compute Motion in Specified Layers.

Operations performed

(1) The acceleration time history is computed at the top of specified sublayers.

(2) The maximum acceleration and times for maxima are printed for the computed motions.

(3) The computed acceleration time histories may be punched--optional.

(4) The acceleration time histories may also be printed or plotted (Option 6, 7 and 14)(a).

Data Cards

1st Card (15I5)

Cols. 1-75	LL5(I)	Array showing the numbers of the sublayers at the top of which the motion is to be computed. Maximum of 15 locations.

2nd Card (15I5)

Cols. 1-75	LT5(I)	Array specifying types of above sublayers. 0 - outcropping (b) sublayer 1 - sublayer within profile

3rd Card (15I5)

Cols. 1-75	LP5(I)(a)	Array with mode of output for the computed motions. 0 - max. acceleration value only printed. 1 - punched cards giving acceleration time history in addition to the printed maximum acceleration value.

Explanatory notes for Option 5

(a) The acceleration time histories can be printed or plotted through the use of Option 7 where a specified motion is set as the new object motion. Subsequent use of Options 6 and 14 give respectively a printed and a plotted output of the acceleration time history of the motion.

(b) See section 2.2.

Option 6. Print or Punch Object Motion.

Operations performed

(1) Maximum acceleration and time at which maximum occurs are found.

(2) The object motion is printed--optional.

(3) The object motion is punched on cards--optional.

Data Cards

1st Card (I5)

Col. 5 K2 Selects mode of output.

K2 = 0 Max. acc. only
1 Punched output
2 Printed and punched output.

Option 7. Change Object Motion.

Operations performed

(1) A motion at the top of a specified sublayer can be set as the new object motion and printed or punched (Option 6) or plotted (Option or used for subsequent computations--optional.

(2) The time step in the object motion can be changed--optional.

(3) The acceleration level in the object motion can be changed--optional.

Data Cards

1st Card (2I5,2F10.0)

Cols. 1-5 LL1 Number of sublayer. Use 0 if object motion originally assigned is to be retained(a).

6-10 LT1 Type of above sublayer
0 - outcropping (c) sublayer
1 - sublayer within profile

11-20 XF Multiplication factor for acceleration values--1. for no change.

21-30 DTNEW New timestep(b).

Explanatory notes for Option 7

(a) The acceleration level and timestep can be changed either on the motion originally set as the object motion, or on the computed motion which is set as the new object motion through Option 7.

(b) A change in time interval will change the predominant period of the motion. If the time interval and predominant period of the original motion are Δt_1 and T_1, respectively, a new predominant period T_2 is obtained by changing the time interval to

$$\Delta t_2 = \frac{T_2}{T_1} \Delta t_1$$

(c) See section 2.2.

Option 8. Read the relation between the Effective Strain and the Dynamic Properties

Operations performed

(1) Effective strain values with corresponding values for damping and moduli are read from cards.

(2) Parameters are computed for interpolation of modulus and damping values using a linear semilogarithmic relation between the given values.

(3) The relationship between the dynamic properties and the strain is plotted--optional.

Data Cards

1st Card (3I5,F10.0,10A6)

Cols.		
1-5	NSOILT	Number of different soil or rock types to be read. Maximum 4.(a)
10	NPL(b)	Set equal to 1 for plot of curves.
11-15	NN(b)	Number of strain-values in each logarithmic unit to be plotted.
16-25	SC	Maximum value of the ordinate in the plotting.
26-80		Title or identification data.

Next follows two sets of cards for each soil or rock type. The first set gives the relationship between the shear modulus parameters (C) and the effective strains; the second set give the relation between the critical damping ratios and the effective strains. Typical data is shown on page D.1-49.

First Set:

1st Card (I5,F5.0,11A6)

Cols.	1-5	NV(L)	Number of strain values to be read. Maximum 20.
	6-10	FPL(L)(b)	Multiplication factor for shear-modulus parameter. Used for plotting only.(b)
	12-76	ID(L,I)	Identification for first data set. Used for plotting only.

2nd and consecutive cards (8F10.0)

Cols.	1-80	X(L,I)	Effective strain values in percent beginning with the lowest value. 8 values per card with maximum of 20 values.

Consecutive cards (8F10.0)

Cols.	1-80	Y(L,I)	Values of the shear modulus parameter (c) corresponding to the strain values given above. Eight values per card with maximum of 20 values.

Second Set:

The input format for the second set is identical to that for the first set with values of critical damping ratios in percent instead of the values for the shear modulus parameter.

Explanatory notes for Option 8.

(a) Three different soil or rock types can be used in the program as described in Option 2. The relationships between effective strains and the dynamic properties must be read in the same sequence as the soil type using the notation:

1 - Clay

2 - Sand

3 - Rock

(b) The values for the shear modulus parameter and the damping can be plotted against the effective strains. If plotting is specified (NPL = 1), values for the shear modulus parameter and damping are computed for a specified number of effective strains (NN) in each logarithmic unit. The computed values should be scaled (FPL(L)) to obtain good representation of all curves on the same plot. The scaled values and the corresponding effective strains are also printed.

(c) The values are used to compute the shear modulus for the different soil types. The relationship for sand and clay used in the program is based on the expressions given by Seed and Idriss (1970):

$$\text{Clay} \qquad K_c(\gamma) = \frac{G_c(\gamma)}{S_u}$$

$$\text{Sand} \qquad K_s(\gamma) = \frac{G_s(\gamma)}{1000 \cdot (\sigma'_m)^{1/2}}$$

The relationship used for rock is the scaled ratio between the shear modulus at low effective strain (10^{-4} percent) and the shear modulus at a specified effective strain:

$$\text{Rock} \qquad K_R(\gamma) = \frac{G(\gamma) \cdot 2000}{G(\gamma 10^{-4})}$$

Option 9. Compute Response Spectra

Operations performed

(1) The motion is computed at the top of a specified sublayer.

(2) Times for maxima in the acceleration, velocity and displacement spectra are computed and printed.

(3) Acceleration and velocity spectra may be plotted and/or punched on cards--optional.

Data Cards

1st Card (2I5)

Cols. 1-5	LL1	Sublayer number. Use 0 if the response spectra are to be computed for the object motion.
10	LT1	Type of sublayer. 0 - outcropping sublayer 1 - sublayer within profile. The response spectra are computed for the motion at the top of the sublayer. May be left blank if LL1 is 0.

2nd Card (5I5)

Col.		
5	ND	Total number of damping values to be used. Maximum 6 values.
10	KP	Set equal to 1 for punched output.
15	KAV	Select plot and punch option: 0 - plot and/or punch velocity spectrum 1 - plot and/or punch acceleration spectrum 2 - plot and/or punch acceleration and velocity spectrum.
20	KPL	Set equal to 1 for plot of spectra according to KAV. All spectra computed since last plotting will be plotted together.
25	KPER	Select periods to be used in the computations: KPER = 0 9 steps from 0.1 sec to 1. sec 5 steps from 1. sec to 2. sec 4 steps from 2. sec to 4. sec KPER = 1 18 steps from 0.1 sec to 1. sec 10 steps from 1. sec to 2. sec 8 steps from 2. sec to 4. sec KPER = 2 38 steps from 0.05 sec to 1. sec 20 steps from 1. sec to 2. sec 30 steps from 2. sec to 5. sec KPER = 3 Logarithmic increments with 10 steps in each log. unit from 0.1 to 5. KPER = 4 Logarithmic increment with 25 steps in each log. unit from 0.05 to 10.

3rd Card (6F10.0)

Cols. 1-60 ZLD(I) Values of critical damping ratios in decimal to be used in the spectral analysis. ND number of values must be given.

Option 10. Increase the Time Interval

Operations performed

The time interval is increased.

Data Cards

1st Card (I5)

Cols. 1-5 IFR(a) Factor for increasing time interval. Must be a power of 2.

Explanatory notes for Option 10

(a) The Fourier Transformation of a given acceleration time history consists of a series of harmonic motions

$$\ddot{u}(t) = \sum_{s=0}^{n/2} (a_s e^{i\omega_s t} + b_s e^{-i\omega_s t})$$

With the harmonic motions given, acceleration values can be computed for any value of the time, t, and a new acceleration time history can be generated with a time interval different from the original.

Suppose, for example an acceleration record is given with 2048 values and a timestep $\Delta t = 0.01$ sec. Through Option 10 with IFR = 2 a new record with 1024 values and timestep 0.02 sec is generated. The acceleration values in the two records are identical at all times $n \cdot .02$ sec., n = 1,2...1024. The new record has a maximum frequency of 25 c/sec. compared to 50 c/sec. in the original records, and frequencies from 25 c/sec. to 50 c/sec. are lost in the operation.

Increasing the time interval reduces the computer time as shown under sect. 4.3. For computation of maximum accelerations a time interval of 0.02 sec. will generally give adequate accuracy while a time interval of 0.04 sec. may be sufficient for the computation of the stresses and strains in a deposit.

The difference in maximum accelerations and strains resulting from the use of different time intervals are shown in Tables 4 and 5 for the example run. The effect may be somewhat higher for earthquakes with lower predominant periods and for stiffer soil systems.

Option 11. Decrease the Time Interval

Operations performed

The time interval is decreased.

Data Cards

1st Card (I5)

Col. 1-5	IFR[a]	Factor for decreasing the time interval; must be a power of 2.

Explanatory notes for Option 11.

(a) See explanation to Option 10. Through Option 11 a new time history is generated with the time interval reduced by a power of 2. Compared with the usual linear interpolation, this method has the advantage of not introducing additional frequencies to the motion.

Option 12. Plot Fourier Spectrum of Object Motion

Operations performed

(1) The Fourier Spectrum of the object motion is plotted.

(2) The spectrum may be smoothed--optional.

Data Cards

1st Card (3I5)

Cols. 5	K1	Select for plotting: 0 - Store spectrum for later plotting. Max. of 2 spectra can be plotted together. 1 - Plot all spectra stored since last plotting.
6-10	NSW[a]	Number of times the spectrum is to be smoothed.
11-15	N	Number of values to be plotted--maximum of 2049.

Table 4. Effect of Time Interval on Maximum Strain.

Depth	Computed Maximum Strain %		
	Δt = .01	Δt = .02	Δt = .04
3.5	.00727	.00725	.00725
13.5	.129	.129	.127
25.	.0392	.0391	.0390
36	.0287	.0287	.0285
52	.00982	.00982	.00981
71	.0505	.0505	.0505
90	.0350	.0349	.0348
110	.0320	.0320	.0316

Table 5. <u>Effect of Time Interval on Maximum Acceleration</u>.

Depth	Maximum Acceleration		
	Δt = .01	Δt = .02	Δt = .04
0	.0971	.0971	.0967
7	.0960	.0958	.0954
20	.0598	.0600	.0590
30	.0554	.0553	.0548
42	.0508	.0508	.0498
62	.0471	.0470	.0462
80	.0317	.0319	.0318
100	.0238	.0239	.0242
120	.0181	.0178	.0178

Explanatory notes to Option 12.

(a) The expression used to smooth the spectrum is:

$$A_i = \frac{A_{i-1} + 2A_i + A_{i+1}}{4}.$$

where A_i is the acceleration amplitude for the i^{th} frequency.

Option 13. Plot Fourier Spectrum (c) of Computed Motions

Operations performed

(1) The motions at the tops of the specified sublayers are computed.

(2) The Fourier Spectra for the computed motions are plotted and printed.

(3) The spectrum may be smoothed--optional.

Data Cards

1st Card (5I5)

Cols.		
1-5	LL(1)	Sublayer number.
10	LT(1)	Type of sublayer: 0 - Outcropping(b) sublayer 1 - Sublayer within profile.
15	LP(1)	Select for plotting: 0 - Store spectrum for later plotting; max. of 2 spectra can be plotted together 1 - Plot all spectra stored since last plotting.
16-20	LNSW(1)(a)	Number of times the spectrum is to be smoothed.
21-25	LLL(1)	Number of values to be plotted. Max. of 2049.

2nd Card (5I5)

As for Card 1 for a second motion. A blank card must be used if only one spectrum is to be computed.

Explanatory notes for Option 13

(a) See Option 12.

(b) See section 2.2.

(c) See section 2.3.

Option 14. Plot Time History of Object Motion(a).

Operations performed

The time history of the object motion is plotted.

Data Cards

1st Card (2I5)

Cols.	1-5	NSKIP	Number of values skipped in the plotting. 0 - every value is plotted 1 - every second value is plotted etc.
	6-10	NN	Number of values to be plotted. Max. of 2049 values.

Explanatory notes to Option 14.

(a) The time history of a computed motion can be plotted by setting this motion as the object motion through Option 7.

Option 15. Compute Amplification Spectrum.

Operations performed

(1) The amplification spectrum between any two sublayers in a given soil system is computed.

(2) The maximum amplification and the corresponding period are printed.

(3) The amplification spectrum may be plotted and printed--optional.

Data Cards

1st Card (5I5, F5.0,8H6)

Cols.	1-5	LIN(a)	Number of first sublayer.
	6	LINT	Type of first sublayer 0 - outcropping(b) sublayer 1 - sublayer within profile
	11-15	LOUT(a)	Number of second sublayer.
	20	LOTP	Type of second sublayer 0 - outcropping sublayer 1 - sublayer within profile.
	25	KP	Select for plotting: 0 - Store spectrum for later plotting. Maximum of 8 spectra can be stored. 1 - Plot all spectra stored since last plotting.

26-30	DFA	Frequency steps. The amplification factor is computed for the first 200 frequencies with interval DFA c/sec. beginning at 0.
32-78	IDAMP(I)	Identification.

Explanatory notes to Option 15.

(a) The amplification factors are computed from the first sublayer to the second.

(b) See section 2.2.

Option 16. Compute Stress or Strain History in the Middle of Specified Sublayers.

Operations performed

(1) The stress and/or strain time history in the middle of any two specified sublayers are computed.

(2) The computed time histories may be plotted or punched on cards.

Data Cards

1st Card (5I5,F10.0,5A6)

Cols.		
1-5	LLL(1)	Sublayer number. The stress or strain history is computed on the middle of the sublayer.
10	LLGS(1)	Select type of response: 0 - strain 1 - stress
15	LLPCH(1)	Set equal to 1 for punched output.
20	LLPL(1)	Set equal to 1 for plotting.
21-25	LNV(1)	Number of values to be plotted; maximum of 2049.
26-35	SK(1)	Scale for plotting--i.e. maximum value of ordinate. If blank, the largest value in the response is set as the maximum value of the ordinate.
37-65	ID(1,)	Identification.

2nd Card. As for Card 1 for second sublayer. Use blank card if only one response is to be computed.

6. EXAMPLE RUN

6.1 Selection of soil system and input motion.

An example problem is shown in Fig. 6. Maximum accelerations, stresses and strains in the soil deposit and response spectra for the surface accelerations are wanted for a magnitude 7.4 earthquake occuring 100 miles from the site.

Based on the relations given by Seed and Idriss (1970), the soil system shown on Fig. 7 was selected for analysis. The factors used for clay are equal to the undrained shear strength in kips/sq. ft. The factors for sand are estimated from relative densities and content of gravel.

The motion in rock for a magnitude 7.4 earthquake 100 miles from the causative fault is estimated to have maximum acceleration of .02g and a predominant period of 0.65 sec (Schnabel and Seed, 1972; Seed et al., 1969). Among the available strong motion records, the Pasadena record from the 1952 Kern County earthquake seems to have characteristics most similar to those desired. The magnitude of the earthquake was 7.7, the record was obtained some 75 miles from the fault, the maximum acceleration was 0.057g and the predominant period was 0.65 sec. Modification of this record to give a maximum acceleration 0.02g gives the desired characteristics for the motion in an outcropping rock formation near the example site.

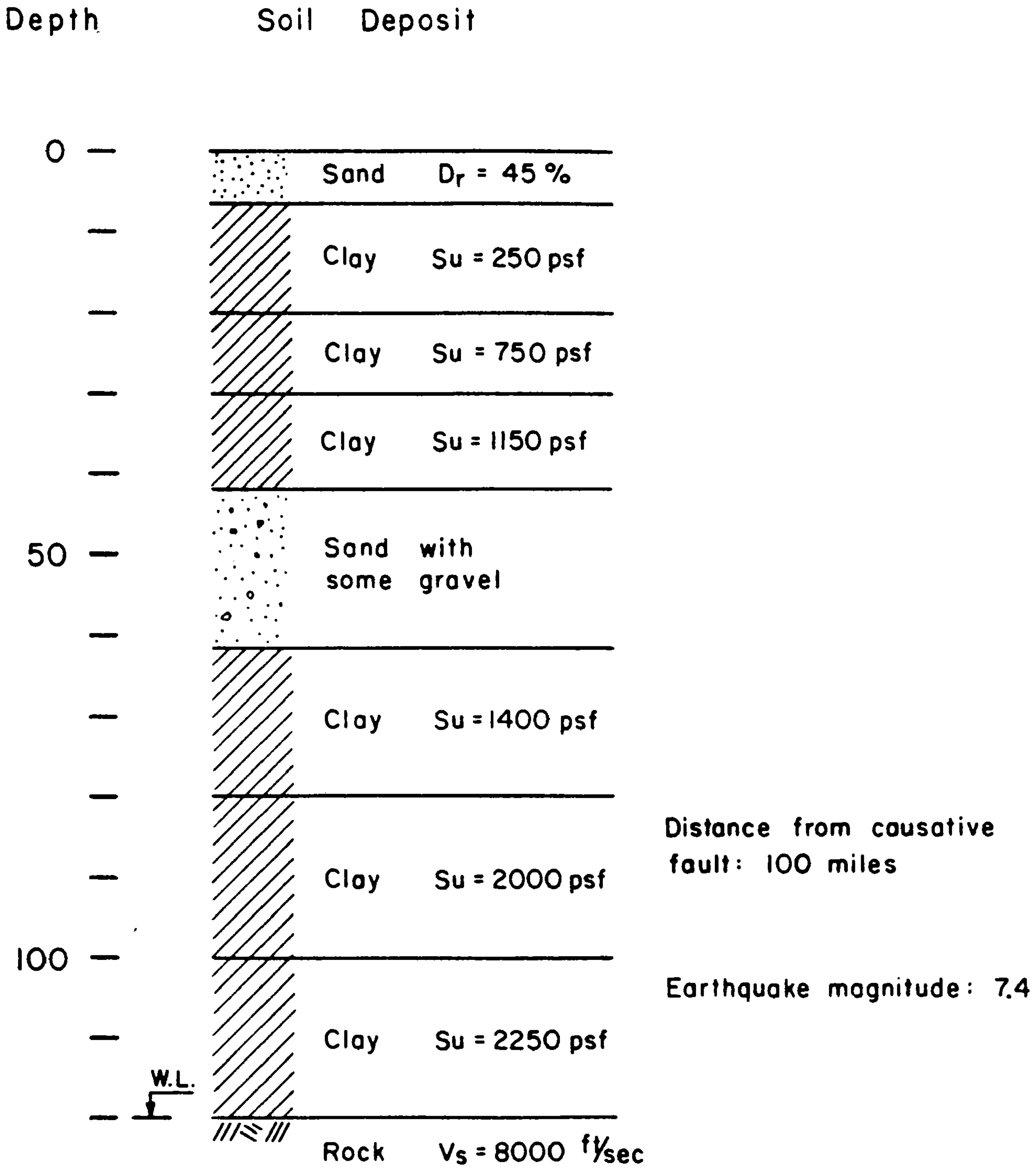
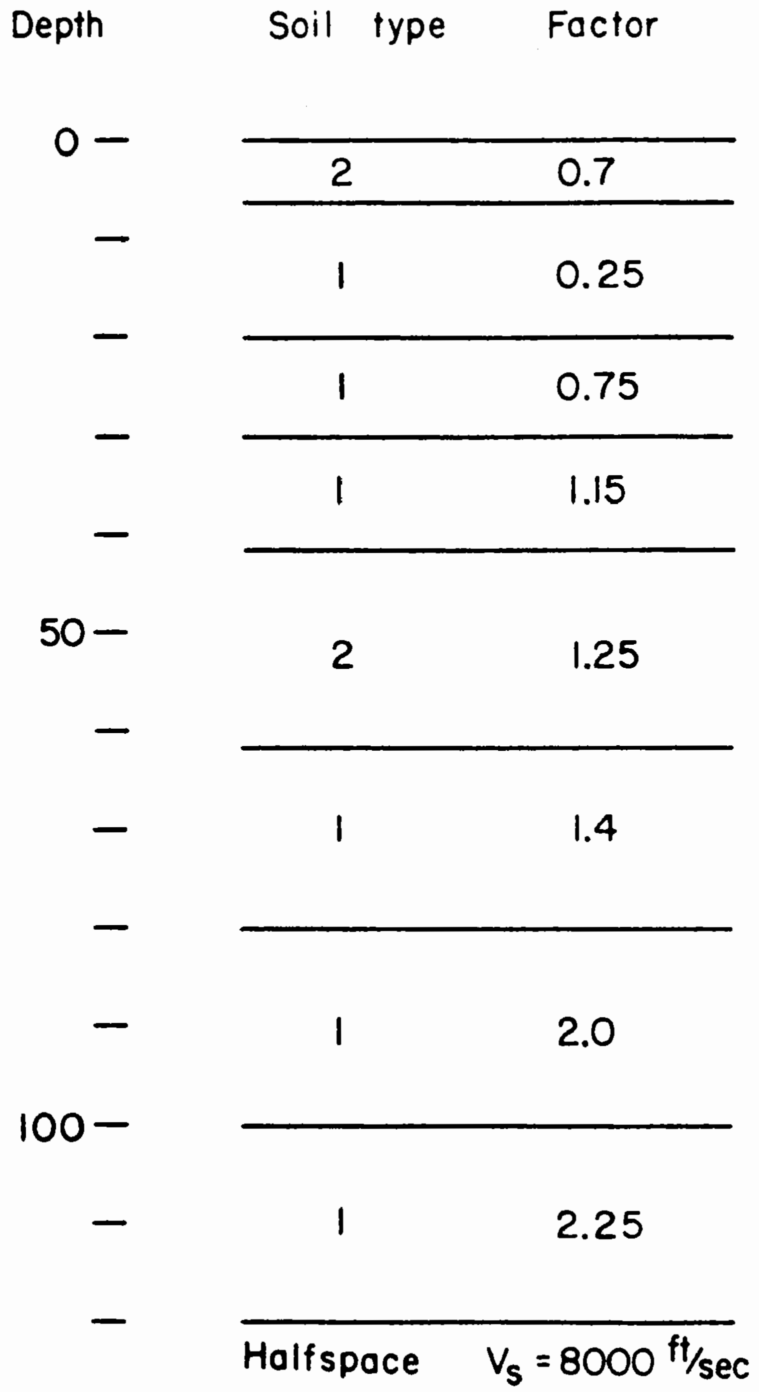


Fig. 6 EXAMPLE PROBLEM



Motion in outcropping rock:

Pasadena record from the 1952 Kern County earthquake scaled to 0.02 g maximum acceleration.

Fig. 7 SYSTEM USED IN THE ANALYSIS OF THE EXAMPLE PROBLEM

6.2 Input data for the analysis.

```
        INITIALIZATION       1 CARD

1.......10........20........30........40........50........60........70........80
 1974


        OPTION 8             28 CARDS

1.......10........20........30........40........50........60........70........80
  8
  3    1   10   50.
 11 .01  SHEAR MODULUS CLAY  MAY 24 - 1972
.0001    .000316   .001     .00316     .01       .0316      .1        .316
 1.        3.16    10.
2300.    2100.     1750.     1300.     920.       600.      350.      175.
94.        30.     10.
 10  1.  DAMPING CLAY  MAY 24 - 1972
  .0001     .001    .00316   .01                .0316    .1        .316       1.
3.16     10.
  2.0    2.5       3.5        4.75      6.5        9.25     13.75      20.
26.      29.
 11 .5   SHEAR MODULUS SAND SQ.ROOT REL. MAY 24-1972
.0001    .000315   .001      .00316    .01       .0316      .1        .316
 1.        3.16    10.
 61.     60.       57.        50.4    40.         27.       15.        7.
 3.      1.        3.
  9  1.  DAMPING SAND  FEBRUARY 1971
  .0001     .001     .003      .01       .03        .1        .3       1.
 10.
1.       1.6       3.12      5.8       9.5        15.4      20.9      25.
 28.9
  8 .01  ATTENUATION OF ROCK    AVERAGE   APRIL 1971
.0001    .0003     .001      .003      .01       .03        .1        1.
2000.    2000.     1975.     1905.     1800.     1620.      1450.     1100.
  5 1.   DAMPING IN ROCK   AVERAGE  9/4  APRIL 1971
.0001    .001      .01       .1        1.
  .4     .8        1.5       3.        4.6
```

```
        OPTION 1             103 CARDS
1.......10........20........30........40........50........60........70........80

  1
800 1024   .02     PASADENA 1952
           .02
-.002493 -.003313 -.002623 -.002526 -.001317 -.000128  .001274  .002382      1
 .002625  .002518  .002590  .002513  .002569  .002502  .002551  .002491      2
 .002533  .002478  .002516  .002465  .002500  .002451  .002486  .002408      3
 .001994  .001425  .000925  .000445  .000425  .000420  .000066 -.000829      4
-.001525 -.002386 -.002381 -.001979 -.001647 -.001658 -.001639 -.001645      5
-.001099 -.000510  .000131 -.000088 -.000763 -.001374 -.002079 -.002893      6
-.003579 -.003281 -.002718 -.002255 -.001509  .000499  .002231  .002675      7
 .003059  .003649  .004454  .005218  .005970  .006832  .006573  .004815      8
 .003349  .000761 -.001550 -.001932 -.001298 -.000592  .000267  .000936      9
 .001844  .001562  .000454 -.000521 -.001442 -.002425 -.003165 -.003689     10
-.004324 -.005168 -.005828 -.005715 -.005336 -.005068 -.003841 -.002305     11
-.001126 -.000120  .001399  .002990  .004500  .004992  .004919  .004081     12
 .002851  .001865  .001481  .001031  .001121  .001382  .001816  .001063     13
-.001154 -.003149 -.005312 -.006564 -.007083 -.007836 -.007772 -.006705     14
-.005669 -.004535 -.003444 -.002324 -.001229 -.000185  .000867  .002097     15
 .003343  .004508  .004360  .003868  .004611  .005548  .006484  .007403     16
 .008347  .009253  .010469  .012118  .012267  .011130  .010349  .008871     17
 .006792  .005192  .004711  .003808  .005427  .011079  .011994  .012237     18
 .012588  .013780  .015396  .016909  .018496  .020026  .021497  .021543     19
 .021695  .019953  .015352  .013395  .013656  .013217  .009669  .005633     20
 .004387  .003030  .002845  .003635  .004411  .004443  .004606  .003431     21
 .000276 -.001939 -.001932 -.006423 -.009774 -.011147 -.012292 -.012660     22
-.013513 -.014858 -.017037 -.018811 -.020927 -.022735 -.024635 -.025974     23
-.027901 -.026475 -.019556 -.014708 -.007272  .002294  .003546  .005332     24
 .004549  .003636  .002636  .002806  .001202 -.000432 -.002572 -.001494     25
-.000616  .000291 -.000496 -.006645 -.014722 -.012778 -.033575 -.042831     26
-.043009 -.045807 -.043294 -.041669 -.037745 -.025278 -.019201  .001040     27
 .013649  .012866  .015824  .015378  .019696  .019668  .021342  .020955     28
 .021309  .022949  .026435  .029246  .031983  .033228  .034739  .031762     29
 .027738  .024080  .020339  .017203  .011562  .007591 -.015693 -.028443     30
-.029791 -.039844 -.042214 -.043695 -.040997 -.038664 -.033649 -.030032     31
-.020213 -.001769  .002023  .007762  .005125  .004710  .007523  .007468     32
 .010956  .011913  .012652  .020094  .028702  .032657  .038837  .040349     33
 .041798  .038336  .032960  .031175  .029425  .026144  .025072  .022677     34
 .021590  .016643  .017778  .015500  .013219 -.005005 -.018653 -.035037     35
-.047254 -.042567 -.045124 -.039513 -.034339 -.020552 -.004749 -.002274     36
-.000997 -.001284 -.000524 -.000064  .000558  .000123 -.001028 -.000021     37
-.000648  .000498  .003457  .006631  .007229  .005943  .005739  .005723     38
 .005276  .003131  .003588  .004395  .001724 -.000949  .000235  .003089     39
 .004174  .002623  .003442  .003680  .004803  .005154  .005346  .007711     40
 .010554  .009195  .008490  .005691  .005005  .000425 -.003943 -.006070     41
-.007428 -.009936 -.010439 -.012590 -.011388 -.019622 -.039587 -.041574     42
-.046392 -.049356 -.052906 -.052793 -.049356 -.045835 -.043181 -.027839     43
-.014535 -.013859  .025815  .043109  .043491  .047904  .047682  .051035     44
 .051602  .054637  .054828  .057238  .055020  .049601  .043950  .040517     45
 .037062  .034520  .031703  .024568  .014474  .011676  .004249 -.007235     46
-.026973 -.041443 -.042865 -.045273 -.045146 -.044133 -.040249 -.033307     47
-.017015 -.004581 -.004544 -.003638 -.004924 -.004706 -.005664 -.005841     48
-.007728 -.003074 -.010774 -.012404 -.014434 -.009151 -.007826  .003369     49
 .024767  .025711  .029998  .032436  .034883  .034808  .035693  .035950     50
 .031252  .022314  .016231  .008574 -.003059 -.007785 -.009396 -.009446     51
-.008794 -.009981 -.009000 -.009324 -.009375 -.009651 -.009853 -.011662     52
-.013760 -.015836 -.017792 -.019124 -.019907 -.016268 -.013307 -.003454     53
```

```
 .002631  .004139  .004371  .001234 -.001186 -.005353 -.008826 -.012269     54
-.013760 -.015331 -.017510 -.022831 -.024390 -.026574 -.024601 -.022072     55
-.018882 -.015641 -.011956 -.012234 -.014489 -.016033 -.017898 -.021668     56
-.027007 -.027043 -.017194 -.007552  .001476  .017401  .023309  .023177     57
 .027040  .028719  .032583  .031626  .030859  .028315  .027878  .023949     58
 .023372  .013397 -.008389 -.017724 -.021337 -.033693 -.037941 -.036294     59
-.036374 -.035359 -.034134 -.030404 -.027147 -.023444 -.020092 -.016374     60
-.012404 -.008319 -.004306 -.000756  .002788  .006267  .010007  .013599     61
 .017748  .020937  .018017  .012943  .010150  .006306  .004310 -.002774     62
-.011569 -.013108 -.012718 -.009093 -.004293  .000696  .001663  .003167     63
 .003833  .005218  .005565  .005307  .004555  .004324  .002268 -.002006     64
-.005485 -.009136 -.010342 -.013972 -.017321 -.020356 -.023507 -.019832     65
-.016115 -.014444 -.012891 -.011308 -.009298 -.007287 -.006066 -.004843     66
-.004426 -.005197 -.005695 -.005885 -.009521 -.010996 -.009946 -.009179     67
-.007934 -.005983 -.004401 -.005072 -.004589 -.006038 -.002838  .000617     68
 .003857  .006366  .007463  .005531  .004975  .002051 -.001933 -.008681     69
-.011511 -.011291 -.011244 -.011473 -.011180 -.012196 -.013164 -.014751     70
-.015533 -.016972 -.018909 -.024014 -.024367 -.021793 -.017011 -.016185     71
 .007041  .017308  .015199  .017725  .017247  .018997  .018977  .019598     72
 .018284  .018230  .017224  .017375  .016698  .015711  .010446  .008182     73
 .009113  .010674  .011417  .011690  .011490  .011663  .011506  .011620     74
 .011817  .012871  .013767  .013843  .009655  .005485  .001648 -.001211     75
-.005169 -.009233 -.013873 -.018128 -.015787 -.010113 -.006007 -.002833     76
-.000194  .002956  .005626  .008930  .011888  .014931  .016787  .018965     77
 .020509  .022943  .025558  .026814  .023618  .021894  .019231  .015899     78
 .006415 -.003205 -.005577 -.014085 -.018955 -.021957 -.024742 -.027639     79
-.030061 -.029838 -.026851 -.025145 -.026483 -.029640 -.032318 -.035157     80
-.035179 -.034894 -.033905 -.032181 -.031237 -.028657 -.025223 -.019982     81
-.014925 -.009185 -.003261  .001908  .005464  .009307  .012673  .014940     82
 .017126  .019247  .021212  .022423  .024027  .025807  .029563  .031823     83
 .030313  .028329  .025858  .022232  .013520  .010163  .006899  .003967     84
 .000597 -.000495  .000120  .000476  .000989  .001265  .003967  .003524     85
 .000241 -.000163 -.000264 -.000584  .000541  .002474  .004356  .005954     86
 .007249  .009512  .009877  .010134  .009723  .009107  .007355  .005186     87
 .004931  .006719  .009156  .009798  .012428  .016216  .018755  .016836     88
 .014459  .011739  .008580  .005413  .002914  .001680  .000421 -.000937     89
-.002750 -.004559 -.006680 -.012332 -.015390 -.015716 -.016604 -.017175     90
-.017957 -.015356 -.014195 -.010472 -.009050 -.006920 -.005341 -.002633     91
-.000012  .003354  .004365  .004209  .004327  .003693  .001851  .000085     92
-.001814 -.003043 -.003466 -.004068 -.004504 -.003832 -.001625 -.000235     93
 .002278  .005874  .014722  .025478  .026123  .028565  .027956  .031019     94
 .029316  .028463  .026981  .026664  .025729  .025371  .024037  .023324     95
 .021865  .021047  .019546  .014894  .012076  .010363  .008051  .005314     96
 .001757 -.002501 -.007163 -.010712 -.015489 -.021659 -.021587 -.019652     97
-.018365 -.016573 -.015225 -.013493 -.011212 -.007342 -.006514 -.007858     98
-.009239 -.010467 -.012825 -.015684 -.015584 -.010756 -.005233  .000573     99
 .005328  .009647  .012139  .015001  .019339  .021552  .019896  .021718    100
```

```
      OPTION 2            10 CARDS

1........10........20........30........40........50........60........70........80

 2
 1     9    9      EXAMPLE SITE
 1     2    1       7.00 1000.        .05        .120                 .700 1.00
 2     1    1      13.00  200.        .1         .100                 .250 1.00
 3     1    1      10.00  1000.       .05        .100                 .750 1.00
 4     1    1      12.00  1000.       .05        .100                1.15  1.00
 5     2    1      20.00  2000.       .05        .125                1.250 1.00
 6     1    1      18.00 1000.        .05        .125                1.4   1.
 7     1    1    20.        2000.     .05        .125                2.000 1.00
 8     1    1    20.        2500.     .05        .125                2.250 1.00
 9                                    0.         .150    8000.


      OPTION 3            2 CARDS

1........10........20........30........40........50........60........70........80

 3
 3     0


      OPTION 4            2 CARDS

1........10........20........30........40........50........60........70........80

 4
 0     5    5.         .65


      OPTION 5            4 CARDS

1........10........20........30........40........50........60........70........80

 5
 1     2    3    4    5    6    7    8    9    9
 1     1    1    1    1    1    1    1    1
 0     0    0    0    0    0    0    0    0
```

```
         OPTION 9                4 CARDS
1........10........20........30........40........50........60........70........80
    9
    1    0
    1    0    2    1    1
   .05

         OPTION 0                1 CARD
1........10........20........30........40........50........60........70........80
    9
```

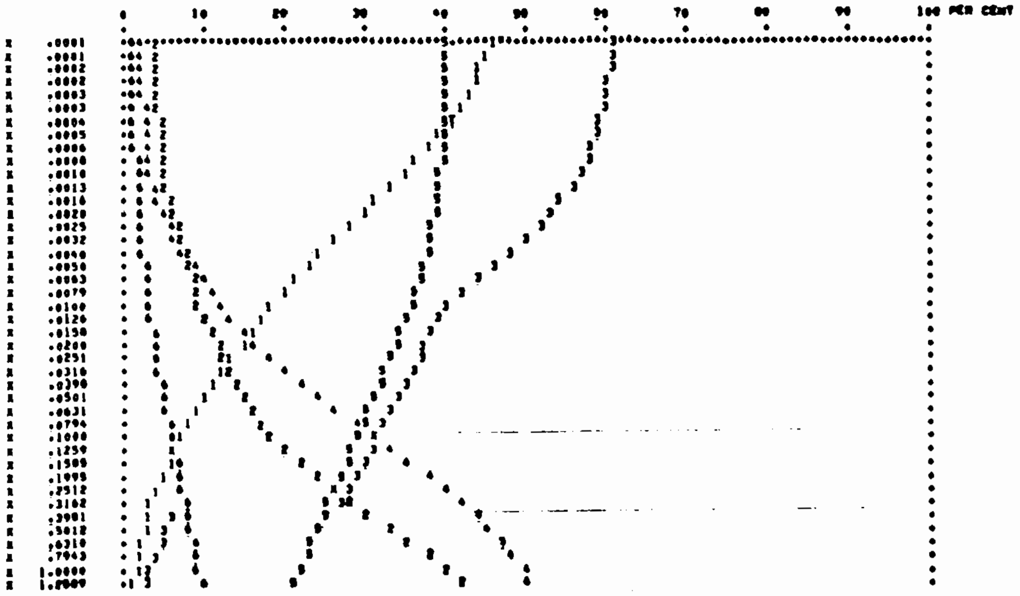
```
MAX. NUMBER OF TERMS IN FOURIER TRANSFORM =        1024
NECESSARY LENGTH OF BLANK COMMON X         =        4419
EARTH PRESSURE AT REST FOR SAND            =        .450
```

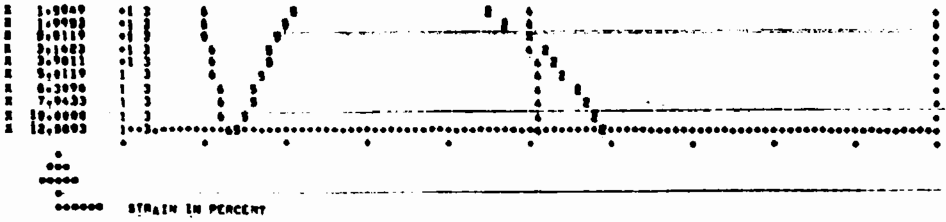
```
******   OPTION  8  ***  READ RELATION BETWEEN SOIL PROPERTIES AND STRAIN

   CURVES FOR RELATION STRAIN VERSUS SHEAR MODULUS AND DAMPING


 MODULUS AND DAMPING VALUES ARE SCALED FOR PLOTTING

SHEAR MODULUS CLAY  MAY 24 - 1972                          MULTIPLICATION FACTOR =    .010
DAMPING CLAY  MAY 24 - 1972                                MULTIPLICATION FACTOR =   1.000
SHEAR MODULUS SAND SQ.ROOT REL. MAY 24-1972                MULTIPLICATION FACTOR =    .500
DAMPING SAND  FEBRUARY 1971                                MULTIPLICATION FACTOR =   1.000
ATTENUATION OF ROCK     AVERAGE    APRIL 1971              MULTIPLICATION FACTOR =    .010
DAMPING IN ROCK    AVERAGE  9/6    APRIL 1971              MULTIPLICATION FACTOR =   1.000
                                                                 100 PER CENT CORRESPONDS TO  30.0000
```





```
CURVE 1  -  SHEAR MODULUS CLAY  MAY 24 - 1972
CURVE 2  -  DAMPING CLAY  MAY 24 - 1972
CURVE 3  -  SHEAR MODULUS SAND SQ.ROOT REL. MAY 24-1972
CURVE 4  -  DAMPING SAND  FEBRUARY 1971
CURVE 5  -  ATTENUATION OF ROCK    AVERAGE   APRIL 1971
CURVE 6  -  DAMPING IN ROCK   AVERAGE  9/4   APRIL 1971

  ABSISSA      CURVE 1    CURVE 2    CURVE 3    CURVE 4    CURVE 5    CURVE 6
    .0001      23.0000     2.0000    30.5000     1.0000    20.0000      .4000
    .0001      22.5997     2.0500    30.3999     1.0000    20.0000      .4400
    .0002      22.1995     2.1000    30.2999     1.1200    20.0000      .4800
    .0002      21.7992     2.1500    30.1999     1.1800    20.0000      .5200
    .0003      21.3990     2.2000    30.0997     1.2400    20.0000      .5600
    .0003      20.9978     2.2500    29.9991     1.3000    19.9891      .6000
    .0004      20.2982     2.3000    29.6992     1.3600    19.9412      .6400
    .0005      19.5987     2.3500    29.3994     1.4200    19.8934      .6800
    .0006      18.8991     2.4000    29.0996     1.4800    19.8456      .7200
    .0008      18.1996     2.4500    28.7998     1.5400    19.7978      .7600
    .0010      17.5000     2.5000    28.5000     1.6000    19.7500      .8000
    .0013      16.5994     2.7001    27.8390     1.9108    19.6033      .8700
    .0016      15.6989     2.9003    27.1792     2.2372    19.4566      .9400
    .0020      14.7983     3.1004    26.5180     2.5557    19.3099     1.0100
    .0025      13.8977     3.3005    25.8583     2.8743    19.1631     1.0800
    .0032      12.9970     3.5000    25.1967     3.2373    19.0041     1.1500
    .0040      12.2381     3.7506    24.1576     3.7498    18.8032     1.2200
    .0050      11.4786     4.0005    23.1180     4.2624    18.6024     1.2900
    .0063      10.7190     4.2503    22.0787     4.7749    18.4016     1.3600
    .0079       9.9585     4.5002    21.0393     5.2875    18.2008     1.4300
    .0100       9.2000     4.7500    20.0000     5.8000    18.0000     1.5000
    .0126       8.5596     5.1002    19.5725     6.5755    17.6227     1.6500
    .0158       7.9192     5.4504    19.1451     7.3510    17.2455     1.8000
    .0200       7.2788     5.8007    18.7176     8.1265    16.8682     1.9500
    .0251       6.6384     6.1509    18.2902     8.9019    16.4909     2.1000
    .0316       5.9980     6.5017    17.8627     9.7582    16.1256     2.2500
    .0398       5.4987     7.0516    17.4352    10.8005    15.8005     2.4000
    .0501       4.9991     7.6010    17.0078    12.0149    15.4754     2.5500
    .0631       4.4994     8.1507    16.5803    13.1433    15.1502     2.7000
    .0794       3.9997     8.7003    16.1529    14.2718    14.8251     2.8500
    .1000       3.5000     9.2500    15.7254    15.4000    14.5000     3.0000
    .1259       3.1498    10.1506    15.2980    16.5527    14.1500     3.1600
    .1585       2.7996    11.0511    14.8705    17.7058    13.8000     3.3200
    .1995       2.4493    11.9517    14.4430    18.8582    13.4500     3.4800
    .2512       2.0991    12.8523    14.0156    20.0110    13.1000     3.6400

    .3162       1.7494    13.7530    13.5881    21.0796    12.7500     3.8000
    .3981       1.5075    15.0031     3.0790    21.8635    12.4000     3.9600
    .5012       1.3057    16.2523     2.6992    22.6476    12.0500     4.1200
    .6310       1.2038    17.5016     2.2995    23.4318    11.7000     4.2800
    .7943       1.0219    18.7508     1.8997    24.2159    11.3500     4.4400
   1.0000        .8400    20.0000     1.5000    25.0000    11.0000     4.6000
   1.2589        .7319    21.2008     1.5000    25.0500    10.6500     4.7600
   1.5849        .6239    22.4015     1.5000    25.1000    10.3000     4.9200
   1.9953        .5158    23.6023     1.5000    25.1500     9.9500     5.0800
   2.5119        .4077    24.8030     1.5000    25.2000     9.6000     5.2400
   3.1623        .2999    26.0019     1.5000    25.2500     9.2500     5.4000
   3.9811        .2599    26.6015     1.5000    25.3000     8.9000     5.5600
   5.0119        .2199    27.2011     1.5000    25.3500     8.5500     5.7200
   6.3096        .1799    27.8008     1.5000    25.4000     8.2000     5.8800
   7.9433        .1400    28.4004     1.5000    25.4500     7.8500     6.0400
  10.0000        .1000    29.0000     1.5000    25.5000     7.5000     6.2000
  12.5893        .0000    29.9996     1.5000    25.5500     7.1500     6.3600
```

```
******  OPTION  1  ***  READ INPUT MOTION

EARTHQUAKE  -  PASADENA 1952

 800 ACCELERATION VALUES AT TIME INTERVAL      .0200

   THE VALUES ARE LISTED ROW BY ROW AS READ FROM CARDS
   TRAILING ZEROS ARE ADDED TO GIVE A TOTAL OF 1024 VALUES

   1        -.002493   -.003313   -.002023   -.002520   -.001317   -.000120    .001276    .002302
   2         .002425    .002516    .002540    .002513    .002569    .002502    .002551    .002491
   3         .002533    .002470    .002516    .002465    .002500    .002451    .002406    .002400
   4         .001994    .001625    .000925    .000445    .000425    .000420    .000086   -.000929
   5        -.001929   -.002308   -.002341   -.001979   -.001647   -.001650   -.001639   -.001445
   6        -.001699   -.000510    .000131   -.000080   -.000763   -.001374   -.002079   -.002093
   7        -.003579   -.003281   -.002718   -.002255   -.001509    .000409    .002231    .002475
   8         .003059    .003640    .004494    .005210    .005970    .006032    .006573    .004815
   9         .003340    .000783   -.001556   -.001032   -.001290   -.000592    .000207    .000936
  10         .001044    .001562    .000456   -.000521   -.001442   -.002425   -.003165   -.003689
  11        -.004326   -.005168   -.005028   -.005715   -.005336   -.005068   -.003841   -.002305
  12        -.001126   -.000120    .001390    .002900    .004500    .004992    .004919    .004001
  13         .002051    .001865    .001461    .001631    .001121    .001382    .001016    .001063
  14        -.001154   -.003140   -.005312   -.006864   -.007983   -.007836   -.007772   -.006705
  15        -.005469   -.004835   -.003440   -.002324   -.001229   -.000185    .000067    .002097
  16         .003343    .004508    .004360    .003060    .004611    .005540    .006484    .007403
  17         .008347    .009253    .010469    .012110    .012267    .011130    .010349    .008871
  18         .006792    .005192    .004711    .003000    .005627    .011079    .011994    .012237
  19         .012508    .013780    .015390    .016909    .010496    .020020    .021497    .021543
  20         .021695    .010953    .015352    .013395    .013656    .013217    .009660    .005633
  21         .004387    .003030    .002045    .003035    .004411    .004443    .004406    .003431
  22         .000276   -.001939   -.001032   -.006423   -.009775   -.011197   -.012292   -.012960
  23        -.013913   -.014050   -.017037   -.010011   -.020927   -.022735   -.024635   -.025974
  24        -.027901   -.026475   -.019096   -.014700   -.007272    .002296    .003546    .006332
  25         .004549    .003636    .002630    .002906    .001282   -.000432   -.002572   -.001484
  26        -.000616    .000291   -.000486   -.006465   -.014722   -.012770   -.033575   -.042031
  27        -.043009   -.045087   -.043294   -.041669   -.037765   -.025270   -.019201    .001040
  28         .013648    .012066    .015824    .016370    .018696    .019669    .021342    .020955
  29         .021309    .022849    .020455    .029249    .031983    .033228    .034739    .031762
  30         .027738    .024080    .020339    .017203    .011562    .007591   -.015690   -.028448
  31        -.029791   -.038844   -.042214   -.043695   -.040997   -.038664   -.033649   -.030902
  32        -.020213   -.001760    .002023    .007762    .006125    .004710    .007523    .009440
  33         .010966    .011913    .012052    .023004    .020702    .032657    .038037    .040949
  34         .041790    .038336    .032960    .031175    .029425    .026144    .025072    .022677
  35         .021590    .016643    .017778    .015908    .019219   -.005005   -.018653   -.035037
  36        -.047254   -.042567   -.045124   -.039513   -.036399   -.020552   -.004749   -.002274
  37        -.000997   -.001204   -.000024   -.000064    .000558    .000123   -.001028   -.000921
  38        -.000640    .000480    .003457    .006031    .007229    .005943    .005730    .005723
  39         .005276    .003131    .003580    .004385    .001724   -.000949    .000235    .003089
  40         .004174    .002023    .003442    .003600    .004003    .005154    .005346    .007711
  41         .018554    .009199    .000490    .005091    .005005    .000429   -.003943   -.006078
  42        -.007420   -.009936   -.010439   -.012590   -.011308   -.019022   -.039507   -.041974
  43        -.040392   -.040356   -.052940   -.052793   -.049356   -.045035   -.043101   -.027839
  44        -.014535   -.013059    .025016    .043108    .043481    .047996    .047682    .051035
  45         .051602    .054637    .054020    .057238    .055020    .049601    .043950    .040517
  46         .037002    .034820    .031703    .024568    .014474    .011676    .004869   -.007235
  47        -.026973   -.041443   -.042069   -.049273   -.048140   -.044333   -.048249   -.033307
  48        -.017015   -.004803   -.004644   -.003630   -.004926   -.004706   -.005666   -.005061
  49        -.007720   -.008076   -.010774   -.012404   -.016439   -.009161   -.007826    .003369
  50         .026767    .025711    .027998    .032436    .034003    .034000    .035693    .035950
  51         .031252    .022314    .016231    .000576   -.003059   -.007705   -.009396   -.009446
  52        -.008796   -.008901   -.009000   -.009326   -.009378   -.009481   -.009053   -.011402


  53        -.013760   -.015036   -.017792   -.010124   -.018907   -.018264   -.010307   -.003494
  54         .002631    .004100    .004371    .001234   -.001186   -.005053   -.008826   -.012249
  55        -.013740   -.019331   -.017818   -.022031   -.024390   -.026574   -.024601   -.022072
  56        -.018082   -.019641   -.011956   -.012234   -.014409   -.018093   -.017098   -.021068
  57        -.027907   -.027963   -.017194   -.007552    .001676    .017401    .023309    .023177
  58         .027040    .028719    .022583    .031426    .030859    .020315    .027870    .023940
  59         .023372    .013997   -.000349   -.017724   -.021337   -.033693   -.037941   -.036296
  60        -.036376   -.035359   -.034194   -.030484   -.027167   -.023444   -.020092   -.014376
  61        -.012404   -.000319   -.004340   -.000759    .002708    .006287    .010007    .013599
  62         .017748    .020897    .018017    .012063    .010190    .006306    .004310   -.002774
  63        -.011969   -.013108   -.012710   -.009093   -.004293    .000696    .001463    .003167
  64         .003833    .005210    .005565    .005367    .004595    .004126    .002260   -.002006
  65        -.005409   -.000136   -.010942   -.013972   -.017321   -.020956   -.023507   -.019032
  66        -.016115   -.014444   -.012891   -.011360   -.009290   -.007287   -.006066   -.004063
  67        -.004426   -.005197   -.005609   -.006009   -.009521   -.010996   -.009946   -.009179
  68        -.007934   -.005903   -.004491   -.005072   -.005909   -.006030   -.002030    .000617
  69         .003857    .006366    .007463    .005531    .004975    .002051   -.001933   -.000601
  70        -.011911   -.011291   -.011264   -.011423   -.011100   -.012156   -.013166   -.014751
  71        -.015533   -.016972   -.010909   -.024014   -.024367   -.021703   -.017011   -.010105
  72         .007961    .017300    .015190    .017729    .017247    .018997    .010977    .019590
  73         .010206    .010230    .017224    .017375    .016690    .015711    .018466    .008102
  74         .009113    .016404    .011417    .011490    .011690    .011643    .011506    .011020
  75         .011817    .012871    .013767    .013043    .009455    .005485    .001640   -.001211
  76        -.005169   -.009233   -.013079   -.010128   -.015707   -.010113   -.000007   -.002033
  77        -.000196    .002056    .005626    .008930    .011000    .014931    .016787    .010965
  78         .020000    .022943    .025656    .020034    .023610    .021806    .018231    .015099
  79         .006415   -.003205   -.005377   -.014900   -.010955   -.021957   -.024742   -.027639
  80        -.030063   -.028030   -.026051   -.025145   -.026489   -.029640   -.032318   -.035157
  81        -.035179   -.034890   -.033905   -.032101   -.030237   -.028057   -.025223   -.019902
  82        -.014925   -.000183   -.003281    .001800    .005464    .009307    .012679    .014960
  83         .017126    .019267    .028912    .022423    .024020    .025809    .029569    .031029
  84         .030313    .028309    .024090    .022232    .013528    .010163    .000899    .003967
  85         .000597   -.000495    .000120    .000478    .000989    .001265    .000967    .000524
  86         .000241   -.000063   -.000204   -.000504    .000541    .002476    .004956    .005954
  87         .007249    .000512    .009077    .010134    .009723    .009107    .007355    .005106
  88         .004931    .006719    .008156    .009798    .012028    .016216    .010755    .010036
  89         .014459    .011738    .006500    .005413    .002914    .001680    .000821   -.000937
  90        -.002750   -.006659   -.006000   -.012308   -.015390   -.015706   -.014606   -.017175
  91        -.017957   -.010358   -.014170   -.010422   -.009050   -.006720   -.005341   -.002633
  92        -.000082    .003094    .004368    .004209    .004327    .003693    .001081    .000005
  93        -.001014   -.003063   -.003466   -.004068   -.004506   -.003032   -.001625   -.000235
  94         .002278    .005074    .014752    .025490    .028123    .028908    .029956    .031018
  95         .029318    .028463    .026701    .026064    .025729    .025371    .024097    .023324
  96         .021065    .021047    .018940    .014096    .012008    .010343    .000053    .005214
  97         .001757   -.002881   -.007363   -.018712   -.015409   -.021050   -.021507   -.019052
  98        -.010366   -.010870   -.018220   -.013493   -.011212   -.007042   -.006516   -.007080
  99        -.009231   -.011407   -.012018   -.015604   -.010004   -.010706   -.005233    .000579
 100         .005328    .009647    .012130    .018901    .018330    .021882    .019096    .021710

MAXIMUM ACCELERATION =    .05724
AT TIME              =  7.10 SEC

THE VALUES WILL BE MULTIPLIED BY A FACTOR =    .349
TO GIVE NEW MAXIMUM ACCELERATION          =    .02000

MEAN SQUARE FREQUENCY =       1.08 C/SEC.
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******   OPTION  2  ***  READ SOIL PROFILE

    NEW SOIL PROFILE NO.  1        IDENTIFICATION         EXAMPLE SITE

NUMBER OF LAYERS                 9          DEPTH TO BEDROCK        120.00
NUMBER OF FIRST SUBMERGED LAYER  9          DEPTH TO WATER LEVEL    120.00

LAYER  TYPE     FACTOR     THICKNESS     DEPTH       EFF. PRESS.     MODULUS        DAMPING    UNIT WEIGTH      SHEAR VEL
              MOD.  DAMP.

  1     2      .70  1.00      7.00        3.50          .42           1500          .056          .1200           510
  2     1      .25  1.00     13.00       13.50         1.49            800          .105          .1000           394
  3     1      .75  1.00     10.00       25.00         2.04           1000          .050          .1000           567
  4     1     1.10  1.00     12.00       36.00         3.74           1000          .050          .1000           507
  5     2     1.25  1.00     20.00       52.00         5.59           2000          .053          .1250           710
  6     1     1.40  1.00     18.00       71.00         7.96           1080          .050          .1250           500
  7     1     2.00  1.00     20.00       90.00        10.30           2000          .050          .1250           710
  8     1     2.25  1.00     20.00      110.00        12.64           2300          .050          .1250           802
  9    BASE                                                         200127        0.            .1400          8000

  PERIOD =  .70 FROM AVERAGE SHEARVEL. =    611

MAXIMUM AMPLIFICATION =  14.10
FOR FREQUENCY         =   1.41 C/SEC.
    PERIOD            =    .71 SEC.




******   OPTION  3  ***  READ WHERE OBJECT MOTION IS GIVEN

  OBJECT MOTION IN LAYER NUMBER    9 OUTCROPPING
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******   OPTION  6  ***  OBTAIN STRAIN COMPATIBLE SOIL PROPERTIES

 MAXIMUM NUMBER OF ITERATIONS                  =     6
 MAXIMUM ERROR IN PERCENT                      =   5.00
 FACTOR FOR EFFECTIVE STRAIN IN TIME DOMAIN    =    .65


    EARTHQUAKE   -  PASADENA 1952
    SOIL PROFILE -        EXAMPLE SITE

 ITERATION NUMBER  1
 THE CALCULATION HAS BEEN CARRIED OUT IN THE TIME DOMAIN WITH EFF. STRAIN = .65* MAX. STRAIN

 LAYER  TYPE       DEPTH        EFF. STRAIN        NEW DAMP.    DAMP USED     ERROR          NEW G      G USED          ERROR

    1     2          3.5          .00223              .027        .050        -84.3        598.089    1000.000          -67.2
    2     1         13.5          .03807              .069        .100        -44.8        139.898     200.000          -43.0
    3     1         25.0          .01287              .050        .050           .7        650.725    1000.000          -53.7
    4     1         36.0          .01577              .054        .050          8.1        912.336    1000.000           -9.6
    5     2         52.0          .01072              .060        .050         17.1       2955.784    2000.000           32.3
    6     1         71.0          .02705              .063        .050         20.2        900.490    1000.000          -11.0
    7     1         90.0          .01542              .054        .050          7.6       1599.092    2000.000          -25.1
    8     1        110.0          .01351              .052        .050          4.0       1881.920    2500.000          -32.8

 VALUES IN TIME DOMAIN

 LAYER  TYPE       THICKNESS          DEPTH       MAX STRAIN     MAX STRESS         TIME
                       FT               FT          PRCNT           PSF            SEC

    1     2            7.0             3.5         .00344          20.56           5.84
    2     1           13.0            13.5         .05857          81.93           5.84
    3     1           10.0            25.0         .01980         128.85           5.82
    4     1           12.0            36.0         .02426         221.33           5.82
    5     2           20.0            52.0         .01649         487.57           5.80
    6     1           10.0            71.0         .04162         374.78           5.80
    7     1           20.0            90.0         .02372         379.30           5.44
    8     1           20.0           110.0         .02078         391.03           5.44




    EARTHQUAKE   -  PASADENA 1952
    SOIL PROFILE -        EXAMPLE SITE

 ITERATION NUMBER  2
 THE CALCULATION HAS BEEN CARRIED OUT IN THE TIME DOMAIN WITH EFF. STRAIN = .65* MAX. STRAIN

 LAYER  TYPE       DEPTH        EFF. STRAIN        NEW DAMP.    DAMP USED     ERROR          NEW G      G USED          ERROR

    1     2          3.5          .00422              .039        .027         30.7        564.684     598.089           -5.9
    2     1         13.5          .06086              .081        .069         13.9        114.441     139.898          -22.2
    3     1         25.0          .01893              .057        .050         12.0        556.935     650.725          -16.8
    4     1         36.0          .01625              .055        .054           .8        902.712     912.336           -1.1
    5     2         52.0          .00637              .046        .060        -29.8       3277.533    2955.784            9.8
    6     1         71.0          .02723              .063        .063           .2        897.949     900.490            -.3
    7     1         90.0          .01797              .056        .054          4.1       1513.983    1599.092           -5.6
    8     1        110.0          .01664              .055        .052          5.7       1751.483    1881.920           -7.4

 VALUES IN TIME DOMAIN

 LAYER  TYPE       THICKNESS          DEPTH       MAX STRAIN     MAX STRESS         TIME
                       FT               FT          PRCNT           PSF            SEC

    1     2            7.0             3.5         .00649          36.65           5.00
    2     1           13.0            13.5         .09363         107.15           5.00
    3     1           10.0            25.0         .02912         162.18           5.00
    4     1           12.0            36.0         .02500         225.68           5.00
    5     2           20.0            52.0         .00980         321.39           7.44
    6     1           10.0            71.0         .04189         376.18           8.08
    7     1           20.0            90.0         .02765         418.59           8.06
    8     1           20.0           110.0         .02560         448.28           8.06
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   EARTHQUAKE   -  PASADENA 1952
   SOIL PROFILE -        EXAMPLE SITE

ITERATION NUMBER  3
THE CALCULATION HAS BEEN CARRIED OUT IN THE TIME DOMAIN WITH EFF. STRAIN = .650 MAX. STRAIN

LAYER  TYPE      DEPTH        EFF. STRAIN        NEW DAMP.    DAMP USED     ERROR          NEW G       G USED          ERROR


  1     2         3.5          .00442             .040         .039          1.0          540.818     544.004           -.6
  2     1        13.5          .06908             .084         .081          3.6          107.569     114.461          -6.4
  3     1        25.0          .02134             .059         .057          3.1          531.858     556.935          -4.7
  4     1        36.0          .01880             .055         .055          1.0          890.965     902.712          -1.3
  5     2        52.0          .00615             .047         .048         -1.7         3301.410    3277.533            .7
  6     1        71.0          .02901             .064         .063          1.5          873.380     887.945          -2.0
  7     1        90.0          .02002             .058         .056          2.0         1453.869    1513.963          -4.1
  8     1       110.0          .01865             .057         .055          3.1         1679.853    1751.403          -4.3

VALUES IN TIME DOMAIN

LAYER  TYPE      THICKNESS         DEPTH     MAX STRAIN     MAX STRESS         TIME
                    FT               FT        PRCNT           PSF              SEC

  1     2           7.0            3.5       .00680          36.77            5.90
  2     1          13.0           13.5       .10698         114.32            5.90
  3     1          10.0           25.0       .03284         174.64            7.66
  4     1          12.0           36.0       .02904         231.80            7.66
  5     2          20.0           52.0       .00946         312.30            7.64
  6     1          10.0           71.0       .04463         389.76            8.10
  7     1          20.0           90.0       .03080         447.08            8.08
  8     1          20.0          110.0       .02890         482.09            8.08
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   EARTHQUAKE   -  PASADENA 1952
   SOIL PROFILE -        EXAMPLE SITE

ITERATION NUMBER  4
THE CALCULATION HAS BEEN CARRIED OUT IN THE TIME DOMAIN WITH EFF. STRAIN = .650 MAX. STRAIN

LAYER  TYPE      DEPTH        EFF. STRAIN        NEW DAMP.    DAMP USED     ERROR          NEW G       G USED          ERROR


  1     2         3.5          .00440             .040         .040           .7          539.481     540.818           -.2
  2     1        13.5          .07590             .086         .084          2.6          102.420     107.569          -5.0
  3     1        25.0          .02304             .061         .059          2.6          510.550     531.858          -4.2
  4     1        36.0          .01780             .056         .055          1.5          873.612     890.965          -2.0
  5     2        52.0          .00626             .048         .047           .9         3289.190    3301.410           -.4
  6     1        71.0          .03108             .065         .064          1.6          846.497     873.380          -3.2
  7     1        90.0          .02163             .059         .058          2.0         1410.880    1453.869          -3.0
  8     1       110.0          .01999             .058         .057          1.0         1636.635    1679.853          -2.6

VALUES IN TIME DOMAIN

LAYER  TYPE      THICKNESS         DEPTH     MAX STRAIN     MAX STRESS         TIME
                    FT               FT        PRCNT           PSF              SEC

  1     2           7.0            3.5       .00689          37.16            7.60
  2     1          13.0           13.5       .11606         110.60            7.70
  3     1          10.0           25.0       .03617         185.67            7.60
  4     1          12.0           36.0       .02718         239.21            7.66
  5     2          20.0           52.0       .00964         316.05            7.66
  6     1          10.0           71.0       .04781         404.72            8.10
  7     1          20.0           90.0       .03398         469.60            8.10
  8     1          20.0          110.0       .03075         503.27            8.08

 PERIOD =  .86 FROM AVERAGE SHEARVEL. =      561

MAXIMUM AMPLIFICATION =  13.15
FOR FREQUENCY         =   1.23 C/SEC.
   PERIOD             =    .82 SEC.
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******   OPTION  5  ***  COMPUTE MOTION IN NEW SUBLAYERS

 EARTHQUAKE - PASADENA 1952
SOIL DEPOSIT -        EXAMPLE SITE
  LAYER            DEPTH          MAX. ACC.          TIME      MEAN SQ. FR.         ACC. RATIO      PUNCHED CARDS
                    FT               G               SEC          C/SEC            QUIET ZONE      ACC. RECORD

  WITHIN              0.           .09377           7.68          1.30               .264               0
  WITHIN             7.0           .09250           7.60          1.30               .262               0
  WITHIN            20.0           .05934           8.18          1.28               .264               0
  WITHIN            30.0           .05487           8.18          1.23               .191               0
  WITHIN            42.0           .05062           8.18          1.22               .181               0
  WITHIN            62.0           .04066           6.88          1.21               .170               0
  WITHIN            80.0           .03195           7.28          1.29               .171               0
  WITHIN           100.0           .02423           7.16          1.49               .120               0
  WITHIN           120.0           .01703           6.78          1.08               .063               0
  OUTCR.           120.0           .02000           7.18          1.50               .000               0
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******   OPTION  9  ***  COMPUTE RESPONSE SPECTRUM

 COMPUTE RESPONSE SPECTRUM IN LAYER  1

 RESPONSE SPECTRUM ANALYSIS FOR LAYER NUMBER
CALCULATED FOR DAMPING           .050


    TIMES AT WHICH MAX. SPECTRAL VALUES OCCUR
         TD = TIME FOR MAX. RELATIVE DISP.
         TV = TIME FOR MAX. RELATIVE VEL.
         TA = TIME FOR MAX. ABSOLUTE ACC.
    DAMPING RATIO =  .05

    PER =  .05     TIMES FOR MAXIMA --   TD =  7.6600   TV =  8.3200   TA =  7.6600
    PER =  .10     TIMES FOR MAXIMA --   TD =  7.6400   TV =  7.7800   TA =  7.6600
    PER =  .15     TIMES FOR MAXIMA --   TD =  7.6400   TV =  7.4200   TA =  7.6400
    PER =  .20     TIMES FOR MAXIMA --   TD =  7.7000   TV =  9.6200   TA =  7.7000
    PER =  .25     TIMES FOR MAXIMA --   TD =  7.2800   TV =  6.0000   TA =  7.2800
    PER =  .30     TIMES FOR MAXIMA --   TD =  7.3000   TV =  8.3400   TA =  7.3000
    PER =  .35     TIMES FOR MAXIMA --   TD =  8.5000   TV =  6.0600   TA =  8.5000
    PER =  .40     TIMES FOR MAXIMA --   TD =  7.6400   TV =  7.5000   TA =  7.6200
    PER =  .45     TIMES FOR MAXIMA --   TD = 12.9600   TV = 12.8400   TA = 12.9400
    PER =  .50     TIMES FOR MAXIMA --   TD =  7.6800   TV =  7.5200   TA =  7.6800
    PER =  .55     TIMES FOR MAXIMA --   TD =  7.7200   TV =  7.0000   TA =  7.7200
    PER =  .60     TIMES FOR MAXIMA --   TD =  8.1600   TV =  7.9200   TA =  8.0800
    PER =  .65     TIMES FOR MAXIMA --   TD =  8.5400   TV =  8.7400   TA =  8.5400
    PER =  .70     TIMES FOR MAXIMA --   TD =  6.4000   TV =  6.2200   TA =  6.3800
    PER =  .75     TIMES FOR MAXIMA --   TD =  6.4600   TV =  6.2800   TA =  6.4400
    PER =  .80     TIMES FOR MAXIMA --   TD =  8.6400   TV =  8.8400   TA =  8.6200
    PER =  .85     TIMES FOR MAXIMA --   TD =  8.7200   TV =  8.9200   TA =  8.7000
    PER =  .90     TIMES FOR MAXIMA --   TD =  8.8000   TV =  8.5800   TA =  8.7800
    PER =  .95     TIMES FOR MAXIMA --   TD =  8.4200   TV =  8.6400   TA =  8.4000
    PER = 1.00     TIMES FOR MAXIMA --   TD =  8.4600   TV =  8.2400   TA =  8.4400
    PER = 1.10     TIMES FOR MAXIMA --   TD =  8.0800   TV =  8.2800   TA =  8.0400
    PER = 1.20     TIMES FOR MAXIMA --   TD =  8.0800   TV =  8.3000   TA =  8.0800
    PER = 1.30     TIMES FOR MAXIMA --   TD =  8.0800   TV =  8.3000   TA =  8.0600
    PER = 1.40     TIMES FOR MAXIMA --   TD =  7.6400   TV =  7.8600   TA =  7.6000
    PER = 1.50     TIMES FOR MAXIMA --   TD =  7.6600   TV =  7.4400   TA =  7.6400
    PER = 1.60     TIMES FOR MAXIMA --   TD = 15.5400   TV =  7.6400   TA = 15.5000
    PER = 1.70     TIMES FOR MAXIMA --   TD = 15.5000   TV = 13.5000   TA = 15.5400
    PER = 1.80     TIMES FOR MAXIMA --   TD = 15.6400   TV = 13.5800   TA = 15.6200
    PER = 1.90     TIMES FOR MAXIMA --   TD =  7.6400   TV =  7.8600   TA =  7.6000
    PER = 2.00     TIMES FOR MAXIMA --   TD =  7.6400   TV =  7.0600   TA =  7.6200
    PER = 2.25     TIMES FOR MAXIMA --   TD = 13.4000   TV =  7.6400   TA = 13.3800
    PER = 2.50     TIMES FOR MAXIMA --   TD =  7.2200   TV =  7.4600   TA =  7.1800
    PER = 2.75     TIMES FOR MAXIMA --   TD =  7.2600   TV =  7.4600   TA =  7.2200
    PER = 3.00     TIMES FOR MAXIMA --   TD =  7.2800   TV =  8.3000   TA =  7.2200
    PER = 3.25     TIMES FOR MAXIMA --   TD =  5.9800   TV =  8.3000   TA =  5.8000
    PER = 3.50     TIMES FOR MAXIMA --   TD =  6.7200   TV =  5.7200   TA =  6.6800
    PER = 3.75     TIMES FOR MAXIMA --   TD =  6.7600   TV =  7.8600   TA =  6.7000
    PER = 4.00     TIMES FOR MAXIMA --   TD = 15.6600   TV =  7.8600   TA =  7.0000
```

```
SPECTRAL VALUES--
     PASADENA 1952                           EXAMPLE SITE                DAMPING RATIO =  .05

NO.    PERIOD      REL. DISP.       REL. VEL.    PSU.REL.VEL.        ABS. ACC.    PSU.ABS.ACC.     FREQ.
        SEC.             FT.        FT./SEC.        FT./SEC.             G.              G.       C/SEC.

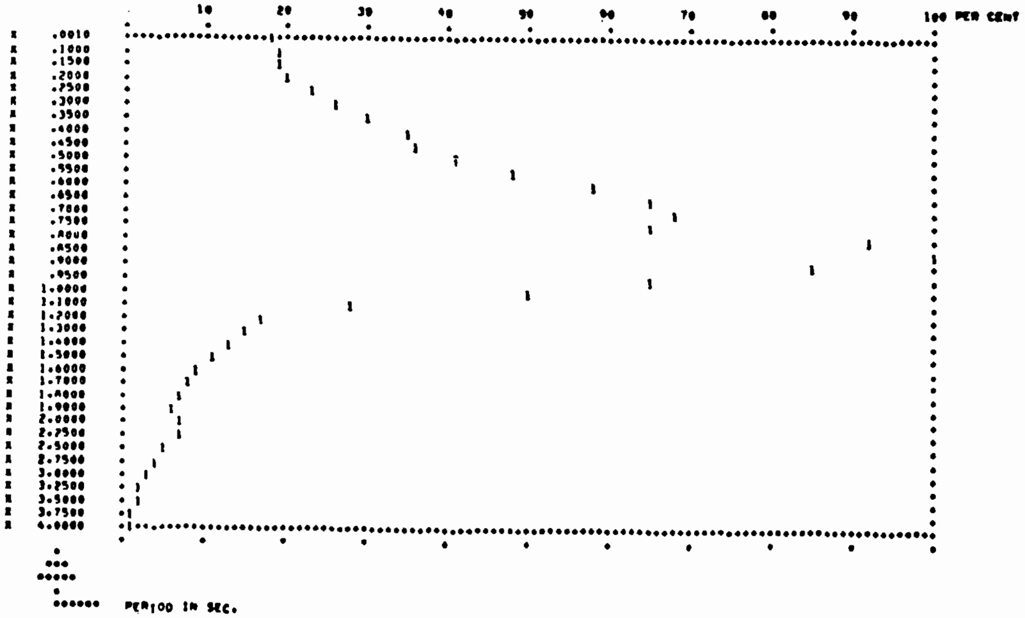
  1       .00          .00000          .00000          .00048           .09377          .09377   1000.00
  2       .10          .00081          .00014          .05072           .09873          .09897     10.00
  3       .15          .00185          .02296          .07735           .10091          .10063      6.67
  4       .20          .00345          .03620          .10836           .10566          .10572      5.00
  5       .25          .00604          .07353          .15182           .11054          .11050      4.00
  6       .30          .00989          .11405          .20716           .13406          .13474      3.33
  7       .35          .01581          .17500          .28386           .15011          .15826      2.86
  8       .40          .02400          .28000          .37693           .18536          .18308      2.50
  9       .45          .03136          .36363          .43780           .18944          .18904      2.22
 10       .50          .04387          .40265          .55135           .21407          .21517      2.00
 11       .55          .06112          .59161          .69825           .24075          .24773      1.82
 12       .60          .08913          .84720          .93333           .30520          .30353      1.67
 13       .65          .11627          .99404         1.12395           .33929          .33741      1.54
 14       .70          .14222         1.21203         1.27660           .35709          .35506      1.43
 15       .75          .15604         1.25725         1.30724           .34888          .34811      1.33
 16       .80          .25047         1.92676         1.96717           .48143          .47982      1.25
 17       .85          .30716         2.25299         2.27049           .52310          .52122      1.18
 18       .90          .29064         2.11149         2.02906           .44272          .44992      1.11
 19       .95          .24876         1.79494         1.64526           .33994          .33794      1.05
 20      1.00          .21183         1.47429         1.33099           .26168          .25972      1.00
 21      1.10          .14183         1.01303          .81012           .16463          .14371       .91
 22      1.20          .10527          .77513          .55120           .09044          .08953       .83
 23      1.30          .11061          .75453          .53460           .09095          .08024       .77
 24      1.40          .11116          .72513          .49890           .07027          .06954       .71
 25      1.50          .10594          .65305          .44375           .05841          .05773       .67
 26      1.60          .09658          .50143          .37928           .04863          .04628       .63
 27      1.70          .10319          .51595          .38137           .04418          .04377       .59
 28      1.80          .09583          .53842          .33170           .03627          .03596       .56
 29      1.90          .09467          .57469          .31307           .03270          .03215       .53
 30      2.00          .11765          .58054          .36901           .03063          .03606       .50
 31      2.25          .14161          .64769          .39545           .03475          .03430       .44
 32      2.50          .11789          .62395          .29629           .02350          .02313       .40
 33      2.75          .12101          .51216          .27832           .02024          .01975       .36
 34      3.00          .10076          .40393          .21102           .01421          .01373       .33
 35      3.25          .08392          .43928          .16224           .01019          .00974       .31
 36      3.50          .08840          .40992          .14447           .00828          .00805       .29
 37      3.75          .08033          .40336          .13450           .00722          .00700       .27
 38      4.00          .07855          .39609          .12338           .00619          .00602       .25

     VALUES IN PERIOD RANGE .1 TO 2.5 SEC.

          AREA OF ACC. RESPONSE SPECTRUM   =        .330
          AREA OF VEL. RESPONSE SPECTRUM   =       1.700
          MAX. ACCELERATION RESPONSE VALUE =        .523
          MAX. VELOCITY     RESPONSE VALUE =       2.253
```

```
PLOT OF ACCELERATION SPECTRA /

                                                    .   100 PER CENT CORRESPONDS TO    .5231
```



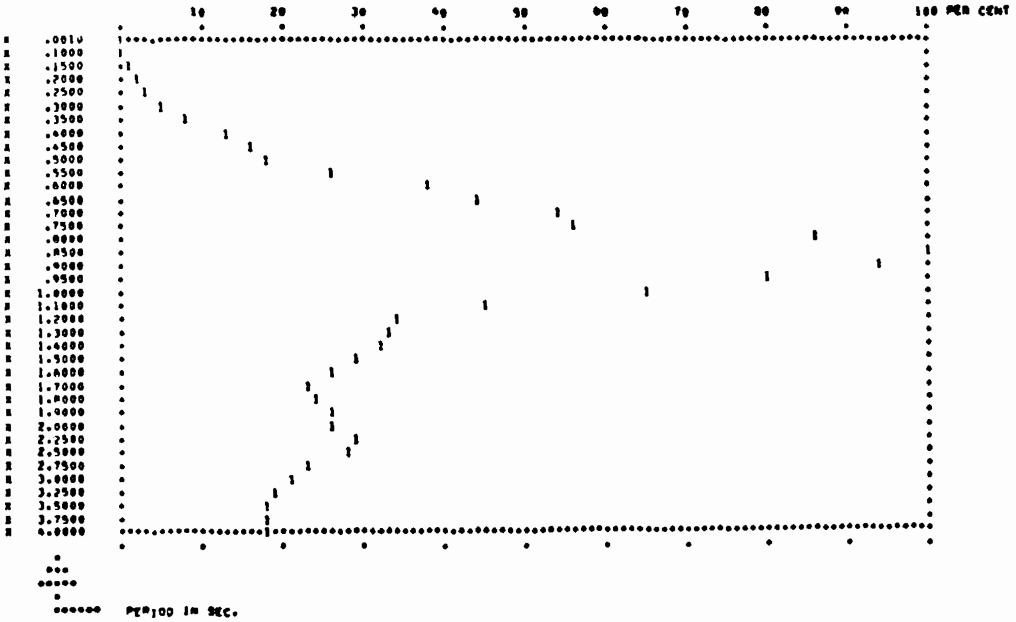
```
 CURVE 1  -   PASADENA 1952                              EXAMPLE SITE

   ABSISSA      CURVE 1
     .0010        .0930
     .1000        .0987
     .1500        .1009
     .2000        .1056
     .2500        .1105
     .3000        .1340
```

```
      .3500          .1581
      .4000          .1854
      .4500          .1894
      .5000          .2149
      .5500          .2487
      .6000          .3052
      .6500          .3393
      .7000          .3571
      .7500          .3400
      .8000          .4814
      .8500          .5231
      .9000          .4427
      .9500          .3399
     1.0000          .2615
     1.1000          .1440
     1.2000          .0984
     1.3000          .0810
     1.4000          .0703
     1.5000          .0584
     1.6000          .0466
     1.7000          .0441
     1.8000          .0363
     1.9000          .0320
     2.0000          .0300
     2.2500          .0340
     2.5000          .0236
     2.7500          .0202
     3.0000          .0142
     3.2500          .0102
     3.5000          .0083
     3.7500          .0072
     4.0000          .0062
```

```
PLOT OF VELOCITY SPECTRA

                                                          100 PER CENT CORRESPONDS TO   2.2530
```



```
 CURVE 1  -   PASADENA 1952                   EXAMPLE SITE
Y

   ABSCISSA      CURVE 1
     .0010         .0000
     .1000         .0001
     .1500         .0230
     .2000         .0383
     .2500         .0735
```

ACKNOWLEDGEMENTS

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Appendix D.2

EXAMPLE SHAKE PROBLEM

Given: Example Boring Log & Data (Fig. D.2.1) 0.35g peak outcrop acceleration. 0.2 sec predominant period in rock motion. (Use rock motion R01--Appendix D.3) For soil property/strain relationships, (Option 8), Use the Standard Option 8 data in Appendix D.1. (28 cards)

(1) Using the SHAKE Program:

  (a) Model the soil column using at least 10 layers.

  (b) Compute 5 percent damped spectra at:

    (1) rock outcrop

    (2) footing level

(2) Compute and plot a smooth amplification spectra (S) at the footing level by dividing the footing level response by the outcrop response:

Plot for at least the following periods (sec):

0.1, 0.15, 0.2, 0.3, 0.4,
0.6, 0.8, 1.0, 1.5, 2.0,
3.0, 4.0 and 5.0 sec.

(3) Compute and plot a smooth elastic design spectra (A.R.S), using A=0.35g, the computed S spectra and the normalized rock spectrum (R) used in AASHTO [Interim 1975 pg.21). Compare this computed spectra with Standard AASHTO elastic spectra.

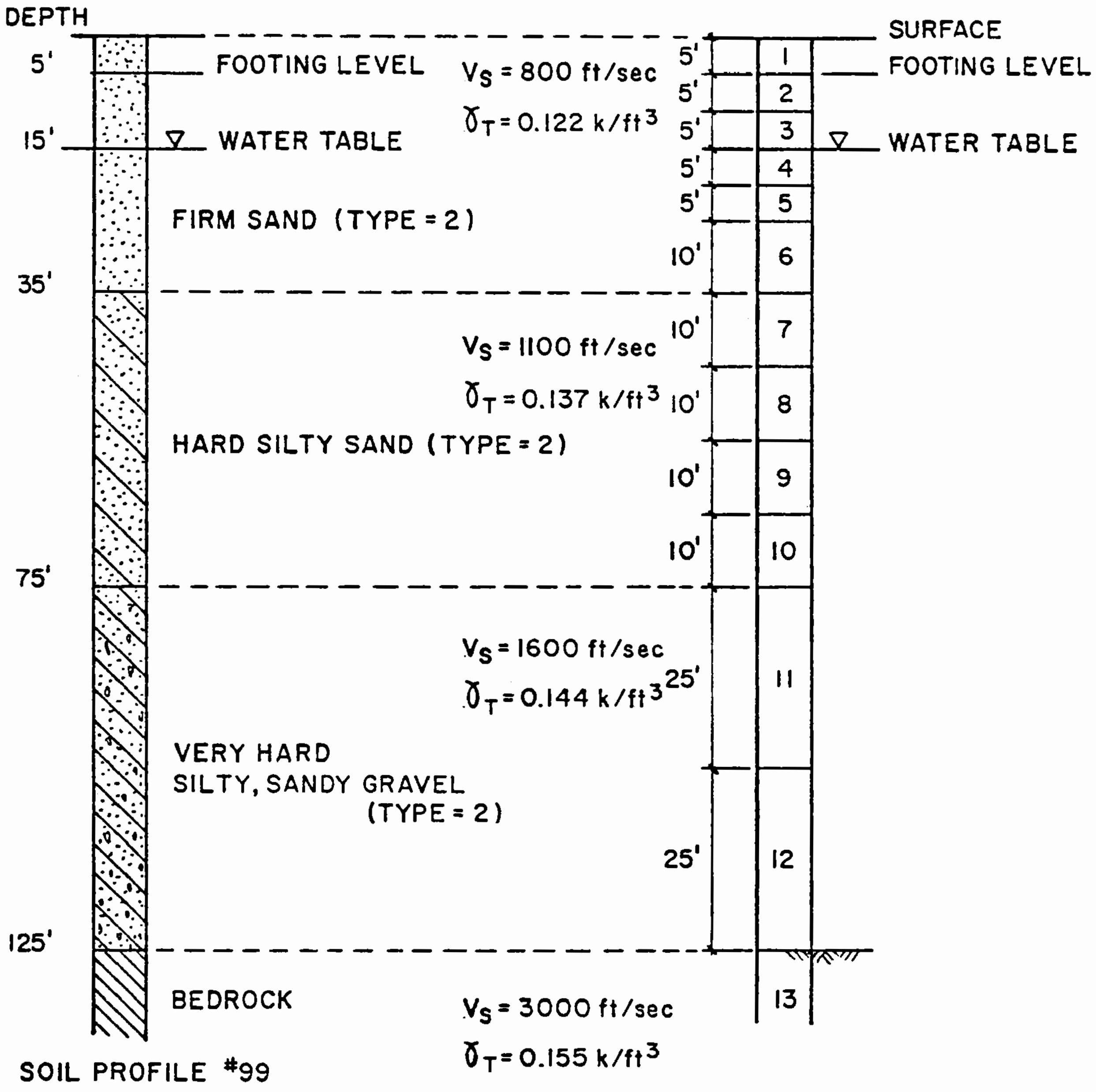

Figure D.2.1

EXAMPLE BORING LOG AND SOIL COLUMN MODEL

```
a)  4096     0.400 EXAMPLE SHAKE PROBLEM
b)                {INSERT OPTION B .. SOIL PROPERTY/STRAIN RELATIONS DATA HERE}
c)     1
     800 4096      0.02 ROCK MOTION R01
                   0.35
                   {INSERT ROCK MOTION TIME HISTORY DATA CARDS HERE}
```

HCS 90 - 062

EXAMPLE SHAKE PROB.

INPUT CODING 1/3

J. GATES
10/78

a) HEADER CARD
b) OPTION 8 (SOIL PROP/STRAIN RELATIONS)
c) OPTION 1 (INPUT MOTIONS)

```
d)
    2
   99   13    3EXAMPLE 125 FT COL
    1    2    1        5.                  .1       .122      800.
    2    2    1        5.                  .1       .122      800.
    3    2    1        5.                  .1       .122      800.
    4    2    1        5.                  .1       .122      800.
    5    2    1        5.                  .1       .122      800.
    6    2    1       10.                  .1       .122      800.
    7    2    1       10.                  .1       .137     1100.
    8    2    1       10.                  .1       .137     1100.
    9    2    1       10.                  .1       .137     1100.
   10    2    1       10.                  .1       .137     1100.
   11    2    1       25.                  .1       .144     1600.
   12    2    1       25.                  .1       .144     1600.
   13                                               .155     3000.
e)
    3
   13    0
f)
    4
    0   10       5.0       0.65
```

HCS 90 – 062

EXAMPLE SHAKE PROBLEM

INPUT CODING 3/3

J. GATES
10/78

d) OPTION 2 -- READ SOIL DATA
e) OPTION 3 -- ROCK MOTION LOCATION
f) OPTION 4 -- ITERATE FOR SOIL PROPS

	1–5	6–10	11–15	16–20	21–25	26–30
g)	5					
	1	2	7	11	13	13
	0	1	1	1	1	0
	0	0	0	0	0	0
h)	9					
	0	0				
	1	0	2	0	2	
		0.05				
i)	9					
	2	1				
	1	0	2	1	2	
		0.05				

HCS 90 – 062

EXAMPLE SHAKE PROBLEM

INPUT CODING 3/3

D. GATES
10/78

g) OPTION 5 COMPUTE MOTIONS
h) OPTION 9 COMPUTE ROCK SPECTRA
i) OPTION 9 COMPUTE SPECTRA AT FOOTING LEVEL

```
4096     0.400 EXAMPLE SHAKE PROBLEM
   8
   3    1    10    50.
  11 .01   SHEAR MODULUS CLAY MAY 24 - 1972
 .0001     .000316     .001     .00316      .01      .0316      .1          .316
  1.          3.16       10.
 2300.      2100.      1750.     1300.       920.       600.       350.       175.
 84.         30.        10.
  10  1.   DAMPING CLAY  MAY 24 - 1972
   .0001      .001      .00316    .01           .0316    .1        .316       1.
 3.16        10.
   2.0       2.5        3.5         4.75      6.5        9.25      13.75      20.
26.         29.
  11 .5    SHEAR MODULUS SAND SQ.ROOT REL. MAY 24-1972
 .0001     .000316     .001      .00316       .01       .0316     .1         .316
 1.           3.16       10.
  61.        60.        57.         50.4     40.         27.       15.         7.
  3.         3.          3.
   9  1.   DAMPING SAND  FEBRUARY 1971
   .0001      .001       .003       .01        .03        .1        .3         1.
  10.
 1.          1.6        3.12       5.8        9.5        15.4      20.9       25.
  25.5
   8 .01   ATTENUATION OF ROCK     AVERAGE   APRIL 1971
.0001      .0003      .001       .003       .01        .03           .1         1.
 2000.     2000.      1975.     1905.      1800.      1620.         1450.   1100.
   5 1.  DAMPING IN ROCK    AVERAGE  9/4    APRIL 1971
.0001     .001       .01          .1         1.
   .4       .8        1.5          3.          4.6
   1
 800 4096        0.02 ROCK MOTION P01
                 0.35
 0.00000 -0.27150 -0.34554 -0.29373 -0.12001  0.06156  0.17035  0.22385        1
 0.27278  0.32047  0.32368  0.24579  0.08893 -0.10570 -0.26241 -0.29245        2
-0.15948  0.05643  0.19371  0.14928 -0.01776 -0.15199 -0.17340 -0.14936        3
-0.17842 -0.24166 -0.22413 -0.08082  0.08188  0.13341  0.09207  0.10247        4
 0.23049  0.34664  0.27265  0.02147 -0.18090 -0.14075  0.08126  0.23995        5
 0.17776 -0.02733 -0.19083 -0.23579 -0.22831 -0.23337 -0.21517 -0.10865        6
 0.06445  0.20982  0.27602  0.29560  0.29679  0.23335  0.05765 -0.16062        7
-0.26212 -0.17117  0.00614  0.10473  0.09638  0.09973  0.18805  0.25907        8
 0.16008 -0.08452 -0.25462 -0.17716  0.06397  0.21711  0.14245 -0.06033        9
-0.20849 -0.24376 -0.22354 -0.15586  0.03907  0.35439  0.56765  0.40821       10
-0.11793 -0.64881 -0.78830 -0.50018 -0.12992 -0.02727 -0.19679 -0.35068       11
-0.26291 -0.00433  0.17454  0.12727 -0.06839 -0.23950 -0.29852 -0.28006       12
-0.24984 -0.22652 -0.18470 -0.09405  0.05623  0.24577  0.42040  0.52367       13
 0.55521  0.58044  0.64406  0.67434  0.52467  0.15201 -0.26841 -0.46615       14
-0.33819 -0.05927  0.09828  0.03564 -0.11588 -0.19093 -0.17528 -0.15943       15
-0.15991 -0.07578  0.15330  0.41151  0.49934  0.35545  0.12821  0.00477       16
 0.01322  0.03492 -0.01685 -0.09863 -0.13019 -0.12133 -0.14344 -0.19392       17
-0.16322  0.02796  0.29833  0.46300  0.41334  0.20050 -0.05644 -0.28632       18
-0.45688 -0.50476 -0.36576 -0.09594  0.11107  0.10253 -0.05049 -0.11858       19
 0.01393  0.19802  0.19565 -0.02197 -0.22867 -0.21678 -0.04966  0.02445       20
-0.11496 -0.31822 -0.36412 -0.23166 -0.09727 -0.08087 -0.08973  0.03506       21
 0.27540  0.42991  0.35933  0.16287  0.05505  0.10337  0.16221  0.07756       22
-0.10940 -0.21689 -0.15289 -0.02344  0.00751 -0.08218 -0.15958 -0.10876       23
 0.03067  0.11233  0.04343 -0.12173 -0.22996 -0.15150  0.12094  0.44780       24
 0.61335  0.47728  0.11114 -0.22620 -0.30613 -0.15351 -0.02599 -0.13592       25
-0.40433 -0.53125 -0.31795  0.11749  0.46673  0.53702  0.38517  0.18939       26
-0.05173 -0.30024 -0.53184 -0.58829 -0.36450  0.02569  0.31664  0.33406       27
 0.13827 -0.07749 -0.18328 -0.17712 -0.10045  0.02916  0.18087  0.27189       28
```

```
 0.23547  0.11987  0.05784  0.11135  0.18779  0.15311  0.00623 -0.11573      29
-0.11338 -0.04892 -0.05181 -0.13725 -0.18907 -0.12981 -0.03480 -0.01976      30
-0.05460  0.01993  0.26487  0.50660  0.50562  0.24604 -0.02498 -0.07487      31
 0.04558  0.09392 -0.04735 -0.22583 -0.23269 -0.09081 -0.03726 -0.21457      32
-0.46365 -0.48644 -0.18609  0.22047  0.43464  0.36417  0.15187 -0.02765      33
-0.12431 -0.16924 -0.16137 -0.06881  0.07579  0.15523  0.08214 -0.09206      34
-0.21773 -0.19291 -0.04755  0.10884  0.18969  0.17542  0.09676  0.01399      35
-0.00164  0.08429  0.22051  0.30124  0.27780  0.22011  0.22465  0.26378      36
 0.18419 -0.10232 -0.45833 -0.61998 -0.47766 -0.20937 -0.08651 -0.16687      37
-0.23997 -0.09199  0.22900  0.46299  0.43317  0.23036  0.08562  0.10807      38
 0.19514  0.18155  0.01879 -0.20385 -0.36045 -0.36656 -0.19442  0.10808      39
 0.38946  0.45110  0.20905 -0.19426 -0.49132 -0.53123 -0.39817 -0.27287      40
-0.21609 -0.14311  0.00190  0.13162  0.13105  0.04078  0.03146  0.17212      41
 0.29965  0.19600 -0.12335 -0.38124 -0.33455 -0.04545  0.20052  0.22174      42
 0.11018  0.05929  0.10502  0.12038  0.04207 -0.00776  0.11181  0.32918      43
 0.39667  0.18199 -0.14476 -0.27736 -0.10819  0.16006  0.24482  0.07559      44
-0.17328 -0.29950 -0.26078 -0.14991 -0.05232  0.01919  0.07320  0.09807      45
 0.03590 -0.07243 -0.19360 -0.28742 -0.32047 -0.25113 -0.05973  0.18853      46
 0.34974  0.31828  0.13782 -0.03847 -0.10849 -0.11139 -0.13931 -0.19575      47
-0.19683 -0.11279 -0.04417 -0.10755 -0.27243 -0.37815 -0.32796 -0.20782      48
-0.15962 -0.16461 -0.03365  0.35087  0.82997  1.00000  0.87879  0.38688      49
-0.09377 -0.35308 -0.40403 -0.37002 -0.32879 -0.28431 -0.22939 -0.17113      50
-0.10295 -0.00837  0.08908  0.12353  0.07203  0.01246  0.03809  0.11426      51
 0.08874 -0.10593 -0.30262 -0.23825  0.13327  0.53128  0.61673  0.35474      52
 0.04591 -0.01608  0.14542  0.27022  0.20515  0.07388  0.08107  0.22991      53
 0.32105  0.21538  0.00327 -0.13241 -0.15710 -0.20006 -0.33460 -0.43738      54
-0.33344 -0.04357  0.19880  0.20190  0.03339 -0.06243  0.05897  0.29998      55
 0.45217  0.41202  0.23457  0.01862 -0.18908 -0.36574 -0.45355 -0.39103      56
-0.20108 -0.00559  0.06898 -0.00937 -0.17127 -0.30640 -0.31565 -0.14573      57
 0.16122  0.43994  0.48672  0.24098 -0.11575 -0.29563 -0.17409  0.09348      58
 0.24752  0.19780  0.08445  0.07539  0.16349  0.20504  0.12226 -0.00547      59
-0.06725 -0.07471 -0.13042 -0.26247 -0.36272 -0.32108 -0.17685 -0.08121      60
-0.11686 -0.20734 -0.22416 -0.14094 -0.03963  0.01389  0.04154  0.08664      61
 0.12712  0.10242  0.00902 -0.08315 -0.11787 -0.10910 -0.08360 -0.01331      62
 0.12854  0.25295  0.18615 -0.11927 -0.44372 -0.46857 -0.12107  0.30736      63
 0.45344  0.27543  0.08522  0.17166  0.46002  0.61060  0.42500  0.06406      64
-0.16835 -0.18408 -0.16978 -0.30496 -0.51911 -0.58688 -0.41447 -0.14668      65
 0.01823  0.04651  0.05291  0.11060  0.15663  0.10009 -0.03933 -0.15277      66
-0.17613 -0.14252 -0.09449 -0.00337  0.17512  0.38598  0.47524  0.33143      67
 0.00590 -0.32528 -0.50340 -0.47843 -0.29383 -0.03889  0.17365  0.24120      68
 0.14692  0.00101 -0.04467  0.04861  0.16305  0.16573  0.07342  0.02895      69
 0.10872  0.20794  0.16324 -0.03370 -0.20656 -0.19452 -0.03232  0.11444      70
 0.16629  0.21324  0.35968  0.54237  0.56863  0.33626 -0.03488 -0.31512      71
-0.38035 -0.29059 -0.18496 -0.15133 -0.19785 -0.28626 -0.34297 -0.26481      72
-0.00843  0.31948  0.49169  0.35922  0.01784 -0.26420 -0.29858 -0.12893      73
 0.07466  0.21584  0.30692  0.33315  0.19211 -0.15803 -0.53888 -0.65484      74
-0.40306 -0.02007  0.15458  0.04728 -0.08995 -0.00755  0.22844  0.32604      75
 0.14439 -0.11873 -0.16540  0.03672  0.21599  0.12878 -0.14794 -0.30909      76
-0.17022  0.14317  0.35556  0.32590  0.13648 -0.05188 -0.15726 -0.18525      77
-0.15413 -0.07159  0.03461  0.12429  0.20051  0.31509  0.48277  0.61178      78
 0.56720  0.31406 -0.03249 -0.31517 -0.46803 -0.52158 -0.50643 -0.41021      79
-0.24345 -0.09427 -0.05979 -0.12848 -0.18072 -0.12682 -0.00960  0.06321      80
 0.06229  0.05685  0.08648  0.07621 -0.06378 -0.27554 -0.36492 -0.21169      81
 0.07051  0.23758  0.17516  0.01102 -0.04671  0.05796  0.18952  0.20434      82
 0.09576 -0.03129 -0.09295 -0.08517 -0.03771  0.02722  0.07576  0.05578      83
-0.04089 -0.12873 -0.09858  0.04851  0.18282  0.19507  0.13052  0.11913      84
 0.18117  0.17876  0.00897 -0.20653 -0.21852  0.04199  0.32920  0.32999      85
 0.02690 -0.26345 -0.23933  0.05873  0.32456  0.16250  0.23679  0.15668      86
 0.25986  0.26021  0.08447 -0.15357 -0.25871 -0.21659 -0.21428 -0.30354      87
-0.60189 -0.61833 -0.35717 -0.02754  0.09431 -0.04607 -0.24018 -0.23825      88
```

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 0.02541  0.40835  0.69521  0.73196  0.48516  0.05211 -0.37988 -0.59051      89
-0.54847 -0.35707 -0.17867 -0.07054  0.02581  0.14777  0.21907  0.13924      90
-0.05672 -0.19376 -0.14201  0.02822  0.12093  0.03514 -0.13179 -0.20981      91
-0.14106 -0.00624  0.09918  0.14975  0.14282  0.03702 -0.19193 -0.44635      92
-0.53382 -0.34878 -0.00504  0.25753  0.30018  0.18950  0.07689  0.02378      93
-0.01749 -0.08199 -0.12003 -0.06858  0.05072  0.15121  0.18374  0.17396      94
 0.16547  0.14597  0.11170  0.11574  0.21826  0.37382  0.43652  0.32484      95
 0.14394  0.07266  0.12611  0.10933 -0.13330 -0.46519 -0.55035 -0.23056      96
 0.25531  0.49909  0.35292  0.06054 -0.05457  0.04214  0.10642 -0.04353      97
-0.30072 -0.42692 -0.36332 -0.28650 -0.35283 -0.47784 -0.43696 -0.15276      98
 0.19469  0.35962  0.27784  0.08704 -0.05730 -0.11909 -0.13656 -0.11369      99
-0.01483  0.15423  0.32075  0.41892  0.45251  0.45668  0.42391  0.31122     100
  2
 99   13    3 EXAMPLE 125 FT COL
  1    2    1         5.                   .1        .122       800.
  2    2    1         5.                   .1        .122       800.
  3    2    1         5.                   .1        .122       800.
  4    2    1         5.                   .1        .122       800.
  5    2    1         5.                   .1        .122       800.
  6    2    1        10.                   .1        .122       800.
  7    2    1        10.                   .1        .137      1100.
  8    2    1        10.                   .1        .137      1100.
  9    2    1        10.                   .1        .137      1100.
 10    2    1        10.                   .1        .137      1100.
 11    2    1        25.                   .1        .144      1600.
 12    2    1        25.                   .1        .144      1600.
 13                                                  .155      3000.
  3
 13    0
  4
  0   10       5.0       0.65
  5
  1    2    7   11   13   13
  0    1    1    1    1    0
  0    0    0    0    0    0
  9
  0    0
  1    0    2    0    2
    0.05
  9
  2    1
  1    0    2    1    2
    0.05
```

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MAX. NUMBER OF TERMS IN FOURIER TRANSFORM =      4096
NECESSARY LENGTH OF BLANK COMMON X        =     25619
EARTH PRESSURE AT REST FOR SAND           =     0.400




****** OPTION  8  ***  READ RELATION BETWEEN SOIL PROPERTIES AND STRAIN

   CURVES FOR RELATION STRAIN VERSUS SHEAR MODULUS AND DAMPING


 MODULUS AND DAMPING VALUES ARE SCALED FOR PLOTTING

SHEAR MODULUS CLAY MAY 24 - 1972                            MULTIPLICATION FACTOR =   0.010
DAMPING CLAY  MAY 24 - 1972                                 MULTIPLICATION FACTOR =   1.000
SHEAR MODULUS SAND SQ.ROOT REL. MAY 24-1972                 MULTIPLICATION FACTOR =   0.500
DAMPING SAND  FEBRUARY 1971                                 MULTIPLICATION FACTOR =   1.000
ATTENUATION OF ROCK     AVERAGE    APRIL 1971               MULTIPLICATION FACTOR =   0.010
AMPING IN ROCK    AVERAGE  9/4     APRIL 1971               MULTIPLICATION FACTOR =   1.000
```

```
                                                                                 100 PER CENT CORRESPONDS TO  50.0000
           0        10        20        30        40        50        60        70        80        90        100 PER CENT
           +         +         +         +         +         +         +         +         +         +         +
 0.0001    +64+2+++++++++++++++++++++++++++++++++++5++++1+++++++++++++++3+++++++++++++++++++++++++++++++++++++++
 0.0002    +64 2                                   5   1                3                                      +
 0.0002    +64 2                                   5   1               3                                       +
 0.0003    +64 2                                   5  1                3                                       +
 0.0003    +6 42                                   5 1                 3                                       +
 0.0004    +6 4 2                                  51                 3                                        +
 0.0005    +6 4 2                                 15                  3                                        +
 0.0006    +6 4 2                                1 5                 3                                         +
 0.0008    + 64 2                              1   5                 3                                         +
 0.0010    + 64 2                             1   5                 3                                          +
 0.0013    + 6 42                           1     5                3                                           +
 0.0016    + 6 4 2                        1       5              3                                             +
 0.0020    + 6  42                       1        5             3                                              +
 0.0025    + 6   42                    1         5             3                                               +
 0.0032    + 6   42                  1           5           3                                                 +
 0.0040    + 6    42               1             5         3                                                   +
 0.0050    +  6    24             1             5        3                                                     +
 0.0063    +  6     24          1               5      3                                                       +
 0.0079    +  6     2 4        1               5     3                                                         +
 0.0100    +  6     2  4     1                 5   3                                                           +
 0.0126    +  6      2  4   1                 5 3                                                              +
 0.0158    +   6      2   41                 53                                                                +
 0.0200    +   6       2  14               3 5
 0.0251    +   6       21    4           3  5                     CURVE 1  -   SHEAR MODULUS CLAY MAY 24 - 1972
 0.0316    +   6       12      4      3    5                      CURVE 2  -   DAMPING CLAY  MAY 24 - 1972
 0.0398    +    6     1  2       4  3      5                      CURVE 3  -   SHEAR MODULUS SAND SQ.ROOT REL. MAY 24-1972
 0.0501    +    6    1    2      3 4      5                       CURVE 4  -   DAMPING SAND  FEBRUARY 1971
 0.0631    +    6   1      2   3     4   5                        CURVE 5  -   ATTENUATION OF ROCK    AVERAGE   APRIL 1971
 0.0794    +     6 1        X           45                        CURVE 6  -  DAMPING IN ROCK   AVERAGE  9/4    APRIL 1971
 0.1000    +     61       3  2          5 4
 0.1259    +     X      3      2       5    4                                                                  +
 0.1585    +     16    3         2     5      4                                                                +
 0.1995    +    1 6  3             2  5          4                                                             +
 0.2512    +   1  6 3                X             4                                                           +
 0.3162    +  1   36                5  2             4                                                         +
 0.3981    +  1  3 6                5    2             4                                                       +
 0.5012    +  1 3  6               5        2           4                                                      +
 0.6310    + 1  3   6             5           2           4                                                    +
 0.7943    + 1 3    6             5              2         4                                                   +
 1.0000    + 13     6            5                 2         4                                                 +
 1.2589    +1 3      6          5                    2       4                                                 +
 1.5849    +1 3      6          5                       2    4                                                 +
 1.9953    +1 3      6         5                          2  4                                                 +
 2.5119    +1 3      6        5                              X                                                 +
 3.1623    +1 3       6      5                               4 2                                               +
 3.9811    +1 3       6      5                                4 2                                              +
 5.0119    1  3       6     5                                 4  2                                             +
 6.3096    1  3        6   5                                  4    2                                           +
 7.9433    1  3        6   5                                  4     2                                          +
10.0000    1  3        6  5                                   4      2                                         +
12.5892    1++3+++++++++65++++++++++++++++++++++++++++++++++++4+++++++2+++++++++++++++++++++++++++++++++++++++++
           +         +         +         +         +         +         +         +         +         +         +
   +
  +++
 +++++
   +
   ++++++   STRAIN IN PERCENT
```

```
ABSISSA     CURVE 1    CURVE 2    CURVE 3    CURVE 4    CURVE 5    CURVE 6
 0.0001     22.5997     2.0500    30.3999     1.0600    20.0000     0.4400
 0.0002     22.1995     2.1000    30.2999     1.1200    20.0000     0.4800
 0.0002     21.7992     2.1500    30.1998     1.1800    20.0000     0.5200
 0.0003     21.3990     2.2000    30.0997     1.2400    20.0000     0.5600
 0.0003     20.9978     2.2500    29.9991     1.3000    19.9891     0.6000
 0.0004     20.2982     2.3000    29.6992     1.3600    19.9412     0.6400
 0.0005     19.5987     2.3500    29.3994     1.4200    19.8934     0.6800
 0.0006     18.8991     2.4000    29.0996     1.4800    19.8456     0.7200
 0.0008     18.1996     2.4500    28.7998     1.5400    19.7978     0.7600
 0.0010     17.5000     2.5000    28.5000     1.6000    19.7500     0.8000
 0.0013     16.5994     2.7001    27.8396     1.9186    19.6033     0.8700
 0.0016     15.6989     2.9002    27.1792     2.2372    19.4566     0.9400
 0.0020     14.7983     3.1004    26.5188     2.5557    19.3099     1.0100
 0.0025     13.8977     3.3005    25.8583     2.8743    19.1631     1.0800
 0.0032     12.9976     3.5008    25.1967     3.2373    19.0041     1.1500
 0.0040     12.2381     3.7506    24.1574     3.7498    18.8032     1.2200
 0.0050     11.4786     4.0005    23.1181     4.2624    18.6024     1.2900
 0.0063     10.7190     4.2503    22.0787     4.7749    18.4016     1.3600
 0.0079      9.9595     4.5002    21.0394     5.2874    18.2008     1.4300
 0.0100      9.2000     4.7500    20.0000     5.8000    18.0000     1.5000
 0.0126      8.5596     5.1002    18.6992     6.5755    17.6227     1.6500
 0.0158      7.9192     5.4504    17.3984     7.3510    17.2455     1.8000
 0.0200      7.2788     5.8007    16.0976     8.1264    16.8682     1.9500
 0.0251      6.6384     6.1509    14.7967     8.9019    16.4909     2.1000
 0.0316      5.9984     6.5017    13.4963     9.7581    16.1256     2.2500
 0.0398      5.4988     7.0514    12.2970    10.8865    15.8005     2.4000
 0.0501      4.9991     7.6010    11.0978    12.0149    15.4754     2.5500
 0.0631      4.4994     8.1507     9.8985    13.1432    15.1502     2.7000
 0.0794      3.9997     8.7003     8.6993    14.2716    14.8251     2.8500
 0.1000      3.5000     9.2500     7.5000    15.4000    14.5000     3.0000
 0.1259      3.1498    10.1506     6.6995    16.5527    14.1500     3.1600
 0.1585      2.7996    11.0511     5.8990    17.7055    13.8000     3.3200
 0.1995      2.4493    11.9517     5.0985    18.8582    13.4500     3.4800
 0.2512      2.0991    12.8522     4.2980    20.0110    13.1000     3.6400
 0.3162      1.7494    13.7539     3.4988    21.0794    12.7500     3.8000
 0.3981      1.5675    15.0031     3.0990    21.8635    12.4000     3.9600
 0.5012      1.3857    16.2523     2.6993    22.6476    12.0500     4.1200
 0.6310      1.2038    17.5016     2.2995    23.4317    11.7000     4.2800
 0.7943      1.0219    18.7508     1.8998    24.2159    11.3500     4.4400
 1.0000      0.8400    20.0000     1.5000    25.0000    11.0000     4.6000
 1.2589      0.7319    21.2007     1.5000    25.0500    10.6500     4.7600
 1.5849      0.6239    22.4015     1.5000    25.1000    10.3000     4.9200
 1.9953      0.5158    23.6022     1.5000    25.1500     9.9500     5.0800
 2.5119      0.4077    24.8030     1.5000    25.2000     9.6000     5.2400
 3.1623      0.2999    26.0019     1.5000    25.2500     9.2500     5.4000
 3.9811      0.2599    26.6015     1.5000    25.3000     8.9000     5.5600
 5.0119      0.2199    27.2011     1.5000    25.3500     8.5500     5.7200
 6.3096      0.1800    27.8007     1.5000    25.4000     8.2000     5.8800
 7.9433      0.1400    28.4004     1.5000    25.4500     7.8500     6.0400
10.0000      0.1000    29.0000     1.5000    25.5000     7.5000     6.2000
12.5892      0.0600    29.5996     1.5000    25.5500     7.1500     6.3600
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******   CPTION  1  ***  READ INPUT MOTION

EARTHQUAKE  -   ROCK MOTION R01

800 ACCELERATION VALUES AT TIME INTERVAL     0.0200

   THE VALUES ARE LISTED ROW BY ROW AS READ FROM CARDS
   TRAILING ZEROS ARE ADDED TO GIVE A TOTAL OF 4096 VALUES
    1          0.000000    -0.271500    -0.345540    -0.293730    -0.120010     0.061560     0.170350     0.223850
    2          0.272760     0.320470     0.323680     0.245790     0.088930    -0.105700    -0.262410    -0.292450
    3         -0.159480     0.056430     0.193710     0.149280    -0.017760    -0.151990    -0.173400    -0.149360
    4         -0.178420    -0.241660    -0.224130    -0.080820     0.061880     0.133410     0.092070     0.102470
    5          0.230490     0.346640     0.272650     0.021470    -0.180900    -0.140750     0.081280     0.239950
    6          0.177760    -0.027330    -0.190830    -0.235790    -0.228310    -0.233370    -0.215170    -0.108650
    7          0.064450     0.209820     0.276020     0.295600     0.296790     0.233350     0.057650    -0.160620
    8         -0.262120    -0.171170     0.006140     0.104730     0.096380     0.099730     0.188050     0.253070
    9          0.160080    -0.084520    -0.254620    -0.177160     0.063970     0.217110     0.142450    -0.560370
   10         -0.208490    -0.243760    -0.223540    -0.155860     0.039070     0.354390     0.567650     0.408210
   11         -0.117030    -0.648810    -0.788300    -0.500180    -0.129920    -0.027270    -0.196790    -0.350600
   12         -0.262910    -0.004330     0.174540     0.127270    -0.068390    -0.239500    -0.298520    -0.280060
   13         -0.249840    -0.226520    -0.184700    -0.094050     0.056230     0.245770     0.420400     0.523670
   14          0.555210     0.560440     0.644060     0.674340     0.524670     0.152010    -0.268410    -0.466150
   15         -0.338190    -0.059270     0.098280     0.035640    -0.115880    -0.190930    -0.175280    -0.159430
   16         -0.159910    -0.075780     0.153300     0.411510     0.499340     0.355450     0.128210     0.004770
   17          0.013220     0.034920    -0.016850    -0.098630    -0.130190    -0.121330    -0.143440    -0.193920
   18         -0.163220     0.027960     0.298330     0.463000     0.413340     0.200500    -0.056440    -0.186320
   19         -0.456860    -0.504760    -0.365760    -0.095940     0.111070     0.102530    -0.050490    -0.118580
   20          0.013930     0.198020     0.195650    -0.021970    -0.228670    -0.216780    -0.049660     0.024450
   21         -0.114960    -0.318220    -0.364120    -0.231660    -0.097270    -0.080870    -0.089730     0.035060
   22          0.275400     0.429910     0.359330     0.162870     0.055050     0.103370     0.162210     0.077560
   23         -0.109400    -0.216890    -0.152890    -0.023440     0.007510    -0.082180    -0.159580    -0.108760
   24          0.030670     0.112330     0.043430    -0.121730    -0.229960    -0.151500     0.120940     0.447800
   25          0.613350     0.477280     0.111140    -0.226200    -0.306130    -0.153510    -0.025990    -0.135920
   26         -0.404330    -0.531250    -0.317950     0.117490     0.466730     0.537020     0.385170     0.168390
   27         -0.051730    -0.300240    -0.531840    -0.588290    -0.364500     0.025690     0.316640     0.334060
   28          0.138270    -0.077490    -0.183280    -0.177120    -0.100450     0.029160     0.180870     0.371890
   29          0.235470     0.119870     0.057840     0.111350     0.187790     0.153110     0.006230    -0.115730
   30         -0.113380    -0.048920    -0.051810    -0.137250    -0.189070    -0.129810    -0.034800    -0.019760
   31         -0.054600     0.019930     0.264870     0.506600     0.505620     0.246040    -0.024280    -0.074870
   32          0.045560     0.093920    -0.047350    -0.225830    -0.232690    -0.090810    -0.037260    -0.214570
   33         -0.463650    -0.486440    -0.186090     0.220470     0.434640     0.364170     0.151870    -0.027650
   34         -0.124310    -0.149240    -0.161370    -0.068810     0.075790     0.155230     0.082140    -0.092000
   35         -0.217730    -0.192910    -0.047550     0.106840     0.189690     0.175420     0.096760     0.013950
   36         -0.001640     0.084290     0.220510     0.301240     0.277800     0.220110     0.224650     0.263780
   37          0.184190    -0.102320    -0.458330    -0.619980    -0.477660    -0.209370    -0.086510    -0.166870
   38         -0.239970    -0.091990     0.229000     0.462490     0.433170     0.230360     0.085620     0.108070
   39          0.155140     0.181550     0.018790    -0.203850    -0.360450    -0.366560    -0.194420     0.108080
   40          0.389460     0.451100     0.209050    -0.194260    -0.491320    -0.531230    -0.396170    -0.272870
   41         -0.216090    -0.143110     0.001900     0.131620     0.131050     0.040780     0.031460     0.172120
   42          0.299650     0.196000    -0.123350    -0.381240    -0.334550    -0.045450     0.200520     0.221740
   43          0.110180     0.059290     0.105020     0.120380     0.042070    -0.007760     0.111810     0.329180
   44          0.396670     0.181590    -0.144760    -0.277360    -0.108190     0.160060     0.244820     0.075550
   45         -0.173280    -0.299500    -0.260780    -0.149910    -0.052320     0.019190     0.073200     0.088070
   46          0.035900    -0.072430    -0.193600    -0.287420    -0.320470    -0.251130    -0.059730     0.188530
   47          0.349740     0.318280     0.137820    -0.038470    -0.108490    -0.111390    -0.139310    -0.195750
   48         -0.196830    -0.112790    -0.044170    -0.107550    -0.272430    -0.378150    -0.327960    -0.207820
   49         -0.159620    -0.164610    -0.033650     0.350870     0.829970     1.000000     0.878790     0.786890
   50         -0.093770    -0.353080    -0.404030    -0.370020    -0.328790    -0.284310    -0.229390    -0.171130
   51         -0.102950    -0.008370     0.089080     0.123530     0.072030     0.012460     0.038090     0.114260
   52          0.068740    -0.105930    -0.302620    -0.238250     0.133270     0.531280     0.616730     0.354740
   53          0.045910    -0.016080     0.145420     0.270220     0.205150     0.073880     0.081070     0.229910
   54          0.321050     0.215380     0.003270    -0.132410    -0.157100    -0.200060    -0.334600    -0.437380
   55         -0.333440    -0.043570     0.198800     0.201900     0.033350    -0.062430     0.059970     0.299980
   56          0.452170     0.412020     0.234570     0.018620    -0.189080    -0.365740    -0.453550    -0.391030
   57         -0.201080    -0.005590     0.068980    -0.009370    -0.171270    -0.306400    -0.315650    -0.145770
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 58      0.161220    0.439940    0.486720    0.240980   -0.115750   -0.295630   -0.174090    0.093480
 59      0.247520    0.197800    0.084450    0.075290    0.163490    0.205040    0.122260   -0.005470
 60     -0.067250   -0.074710   -0.130420   -0.262470   -0.362720   -0.321080   -0.176850   -0.081210
 61     -0.116860   -0.207340   -0.224160   -0.140940   -0.039630    0.013890    0.041540    0.086640
 62      0.127120    0.102420    0.009020   -0.083150   -0.117870   -0.109100   -0.083600   -0.013310
 63      0.128540    0.252950    0.186150   -0.119270   -0.443720   -0.468570   -0.121070    0.307360
 64      0.453440    0.275430    0.085220    0.171660    0.460020    0.610600    0.425000    0.064060
 65     -0.168350   -0.184080   -0.169780   -0.304560   -0.519110   -0.586880   -0.414470   -0.146680
 66      0.018230    0.046510    0.052910    0.110600    0.156630    0.100090   -0.039330   -0.152770
 67     -0.176130   -0.142520   -0.094490   -0.007370    0.175120    0.385980    0.475240    0.331430
 68      0.005900   -0.325280   -0.503400   -0.478430   -0.293830   -0.038850    0.173650    0.241200
 69      0.146920    0.001010   -0.044670    0.048610    0.163050    0.165730    0.073420    0.028950
 70      0.108720    0.207940    0.163240   -0.033700   -0.206560   -0.194520   -0.032320    0.114440
 71      0.166290    0.213240    0.359680    0.542370    0.568630    0.336260   -0.034880   -0.315120
 72     -0.380350   -0.290590   -0.184960   -0.151330   -0.197850   -0.286260   -0.342070   -0.264810
 73     -0.008430    0.319480    0.491690    0.359220    0.017840   -0.264200   -0.298580   -0.128930
 74      0.074660    0.215840    0.306920    0.333150    0.192110   -0.158030   -0.538880   -0.654840
 75     -0.403060   -0.020070    0.154580    0.047280   -0.089950   -0.007550    0.228440    0.326040
 76      0.144390   -0.118730   -0.165400    0.036720    0.215990    0.128780   -0.147940   -0.309090
 77     -0.170220    0.143170    0.355560    0.325500    0.136480   -0.051880   -0.157260   -0.185250
 78     -0.154130   -0.071590    0.034610    0.124290    0.200510    0.315090    0.482770    0.611780
 79      0.567200    0.314060   -0.032490   -0.315170   -0.468030   -0.521580   -0.506430   -0.410210
 80     -0.243450   -0.094270   -0.059790   -0.128480   -0.180720   -0.126820   -0.009600    0.063210
 81      0.062290    0.056850    0.086480    0.076210   -0.063780   -0.275540   -0.364920   -0.211690
 82      0.070510    0.237580    0.175160    0.011020   -0.046710    0.057060    0.189520    0.204340
 83      0.095760   -0.031290   -0.092950   -0.085170   -0.027710    0.027220    0.075760    0.055780
 84     -0.040890   -0.128730   -0.098580    0.048510    0.182820    0.195070    0.130520    0.119130
 85      0.181170    0.178760    0.008970   -0.206530   -0.218520    0.041990    0.329200    0.325990
 86      0.026900   -0.263450   -0.239330    0.058730    0.324560    0.352580    0.238790    0.196680
 87      0.255860    0.260210    0.084470   -0.153570   -0.258710   -0.216590   -0.214280   -0.383540
 88     -0.601890   -0.618330   -0.357170   -0.027540    0.094310   -0.046070   -0.240180   -0.238250
 89      0.025410    0.408350    0.695210    0.731960    0.485160    0.052110   -0.370880   -0.590510
 90     -0.548470   -0.357070   -0.178670   -0.070540    0.025810    0.147770    0.219070    0.139240
 91     -0.056720   -0.193760   -0.142010    0.028220    0.120930    0.035140   -0.131790   -0.209810
 92     -0.141060   -0.006240    0.099180    0.149750    0.142820    0.037020   -0.191930   -0.446350
 93     -0.523820   -0.348780   -0.005040    0.257530    0.300180    0.189500    0.076890    0.023780
 94     -0.017490   -0.081990   -0.120030   -0.068580    0.050780    0.151210    0.183740    0.175960
 95      0.165470    0.145970    0.111700    0.115740    0.218260    0.373820    0.436520    0.324840
 96      0.143940    0.072660    0.126110    0.109330   -0.133300   -0.465190   -0.550350   -0.230560
 97      0.255310    0.499090    0.352920    0.060540   -0.054570    0.042140    0.106420   -0.043530
 98     -0.300720   -0.426920   -0.363320   -0.286500   -0.352830   -0.477840   -0.436960   -0.152760
 99      0.194690    0.359620    0.277840    0.087040   -0.057300   -0.119090   -0.136560   -0.113690
100     -0.014830    0.154230    0.320750    0.418920    0.452510    0.456680    0.423910    0.311220

MAXIMUM ACCELERATION =  1.00000
AT TIME              =  7.78 SEC

THE VALUES WILL BE MULTIPLIED BY A FACTOR =  0.350
TO GIVE NEW MAXIMUM ACCELERATION          =  0.35000
MEAN SQUARE FREQUENCY =        4.65 C/SEC.
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******    OPTION  2  ***  READ SOIL PROFILE

    NEW SOIL PROFILE NO. 99       IDENTIFICATION   EXAMPLE 125 FT COL

NUMBER OF LAYERS                    13          DEPTH TO BEDROCK        125.00
NUMBER OF FIRST SUBMERGED LAYER      3          DEPTH TO WATER LEVEL     10.00

LAYER   TYPE      FACTOR      THICKNESS      DEPTH        EFF. PRESS.     MODULUS          DAMPING     UNIT WEIGTH        SHEAR VEL
               MOD.  DAMP.

   1       2    2.97  1.41       5.00          2.50          0.30          2425.           0.100         0.1220            800.
   2       2    1.71  1.20       5.00          7.50          0.91          2425.           0.100         0.1220            800.
   3       2    1.40  1.12       5.00         12.50          1.37          2425.           0.100         0.1220            800.
   4       2    1.27  1.08       5.00         17.50          1.67          2425.           0.100         0.1220            800.
   5       2    1.17  1.05       5.00         22.50          1.96          2425.           0.100         0.1220            800.
   6       2    1.06  1.01      10.00         30.00          2.41          2425.           0.100         0.1220            800.
   7       2    1.98  0.96      10.00         40.00          3.08          5148.           0.100         0.1370           1100.
   8       2    1.78  0.92      10.00         50.00          3.83          5148.           0.100         0.1370           1100.
   9       2    1.63  0.88      10.00         60.00          4.57          5148.           0.100         0.1370           1100.
  10       2    1.51  0.85      10.00         70.00          5.32          5148.           0.100         0.1370           1100.
  11       2    2.99  0.81      25.00         87.50          6.71         11448.           0.100         0.1440           1600.
  12       2    2.62  0.76      25.00        112.50          8.75         11448.           0.100         0.1440           1600.
  13    BASE                                                            43323.           0.000         0.1550           3000.

 PERIOD = 0.41 FROM AVERAGE SHEARVEL. =    1216.
MAXIMUM AMPLIFICATION =    7.27
FOR FREQUENCY          =    2.77 C/SEC.
   PERIOD              =    0.36 SEC.



 ******    OPTION  3  ***  READ WHERE OBJECT MOTION IS GIVEN

   OBJECT MOTION IN LAYER NUMBER  13 OUTCROPPING
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******   OPTION  4  ***  OBTAIN STRAIN COMPATIBLE SOIL PROPERTIES

 MAXIMUM NUMBER OF ITERATIONS                =    10
 MAXIMUM ERROR IN PERCENT                    =   5.00
 FACTOR FOR EFFECTIVE STRAIN IN TIME DOMAIN  =   0.65


    EARTHQUAKE    -  ROCK MOTION R01
    SOIL PROFILE -   EXAMPLE 125 FT COL

 ITERATION NUMBER  1
 THE CALCULATION HAS BEEN CARRIED OUT IN THE TIME DOMAIN WITH EFF. STRAIN = $0.* MAX. STRAIN
 LAYER  TYPE       DEPTH        EFF. STRAIN         NEW DAMP.    DAMP USED       ERROR          NEW G      G USED           ERROR

    1      2         2.5          0.00467            0.058        0.100         -72.4        1881.433    2424.845          -28.9
    2      2         7.5          0.01388            0.083        0.100         -21.0        1457.129    2424.845          -66.4
    3      2        12.5          0.02265            0.096        0.100          -4.5        1235.002    2424.845          -96.3
    4      2        17.5          0.03070            0.104        0.100           3.7        1097.112    2424.845         -121.0
    5      2        22.5          0.03775            0.111        0.100          10.2        1009.534    2424.845         -140.2
    6      2        30.0          0.04605            0.117        0.100          14.5         926.469    2424.845         -161.7
    7      2        40.0          0.02549            0.086        0.100         -16.4        2508.257    5148.137         -105.2
    8      2        50.0          0.02910            0.086        0.100         -16.0        2380.651    5148.137         -116.2
    9      2        60.0          0.03154            0.086        0.100         -16.2        2303.193    5148.137         -123.5
   10      2        70.0          0.03415            0.086        0.100         -15.6        2232.318    5148.137         -130.6
   11      2        87.5          0.01783            0.063        0.100         -59.8        6342.715   11448.447          -80.5
   12      2       112.5          0.02008            0.062        0.100         -62.3        6088.742   11448.447          -88.0


 VALUES IN TIME DOMAIN

 LAYER  TYPE      THICKNESS          DEPTH     MAX STRAIN     MAX STRESS         TIME
                     FT               FT        PRCNT          PSF              SEC
    1      2          5.0             2.5      0.00719        135.32            7.90
    2      2          5.0             7.5      0.02135        311.16            7.90
    3      2          5.0            12.5      0.03485        430.35            7.90
    4      2          5.0            17.5      0.04723        518.12            7.90
    5      2          5.0            22.5      0.05806        586.38            7.90
    6      2         10.0            30.0      0.07085        656.38            7.90
    7      2         10.0            40.0      0.03921        983.61            7.88
    8      2         10.0            50.0      0.04477       1065.84            7.88
    9      2         10.0            60.0      0.04852       1117.53            7.88
   10      2         10.0            70.0      0.05254       1172.96            2.20
   11      2         25.0            87.5      0.02744       1740.30            2.20
   12      2         25.0           112.5      0.03089       1880.96            2.18
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   EARTHQUAKE    -  ROCK MOTION R01
   SOIL PROFILE -  EXAMPLE 125 FT COL

ITERATION NUMBER  2
THE CALCULATION HAS BEEN CARRIED OUT IN THE TIME DOMAIN WITH EFF. STRAIN = $0.* MAX. STRAIN
LAYER  TYPE       DEPTH        EFF. STRAIN          NEW DAMP.    DAMP USED      ERROR          NEW G       G USED          ERROR

   1     2          2.5         0.00622              0.067        0.058        13.4        1777.928    1881.433          -5.8
   2     2          7.5         0.02363              0.104        0.083        20.6        1215.801    1457.129         -19.8
   3     2         12.5         0.04505              0.129        0.096        25.6         935.695    1235.002         -32.0
   4     2         17.5         0.06768              0.146        0.104        28.7         765.468    1097.112         -43.3
   5     2         22.5         0.08823              0.155        0.111        28.1         654.552    1009.534         -54.2
   6     2         30.0         0.11090              0.160        0.117        27.1         573.315     926.469         -61.6
   7     2         40.0         0.04522              0.110        0.086        22.2        1983.131    2508.257         -26.5
   8     2         50.0         0.05540              0.115        0.086        24.9        1802.800    2380.651         -32.1
   9     2         60.0         0.06426              0.117        0.086        26.4        1671.138    2303.193         -37.8
  10     2         70.0         0.06987              0.116        0.086        25.7        1596.625    2232.318         -39.8
  11     2         87.5         0.02626              0.073        0.063        14.4        5514.477    6342.715         -15.0
  12     2        112.5         0.02984              0.072        0.062        14.1        5240.651    6088.742         -16.2


VALUES IN TIME DOMAIN

LAYER  TYPE       THICKNESS          DEPTH     MAX STRAIN      MAX STRESS          TIME
                      FT              FT        PRCNT            PSF              SEC
   1     2            5.0             2.5      0.00957          170.14           7.94
   2     2            5.0             7.5      0.03635          441.98           7.94
   3     2            5.0            12.5      0.06930          648.45           7.94
   4     2            5.0            17.5      0.10412          796.99           7.94
   5     2            5.0            22.5      0.13574          808.50           7.94
   6     2           10.0            30.0      0.17062          978.17           7.94
   7     2           10.0            40.0      0.06957         1379.65           2.26
   8     2           10.0            50.0      0.08524         1536.65           2.24
   9     2           10.0            60.0      0.09886         1652.13           2.24
  10     2           10.0            70.0      0.10749         1716.48           2.24
  11     2           25.0            87.5      0.04039         2227.47           2.22
  12     2           25.0           112.5      0.04590         2405.59           2.20
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   EARTHQUAKE    -  ROCK MOTION R01
   SOIL PROFILE  -  EXAMPLE 125 FT COL

ITERATION NUMBER  3
THE CALCULATION HAS BEEN CARRIED OUT IN THE TIME DOMAIN WITH EFF. STRAIN = $0.* MAX. STRAIN
LAYER  TYPE      DEPTH        EFF. STRAIN        NEW DAMP.   DAMP USED      ERROR           NEW G      G USED          ERROR

   1     2         2.5          0.00520            0.061       0.067        -9.2         1843.149    1777.928          3.5
   2     2         7.5          0.02270            0.103       0.104        -1.6         1233.942    1215.801          1.5
   3     2        12.5          0.04763            0.132       0.129         2.3          912.328     935.695         -2.6
   4     2        17.5          0.07665            0.152       0.146         4.3          713.375     765.468         -7.3
   5     2        22.5          0.11237            0.168       0.155         7.5          569.630     654.552        -14.9
   6     2        30.0          0.15874            0.179       0.160        10.1          473.212     573.315        -21.2
   7     2        40.0          0.05403            0.119       0.110         7.0         1825.037    1983.131         -8.7
   8     2        50.0          0.06687            0.123       0.115         6.9         1635.774    1802.800        -10.2
   9     2        60.0          0.07730            0.125       0.117         6.4         1507.083    1671.138        -10.9
  10     2        70.0          0.08504            0.125       0.116         6.6         1422.436    1596.825        -12.3
  11     2        87.5          0.02721            0.074       0.073         1.3         5438.363    5514.477         -1.4
  12     2       112.5          0.03470            0.077       0.072         7.2         4932.914    5240.651         -6.2


VALUES IN TIME DOMAIN

LAYER  TYPE       THICKNESS          DEPTH     MAX STRAIN     MAX STRESS          TIME
                      FT              FT          PRCNT           PSF             SEC
   1     2            5.0            2.5        0.00799         147.33           7.98
   2     2            5.0            7.5        0.03493         430.99           7.98
   3     2            5.0           12.5        0.07328         668.59           7.98
   4     2            5.0           17.5        0.11793         841.29           7.98
   5     2            5.0           22.5        0.17288         984.80           2.30
   6     2           10.0           30.0        0.24421        1155.63           2.30
   7     2           10.0           40.0        0.08313        1517.13           2.28
   8     2           10.0           50.0        0.10288        1682.88           2.26
   9     2           10.0           60.0        0.11893        1792.32           2.26
  10     2           10.0           70.0        0.13062        1860.88           2.24
  11     2           25.0           87.5        0.04185        2276.20           2.20
  12     2           25.0          112.5        0.05338        2633.42           2.18
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    EARTHQUAKE    -  ROCK MOTION R01
    SOIL PROFILE -  EXAMPLE 125 FT COL

 ITERATION NUMBER  4
 THE CALCULATION HAS BEEN CARRIED OUT IN THE TIME DOMAIN WITH EFF. STRAIN = 50.* MAX. STRAIN
 LAYER  TYPE        DEPTH        EFF. STRAIN          NEW DAMP.    DAMP USED      ERROR              NEW G       G USED             ERROR

    1     2          2.5           0.00460              0.057        0.061        -6.7           1887.563     1843.149               2.4
    2     2          7.5           0.02034              0.098        0.103        -4.5           1283.845     1233.942               3.9
    3     2         12.5           0.04600              0.130        0.132        -1.5            926.919      912.328               1.6
    4     2         17.5           0.08120              0.155        0.152         2.0            689.292      713.375              -3.5
    5     2         22.5           0.12639              0.174        0.168         3.6            536.821      569.630              -6.1
    6     2         30.0           0.18958              0.187        0.179         4.8            423.645      473.212             -11.7
    7     2         40.0           0.05637              0.121        0.119         1.6           1787.451     1825.037              -2.1
    8     2         50.0           0.06918              0.125        0.123         1.2           1605.583     1635.774              -1.9
    9     2         60.0           0.08207              0.127        0.125         2.0           1453.901     1507.083              -3.7
   10     2         70.0           0.09249              0.128        0.125         2.7           1347.858     1422.436              -5.5
   11     2         87.5           0.02752              0.074        0.074         0.4           5414.105     5438.363              -0.4
   12     2        112.5           0.03676              0.079        0.077         2.7           4818.875     4932.914              -2.4


 VALUES IN TIME DOMAIN

 LAYER  TYPE        THICKNESS          DEPTH     MAX STRAIN     MAX STRESS        TIME
                       FT               FT         PRCNT           PSF             SEC
    1     2            5.0              2.5       0.00707         133.48          7.98
    2     2            5.0              7.5       0.03129         401.70          2.32
    3     2            5.0             12.5       0.07077         655.99          2.32
    4     2            5.0             17.5       0.12492         861.07          2.32
    5     2            5.0             22.5       0.19445        1043.83          2.32
    6     2           10.0             30.0       0.29166        1235.62          2.30
    7     2           10.0             40.0       0.08672        1550.14          2.28
    8     2           10.0             50.0       0.10644        1708.95          2.26
    9     2           10.0             60.0       0.12627        1835.61          2.26
   10     2           10.0             70.0       0.14229        1917.83          2.24
   11     2           25.0             87.5       0.04233        2291.86          2.20
   12     2           25.0            112.5       0.05656        2725.50          2.18
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    EARTHQUAKE    -  ROCK MOTION R01
    SOIL PROFILE -  EXAMPLE 125 FT COL

 ITERATION NUMBER  5
 THE CALCULATION HAS BEEN CARRIED OUT IN THE TIME DOMAIN WITH EFF. STRAIN = 50.* MAX. STRAIN
 LAYER  TYPE        DEPTH         EFF. STRAIN           NEW DAMP.    DAMP USED      ERROR              NEW G       G USED            ERROR

    1     2           2.5           0.00434               0.056       0.057        -3.3            1908.696     1887.563             1.1
    2     2           7.5           0.01948               0.096       0.098        -1.8            1303.498     1283.845             1.5
    3     2          12.5           0.04505               0.129       0.130        -0.5             935.671      926.919             0.9
    4     2          17.5           0.08375               0.157       0.155         1.0             676.340      689.292            -1.9
    5     2          22.5           0.13406               0.177       0.174         1.7             520.380      536.821            -3.2
    6     2          30.0           0.20631               0.192       0.187         2.2             400.038      423.645            -5.9
    7     2          40.0           0.05579               0.120       0.121        -0.4            1796.557     1787.451             0.5
    8     2          50.0           0.06884               0.125       0.125        -0.2            1609.967     1605.583             0.3
    9     2          60.0           0.08350               0.128       0.127         0.6            1438.662     1453.901            -1.1
   10     2          70.0           0.09660               0.130       0.128         1.4            1309.198     1347.858            -3.0
   11     2          87.5           0.02764               0.075       0.074         0.2            5404.153     5414.105            -0.2
   12     2         112.5           0.03768               0.080       0.079         1.1            4770.011     4818.875            -1.0


 VALUES IN TIME DOMAIN

 LAYER  TYPE        THICKNESS            DEPTH     MAX STRAIN      MAX STRESS           TIME
                        FT                FT         PRCNT            PSF               SEC
    1     2            5.0               2.5       0.00667          127.33             2.32
    2     2            5.0               7.5       0.02996          390.56             2.32
    3     2            5.0              12.5       0.06931          648.47             2.32
    4     2            5.0              17.5       0.12885          871.47             2.32
    5     2            5.0              22.5       0.20624         1073.25             2.32
    6     2           10.0              30.0       0.31740         1269.73             2.30
    7     2           10.0              40.0       0.08584         1542.13             2.28
    8     2           10.0              50.0       0.10591         1705.18             2.28
    9     2           10.0              60.0       0.12845         1848.02             2.26
   10     2           10.0              70.0       0.14862         1945.72             2.24
   11     2           25.0              87.5       0.04253         2298.31             2.22
   12     2           25.0             112.5       0.05798         2765.47             2.20
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   EARTHQUAKE    -  ROCK MOTION R01
   SOIL PROFILE -  EXAMPLE 125 FT COL

ITERATION NUMBER  6
THE CALCULATION HAS BEEN CARRIED OUT IN THE TIME DOMAIN WITH EFF. STRAIN = $0.* MAX. STRAIN
LAYER  TYPE     DEPTH        EFF. STRAIN         NEW DAMP.    DAMP USED     ERROR           NEW G      G USED          ERROR

   1     2         2.5         0.00428             0.055        0.056       -0.8         1913.492   1908.696          0.3
   2     2         7.5         0.01909             0.096        0.096       -0.8         1312.490   1303.498          0.7
   3     2        12.5         0.04437             0.128        0.129       -0.7          942.047    935.671          0.7
   4     2        17.5         0.08488             0.158        0.157        0.4          670.774    676.340         -0.6
   5     2        22.5         0.13762             0.178        0.177        0.8          513.054    520.380         -1.4
   6     2        30.0         0.21638             0.194        0.192        1.2          386.734    400.036         -3.4
   7     2        40.0         0.05458             0.119        0.120       -0.9         1816.096   1796.557          1.1
   8     2        50.0         0.06783             0.124        0.125       -0.5         1623.084   1609.967          0.8
   9     2        60.0         0.08372             0.128        0.128        0.1         1436.245   1438.662         -0.2
  10     2        70.0         0.09910             0.131        0.130        0.8         1286.558   1309.198         -1.8
  11     2        87.5         0.02772             0.075        0.075        0.1         5398.329   5404.153         -0.1
  12     2       112.5         0.03806             0.081        0.080        0.5         4750.528   4770.011         -0.4


VALUES IN TIME DOMAIN

LAYER  TYPE       THICKNESS           DEPTH     MAX STRAIN     MAX STRESS         TIME
                      FT                FT         PRCNT          PSF             SEC
   1     2           5.0               2.5       0.00658        125.97           2.32
   2     2           5.0               7.5       0.02937        385.53           2.32
   3     2           5.0              12.5       0.06826        643.01           2.32
   4     2           5.0              17.5       0.13058        875.88           2.32
   5     2           5.0              22.5       0.21173       1086.28           2.32
   6     2          10.0              30.0       0.33290       1287.42           2.32
   7     2          10.0              40.0       0.08397       1524.97           2.28
   8     2          10.0              50.0       0.10436       1693.66           2.28
   9     2          10.0              60.0       0.12680       1849.94           2.26
  10     2          10.0              70.0       0.15246       1961.46           2.24
  11     2          25.0              87.5       0.04264       2302.08           2.22
  12     2          25.0             112.5       0.05855       2781.48           2.20

 PERIOD = 0.67 FROM AVERAGE SHEARVEL. =     747.
MAXIMUM AMPLIFICATION =     5.66
FOR FREQUENCY          =    1.54 C/SEC.
    PERIOD             =    0.65 SEC.
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******   OPTION  5  ***  COMPUTE MOTION IN NEW SUBLAYERS

  EARTHQUAKE - ROCK MOTION R01
 SOIL DEPOSIT -  EXAMPLE 125 FT COL

    LAYER              DEPTH          MAX. ACC.           TIME        MEAN SQ. FR.         ACC. RATIO       PUNCHED CARDS
                        FT              G                  SEC           C/SEC             QUIET ZONE        ACC. RECORD

    OUTCR.              0.0           0.40137             2.30           2.73               0.000                   0
    WITHIN              5.0           0.40008             2.30           2.70               0.000                   0
    WITHIN             35.0           0.29251             1.76           3.92               0.000                   0
    WITHIN             75.0           0.28949             7.82           4.52               0.000                   0
    WITHIN            125.0           0.26957             7.78           4.59               0.000                   0
    OUTCR.            125.0           0.34996             7.78           4.65               0.000                   0




******   OPTION  9  ***  COMPUTE RESPONSE SPECTRUM


COMPUTE RESPONSE SPECTRUM OF OBJECT MOTION
 RESPONSE SPECTRUM ANALYSIS FOR LAYER NUMBER        13
CALCULATED FOR DAMPING         0.050
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SPECTRAL VALUES--
     ROCK MOTION R01                                                                  DAMPING RATIO = 0.05

NO.     PERIOD      REL. DISP.         REL. VEL.    PSU.REL.VEL.       ABS. ACC.   PSU.ABS.ACC.       FREQ.
          SEC.             FT.         FT./SEC.         FT./SEC.             G.             G.       C/SEC.

  1       0.00         0.00000          0.00001          0.00179          0.34996        0.34991    1000.00
  2       0.05         0.00075          0.02389          0.09438          0.36750        0.36833      20.00
  3       0.07         0.00197          0.06704          0.16521          0.43046        0.42984      13.33
  4       0.10         0.00519          0.25180          0.32626          0.64604        0.63664      10.00
  5       0.12         0.01014          0.41008          0.50952          0.79455        0.79538       8.00
  6       0.15         0.01585          0.58765          0.66395          0.85533        0.86371       6.67
  7       0.17         0.02063          0.64683          0.74087          0.84576        0.82609       5.71
  8       0.20         0.02748          0.80687          0.86343          0.85967        0.84240       5.00
  9       0.22         0.03260          0.95416          0.91608          0.80390        0.79447       4.44
 10       0.25         0.04074          0.90666          1.02390          0.81457        0.79918       4.00
 11       0.27         0.04935          1.11575          1.12762          0.79082        0.80012       3.64
 12       0.30         0.05218          1.04896          1.09281          0.72182        0.71080       3.33
 13       0.32         0.06000          1.15045          1.15999          0.71046        0.69646       3.08
 14       0.35         0.06586          1.25802          1.18223          0.65989        0.65911       2.86
 15       0.37         0.07134          1.29108          1.19532          0.62704        0.62198       2.67
 16       0.40         0.07070          1.16544          1.11060          0.54706        0.54178       2.50
 17       0.42         0.08406          1.32569          1.24269          0.57265        0.57056       2.35
 18       0.45         0.08342          1.20487          1.16482          0.51074        0.50509       2.22
 19       0.47         0.08311          1.20348          1.09933          0.45407        0.45161       2.11
 20       0.50         0.09069          1.11375          1.13961          0.44681        0.44474       2.00
 21       0.52         0.09430          1.14556          1.12852          0.42242        0.41944       1.90
 22       0.55         0.09722          1.26475          1.11066          0.39397        0.39404       1.82
 23       0.57         0.09530          1.21799          1.04134          0.35412        0.35339       1.74
 24       0.60         0.09745          1.32048          1.02050          0.33366        0.33188       1.67
 25       0.62         0.10562          1.16917          1.06184          0.33151        0.33152       1.60
 26       0.65         0.11001          1.11490          1.06343          0.32044        0.31924       1.54
 27       0.67         0.11369          1.12384          1.05831          0.30906        0.30594       1.48
 28       0.70         0.11569          1.10447          1.03843          0.29066        0.28947       1.43
 29       0.72         0.13629          1.09381          1.18119          0.31867        0.31791       1.38
 30       0.75         0.14818          1.15138          1.24140          0.32529        0.32298       1.33
 31       0.77         0.13971          1.17680          1.13266          0.28690        0.28518       1.29
 32       0.80         0.14679          1.21753          1.15285          0.28272        0.28120       1.25
 33       0.82         0.17241          1.25493          1.31310          0.31233        0.31058       1.21
 34       0.85         0.17961          1.36251          1.32769          0.30615        0.30479       1.18
 35       0.87         0.17127          1.29448          1.22968          0.27619        0.27427       1.14
 36       0.90         0.16923          1.25088          1.18147          0.25717        0.25616       1.11
 37       0.92         0.18521          1.35024          1.25808          0.26672        0.26539       1.08
 38       0.95         0.19486          1.39971          1.28877          0.26624        0.26471       1.05
 39       0.97         0.19208          1.34154          1.23781          0.24866        0.24773       1.03
 40       1.00         0.17734          1.30812          1.11427          0.21837        0.21743       1.00
 41       1.05         0.18295          1.30719          1.09480          0.20555        0.20346       0.95
 42       1.10         0.21812          1.41381          1.24591          0.22342        0.22101       0.91
 43       1.15         0.20474          1.25620          1.11864          0.19136        0.18981       0.87
 44       1.20         0.17838          1.22047          0.93401          0.15414        0.15188       0.83
 45       1.25         0.22628          1.30915          1.13741          0.17845        0.17755       0.80
 46       1.30         0.24145          1.27503          1.16696          0.17653        0.17516       0.77
 47       1.35         0.24957          1.42505          1.16157          0.16956        0.16789       0.74
 48       1.40         0.23494          1.41712          1.05440          0.14821        0.14696       0.71
 49       1.45         0.24272          1.17361          1.05176          0.14287        0.14154       0.69
 50       1.50         0.24474          1.13674          1.02515          0.13448        0.13336       0.67
 51       1.55         0.24223          1.16972          0.98192          0.12452        0.12361       0.65
 52       1.60         0.25596          1.19843          1.00514          0.12313        0.12258       0.63
 53       1.65         0.26837          1.10965          1.02196          0.12183        0.12086       0.61
 54       1.70         0.25953          1.03731          0.95924          0.11142        0.11010       0.59
 55       1.75         0.26257          1.05605          0.94271          0.10631        0.10512       0.57
 56       1.80         0.26007          1.09672          0.90781          0.09953        0.09841       0.56
 57       1.85         0.22747          1.03154          0.77257          0.08256        0.08149       0.54
 58       1.90         0.21975          1.12296          0.72670          0.07519        0.07463       0.53
 59       1.95         0.24797          1.16385          0.79900          0.08035        0.07995       0.51
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 60     2.00      0.26572     1.11050     0.83478     0.08193     0.08145    0.50
 61     2.10      0.27222     1.00555     0.81449     0.07661     0.07568    0.48
 62     2.20      0.26184     1.13099     0.74783     0.06744     0.06633    0.45
 63     2.30      0.26508     1.01597     0.72414     0.06190     0.06144    0.43
 64     2.40      0.27002     0.91339     0.70692     0.05805     0.05748    0.42
 65     2.50      0.28805     0.85359     0.72395     0.05679     0.05651    0.40
 66     2.60      0.30115     0.92937     0.72777     0.05490     0.05462    0.38
 67     2.70      0.30852     1.03994     0.71796     0.05240     0.05189    0.37
 68     2.80      0.34304     1.11392     0.76979     0.05413     0.05365    0.36
 69     2.90      0.36409     1.12150     0.78885     0.05405     0.05308    0.34
 70     3.00      0.35776     1.17028     0.74929     0.04962     0.04874    0.33
 71     3.10      0.33815     1.16216     0.68537     0.04346     0.04314    0.32
 72     3.20      0.33560     1.11029     0.65890     0.04094     0.04018    0.31
 73     3.30      0.33672     1.03013     0.64112     0.03850     0.03791    0.30
 74     3.40      0.34382     0.93589     0.63537     0.03740     0.03646    0.29
 75     3.50      0.39375     0.89142     0.70687     0.03961     0.03941    0.29
 76     3.60      0.42503     0.84370     0.74181     0.04041     0.04021    0.28
 77     3.70      0.44026     0.80465     0.74763     0.03963     0.03943    0.27
 78     3.80      0.44269     0.77250     0.73197     0.03778     0.03759    0.26
 79     3.90      0.43594     0.78361     0.70233     0.03532     0.03514    0.26
 80     4.00      0.42337     0.78045     0.66503     0.03260     0.03244    0.25
 81     4.10      0.40789     0.76815     0.62509     0.02990     0.02975    0.24
 82     4.20      0.39197     0.75069     0.58638     0.02738     0.02724    0.24
 83     4.30      0.37730     0.73108     0.55131     0.02514     0.02502    0.23
 84     4.40      0.36496     0.71149     0.52116     0.02323     0.02311    0.23
 85     4.50      0.35559     0.69307     0.49650     0.02164     0.02153    0.22
 86     4.60      0.34889     0.67628     0.47655     0.02032     0.02022    0.22
 87     4.70      0.34494     0.66169     0.46113     0.01924     0.01914    0.21
 88     4.80      0.34312     0.65405     0.44914     0.01835     0.01826    0.21
 89     4.90      0.34289     0.65274     0.43968     0.01760     0.01751    0.20
 90     5.00      0.34412     0.65161     0.43243     0.01696     0.01688    0.20


      VALUES IN PERIOD RANGE .1 TO 2.5 SEC.

           AREA OF ACC. RESPONSE SPECTRUM    =      0.607
           AREA OF VEL. RESPONSE SPECTRUM    =      2.719
           MAX. ACCELERATION RESPONSE VALUE =       0.860
           MAX. VELOCITY     RESPONSE VALUE =       1.425



******   OPTION  9  ***  COMPUTE RESPONSE SPECTRUM

 COMPUTE RESPONSE SPECTRUM IN LAYER  2
 RESPONSE SPECTRUM ANALYSIS FOR LAYER NUMBER         2
CALCULATED FOR DAMPING        0.050
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SPECTRAL VALUES--
     ROCK MOTION R01                EXAMPLE 125 FT COL              DAMPING RATIO = 0.05

NO.     PERIOD      REL. DISP.       REL. VEL.   PSU.REL.VEL.      ABS. ACC.  PSU.ABS.ACC.     FREQ.
          SEC.             FT.       FT./SEC.       FT./SEC.           G.             G.      C/SEC.

  1       0.00       0.00000         0.00001        0.00205        0.40008       0.40007    1000.00
  2       0.05       0.00082         0.01655        0.10271        0.40126       0.40083      20.00
  3       0.07       0.00193         0.04073        0.16143        0.42079       0.41999      13.33
  4       0.10       0.00398         0.10455        0.24996        0.49605       0.48774      10.00
  5       0.12       0.00677         0.20637        0.34023        0.53860       0.53111       8.00
  6       0.15       0.01054         0.34451        0.44150        0.58545       0.57433       6.67
  7       0.17       0.01622         0.44450        0.58237        0.66368       0.64936       5.71
  8       0.20       0.02583         0.71431        0.81149        0.78112       0.79173       5.00
  9       0.22       0.03784         0.84647        1.05669        0.91893       0.91641       4.44
 10       0.25       0.05094         1.04120        1.28016        0.99260       0.99919       4.00
 11       0.27       0.07374         1.55704        1.68491        1.20972       1.19555       3.64
 12       0.30       0.07537         1.42290        1.57856        1.02332       1.02675       3.33
 13       0.32       0.08349         1.60170        1.61411        0.96037       0.96911       3.08
 14       0.35       0.09808         1.81784        1.76070        0.98150       0.98162       2.86
 15       0.37       0.11375         1.87247        1.90596        0.98610       0.99176       2.67
 16       0.40       0.11797         1.84415        1.85312        0.90476       0.90400       2.50
 17       0.42       0.14150         2.14170        2.09199        0.97291       0.96049       2.35
 18       0.45       0.14516         2.04396        2.02688        0.88310       0.87890       2.22
 19       0.47       0.17279         2.29647        2.28558        0.94186       0.93892       2.11
 20       0.50       0.18008         2.26398        2.26297        0.88886       0.88315       2.00
 21       0.52       0.18927         2.44575        2.26513        0.85156       0.84190       1.90
 22       0.55       0.23249         2.76405        2.65595        0.94445       0.94228       1.82
 23       0.57       0.22477         2.53719        2.45616        0.83933       0.83351       1.74
 24       0.60       0.24058         2.91806        2.51933        0.82091       0.81933       1.67
 25       0.62       0.24980         2.71889        2.51130        0.78992       0.78405       1.60
 26       0.65       0.25056         2.51369        2.42206        0.73019       0.72710       1.54
 27       0.67       0.25081         2.46368        2.40908        0.70225       0.69642       1.48
 28       0.70       0.25693         2.42523        2.30617        0.64744       0.64286       1.43
 29       0.72       0.31619         2.69055        2.74028        0.74030       0.73753       1.38
 30       0.75       0.34230         2.87731        2.86761        0.74996       0.74608       1.33
 31       0.77       0.31443         2.71719        2.54917        0.64499       0.64183       1.29
 32       0.80       0.33242         2.83554        2.61085        0.64008       0.63682       1.25
 33       0.82       0.36583         2.88311        2.78612        0.66233       0.65898       1.21
 34       0.85       0.38060         2.95574        2.81342        0.64856       0.64586       1.18
 35       0.87       0.36510         2.74397        2.62169        0.58705       0.58465       1.14
 36       0.90       0.35041         2.57164        2.44632        0.53476       0.53039       1.11
 37       0.92       0.37979         2.67195        2.57977        0.54741       0.54421       1.08
 38       0.95       0.40298         2.71445        2.66525        0.55055       0.54744       1.05
 39       0.97       0.39882         2.80183        2.57014        0.51671       0.51437       1.03
 40       1.00       0.36939         2.72258        2.32092        0.45618       0.45288       1.00
 41       1.05       0.32258         2.20550        1.93033        0.36198       0.35873       0.95
 42       1.10       0.34847         2.24481        1.99046        0.35529       0.35309       0.91
 43       1.15       0.31785         2.25478        1.73662        0.29714       0.29467       0.87
 44       1.20       0.31122         2.24652        1.62954        0.26729       0.26498       0.83
 45       1.25       0.37152         2.25140        1.86746        0.29343       0.29152       0.80
 46       1.30       0.37265         2.44932        1.80109        0.27200       0.27034       0.77
 47       1.35       0.38042         2.27922        1.77057        0.25766       0.25592       0.74
 48       1.40       0.37359         2.12843        1.67666        0.23554       0.23369       0.71
 49       1.45       0.30857         2.08647        1.33712        0.18093       0.17994       0.69
 50       1.50       0.32226         2.04675        1.34989        0.17696       0.17560       0.67
 51       1.55       0.32939         2.00271        1.33525        0.16976       0.16810       0.65
 52       1.60       0.33975         2.02054        1.33418        0.16451       0.16271       0.63
 53       1.65       0.34644         1.90929        1.31926        0.15733       0.15602       0.61
 54       1.70       0.33176         1.85447        1.22619        0.14136       0.14074       0.59
 55       1.75       0.33431         1.80294        1.20029        0.13447       0.13384       0.57
 56       1.80       0.33613         1.75179        1.17332        0.12794       0.12719       0.56
 57       1.85       0.30585         1.70170        1.03876        0.11047       0.10956       0.54
 58       1.90       0.29318         1.65321        0.96953        0.10124       0.09957       0.53
 59       1.95       0.31244         1.60676        1.00672        0.10253       0.10074       0.51
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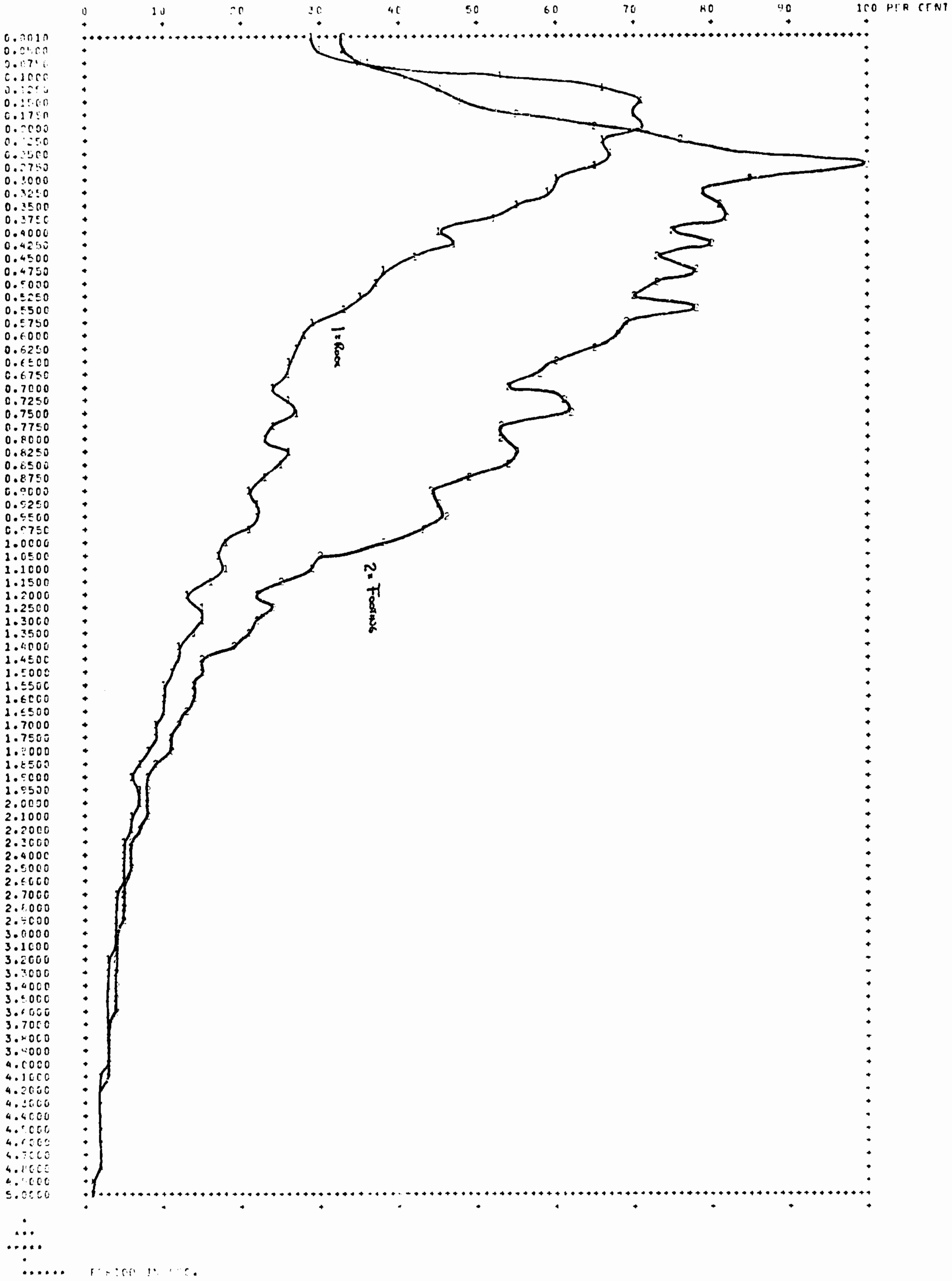
60     2.00      0.31439     1.57224     0.98767     0.09720     0.09636     0.50
61     2.10      0.34065     1.48202     1.01923     0.09549     0.09471     0.48
62     2.20      0.35089     1.57881     1.00215     0.09018     0.08889     0.45
63     2.30      0.30398     1.51797     0.83043     0.07124     0.07045     0.43
64     2.40      0.31220     1.41064     0.81734     0.06738     0.06645     0.42
65     2.50      0.34979     1.28719     0.87912     0.06950     0.06862     0.40
66     2.60      0.35130     1.22629     0.84896     0.06436     0.06371     0.38
67     2.70      0.37509     1.30362     0.87288     0.06440     0.06308     0.37
68     2.80      0.39384     1.28242     0.88378     0.06292     0.06159     0.36
69     2.90      0.39569     1.31702     0.85732     0.05858     0.05769     0.34
70     3.00      0.37749     1.33821     0.79061     0.05206     0.05142     0.33
71     3.10      0.36397     1.38340     0.73771     0.04733     0.04644     0.32
72     3.20      0.37796     1.37473     0.74212     0.04602     0.04525     0.31
73     3.30      0.37905     1.32603     0.72170     0.04361     0.04267     0.30
74     3.40      0.39384     1.25973     0.72781     0.04255     0.04177     0.29
75     3.50      0.44315     1.29154     0.79554     0.04504     0.04435     0.29
76     3.60      0.46054     1.26808     0.80380     0.04414     0.04357     0.28
77     3.70      0.45325     1.22006     0.76969     0.04101     0.04059     0.27
78     3.80      0.45129     1.27802     0.74620     0.03850     0.03832     0.26
79     3.90      0.45302     1.29893     0.72984     0.03677     0.03652     0.26
80     4.00      0.44431     1.29322     0.69793     0.03432     0.03405     0.25
81     4.10      0.42923     1.26983     0.65778     0.03158     0.03131     0.24
82     4.20      0.41163     1.23817     0.61579     0.02887     0.02861     0.24
83     4.30      0.39439     1.20458     0.57628     0.02638     0.02615     0.23
84     4.40      0.37934     1.17335     0.54170     0.02423     0.02402     0.23
85     4.50      0.36734     1.14630     0.51290     0.02243     0.02224     0.22
86     4.60      0.35853     1.12402     0.48971     0.02094     0.02077     0.22
87     4.70      0.35271     1.10636     0.47153     0.01972     0.01958     0.21
88     4.80      0.34931     1.09250     0.45725     0.01871     0.01859     0.21
89     4.90      0.34798     1.08185     0.44622     0.01786     0.01777     0.20
90     5.00      0.34831     1.07356     0.43770     0.01714     0.01708     0.20


   VALUES IN PERIOD RANGE .1 TO 2.5 SEC.

        AREA OF ACC. RESPONSE SPECTRUM   =      0.946
        AREA OF VEL. RESPONSE SPECTRUM   =      4.652
        MAX. ACCELERATION RESPONSE VALUE =      1.210
        MAX. VELOCITY     RESPONSE VALUE =      2.956
```

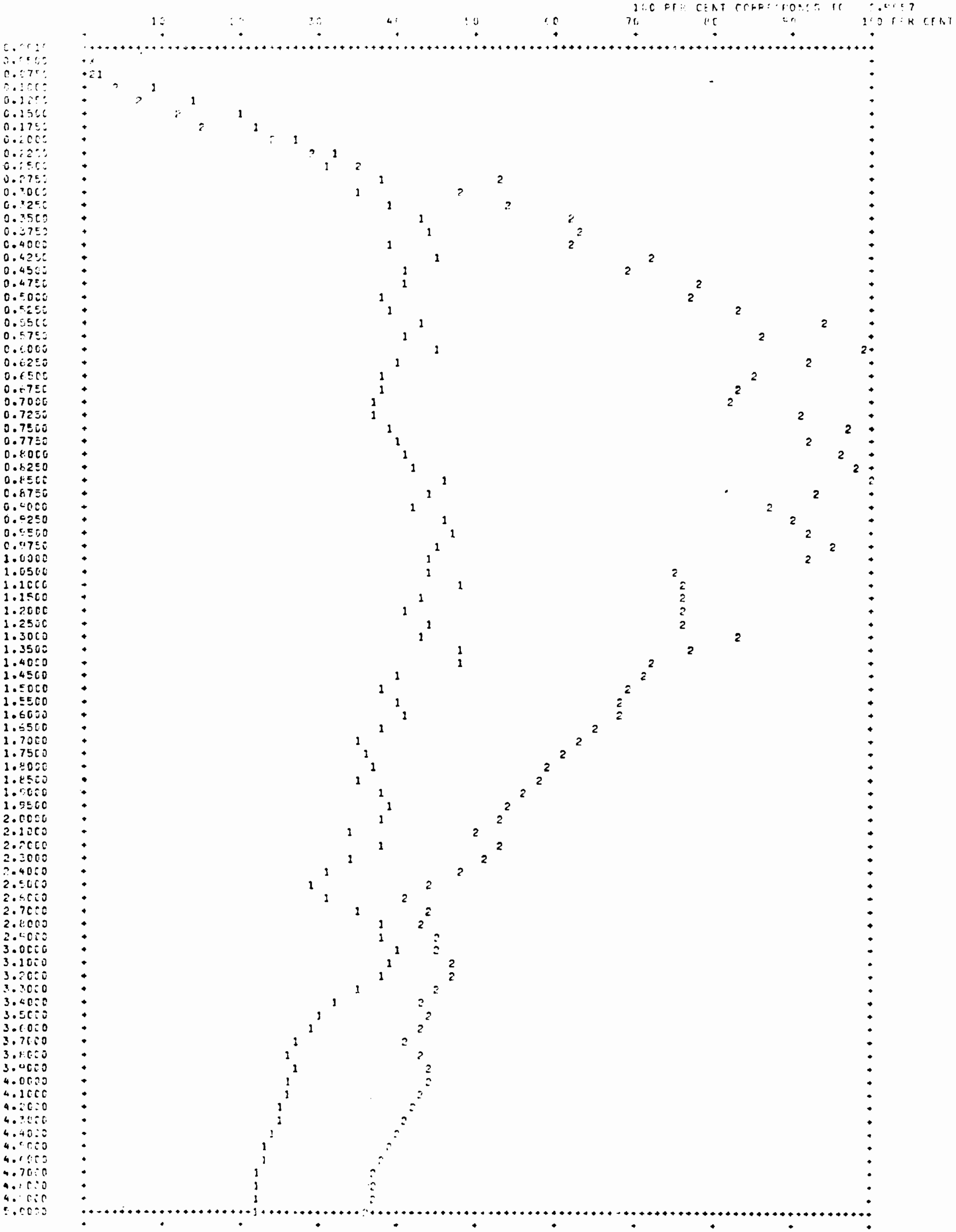
PLOT OF ACCELERATION SPECTRA %



```
CURVE 1  -   ROCK MOTION R01
CURVE 2  -   ROCK MOTION R01                    EXAMPLE 125 FT COL

  ABSISSA      CURVE 1   CURVE 2                    ABSISSA     CURVE 1   CURVE 2
   0.0010       0.3500    0.4001
   0.0500       0.3675    0.4013                     2.1000      0.0766    0.0955
   0.0750       0.4305    0.4208                     2.2000      0.0674    0.0902
   0.1000       0.6460    0.4960                     2.3000      0.0619    0.0712
   0.1250       0.7946    0.5386                     2.4000      0.0581    0.0674
   0.1500       0.8553    0.5855                     2.5000      0.0568    0.0695
   0.1750       0.8458    0.6637                     2.6000      0.0549    0.0644
   0.2000       0.8597    0.7811                     2.7000      0.0524    0.0644
   0.2250       0.8039    0.9189                     2.8000      0.0541    0.0629
   0.2500       0.8146    0.9926                     2.9000      0.0540    0.0586
   0.2750       0.7908    1.2097                     3.0000      0.0496    0.0521
   0.3000       0.7218    1.0233                     3.1000      0.0435    0.0473
   0.3250       0.7105    0.9604                     3.2000      0.0409    0.0460
   0.3500       0.6599    0.9815                     3.3000      0.0385    0.0436
   0.3750       0.6270    0.9861                     3.4000      0.0374    0.0426
   0.4000       0.5471    0.9048                     3.5000      0.0396    0.0450
   0.4250       0.5726    0.9729                     3.6000      0.0404    0.0441
   0.4500       0.5107    0.8931                     3.7000      0.0396    0.0410
   0.4750       0.4541    0.9419                     3.8000      0.0378    0.0385
   0.5000       0.4468    0.8889                     3.9000      0.0353    0.0368
   0.5250       0.4224    0.8516                     4.0000      0.0326    0.0343
   0.5500       0.3940    0.7444                     4.1000      0.0299    0.0316
   0.5750       0.3541    0.8393                     4.2000      0.0274    0.0269
   0.6000       0.3337    0.8209                     4.3000      0.0251    0.0264
   0.6250       0.3315    0.7899                     4.4000      0.0232    0.0242
   0.6500       0.3204    0.7302                     4.5000      0.0216    0.0224
   0.6750       0.3091    0.7022                     4.6000      0.0203    0.0209
   0.7000       0.2909    0.6474                     4.7000      0.0192    0.0197
   0.7250       0.3187    0.7403                     4.8000      0.0183    0.0187
   0.7500       0.3253    0.7500                     4.9000      0.0176    0.0179
   0.7750       0.2869    0.6450                     5.0000      0.0170    0.0171
   0.8000       0.2827    0.6401
   0.8250       0.3123    0.6623
   0.8500       0.3062    0.6486
   0.8750       0.2762    0.5870
   0.9000       0.2572    0.5348
   0.9250       0.2667    0.5474
   0.9500       0.2662    0.5506
   0.9750       0.2487    0.5167
   1.0000       0.2184    0.4562
   1.0500       0.2055    0.3620
   1.1000       0.2234    0.3553
   1.1500       0.1914    0.2971
   1.2000       0.1541    0.2673
   1.2500       0.1784    0.2924
   1.3000       0.1765    0.2720
   1.3500       0.1696    0.2577
   1.4000       0.1482    0.2355
   1.4500       0.1429    0.1809
   1.5000       0.1345    0.1770
   1.5500       0.1245    0.1698
   1.6000       0.1231    0.1645
   1.6500       0.1218    0.1573
   1.7000       0.1114    0.1414
   1.7500       0.1063    0.1345
   1.8000       0.0995    0.1279
   1.8500       0.0826    0.1105
   1.9000       0.0752    0.1012
   1.9500       0.0803    0.1025
   2.0000       0.0819    0.0972
```

PLOT OF VELOCITY SPECTRA



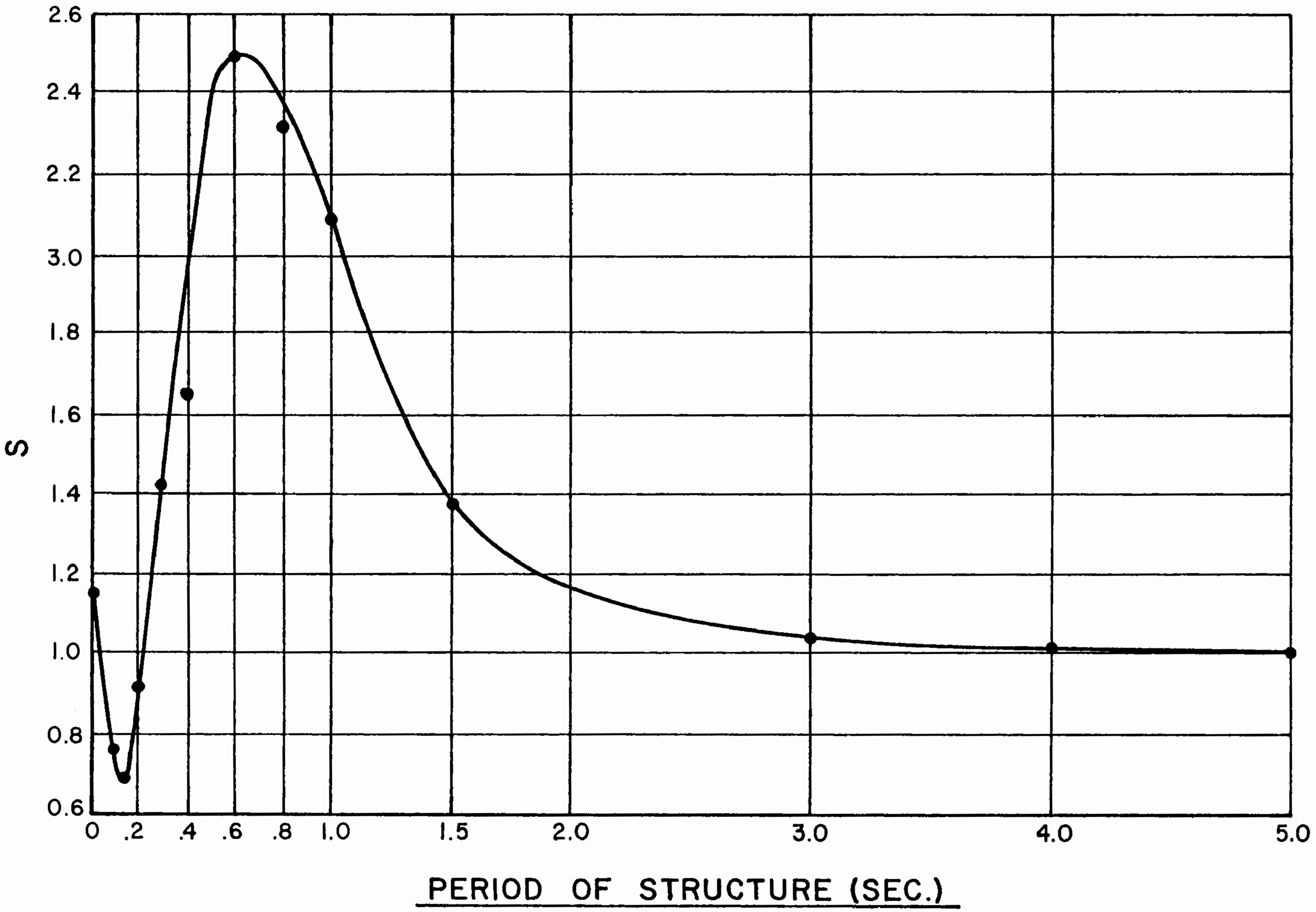
```
CURVE 1  -   ROCK MOTION R01
CURVE 2  -   ROCK MOTION R01                      EXAMPLE 125 FT COL

  ABSISSA       CURVE 1    CURVE 2             ABSISSA       CURVE 1    CURVE 2
   0.0010        0.0000     0.0000
   0.0500        0.0239     0.0165              2.1000        1.0056     1.4820
   0.0750        0.0670     0.0407              2.2000        1.1310     1.5788
   0.1000        0.2518     0.1045              2.3000        1.0160     1.5180
   0.1250        0.4101     0.2064              2.4000        0.9134     1.4106
   0.1500        0.5877     0.3445              2.5000        0.8536     1.2872
   0.1750        0.6468     0.4445              2.6000        0.9294     1.2263
   0.2000        0.8069     0.7143              2.7000        1.0399     1.3036
   0.2250        0.9542     0.8465              2.8000        1.1139     1.2824
   0.2500        0.9067     1.0412              2.9000        1.1215     1.3170
   0.2750        1.1157     1.5570              3.0000        1.1703     1.3382
   0.3000        1.0490     1.4229              3.1000        1.1622     1.3834
   0.3250        1.1505     1.6017              3.2000        1.1103     1.3747
   0.3500        1.2580     1.8178              3.3000        1.0301     1.3260
   0.3750        1.2911     1.8725              3.4000        0.9359     1.2597
   0.4000        1.1654     1.8442              3.5000        0.8914     1.2915
   0.4250        1.3257     2.1417              3.6000        0.8437     1.2681
   0.4500        1.2049     2.0440              3.7000        0.8047     1.2201
   0.4750        1.2035     2.2965              3.8000        0.7725     1.2780
   0.5000        1.1137     2.2640              3.9000        0.7836     1.2989
   0.5250        1.1456     2.4458              4.0000        0.7804     1.2932
   0.5500        1.2647     2.7641              4.1000        0.7681     1.2698
   0.5750        1.2180     2.5372              4.2000        0.7507     1.2382
   0.6000        1.3205     2.9181              4.3000        0.7311     1.2046
   0.6250        1.1692     2.7189              4.4000        0.7115     1.1733
   0.6500        1.1149     2.5137              4.5000        0.6931     1.1463
   0.6750        1.1238     2.4637              4.6000        0.6763     1.1240
   0.7000        1.1045     2.4252              4.7000        0.6617     1.1064
   0.7250        1.0938     2.6906              4.8000        0.6541     1.0925
   0.7500        1.1514     2.8773              4.9000        0.6527     1.0818
   0.7750        1.1768     2.7172              5.0000        0.6516     1.0736
   0.8000        1.2175     2.8355
   0.8250        1.2549     2.8931
   0.8500        1.3625     2.9557
   0.8750        1.2945     2.7440
   0.9000        1.2509     2.5716
   0.9250        1.3502     2.6720
   0.9500        1.3997     2.7145
   0.9750        1.3415     2.8018
   1.0000        1.3081     2.7226
   1.0500        1.3072     2.2055
   1.1000        1.4138     2.2448
   1.1500        1.2562     2.2548
   1.2000        1.2205     2.2465
   1.2500        1.3091     2.2514
   1.3000        1.2750     2.4493
   1.3500        1.4250     2.2792
   1.4000        1.4171     2.1284
   1.4500        1.1736     2.0865
   1.5000        1.1367     2.0468
   1.5500        1.1697     2.0027
   1.6000        1.1984     2.0205
   1.6500        1.1097     1.9093
   1.7000        1.0373     1.8545
   1.7500        1.0560     1.8029
   1.8000        1.0967     1.7518
   1.8500        1.0315     1.7017
   1.9000        1.1230     1.6532
   1.9500        1.1639     1.6068
   2.0000        1.1105     1.5722
```

EXAMPLE SHAKE PROBLEM
PART 2 COMPUTE AMPLIFICATION FACTOR [S]

Period (sec)	OUTCROP Response (g)	Footing Response (g)	S Computed (Footing/Outcrop	S Smooth
0.01	.35	.40	1.14	1.14
0.10	.65	.50	0.77	0.77
0.15	.86	.59	0.69	0.69
0.20	.86	.78	0.91	0.91
0.30	.72	1.02	1.42	1.42
0.40	.55	.90	1.64	1.98
0.60	.33	.82	2.48	2.48
0.80	.28	.64	2.29	2.38
1.00	.22	.46	2.09	2.09
1.50	.13	.18	1.38	1.38
2.00	.082	.097	1.18	1.18
3.00	.050	.052	1.04	1.04
4.00	.033	.034	1.03	1.03
5.00	.017	.017	1.00	1.00

Soil Period From SHAKE Run = 0.65 sec.



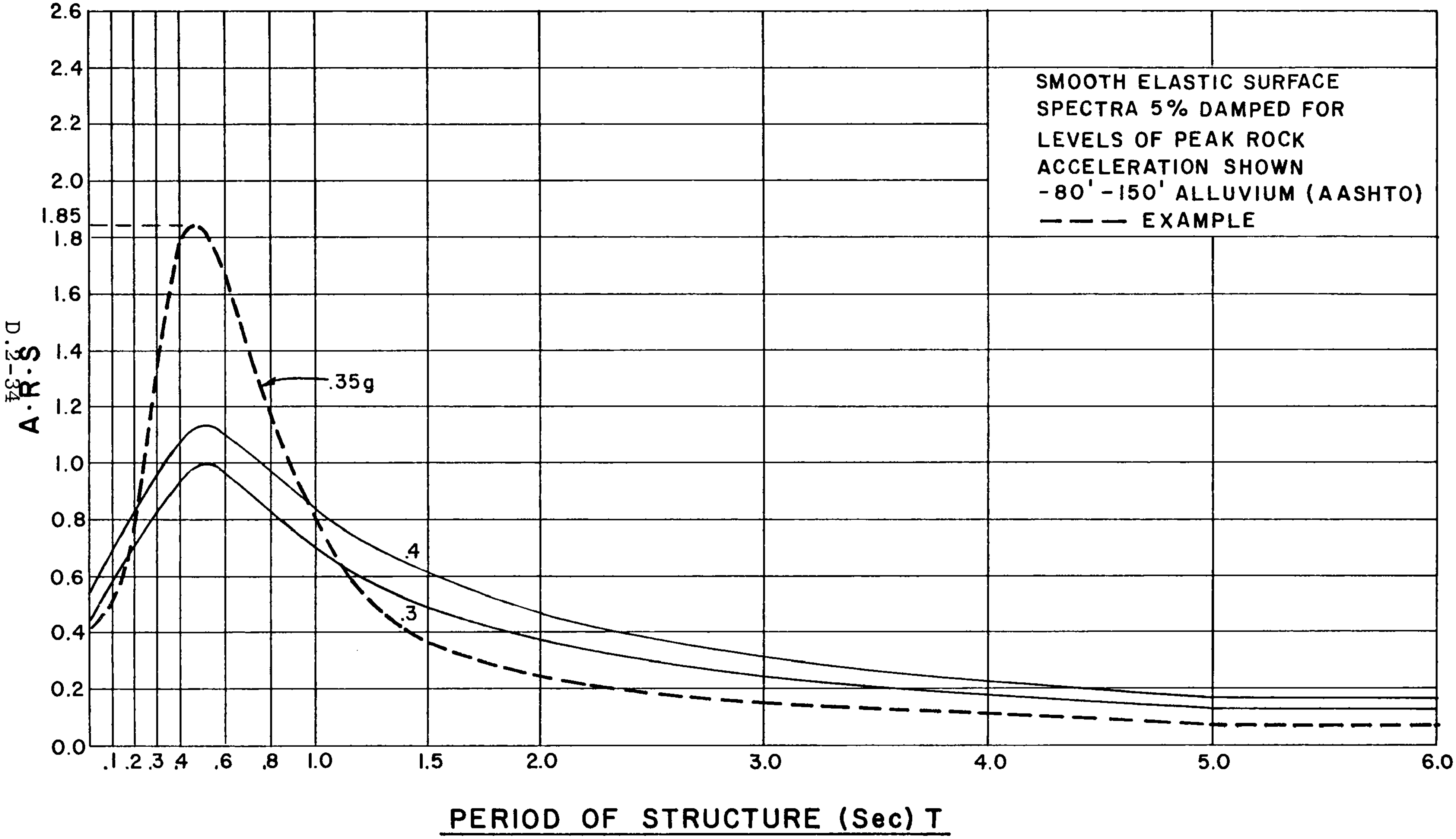
SOIL AMPLIFICATION FACTOR (S)

Period [sec]	A<0.15g	A = .15-.3g	A>.3g
0.1	1.9	1.9	1.9
0.15	2.35	2.35	2.35
0.2	2.6	2.6	2.6
0.3	2.6	2.6	2.6
0.4	2.6	2.6	2.6
0.6	2.6	2.3	1.9
0.8	2.0	1.7	1.4
1.0	1.6	1.35	1.15
1.5	1.1	0.90	0.75
2.0	0.85	0.70	0.60
3.0	0.55	0.45	0.40
4.0	0.40	0.35	0.30
5.0	0.30	0.25	0.20

AASHTO [1975] R CURVES DIGITIZED AT SPECIFIC POINTS

PART 3 COMPUTE ELASTIC DESIGN SPECTRA [A.R.S]

Period [sec]	A	R	[Smoothed] S	A.R.S
0.01	0.35	1.00	1.14	0.40
0.10	0.35	1.90	0.77	0.51
0.15	0.35	2.35	0.69	0.57
0.20	0.35	2.60	0.91	0.83
0.30	0.35	2.60	1.42	1.29
0.40	0.35	2.60	1.98	1.80
0.60	0.35	1.90	2.48	1.65
0.80	0.35	1.40	2.38	1.17
1.00	0.35	1.15	2.09	0.84
1.50	0.35	0.75	1.38	0.36
2.00	0.35	0.60	1.18	0.25
3.00	0.35	0.40	1.04	0.15
4.00	0.35	0.30	1.03	0.11
5.00	0.35	0.20	1.00	0.07


SMOOTH ELASTIC SURFACE
SPECTRA 5% DAMPED FOR
LEVELS OF PEAK ROCK
ACCELERATION SHOWN
-80'-150' ALLUVIUM (AASHTO)
— — — EXAMPLE
.35g
.4
.3
1.85
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
A·R·S
.1 .2 .3 .4 .6 .8 1.0 1.5 2.0 3.0 4.0 5.0 6.0
PERIOD OF STRUCTURE (Sec) T

DISCUSSION

The comparison plot of the computed design spectra at the site for the .35g peak outcrop acceleration when compared to the AASHTO curves for 80'to 150' of alluvium reveals the following:

[1] In the period range between 0.2 and 1.0 sec., the response is larger than AASHTO.

[2] In the regions below 0.2 sec and above 1.0 sec., the response is less than AASHTO.

[3] The predominant period aligns well with the AASHTO value [approx. 0.5 sec.].

This site is stiffer than the standard alluvium site used in the AASHTO curves, which would account for these differences. When utilizing the curve for design, it is recommended that the curve be extended to the left of 0.5 sec. at 1.85g as shown to provide a conservative spectra for the lengthening of period in the inelastic range.

Appendix D.3

ROCK MOTION RO1

The spectrum compatible rock motion developed by Romstad [4.5] provides a smoothed response spectra and eliminates the problem of large random vibrations in frequency content. This particular rock motion reproduces the Seed, Ugas and Lysmer spectrum [4.3] which was based on analysis of 28 rock site records.

ROCK MOTION RO1 STATISTICS

TIME INTERVAL = 0.02 sec.

MAXIMUM ACCELERATION = 1.00 g

PREDOMINANT PERIOD = 0.20 sec.

NUMBER OF TIME POINTS = 800

A complete listing of the rock motion is included in Appendix D.2 (input).

APPENDIX D.4

SOIL MODULI AND DAMPING FACTORS FOR DYNAMIC RESPONSE ANALYSES

by

H. Bolton Seed[1] and I. M. Idriss

(EERC 70-10)

1. Introduction

Much progress has been made in recent years in the development of analytical procedures for evaluating the response of soil deposits under seismic loading conditions. Successful application of such procedures for determining ground response in specific cases, however, is essentially dependent on the incorporation of representative soil properties in the analyses. Thus considerable effort has also been directed toward the determination of soil properties for use in these analytical procedures.

In cases of ground response involving no residual soil displacements, the response is determined mainly by the shear modulus and damping characteristics of the soil under symmetrical cyclic loading conditions. Because most soils have curvilinear stress-strain relationships as shown in Fig. 1, the shear modulus is usually expressed as the secant modulus determined by the extreme points on the hysteresis loop while the damping factor is proportional to the area inside the hysteresis loop. It is readily apparent that each of these properties will depend on the magnitude of the strain for which the hysteresis loop is determined (see Fig. 1) and thus both shear moduli and damping factors must be determined as functions of the induced strain in a soil specimen or soil deposit.

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[1] Professor of Civil Engineering, University of California, Berkeley, Calif.

[2] Assistant Research Engineer, University of California, Berkeley, Calif.

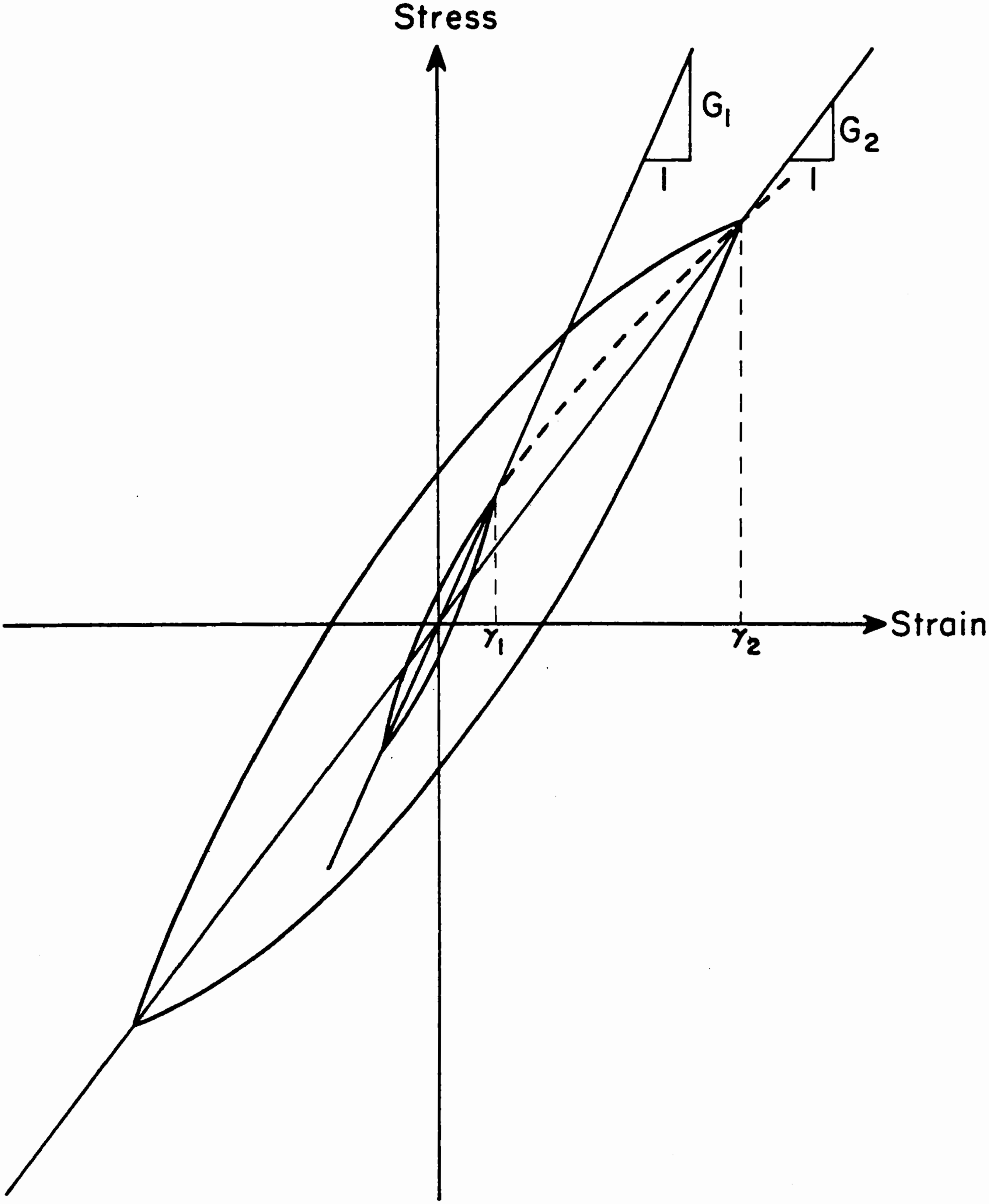

Fig. 1 HYSTERETIC STRESS-STRAIN RELATIONSHIPS AT DIFFERENT STRAIN AMPLITUDES.

It is the purpose of this report to summarize available data on the dynamic shear moduli and damping factors for soils under loading conditions similar to those illustrated in Fig. 1 and to present the results in a form which will provide a useful guide in the selection of soil characteristics for analysis purposes. Since most of the data available to date have been developed for sands and saturated clays, the report will deal primarily with these two types of materials, though limited data for gravelly soils and peats is also included.

2. Methods of Determining Shear Moduli and Damping Characteristics

A wide variety of procedures, including laboratory and field tests have been used to determine both shear moduli and damping characteristics. The main procedures may be summarized as follows:

(a) Direct determination of stress-strain relationships

Hysteretic stress-strain relationships of the type shown in Fig. 1 may be determined in the laboratory by means of triaxial compression tests, simple shear tests or torsional shear tests conducted under cyclic loading conditions. In general these procedures are useful for measuring moduli and damping factors under moderate to relatively high strains.

(b) Forced vibration tests

Forced vibration tests, involving the determination of resonant frequencies and measurement of response at other frequencies have been used to determine both moduli and damping factors. Test conditions in the laboratory have included the application of longitudinal vibrations and torsional vibrations to cylindrical samples or shear vibrations to layers of soil placed on a shaking table; in the field, shear vibrations of dams have been induced by large shaking machines but it is difficult to interpret the results of field tests to determine damping factors. In general these procedures are useful for determining properties at relatively low to moderate strain levels.

(c) Free vibration tests

Free vibration tests, in which measurements are made of the decay in response of a soil sample or soil deposit, have been used to measure both moduli and damping factors for soils. Methods of excitation are essentially similar to those used for forced vibration tests, but the procedures can be used for measurement of soil characteristics at relatively low to moderately high strain levels.

(d) Field measurement of wave velocities

Field tests have been used to measure the velocity of propagation of compression waves, shear waves, and Rayleigh waves from which values of soil modulus can readily be determined for low strain conditions. These procedures have not provided values of damping factors however.

(e) Analysis of ground response during earthquakes

In a few cases where motions have been determined at different depths in a soil profile during earthquakes, computations have been made to determine the effective moduli and damping factors controlling the response of the deposit.

Details of the different test procedures are described elsewhere (Shannon

Table 1. Test Procedures for Measuring Moduli and Damping Characteristics

General Procedure	Test Condition	Approximate Strain Range	Properties Determined
Determination of hysteretic stress-strain relationships	Triaxial compression	10^{-2} to 5%	Modulus; damping
	Simple Shear	10^{-2} to 5%	Modulus; damping
	Torsional shear	10^{-2} to 5%	Modulus; damping
Forced vibration	Longitudinal vibrations	10^{-4} to 10^{-2}%	Modulus; damping
	Torsional vibrations	10^{-4} to 10^{-2}%	Modulus; damping
	Shear vibrations - lab	10^{-4} to 10^{-2}%	Modulus; damping
	Shear vibrations - field		Modulus
Free vibration tests	Longitudinal vibrations	10^{-3} to 1%	Modulus; damping
	Torsional vibrations	10^{-3} to 1%	Modulus; damping
	Shear vibrations - lab	10^{-3} to 1%	Modulus; damping
	Shear vibrations - field	10^{-3} to 1%	Modulus
Field wave velocity measurements	Compression waves	$\simeq 5 \times 10^{-4}$%	Modulus
	Shear waves	$\simeq 5 \times 10^{-4}$%	Modulus
	Rayleigh waves	$\simeq 5 \times 10^{-4}$%	Modulus
Field seismic response	Measurement of motions at different levels in deposit		Modulus; damping

and Wilson, 1970) but a summary of the procedures and the approximate ranges of strain within which they have been used is presented in Table 1.

3. Previous Study by Hardin and Drnevich

A comprehensive survey of the factors affecting the shear moduli and damping factors of soils and expressions for determining these properties have recently been presented by Hardin and Drnevich (1970). In this study it was suggested that the primary factors affecting moduli and damping factors are:

Strain amplitude, γ

Effective mean principal stress, σ'_m

Void ratio, e

Number of cycles of loading, N

Degree of saturation for cohesive soils, S

and that less important factors include:

Octahedral shear stress

Overconsolidation ratio, OCR

Effective stress strength parameters, c' and ϕ'

Time effects

Relationships were presented to determine the values of maximum shear modulus (at essentially zero strain) and the variations of modulus values with strain for all soils. The expression for evaluating the maximum shear modulus is:

$$G_{max} = 14760 \times \frac{(2.973 - e)^2}{1 + e} (OCR)^a (\sigma'_m)^{\frac{1}{2}}$$

where G_{max} = maximum shear modulus in psf, e = void ratio

OCR = overconsolidation ratio

a = a parameter that depends on the plasticity index of the soil, and

σ'_m = mean principal effective stress in psf.

The value of a can be obtained from the following table:

PI	a
0	0
20	0.18
40	0.30
60	0.41
80	0.48
$\geq$100	0.50

The modulus value, G, at a strain level, γ, is then evaluated from the relationship:

$$G = \frac{G_{max}}{1 + \gamma/\gamma_r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

where
$$\gamma_r = \frac{\tau_{max}}{G_{max}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3a)$$

$$\tau_{max} = \left\{ \left(\frac{1 + K_o}{2} \sigma_v' \sin\phi' + c' \cos\phi' \right)^2 - \left(\frac{1 - K_o}{2} \sigma_v' \right)^2 \right\}^{1/2} \qquad (3b)$$

K_o = coefficient of lateral stress at rest,

σ_v' = vertical effective stress, and

c', ϕ' = static strength parameters in terms of effective stress.

Similar relationships were also presented for evaluating the damping ratio. The damping ratio, λ, at a strain level, γ, is given by:

$$\lambda = \frac{\lambda_{max} \cdot \gamma/\gamma_r}{1+\gamma/\gamma_r} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

where λ_{max} is the maximum damping ratio corresponding to very large strains. For clean sands, λ_{max} (in percent) is evaluated by:

$$\lambda_{max} = D - 1.5 \log_{10} N \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5a)$$

where D = 33 percent for clean dry sands or D = 28 percent for clean saturated sands, and N = number of cycles. For saturated cohesive soils, λ_{max} is given by:

$$\lambda_{max} = 31 - (3+0.03f)(\sigma_m')^{\frac{1}{2}} + 1.5\ f^{\frac{1}{2}} - 1.5 \log N \quad . \; . \; . \quad (5b)$$

where f = frequency of applied cyclic load in cycles per second, and

σ_m' = mean principal effective stress in kg/cm^2.

The significance of the factors involved in these relationships is discussed in the following section.

4. Shear Modulus Values for Sands

All investigations have shown that modulus values for sands are strongly influenced by the confining pressure, the strain amplitude and the void ratio (or relative density) but not significantly by variations in grain size characteristics. It has been found that in general, the shear modulus and confining pressure are related by the equation

$$G = 100\ K_2\ (\sigma_m')^{\frac{1}{2}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

so that the influence of void ratio and strain amplitude can be expressed through their influence on the parameter K_2.

The influence of other factors on K_2, may be illustrated by the results in Fig. 2 which were computed using the relationships suggested by Hardin and Drnevich. Plots are presented to show the influence of ϕ', effective vertical stress (σ_v'), K_o, and void ratio on the computed relationships between K_2 and strain amplitude. It may be seen that:

(a) At very low strains ($\gamma \leq 10^{-3}$ percent), K_2 depends only on the void ratio, e.

(b) At intermediate strains ($10^{-3} < \gamma < 10^{-1}$ percent) the variation of K_2 with strain is only slightly influenced by the vertical stress, and very slightly by variations in ϕ' and K_o. The values of K_2 are still influenced strongly by the void ratio however.

(c) At very high strains ($\gamma > 10^{-1}$ percent), the values of K_2 are slightly influenced by the vertical stress but they are essentially independent of K_o, ϕ' and e.

Thus for practical purposes, values of K_2 may be considered to be determined mainly by the void ratio or relative density and the strain amplitude of the motions.

A number of investigators, using different laboratory testing procedures, have presented data on the relationships between these factors. The test conditions used in these investigations are summarized in Table 2 and the results are presented in Fig. 3, for samples having a relative density of about 75 percent, and in Fig. 4 for samples having a relative density of about 40%. Average relationships between K_2 and strain for these two relative density conditions are shown in Figs. 3 and 4, and they are compared in Fig. 5. Values of K_2 at other relative densities can be estimated by interpolation, as shown in Fig. 5.

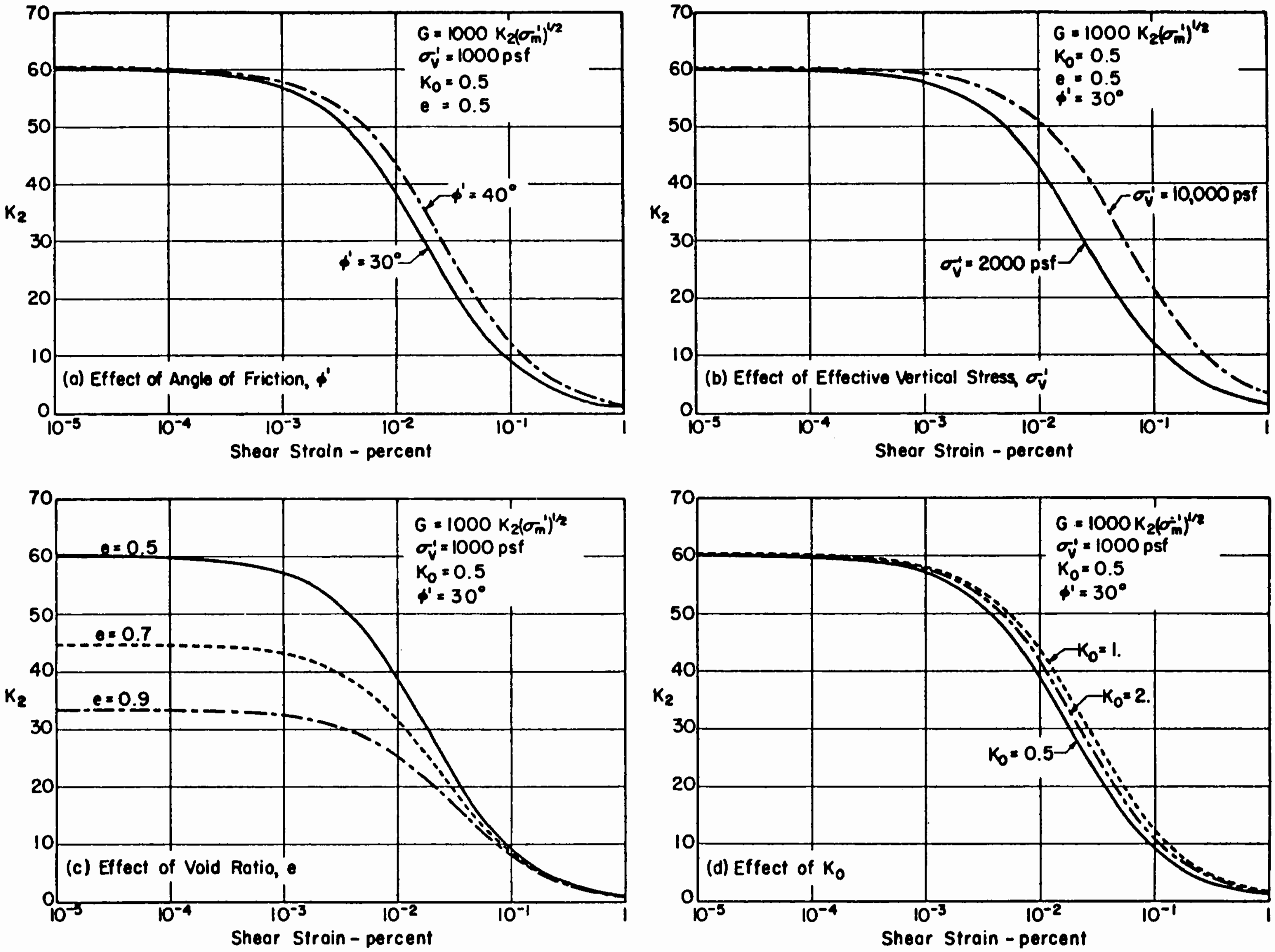

Fig. 2 INFLUENCE OF VARIOUS FACTORS ON THE SHEAR MODULI OF SANDS.
(based on Hardin and Drnevich expressions)

Table 2. Summary of Laboratory Investigations of Shear Moduli and Damping Ratios for Sandy Soils.

Type of Test	Soil Tested	Range of Strain		Range of Confining Pressure	Reference
		Shear	Axial		
Forced Vibration: Longitudinal Vibration	Sand		$<5\times10^{-3}\%$	600 to 7200 psf	Richart, Hall and Lysmer (1962)
	"		"	450 to 7500	Hardin and Richart (1963)
	"		"	600 to 7400	Hall and Richart (1963)
	"		"	600 to 7200	Hardin (1965)
	Sand, silty sand and clayey sand		"	1000 to 3500	Donovan (1968, 1969)
Forced Vibration: Torsional Vibrations Solid Sample	Sand	$<10^{-2}\%$		600 to 7200	Richart, Hall and Lysmer (1962)
	"	"		600 to 7400	Hall and Richart (1963)
	"	"		450 to 7500	Hardin and Richart (1963)
	"	"		600 to 7200	Hardin (1965)
	"	"		600 to 8500	Drnevich, Hall and Richart (1966)
Forced Vibration Torsional Vibration Hollow Sample	Sand	10^{-3} to $6\times10^{-2}\%$		600 to 8500	Drnevich, Hall and Richart (1966)
	"	"		500 to 1800	Hardin and Drnevich (1970)
Free Vibration: Cylindrical Sample	Sand		10^{-3} to $10^{-4}\%$	400 to 6400	Kishida and Takano (1970)
Triaxial Compression	Sand and gravel, silt and sand		2×10^{-3} to $5\times10^{-3}\%$	400 to 1800	Weissman and Hart (1961)
	Sand, silty sand and clayey sand		5×10^{-3} to 0.1%	1000 to 3500	Donovan (1968, 1969)
	Sand		10^{-1} to 1%	3000 to 3400	Matsushita, Kishida and Kyo (1967)
Simple Shear	Sand	3×10^{-2} to 0.5%		2000	Seed (1968)
	"	10^{-2} to 0.5%		500 to 4000	Silver and Seed (1969)

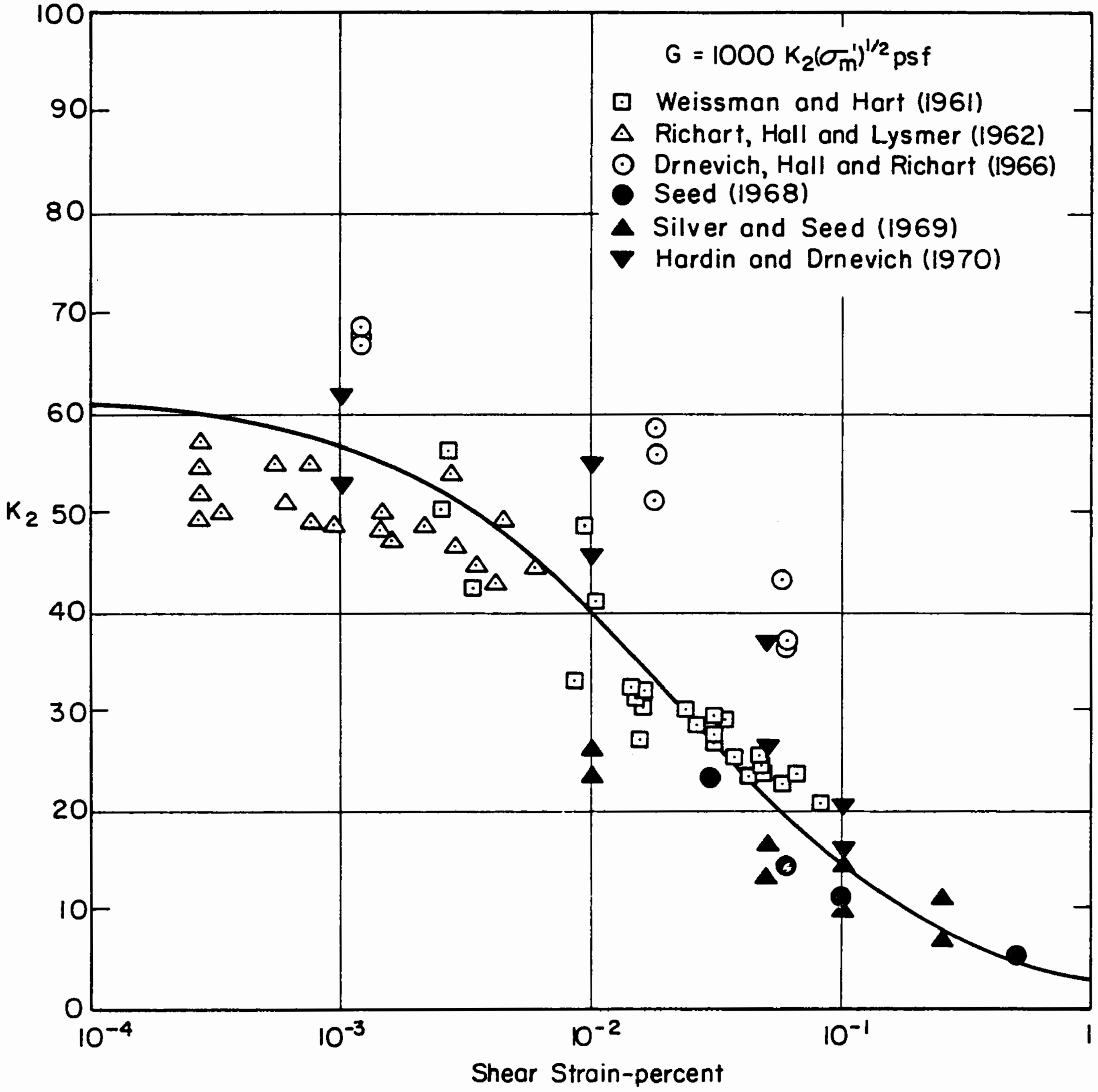

Fig. 3 SHEAR MODULI OF SANDS AT RELATIVE DENSITY OF ABOUT 75 %.

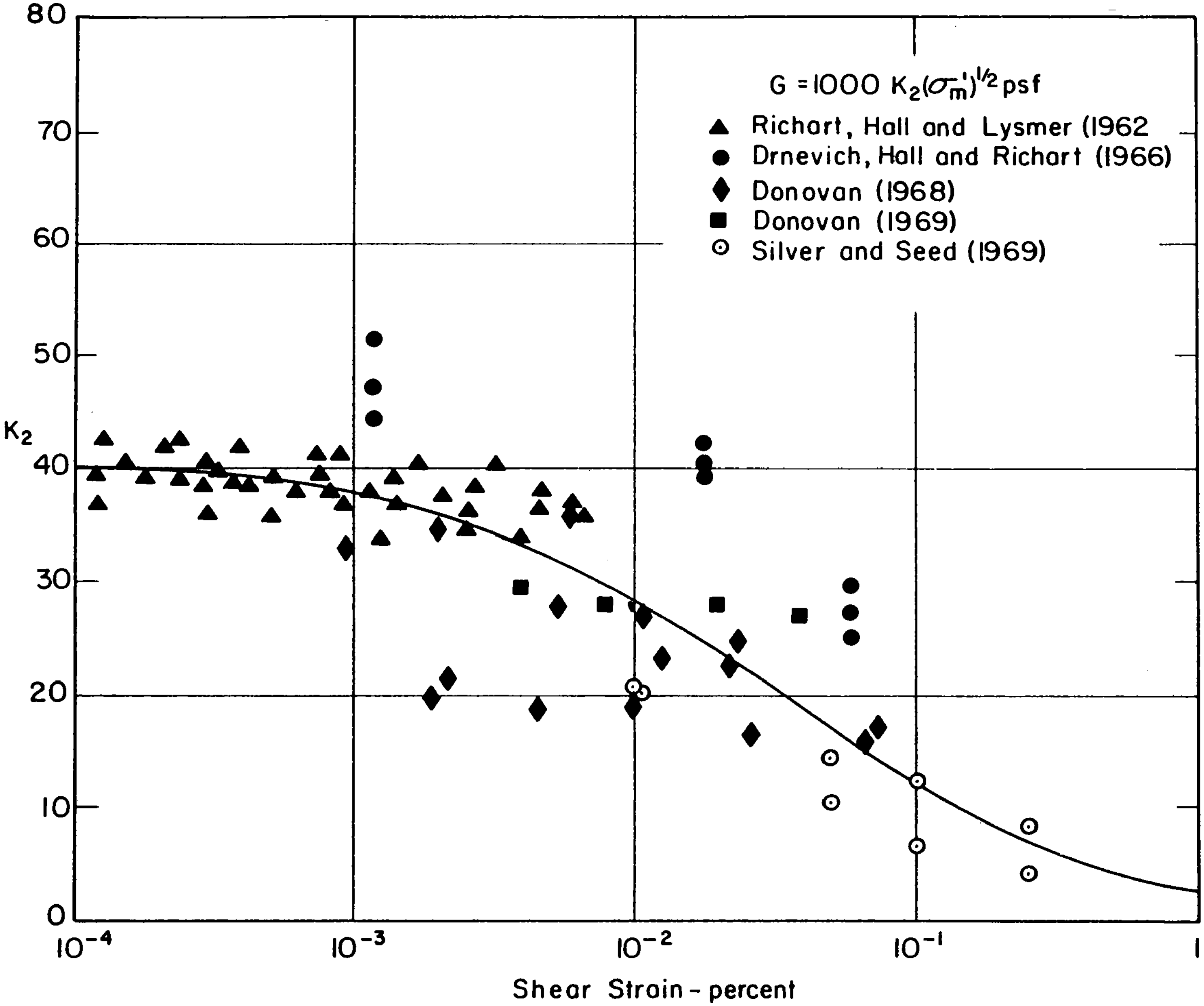

Fig. 4 SHEAR MODULI OF SANDS AT RELATIVE DENSITY OF ABOUT 40%.

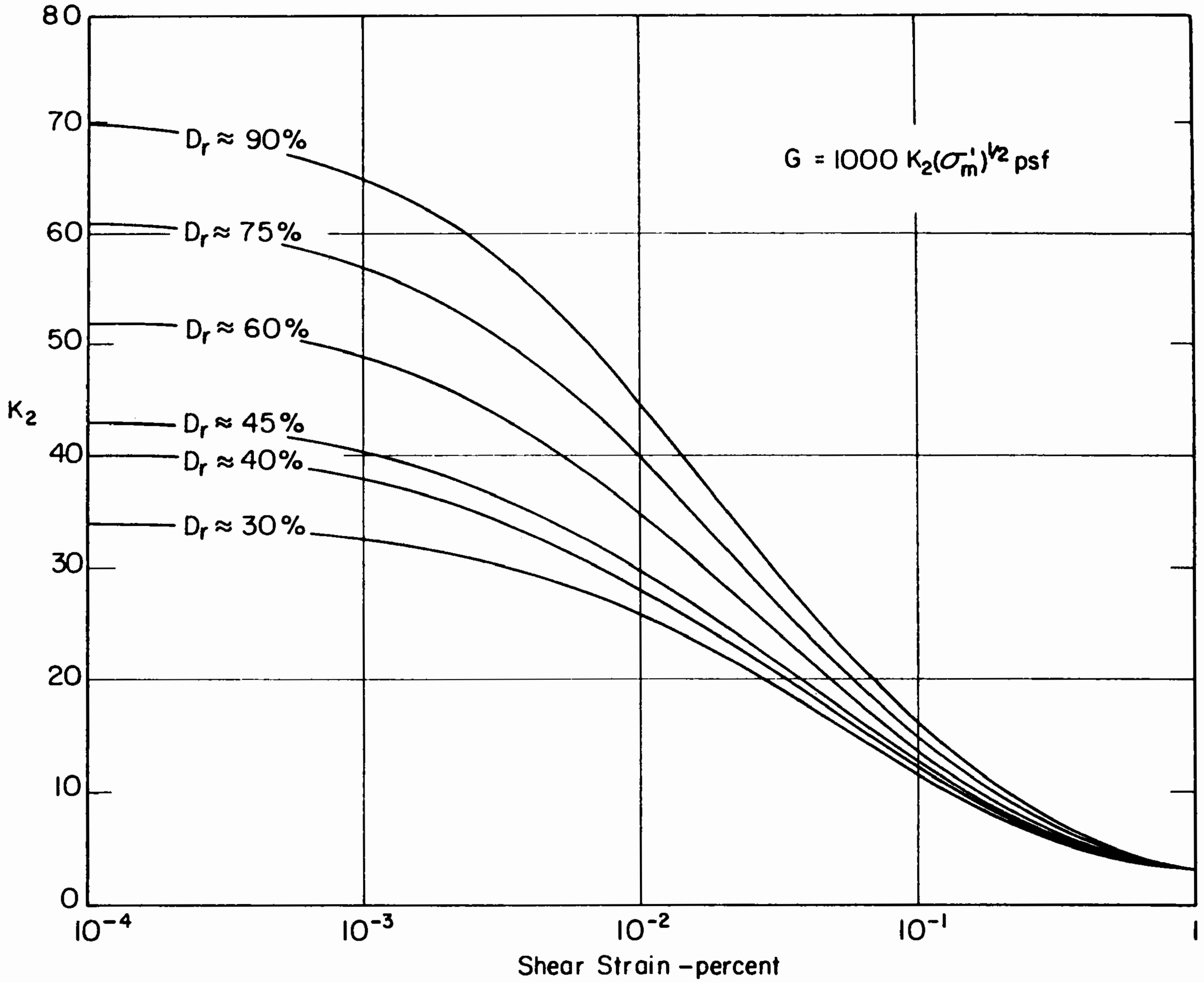

Fig. 5 SHEAR MODULI OF SANDS AT DIFFERENT RELATIVE DENSITIES.

It may be seen that for relatively dense samples, the values of K_2 determined at very low strains for laboratory test specimens are typically in the range of 50 to 75. The results of a number of determinations of shear moduli for sands at very low strain levels by means of in-situ shear wave velocity measurements are summarized in Table 3; the six investigations for dense to extremely dense sands (excluding clayey and partly cemented sands) give values for K_2 ranging from 44 to 86. Thus there appears to be good general agreement between the results of laboratory and in-situ investigations.

For purposes of comparison, representative values of the relationship between K_2 and strain at different void ratios determined by the Hardin-Drnevich relationship for an effective vertical stress of 3000 psf, $K_o = 0.5$ and $\phi' = 36°$ are plotted in Fig. 6.

The good agreement between the results in Figs. 5 and 6 indicates that reasonable values for the shear moduli of sands may be obtained either by use of the curves in Fig. 5 or by use of the Hardin-Drnevich equations. Where field data is obtained in terms of the standard penetration resistance, the data in Fig. 5 is likely to be more convenient but for other purposes, direct computation from equations 1, 2 and 3 may be desirable.

It may be noted that if each of the relationships shown in Figs. 5 and 6 is replotted to show the variation with shear strain of the ratio of shear modulus at strain γ to shear modulus at a shear strain of 10^{-4} percent, the results fall within the relatively narrow band shown in Fig. 7. Thus a close approximation to the modulus vs shear strain relationship for any sand can be obtained by determining the modulus at a very low strain level, say by wave propagation methods in the field, and then reducing this value for other strain levels in accordance with the results indicated by the average (dashed) line in Fig. 7.

Table 3. Shear Moduli* of Sands Based on In-Situ Shear Wave Velocity Measurements

Soil	Location	Depth ft.	K_2
Loose moist sand	Minnesota	10	34
Dense dry sand	Washington	10	44
Dense saturated sand	So. California	50	58
Dense saturated sand	Georgia	200	60
Dense saturated silty sand	Georgia	60	65
Dense saturated sand	So. California	300	72
Extremely dense silty sand	So. California	125	86
Dense dry sand (slightly cemented)	Washington	65	166
Moist clayey sand	Georgia	30	119

*Shear modulus, $G = 1000\ K_2\ (\sigma_m')^{\frac{1}{2}}$ psf

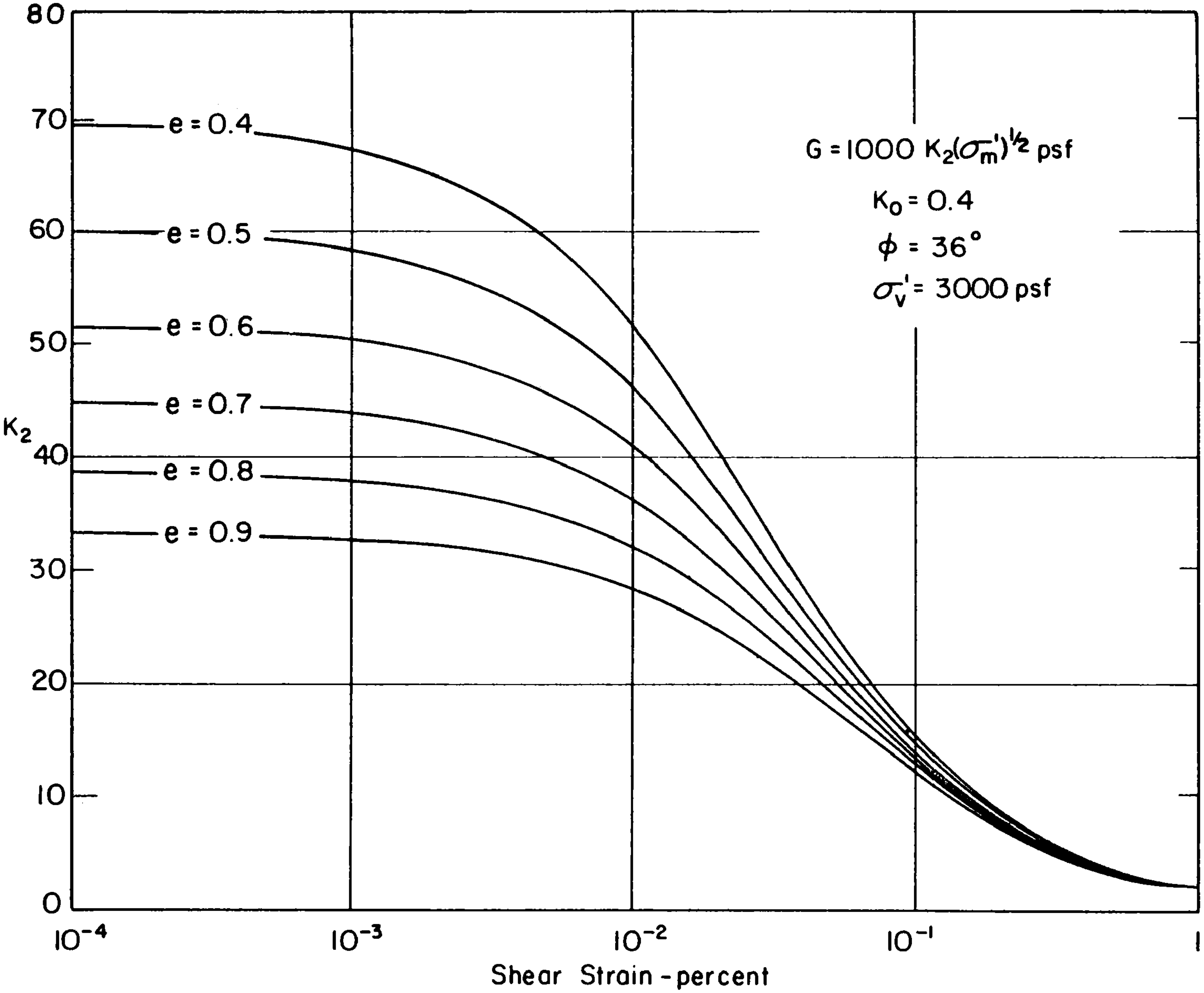

Fig. 6 SHEAR MODULI OF SANDS AT DIFFERENT VOID RATIOS.
(based on Hardin – Drnevich expressions)

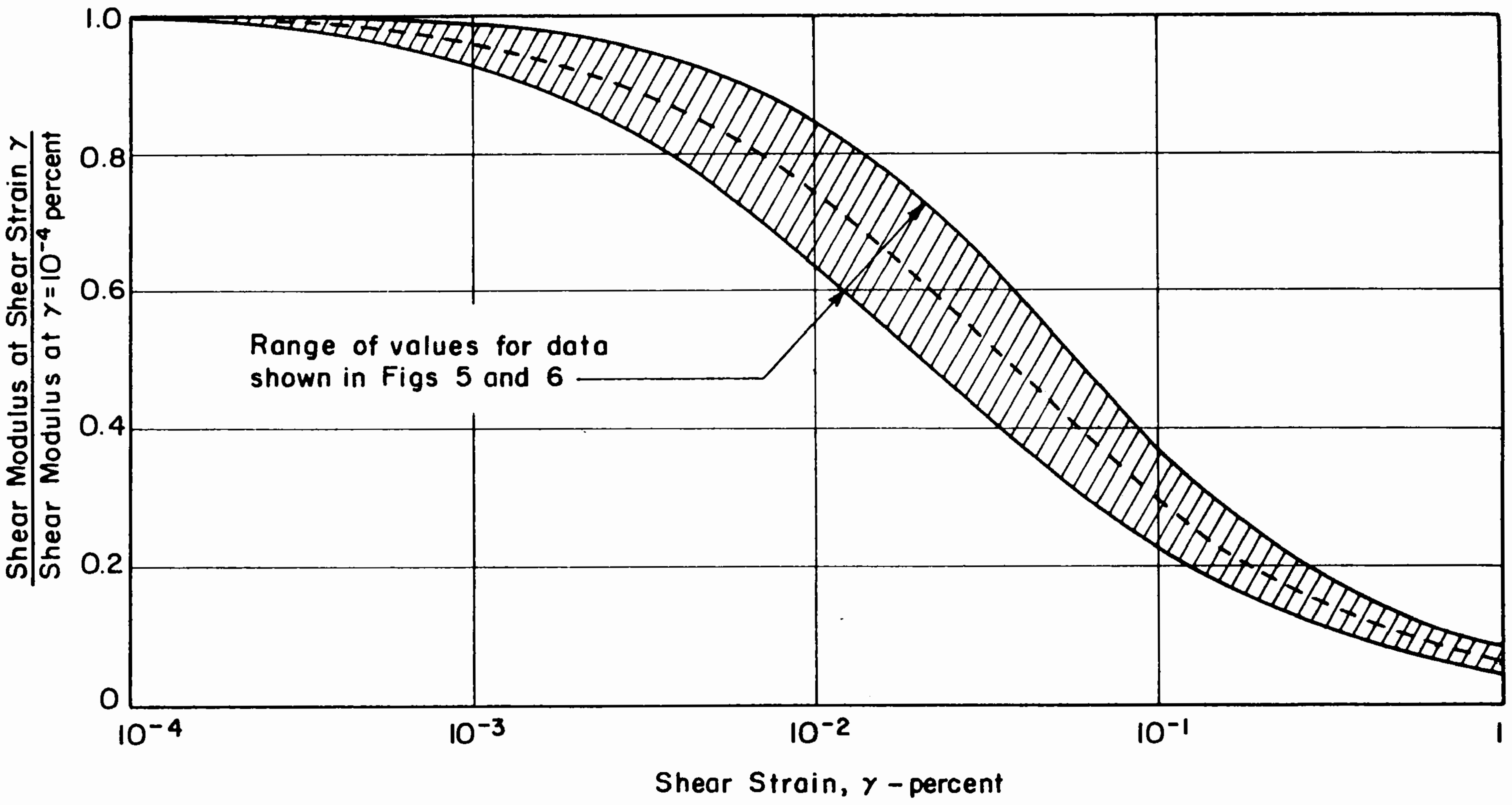

Fig. 7 VARIATION OF SHEAR MODULUS WITH SHEAR STRAIN FOR SANDS.

5. Damping Ratios for Sands

From their study of factors influencing the damping ratios of sands, Hardin and Drnevich concluded that shear strain, effective mean principal stress (or σ_v' and K_o), void ratio and number of cycles were very important, while octahedral shear stress, angle of friction and degree of saturation had lesser effects. As in the case of moduli, the effects of variations in grain size characteristics were considered to be relatively insignificant.

Computations of the effects of the above factors on the relationship between damping ratio and shear strain amplitude, as determined by the Hardin-Drnevich relationships are shown in Fig. 8. It is apparent that the effects of ϕ', K_o, void ratio and degree of saturation are relatively minor, and it can readily be seen from the equation for maximum damping ratio

$$\lambda_{max} \simeq 30 - 1.5 \log_{10} N$$

that if values of λ are determined for about $N = 5$ cycles, values for other numbers of cycles in the range of interest (say 5 to 30) will not be significantly different.

Thus the main factor affecting the relationship between damping ratio and shear strain is the vertical confining pressure σ_v'. The influence of this factor, as determined by two studies is shown in Fig. 9. For pressures less than about 500 psf, the effect of pressure changes may be significant but excluding these very low pressures, which represent conditions in the top few feet of soils, the effect of variations in pressure is very small compared with the effect of shear strain, and an average damping ratio vs shear strain relationship determined for an effective vertical stress of

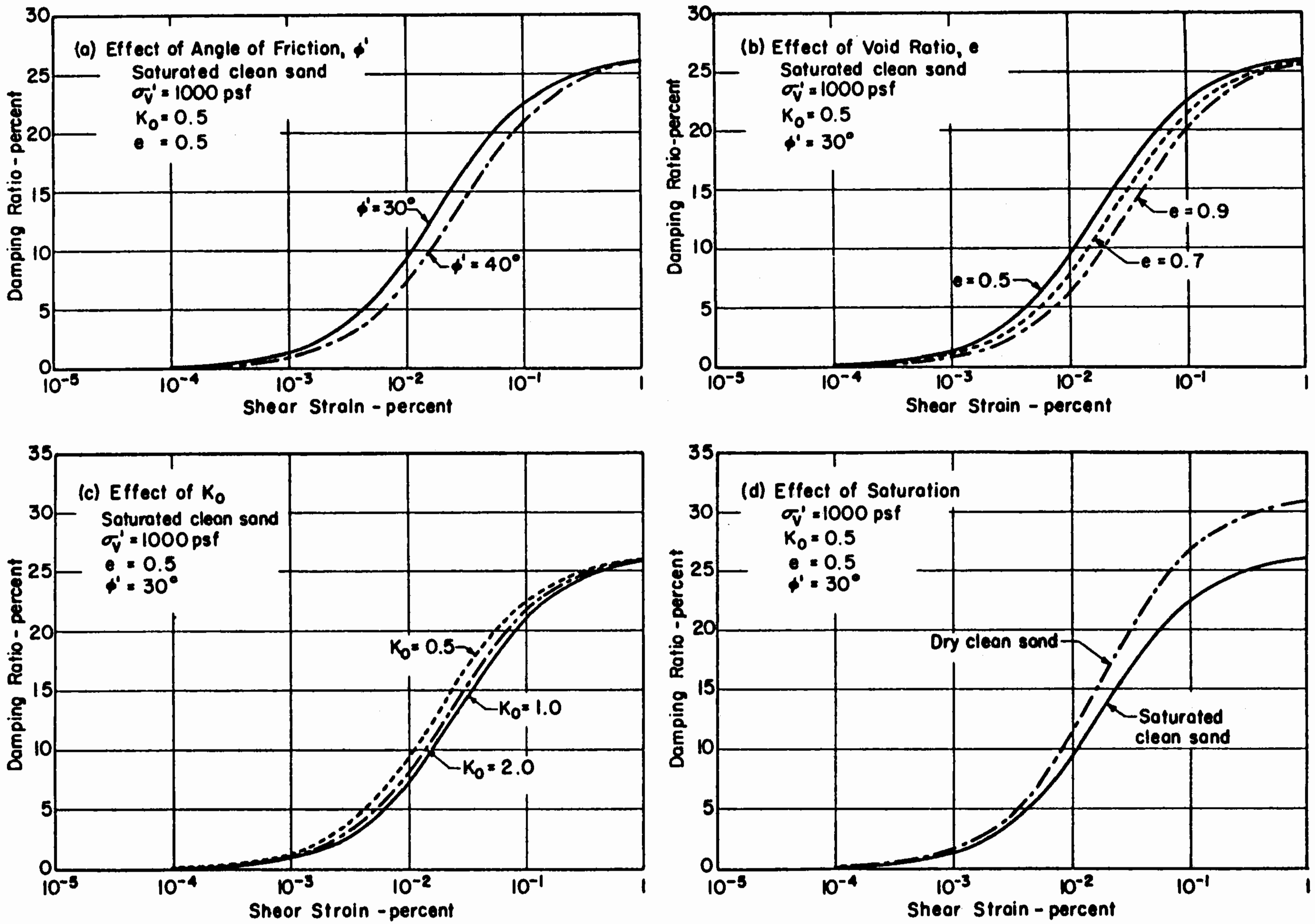

Fig. 8 INFLUENCE OF VARIOUS FACTORS ON THE DAMPING RATIOS FOR SANDS.
(based on Hardin and Drnevich expressions)

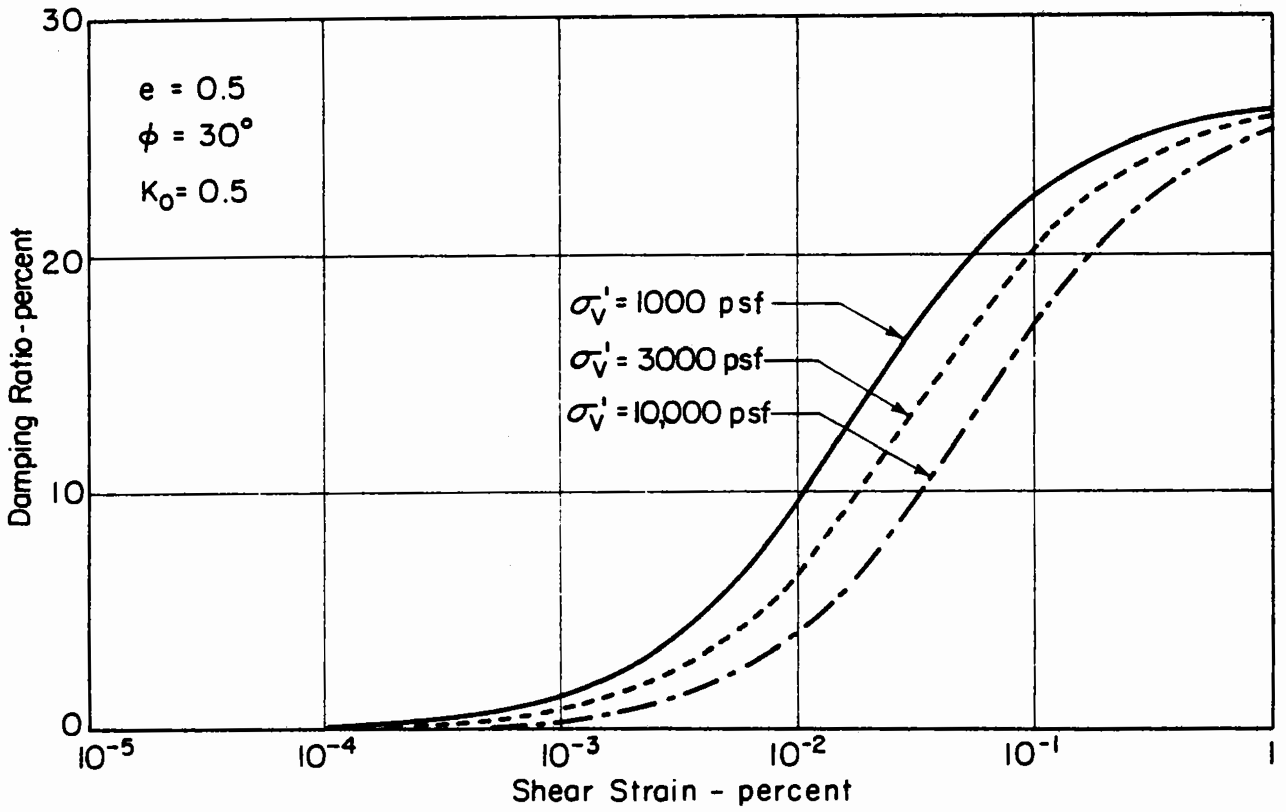

Fig. 9a INFLUENCE OF CONFINING PRESSURE ON DAMPING RATIO OF SATURATED SAND.
(based on Hardin and Drnevich expressions)

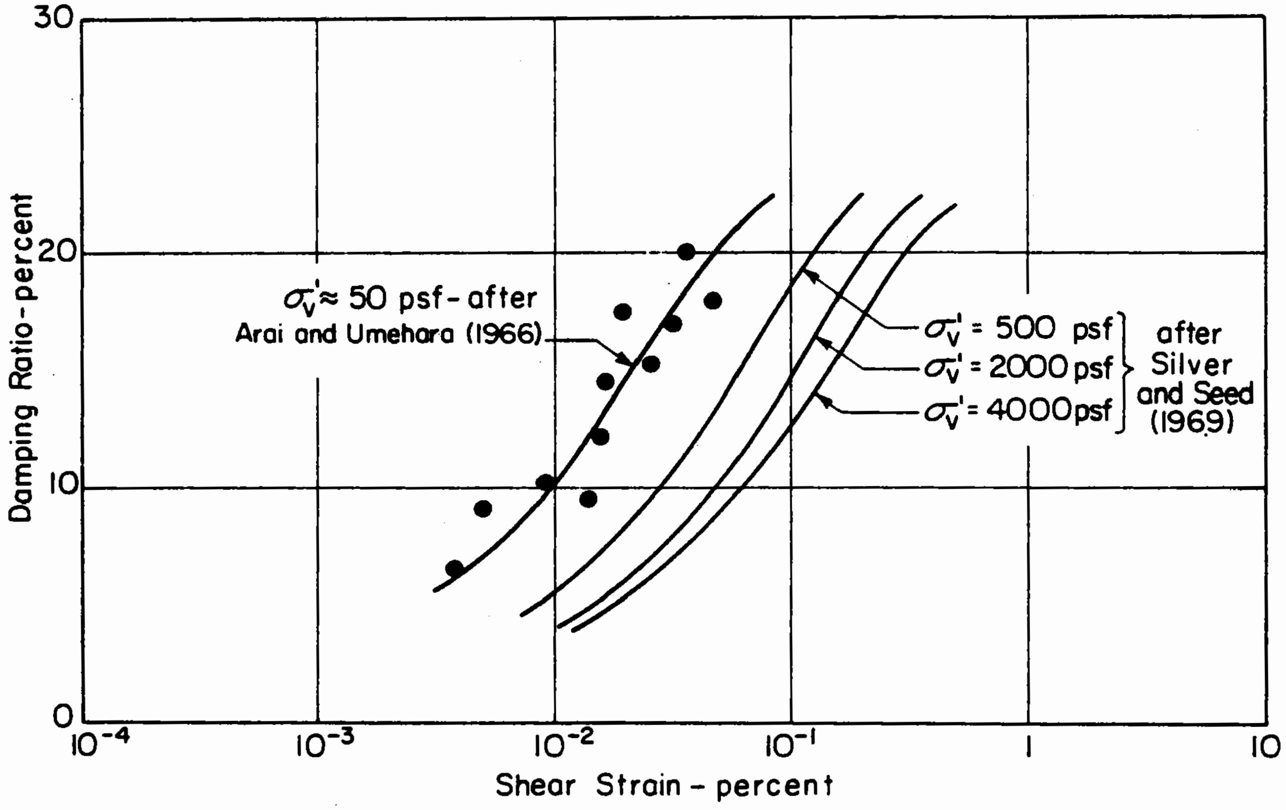

Fig. 9b INFLUENCE OF CONFINING PRESSURE ON DAMPING RATIO OF DRY SAND.

2000 to 3000 psf would appear to be adequate for many practical purposes. Considering the potential scatter of test data for damping ratios, even those obtained by the same investigator using the same test procedure, the adoption of such an average relationship may be even more justified.

A list of previous investigations of damping ratios of sands is presented in Table 2, and the results of these studies are summarized in Fig. 10. Approximate upper and lower bound relationships are shown by dashed lines and a representative average relationship for all of the test data is shown by the solid line. This average relationship is likely to provide values of damping ratio with sufficient accuracy for many practical purposes.

The curves in Fig. 10 also provide a basis for evaluating the relationship between damping ratio and strain for particular sands for which limited test data is available. If the value of damping ratio at a strain level of 0.1 to 0.5 percent is determined, the probable damping ratios at other strains can be closely approximated by drawing a line through the known data point parallel to the curves shown in Fig. 10.

6. Shear Modulus Values for Saturated Clays

Accurate determination of the shear moduli of saturated clays is enormously complicated by the large effects of strain amplitude and sample disturbance on modulus values. In-situ measurements eliminate the problems raised by sample disturbance, but to date no techniques have been developed for inducing large controlled strain amplitudes in natural deposits and thus moduli can only be determined at very small strain levels. In the laboratory, on the other hand, samples may be tested under a wide range of strains but for test specimens from natural deposits, the moduli

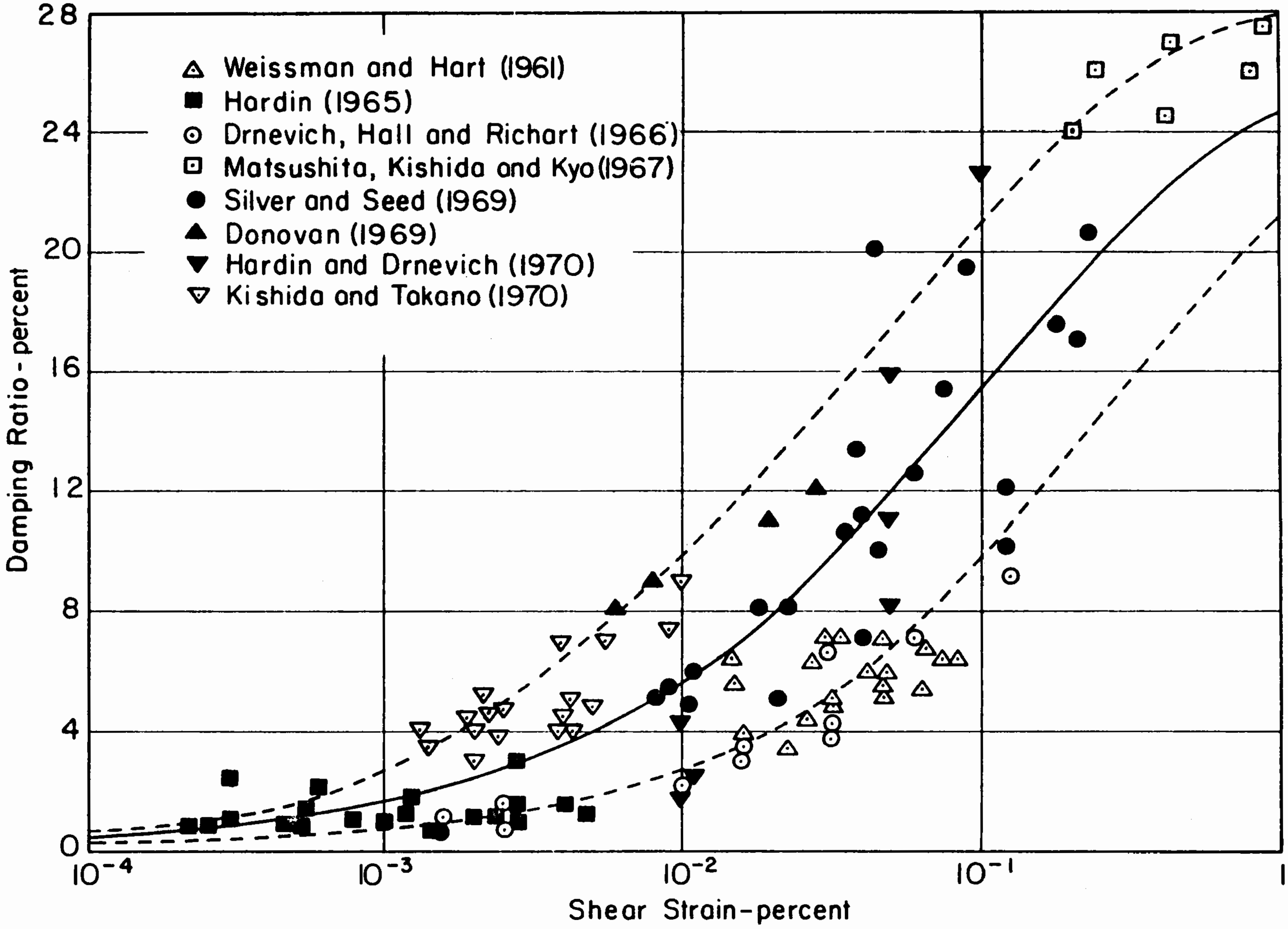

Fig. 10 DAMPING RATIOS FOR SANDS.

determined will inevitably be influenced by the effects of sample disturbance.

The joint influence of these effects is illustrated by the data presented in Figs. 11 and 12. Fig. 11 shows values of shear moduli for San Francisco Bay mud at a depth of about 25 ft determined by in-situ shear wave velocity measurements by Aisiks and Tarshansky (1968) and values determined by cyclic loading simple shear tests on undisturbed samples by Thiers (1965). Projecting the laboratory test data to the strain level corresponding to the field test conditions, it may be seen that the laboratory test values are only about 40 percent of those for the in-situ clay. This result is not surprising in the light of previous studies of the influence of disturbance on the moduli of natural clays (Ladd, 1964) and it emphasizes the magnitude of the correction which may have to be made for this effect.

The influence of strain amplitude on shear modulus is also apparent from the data in Fig. 11, the values at strains of about 0.5 percent being only about 12 percent of those corresponding to strains of the order of 10^{-3} or 10^{-4} percent.

Fig. 12 shows similar data for Union Bay clay. In this case values of shear moduli for in-situ conditions were determined from seismic wave velocity measurements and from observations of the response of the clay during an earthquake; modulus values for undisturbed samples were determined by resonant frequency tests and cyclic loading tests in the laboratory. Again the in-situ moduli are two or three times greater than the laboratory test values at comparable strains, and the modulus decreases enormously with increasing strain amplitude.

In addition to the effects of strain amplitude and disturbance, the shear moduli of different clays will clearly depend on their relative

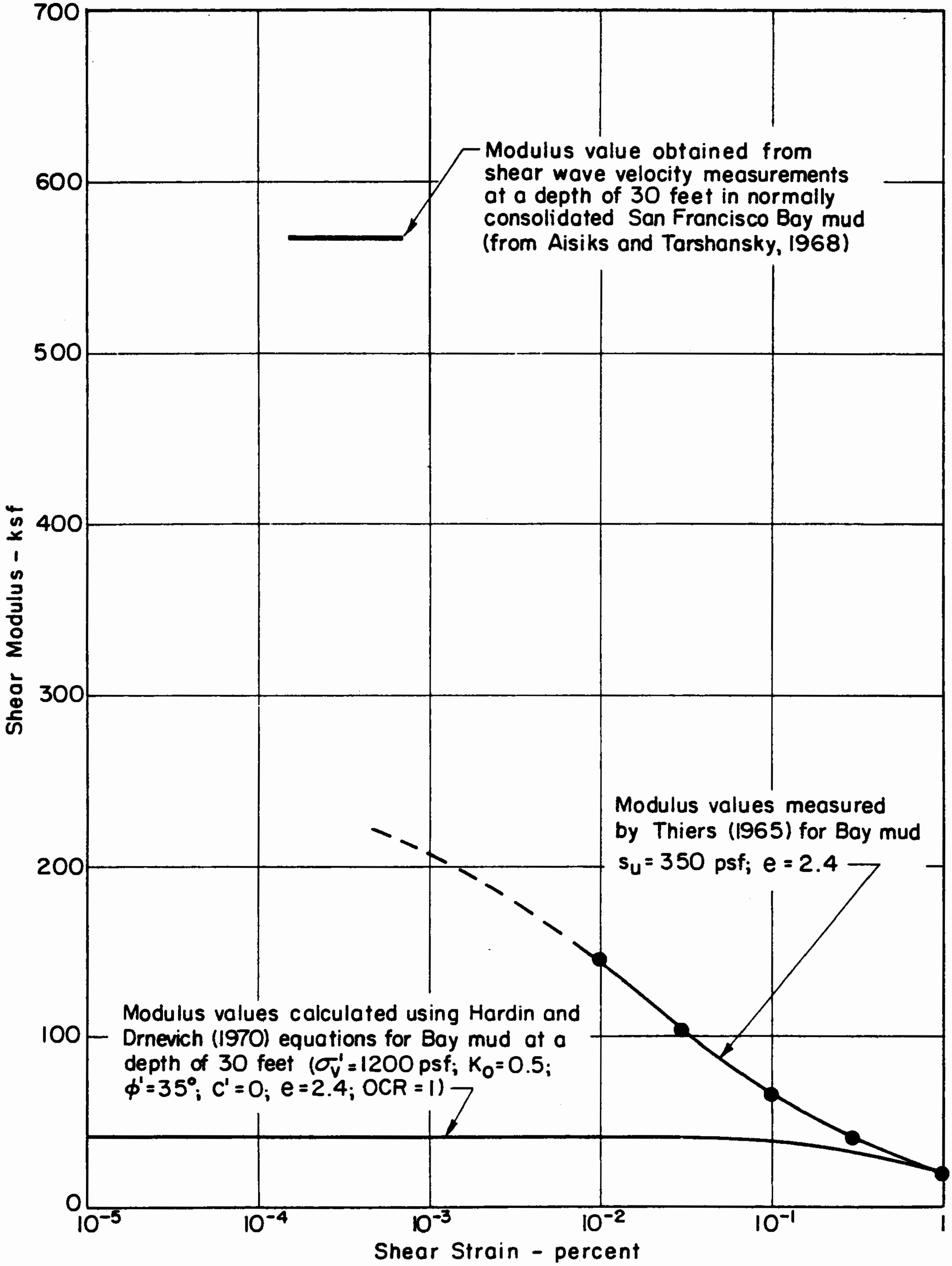

Fig.11 SHEAR MODULUS DETERMINATIONS FOR SAN FRANCISCO BAY MUD.

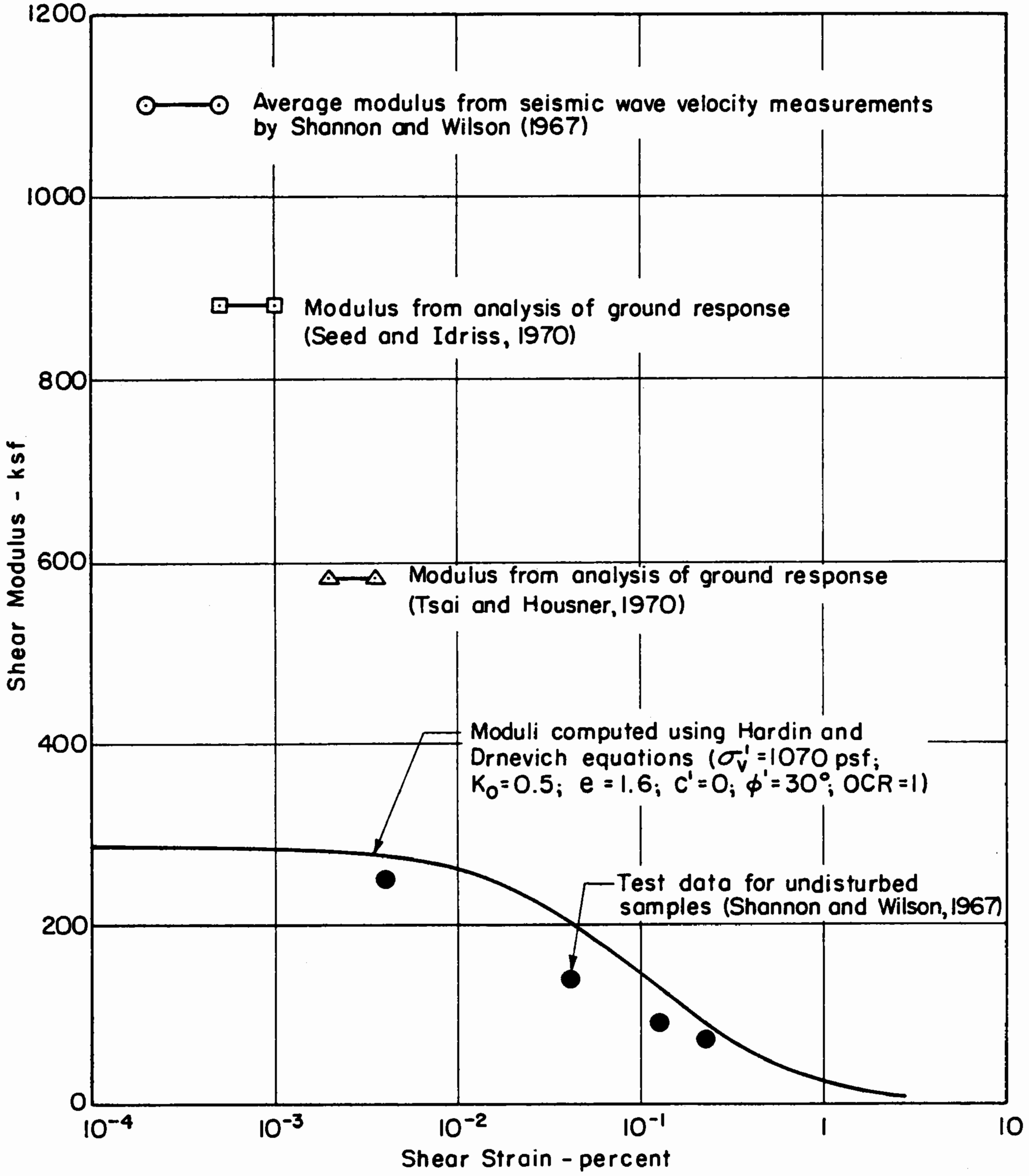

Fig. 12 SHEAR MODULUS DETERMINATIONS FOR UNION BAY CLAY AT DEPTH OF ABOUT 80 FT.

strengths and stiffnesses. Hardin and Drnevich express these effects in terms of the effective mean principal stress, void ratio, overconsolidation ratio and effective stress strength parameters, but the resulting relationships do not always provide reasonable evaluations of shear moduli for in-situ conditions, as evidenced by the results shown in Figs. 11 and 12.

However in view of the facts that (1) stiffness increases in general with soil strength, (2) for static load conditions, the ratio E/s_u for saturated clays does not vary widely from one soil to another, and (3) test data at very low strain levels indicates an approximately linear relationship between the shear modulus and shear strength for a number of clays (Wilson and Dietrich, 1960), it seems reasonable to expect that variations in clay characteristics might be taken into account with a reasonable degree of accuracy by normalizing the shear modulus, G, with respect to the undrained shear strength, s_u, and expressing the relationship G/s_u as a function of shear strain.

Test data obtained by a number of investigators and expressed in this form are summarized in Table 4 and plotted in Fig. 13. For test data obtained in laboratory tests under unconsolidated-undrained test conditions, the measured moduli were multiplied by a factor of 2.5 to make an approximate allowance for sample disturbance. Clearly the effects of disturbance will vary from one study to another but in the absence of detailed information on sampling and testing conditions it was considered that a factor of 2.5 would represent a reasonable average correction factor for these effects. For in-situ and laboratory consolidated-undrained test conditions, no correction was applied to the test results.

While there is considerable scatter in the data, most of the test results fall within the dashed lines in Fig. 13; that is, within ±50% of the average values shown by the solid line in the figure. Thus the average values are likely to provide reasonable estimates of the in-situ moduli for many clays.

Table 4. Summary of Investigations of Shear Moduli and Damping Ratios for Saturated Clays.

Type of Test	Soil Tested	Range of Strain		Range of Shear Strength	Data Correction Factor*	Reference
		Shear Strain	Axial Strain			
Field shear wave velocity measurements	S.F. Bay mud	$<10^{-3}\%$		200 to 500 psf	1.0	Aisiks and Tarshansky (1968)
Field compression wave velocity measurements	Union Bay clay		$<10^{-3}\%$		1.0	Shannon and Wilson (1967)
Lab. Free Vibration Tests: Longitudinal Vibrations	Elkhorn Slough silty clay		3×10^{-2} to 2%	300 to 1100 psf	2.5	Parmalee et al. (1964); Idriss (1966)
Lab. Free Vibration Tests: Shear Vibrations	S.F. Bay mud	2×10^{-2} to 0.5%		300 psf	2.5	Kovacs (1968)
	Kaolinite/Bentonite mixture	5×10^{-2} to 2%		44 to 85 psf	2.5	Kovacs (1968)
Lab. Forced Vibration Tests: Longitudinal Vibrations	Cambridge clay		$\approx2.5\times10^{-3}\%$	1080 psf	2.5	Wilson and Dietrich (1960)
	Mississippi gravels		$\approx2.5\times10^{-3}\%$	520 psf	2.5	Wilson and Dietrich (1960)
Lab. Forced Vibration Tests: Torsional Vibrations	Birch Bay clay	$\approx2.5\times10^{-3}\%$		1000 to 2420 psf	2.5	Wilson and Dietrich (1960)
	Montana clay	$\approx2.5\times10^{-3}\%$		6000 psf	2.5	Wilson and Dietrich (1960)
Lab. Forced Vibration Tests: Torsional Vibrations (consol. samples)	Whidbey Bay clay	$\approx2.5\times10^{-3}\%$		230 to 1800 psf	1.5**	Wilson and Dietrich (1960)
	Silty clay	0.125%		800 to 1500 psf	1.0	Zeevaert (1967)
	Edgar Plastic Kaolin	$\approx2.5\times10^{-3}\%$		1400 to 1800 psf	1.0	Hardin and Black (1968)
Lab. Triaxial Comp. Tests	Ardmore clay		0.1 to 0.5%	-	-	Taylor and Menzies (1963)
	Ardmore clay		0.5 to 1%	-	-	Taylor and Hughes (1965)
	Union Bay clay		3×10^{-3} to 0.3%	200 to 880 psf	2.5	Shannon and Wilson (1967)
	Silty clay		10^{-2} to 0.1%	-	-	Donovan (1969)
	Webb Mark IV clay		0.2 to 1%	-	-	Taylor and Bacchus (1969)
Lab. Torsional Shear Tests	Georgia Kaolinite	3×10^{-2} to 0.2%		-	-	Krizek and Franklin (1967) Hardin and Drnevich (1970)
Lab. Simple Shear Tests	S.F. Bay mud	0.2 to 4%		300 to 400 psf	2.5	Thiers (1965), Thiers & Seed (1968)
	Kaolinite/Bentonite mixture	0.1 to 2.5%		44 to 85 psf	2.5	Kovacs
	S.F. Bay mud	0.1 to 3%		300 psf	2.5	Kovacs

*Applied to modulus values to allow for sample disturbance.
**Sample disturbed slightly after consolidation.

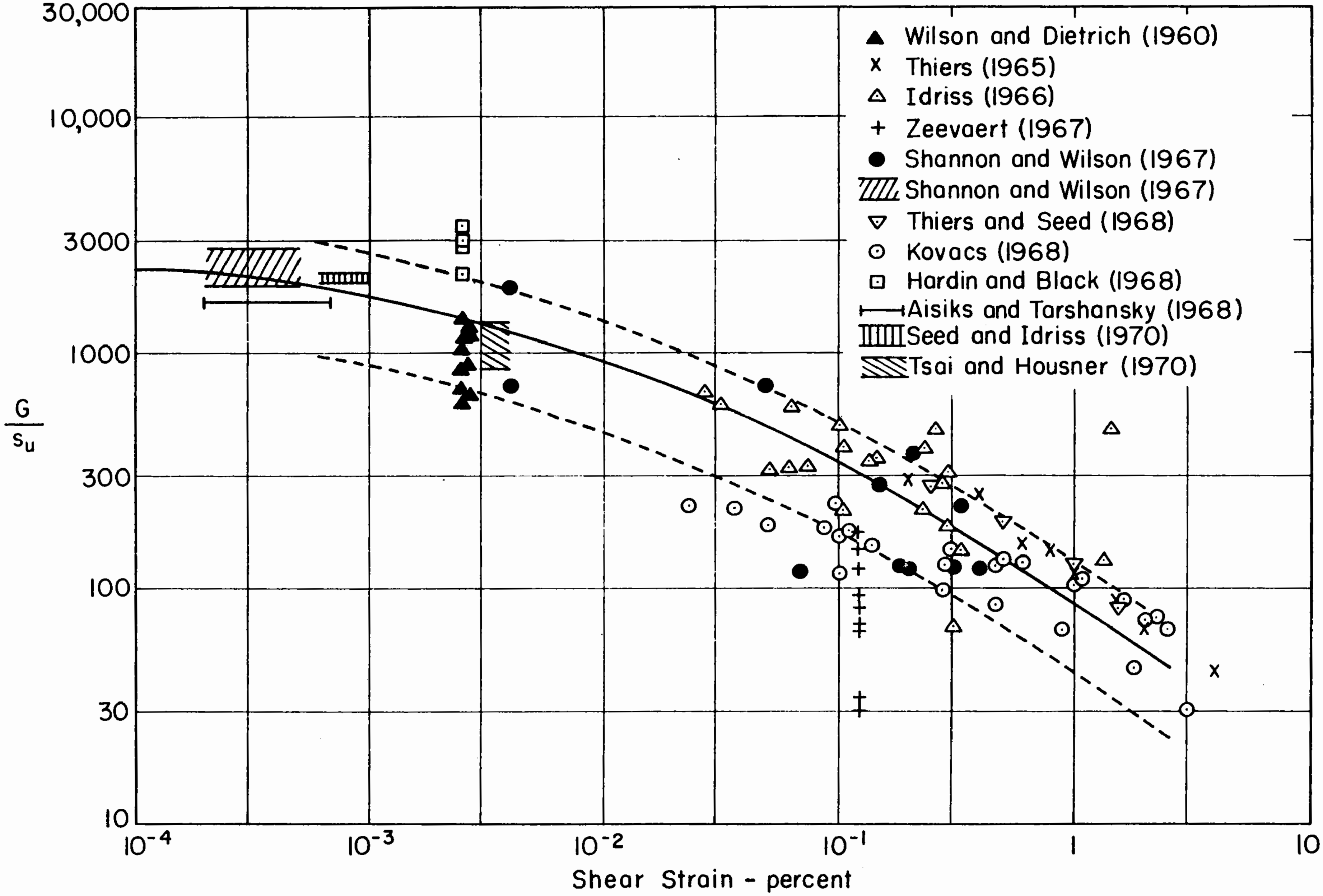

Fig. 13 IN-SITU SHEAR MODULI FOR SATURATED CLAYS.

Alternatively the data in Fig. 13 might be used to assess the influence of strain amplitude on the shear modulus of natural clays, by expressing the ordinates in terms of the ratio of shear modulus at shear strain γ to shear modulus at a shear strain of 3×10^{-4} percent. This ratio for the average values shown in Fig. 13, is plotted as a function of shear strain in Fig. 14. Reasonable estimates of the shear modulus of a clay at any strain amplitude can be obtained by determining the in-situ value at strains of the order of 3×10^{-4} percent by means of shear wave velocity measurements and applying the reduction factors shown in Fig. 14 to determine values at other shear strains.

7. Damping Values for Saturated Clays

Test data for damping ratios for saturated clays are so limited and the results vary to such an extent that it is difficult to determine the main factors influencing the damping ratios of these soils. A list of previous investigations of damping ratios for saturated clays is presented in Table 3 and the results of these studies are summarized in Fig. 15. Approximate upper and lower bound relationships between damping ratio and shear strain are shown by the dashed lines and a representative average relationship for all of the test data is shown by the solid line. This average relationship may well provide values of damping ratio with sufficient accuracy for many practical purposes.

The curves in Fig. 15 also provide a basis for evaluating the relationship between damping ratio and strain for any particular clay. If the value of damping ratio at a strain level of 0.1 to 0.5 percent is determined, the probable damping ratios at other strains can be estimated by drawing a line through the known data point parallel to the curves shown in Fig. 15.

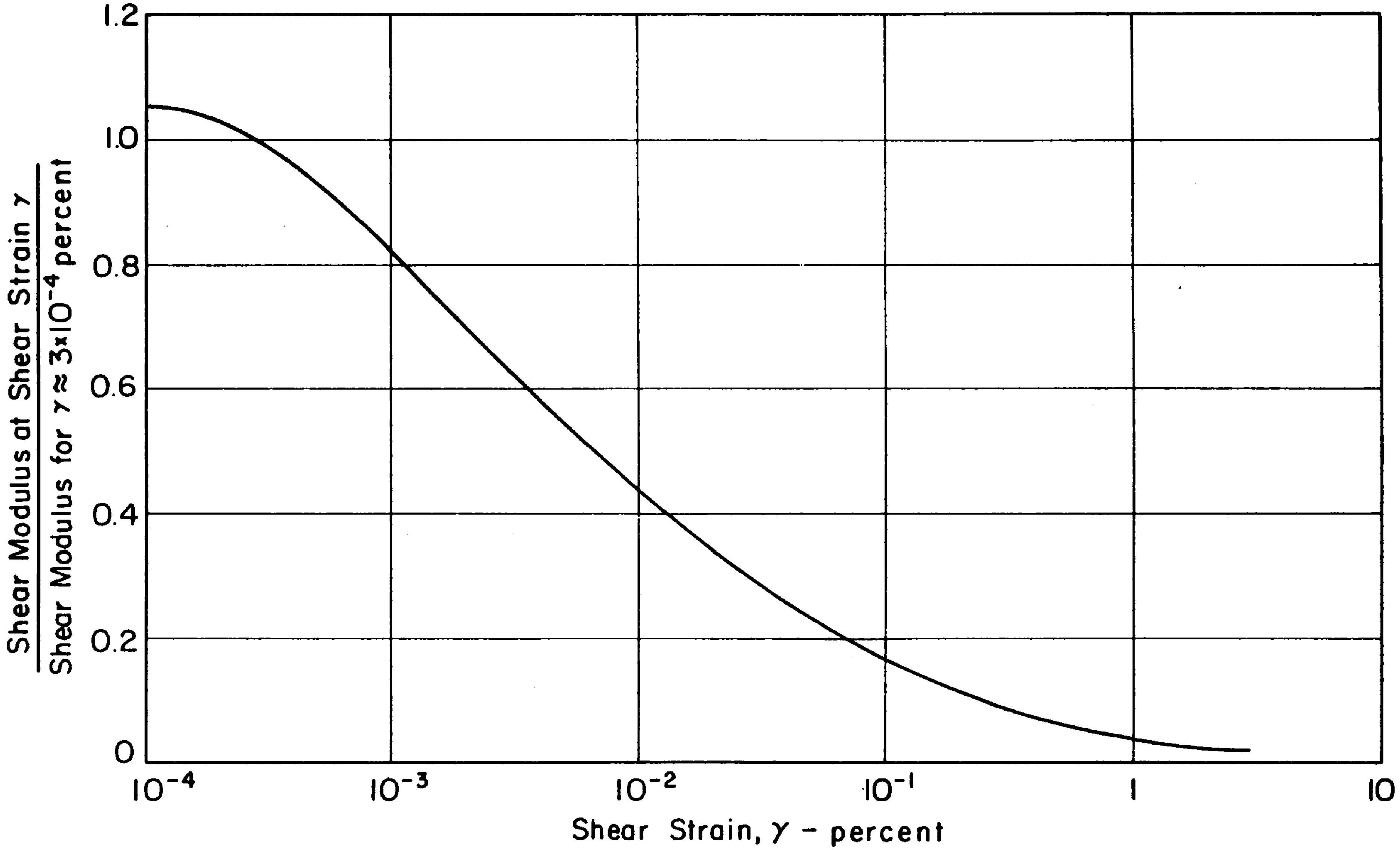

Fig. 14 TYPICAL REDUCTION OF SHEAR MODULUS WITH SHEAR STRAIN FOR SATURATED CLAYS.

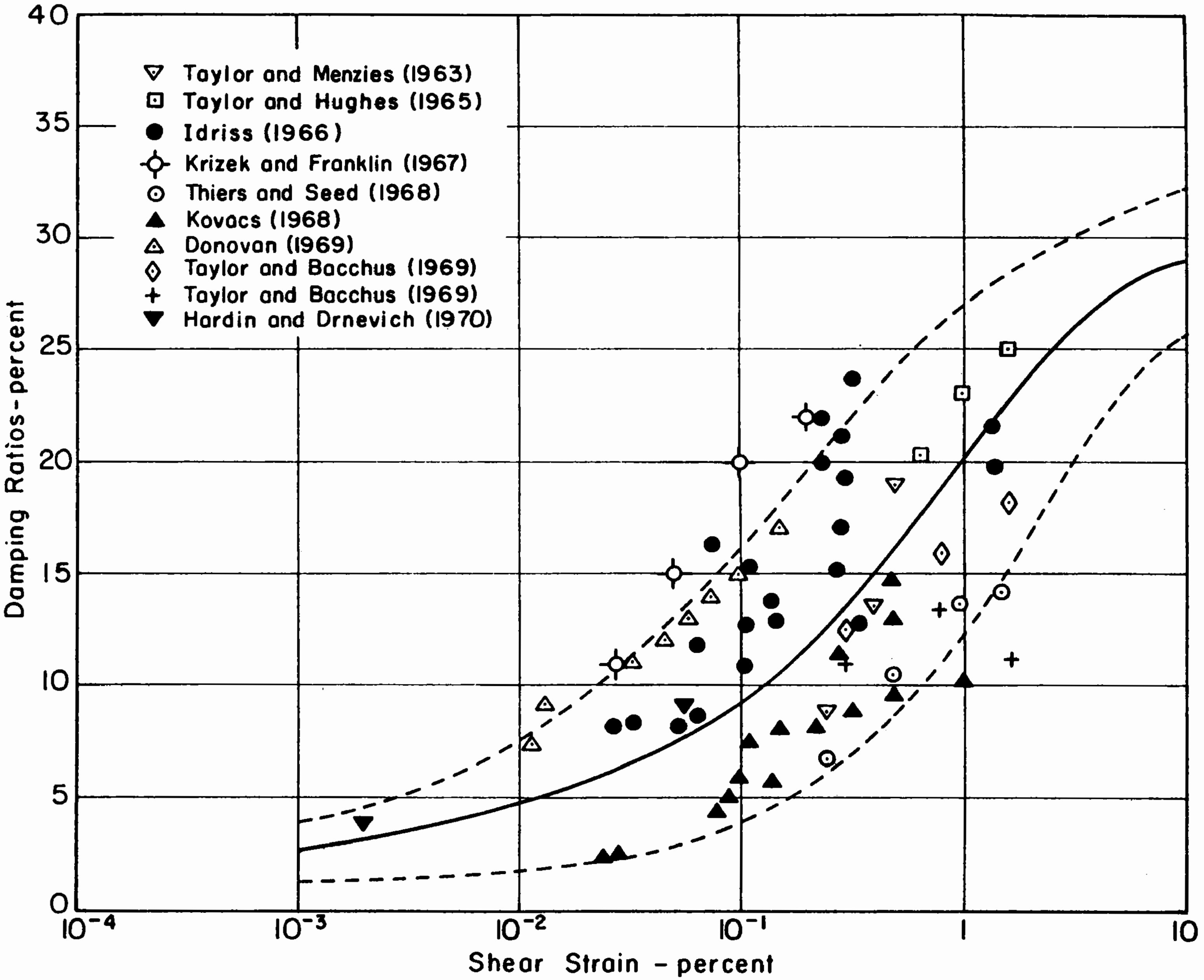

Fig. 15 DAMPING RATIOS FOR SATURATED CLAYS.

Further studies are required of the factors influencing the damping ratios of saturated clays to permit more detailed assessments of this characteristic for analysis purposes.

8. Shear Moduli and Damping Ratios for Gravelly Soils

Probably because of the large diameter of test specimens required, there do not appear to have been any laboratory investigations of the shear moduli and damping ratios for gravelly soils. The results of a limited number of moduli determinations for these type of soil, based on in-situ shear wave velocity measurements, are summarized in Table 5, from which it may be seen that at small strain levels, modulus values are between 1.25 and 2.5 times greater than those for dense sands.

At higher strains, it seems likely that moduli for gravelly soils will decrease in a manner similar to that for sands. Thus by applying the moduli reduction factors shown in Fig. 7 to the data in Table 5, variations in shear moduli with strain might be estimated as shown in Fig. 16. Additional data on modulus values and damping ratios for gravelly soils is badly needed; however approximate values for use in some types of response analyses can be estimated by the procedure shown in Fig 16 and the assumption that damping is approximately the same as for sandy materials.

9. Shear Moduli and Damping Ratios for Peats

The results of several investigations which have provided data on the shear moduli and damping ratios for peats are summarized in Fig. 17. Shear moduli are shown as the ratio of G/s_u, with values determined by in-situ seismic wave velocity measurements, evaluation of ground response to earthquake excitation, and laboratory investigations. As in the case of clays, moduli determined by laboratory unconsolidated-undrained tests were multiplied

Table 5. Shear Moduli* of Gravelly Soils Based on In-Situ Shear Wave Velocity Measurements

Soil	Location	Depth ft.	K_2
Sand, gravel, and cobbles with little clay	Caracas	200	90
Dense sand and gravel	Washington	150	122
Sand, gravel and cobbles with little clay	Caracas	255	123
Dense sand and sandy gravel	So. California	175	188

*Shear modulus $G = 1000\ K_2\ (\sigma_m')^{\frac{1}{2}}$ psf

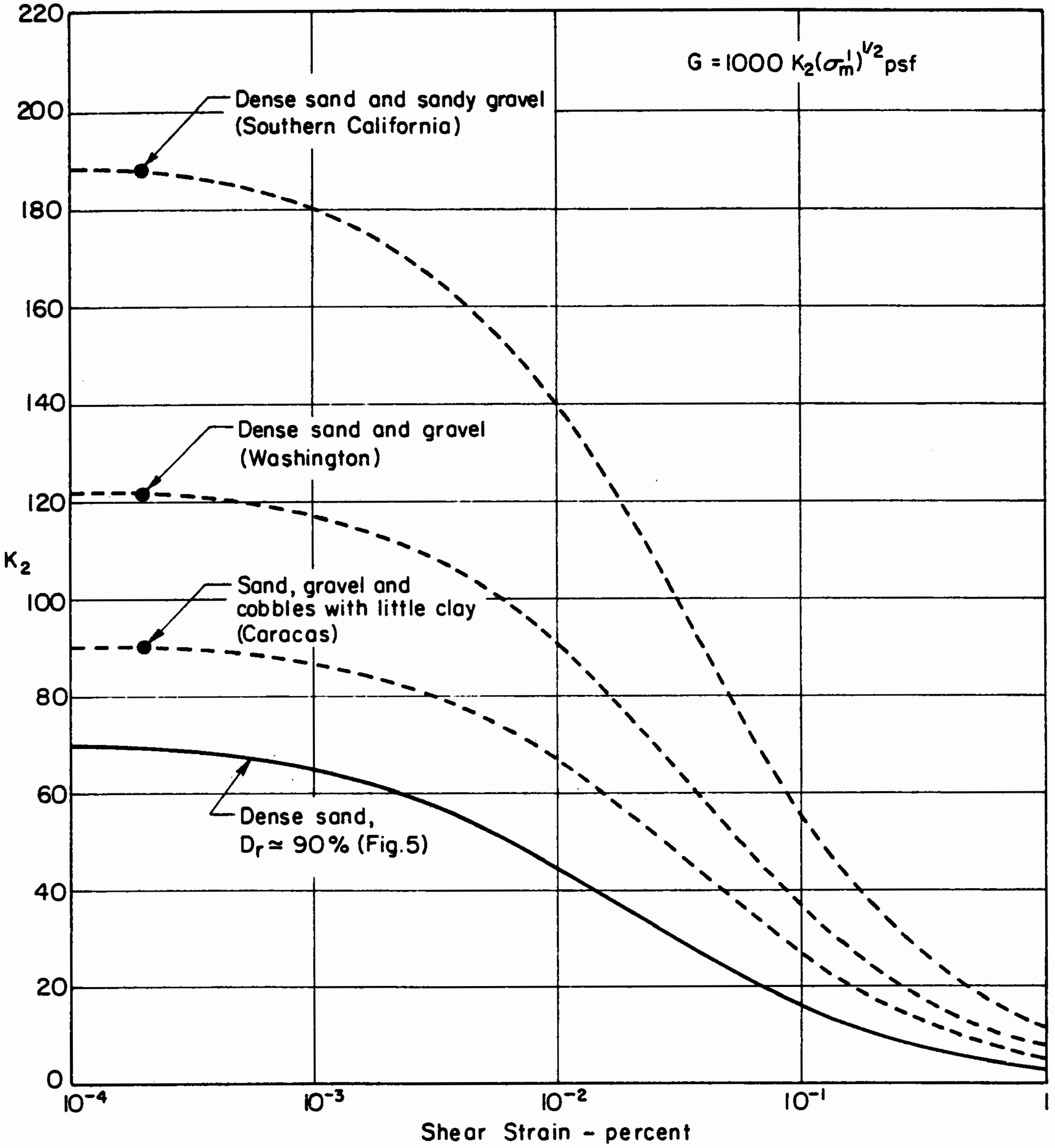

Fig. 16 MODULI DETERMINATIONS FOR GRAVELLY SOILS.

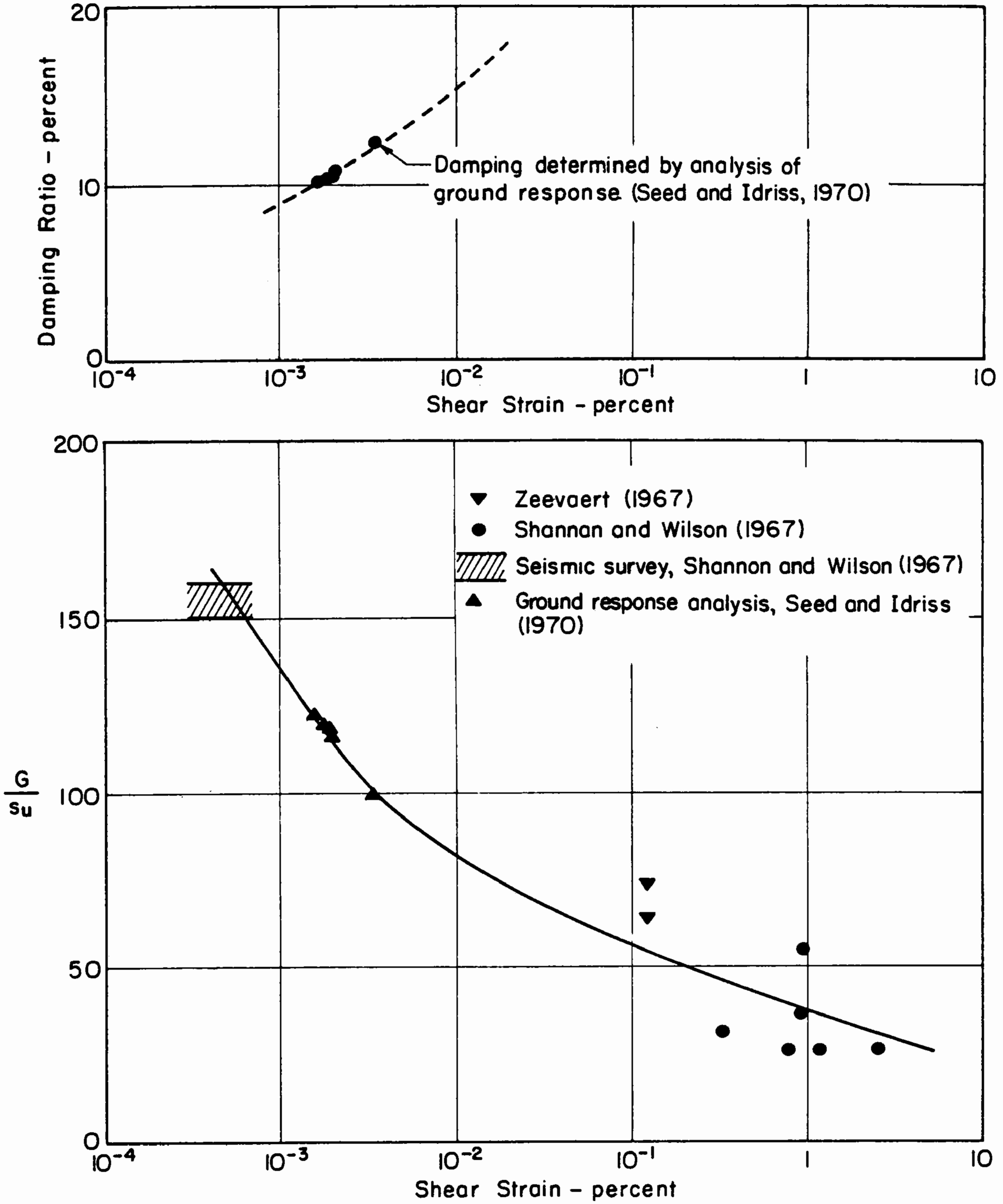

Fig. 17 MODULI AND DAMPING DETERMINATIONS FOR PEATS.

by a factor of 2.5 before being plotted in Fig. 17. While the nature of peaty soils is likely to vary considerably from one location to another, the data in Fig. 17 may provide some indication of the dynamic characteristics of this type of soil.

10. Conclusion

In the preceding pages an attempt has been made to summarize in a convenient form, the available data concerning the shear moduli and damping ratios for soils. Clearly more data on these dynamic characteristics is required, particularly for silts, clays and gravelly soils. However it is hoped that the data presented will serve as a useful guide in the selection of soil properties for dynamic response analyses and that other engineers might be encouraged to make available any additional data which would supplement that presented above.

Acknowledgement

The authors gratefully acknowledge the cooperation of many engineers who contributed data for use in this report; appreciation is due particularly to the Bechtel Corporation and Shannon and Wilson Inc. who provided previously unpublished field data and to I. Arango who assisted in the data reduction.

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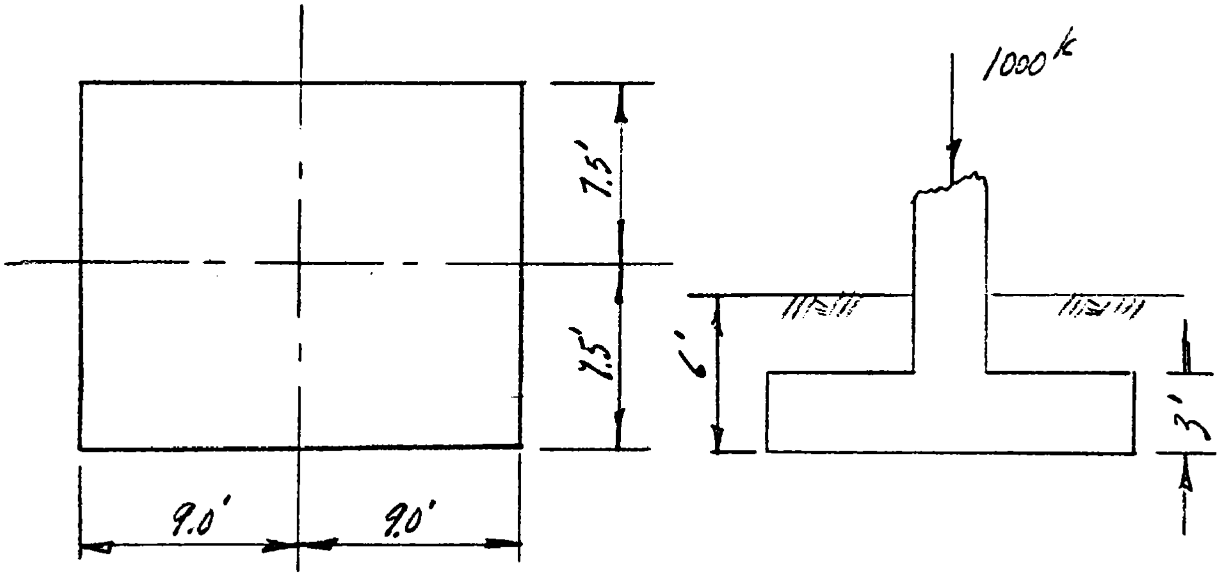
Appendix E.1

SAMPLE PROBLEM 10

Determine soil spring and damping coefficients for the spread footing shown in Figure E.1.1. Assume the footing is resting on a cohesionless soil with a density of 120 pcf and a shear wave velocity of 1500 ft/sec. The column deadload reaction is 1000 kips.

Sample Problem 10

Determine soil spring and damping coefficients for the spread footing shown in the sketch below. Assume a cohesionless soil with a density of 120 pcf and a shear wave velocity of 1500 ft/sec. The column deadload reaction is 1000 kips.



Poisson's Ratio $\nu = 0.33$ $^{*}(6.3.5.1)$

Shear Modulus $G = \rho V_s^2$ Eq. 6.3.34

$$= \frac{120}{32.2}(1500)^2$$

$$= 8.41 \times 10^3 \text{ KSF}$$

Vertical Stiffness k_z (6.3.3.1)

$$k_z = \frac{\beta_z G \sqrt{A}}{1-\nu}$$

* Section No "Seismic Design of Highway Bridges Manual"

$\beta_z = 2.13$ for $\frac{L}{W} = \frac{18}{15} = 1.20$ Table 6.3.1

$$k_z = \frac{2.13 \times 8.41 \times 10^3 \sqrt{18 \times 15}}{1-.33} = 4.4 \times 10^5 \text{ Kips/Ft.}$$

Embedment Factor (6.3.3.5)

$$r_o = r_e = \sqrt{\frac{A}{\pi}} = 9.27 \text{ Ft}$$

For $H = 6'$ $\frac{H}{r_o} = 0.65$

From graph shown in Figure 6.3.4 the added stiffness due to the embedment is approximately 1.35 times that of a footing resting on an elastic half space. Thus the vertical stiffness is modified to yield

$$k_{zH} = 1.35 \times k_z = 1.35 \times 4.4 \times 10^5 = 5.94 \times 10^5 \text{ Kips/Ft}$$

Horizontal Stiffness k_x (6.3.3.2)

$$k_x = \beta_x (1+\nu) G \sqrt{A}$$

$$\beta_x \cong 2.0 \text{ for } \frac{L}{W} \leq 5$$

$$k_x = 2.0(1+.33) \times 8.41 \times 10^3 \sqrt{13 \times 15} = 3.7 \times 10^5 \text{ kips/Ft}$$

Embedment effect is assumed to be approximately the same as vertical vibration.

$$k_{xH} = 3.7 \times 10^5 \times 1.35 = 5.0 \times 10^5 \text{ kips/Ft}$$

<u>Rocking Stiffness</u> (6.3.3.3)

Case A: $c = 7.5'$ $d = 9.0'$ rocking axis parallel to c

$$\frac{d}{c} = 1.20 \qquad \beta_\psi = 0.52$$

$$k_\psi = \beta_\psi \frac{8Gcd^2}{1-\nu} = \frac{0.52 \times 8 \times 8.41 \times 10^3 \times 7.5 \times 9^2}{1-.33}$$

$$= 3.2 \times 10^7 \text{ Kip ft/rad}$$

$$k_{\psi H} = 1.35\, k_\psi = 4.32 \times 10^7 \text{ kip ft/rad.}$$

Case B: $c = 9.0'$ $d = 7.5'$

$$\frac{d}{c} = 0.83 \qquad \beta_\psi = 0.48$$

$$K_\psi = \beta_\psi \frac{8Gcd^2}{1-\nu} = \frac{0.48 \times 8 \times 8.41 \times 10^3 \times 9.0 \times 7.5^2}{1-.33}$$

$$= 2.4 \times 10^7 \text{ kip ft/rad.}$$

$$k_{\psi H} = 1.35\, k_\psi = 3.24 \times 10^7 \text{ kip ft/rad.}$$

<u>Twisting Stiffness</u> (6.3.3.4)

$$k_\theta = \frac{16}{3} G r_e^3 \;\&\; r_e = \sqrt[4]{\frac{16cd(c^2+d^2)}{6\pi}}$$

$$r_e = \sqrt[4]{\frac{16 \times 7.5 \times 9.0\,(7.5^2 + 9.0^2)}{6\pi}} = 9.42'$$

$$k_\theta = \frac{16}{3} \times 8.41 \times 10^3 \times 9.42^3 = 3.7 \times 10^7 \text{ kipft/rad}$$

$$k_{\theta H} = 1.35\,k_\theta = 5.0 \times 10^7 \text{ kipft/rad}$$

Vertical Damping (6.3.4.1)

$$\beta_z = \frac{(1-\nu)m}{4\rho r_0^3} \qquad r_0 = 9.27'$$

$$m = \frac{1000 + .150 \times 3 \times 18 \times 15}{32.2}$$

$$= 34.83 \text{ mugs}$$

$$\beta_z = \frac{(1-.33)\,34.83}{4 \times \frac{.120}{32.2}\,(9.27)^3} = \frac{.67 \times 34.83}{4 \times 3.73 \times 10^{-3} \times 9.27^3}$$

$$= 1.97$$

Geometrical damping (Assume 5% Internal Damping)

$$D_z = \frac{0.45}{\sqrt{\beta_z}} = \frac{0.45}{\sqrt{1.97}} = 0.32$$

Total damping ratio = 0.35 + 0.05 = 0.37

Horizontal Damping

$$B_x = \frac{(7-8\nu)m}{32(1-\nu)\rho r_0^3} \qquad r_0 = 9.27'$$

m = 34.83 mugs

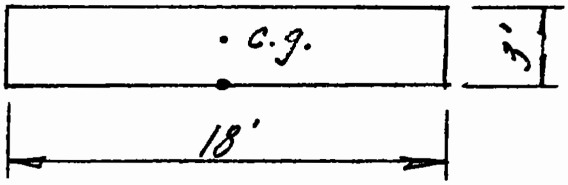
$$\beta_x = \frac{(7-8\times.33)\times 34.83}{32(1-.33)3.73\times10^{-3}\times9.27^3} = 2.38$$

$$D_x = \frac{0.288}{\sqrt{B_x}} = \frac{0.288}{\sqrt{2.38}} = 0.19 \text{ Geometric damping}$$

Total damping = $0.19+0.05 = 0.24$

Rocking Damping

Case A:



$$I\psi = I_{c.g.} + md^2$$

$$= \frac{1}{12} m\left(a^2+b^2\right)+md^2$$

$$= \frac{1}{12}\, 34.83\left(3^2+18^2\right)+34.83(1.5)^2$$

$$= 1045$$

$$r_e = \sqrt[4]{\frac{16cd^3}{3\pi}} = \sqrt[4]{\frac{16\times7.5\times9^3}{3\pi}} = 9.82 \text{ Ft.}$$

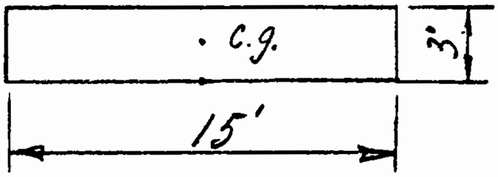
$$\beta_\psi = \frac{3(1-\nu)I\psi}{8\rho\, r_e^5} = \frac{3(1-.33)1045}{8\times3.73\times10^{-3}\; 9.82^5}$$

$$= 0.77$$

$$D\psi = \frac{0.15}{(1+B\psi)\sqrt{B\psi}} = \frac{0.15}{(1.77)\sqrt{.77}} = 0.10$$

Total damping $= 0.10 + 0.05 = 0.15$

Case B:



$$I_\psi = I_{c.g} + md^2$$

$$= \frac{1}{12} m(a^2+b^2) + md^2$$

$$= \frac{1}{12} 34.83(3^2+15^2) + 34.83(1.5)^2$$

$$= 758$$

$$r_e = \sqrt[4]{\frac{16cd^3}{3\pi}} = \sqrt[4]{\frac{16 \times 9 \times 7.5^3}{3\pi}} = 8.96 \text{ Ft.}$$

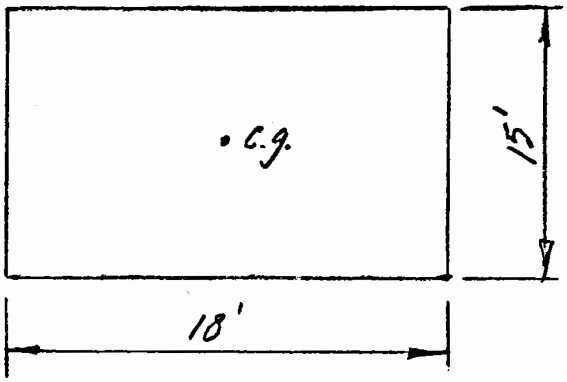
$$B_\psi = \frac{3(1-\nu)I_\psi}{8\rho r_e^5} = \frac{3(1-.33)758}{8 \times 3.73 \times 10^{-3}\, 9.82^5} = 0.56$$

$$D_\psi = \frac{0.15}{(1+B_\psi)\sqrt{B_\psi}} = \frac{0.15}{(1+.56)\sqrt{.56}} = 0.13$$

Total Damping $= 0.13 + 0.05 = 0.18$

Torsional Damping

$$B_\theta = \frac{I_x}{\rho r_0^5}$$



$$I_x = \frac{1}{12} m(a^2+b^2)$$

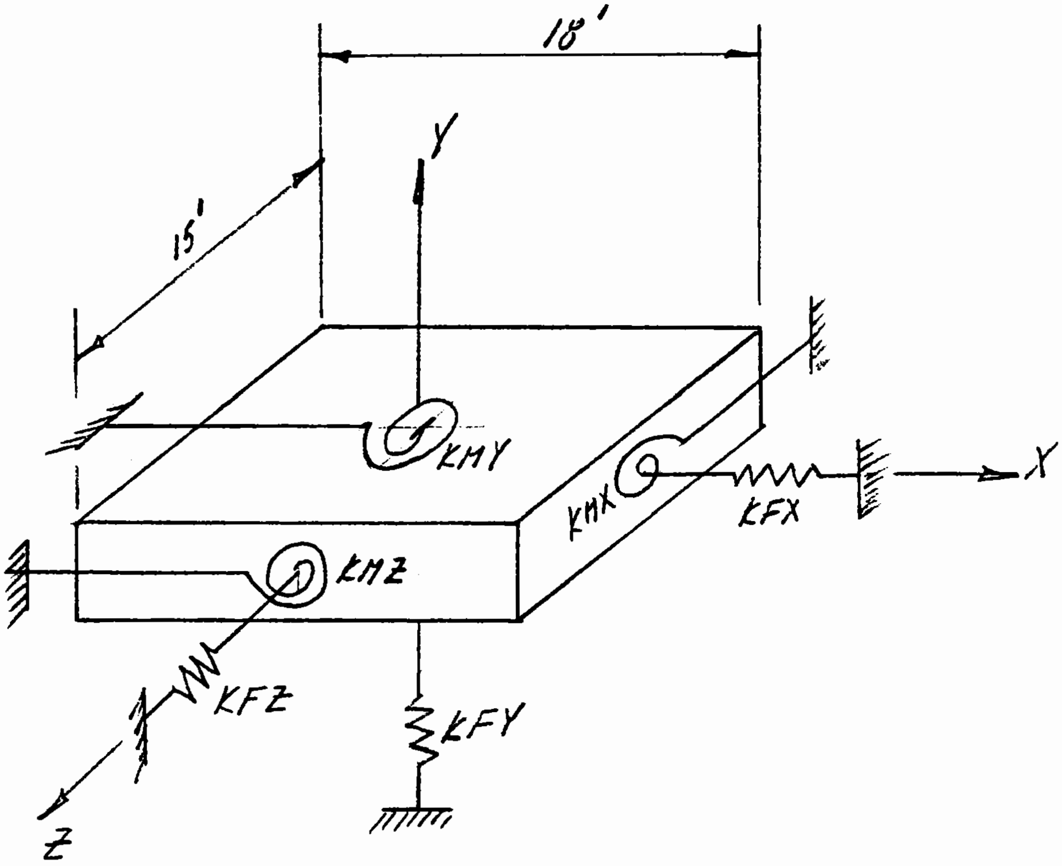
$$= \frac{1}{12} 34.83(15^2+18^2)$$

$$= 1593$$

$$B_\theta = \frac{I_x}{\rho r_e^5} = \frac{1593}{3.73 \times 10^{-3} \times 9.42^5} = 5.76$$

$$D_\theta = \frac{0.5}{1+2B_\theta} = \frac{0.5}{1+2 \times 5.76} = 0.04$$

Total Damping = 0.04 + 0.05 = 0.09



KFX	KFY	KFZ	KMX	KMY	KMZ
5.0×10^5	0	0	0	0	0
0	5.9×10^5	0	0	0	0
0	0	5.0×10^5	0	0	0
0	0	0	4.3×10^7	0	0
0	0	0	0	5.0×10^7	0
0	0	0	0	0	3.2×10^7

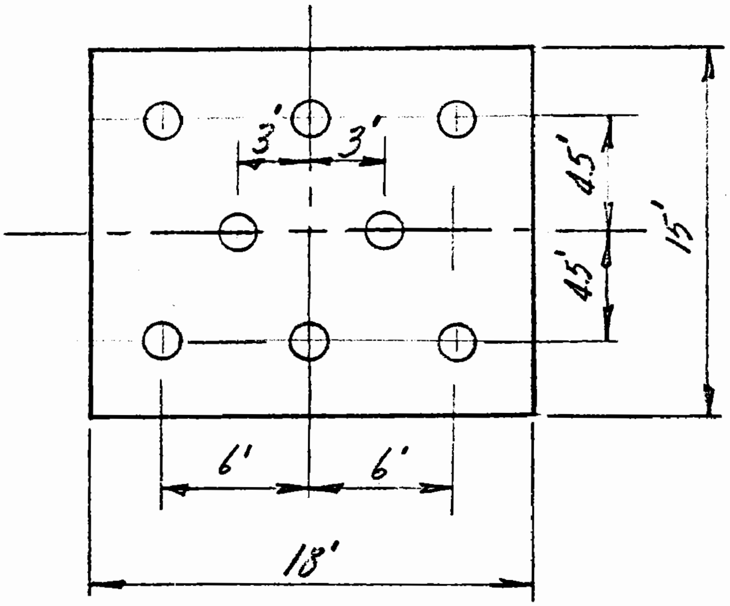
Appendix E.2

SAMPLE PROBLEM 11

Calculate stiffness coefficients for the pile group shown in Figure E.2.1. Assume that the soil is a medium stiff clay and the pile heads are fixed to the pile cap. Use the pile properties shown in the figure.

Sample Problem 11

Calculate stiffness coefficients for the pile group shown in the figure below. Assume that the soil is a medium stiff clay and the pile heads are fixed to the pile cap. The piles are 50 feet long.



Pile Prop.
Dia = 15"
$A = 1.23\ Ft^2$
$I = 0.12\ Ft^4$
$E = 432{,}000$ KSF
$L = 50$ Ft
$N = 8$ piles

Vertical Stiffness - Individual Pile

For soft to medium stiff clays use constant skin friction, as shown in Figure 6.3.1b, which is expressed mathematically by Equation 6.3.2.

$$k_v = 2\frac{AE}{L} = \frac{2 \times 1.23 \times 432000}{50} = 21300\ \frac{\text{kips}}{\text{Ft}}$$

Lateral Stiffness - Individual Pile

For very soft to medium stiff clays and cohesionless soils the procedures outlined in Section 6.3.2.2 - Case B apply. Thus from Equation 6.3.7

$$k_L = \frac{f^{.6}(EI)^{.4}}{A_\delta} \quad \text{and} \quad A_\delta = 0.93$$

From Figure 6.3.3 Curve 2 $f \cong 8$ tons/Ft^3 or 16 kips/Ft^3

$$k_L = \frac{(16)^{.6}(432000 \times .12)^{.4}}{0.93} = 436 \text{ kips/ft.}$$

Check $L > 5T$ From Equation 6.3.9

$$T = \left(\frac{EI}{f}\right)^{.2} = \left(\frac{432000 \times .12}{16}\right)^{.2}$$

$$= 5.0 \quad \text{OK} \quad 50 > 25$$

Having obtained the stiffness coefficients for the individual piles the next step is to combine these effects so they may be represented by a single set of spring coefficients. The following assumptions are made relative to the combined behavior of the pile group:

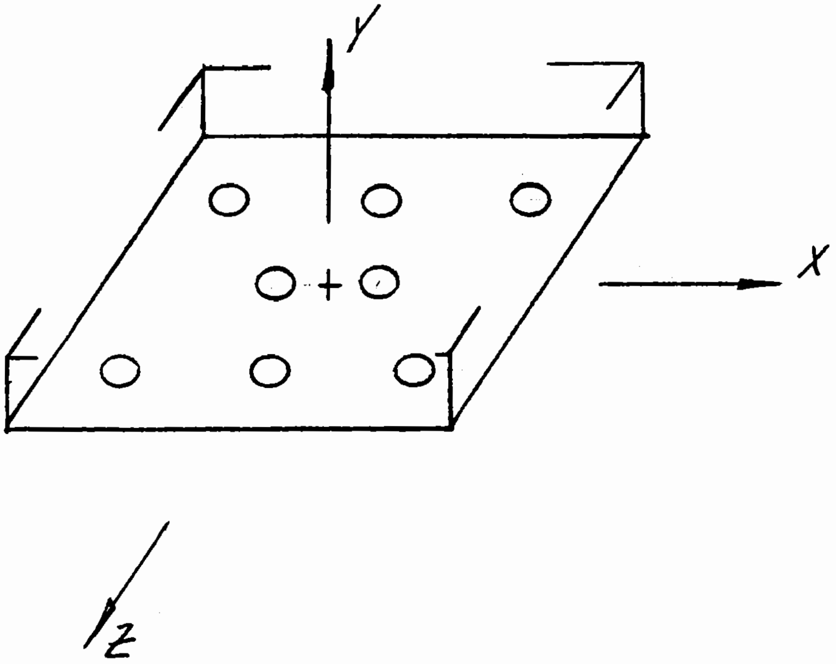
1.) The pile cap is infinitely rigid.

2.) The soil surrounding the pile cap is inactive in resisting the applied loads

3.) Load transfer to the soil is linear-elastic

4.) Shear and moment coupling effects are small and can be ignored (i.e. off diagonal terms of the stiffness matrix are not included).

Vertical Stiffness - Pile Group

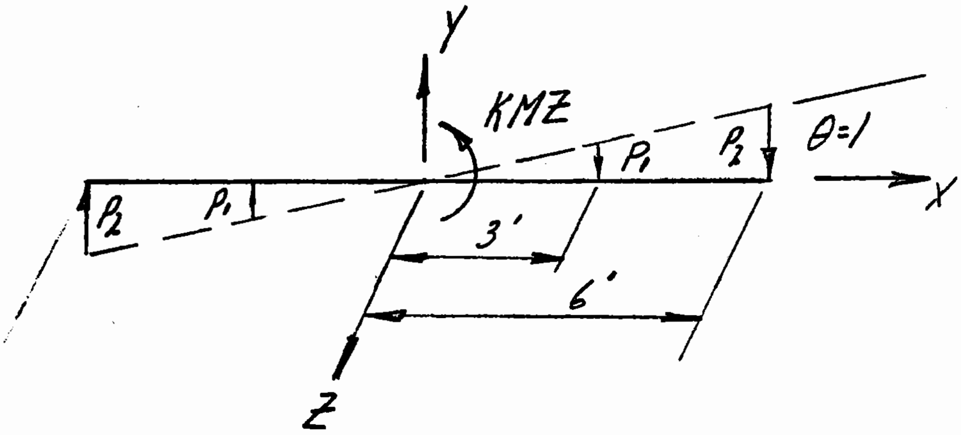


$KFY = N k_V = 8 \times 21300 = 1.7 \times 10^5$ kips/ft.

Horizontal Stiffness - Pile Group

$KFX = KFZ = N k_L = 8 \times 436 = 3.5 \times 10^3$ kips/ft.

Rocking Stiffness - Pile Group



$KMZ = 2N_1P_1(3) + 2N_2P_2(6)$

where N_1 = Number of Piles in Row 1

and $P_1 = k_V(3)$ and $P_2 = k_V(6)$

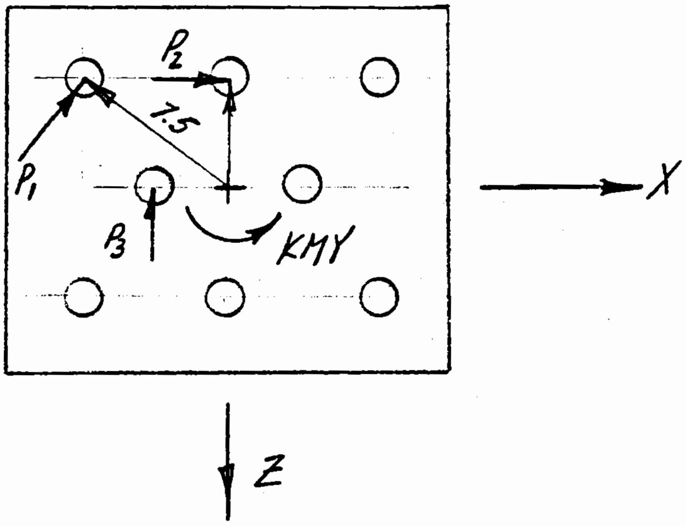
Thus $KMZ = 2(1)(21300 \times 3) \times 3 + 2(2)(21300 \times 6) \times 6$

$= 3.45 \times 10^6$ kip-ft/rad.

KHX may be computed in a similar manor yielding:

$$KMX = 2(3)(21300 \times 4.5) \times 4.5 = 2.59 \times 10^6 \; \frac{Kip\text{-}ft}{rad}$$

Twisting Stiffness - Pile Group



$$KMY = 4P_1(7.5) + 2P_2(4.5) + 2P_3(3)$$

$P_1 = k_L\, 7.5$

$P_2 = k_L\, 4.5$

$P_3 = k_L\, 3.0$

$$KMY = 436\,(4 \times 7.5^2 + 2 \times 4.5^2 + 2 \times 3^2) = 1.24 \times 10^5$$

$$KMY = 1.24 \times 10^5 \; Kip\text{-}ft\,/rad.$$

Appendix F.1

MATRIX ALGEBRA

F.1.1 Introduction:

Fundamental to the intelligent use of computer programs for structural analysis and design is at least a basic knowledge of the methodology upon which these programs are based. Since approximately 1958 the language of structural analysis and design has undergone a most profound change. The new language is that of matrix algebra and corresponding to it matrix notation. This is readily obvious to anyone attempting to follow research publications in this area and should serve to motivate the practicing engineer to understand the new language in order to implement the vast amount of new knowledge wisely. Actually the basic methodology for structural analysis has not changed but only become more fundamental and compact. The primary reason for the change in the language is that computers can easily manipulate large blocks of numbers and solve large numbers of simultaneous equations much more directly than they can operate on methods such as moment distribution. Hence, a brief definition of matrix notation, a description of basic operations employing matrix algebra and some example applications of matrix techniques follow. These methods form the basis for STRUDL's internal operation.

Definition and Notation:

A matrix is defined as a <u>rectangular</u> block, or array, of numbers composed of m rows and n columns. For example:

$$[A]_{3\times 2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} = \begin{bmatrix} 6 & 1 \\ 2 & -8 \\ 1 & -3 \end{bmatrix}$$

Here we have the matrix A composed of m = 3 rows and n = 2 columns containing six coefficients a_{ij}. The first index on the coefficient within the array defines its row position and the second index defines its column position or $a_{31} = 1$, where a_{31} is the element in the third row and first column.

F.1.2 Matrix Operations:

I Addition and Subtraction:

Two or more matrices of the same size (those having the same number of rows and columns) may be added or subtracted.

$$[C]_{2\times 2} = [A]_{2\times 2} + [B]_{2\times 2} = \begin{bmatrix} 6 & 1 \\ 2 & -8 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 9 & 5 \\ -2 & -6 \end{bmatrix}$$

$$[D]_{2\times 2} = [A]_{2\times 2} - [B]_{2\times 2} = \begin{bmatrix} 6 & 1 \\ 2 & -8 \end{bmatrix} - \begin{bmatrix} 3 & 4 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 6 & -10 \end{bmatrix}$$

$$\text{or } c_{ij} = a_{ij} + b_{ij} \text{ , } d_{ij} = a_{ij} - b_{ij}$$

II Multiplication:

The product of two matrices (A) (B) is equal to a matrix (C) having the same number or rows as (A) and the same number of columns as (B). The product (C) can only exist when the number of rows in (B) is the same as the number of columns in (A). For example:

$$\overset{[A]}{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix}}_{4X2} \quad \overset{[B]}{\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}}_{2x3} = \overset{[C]}{\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{bmatrix}}_{4x3}$$

The coefficients c_{ij} in matrix (C) are determined from the relation:

$$c_{ij} = \sum_{k=1}^{n} a_{ik}\, b_{kj}$$

where n is the number of rows in (B) or the number of columns in (A). As an example, in the above product

$$c_{12} = a_{11}\, b_{12} + a_{12}\, b_{22} \; \underline{\text{or}} \; c_{23} = a_{21}\, b_{13} + a_{22}\, b_{23}$$

$$c_{ij} = (i^{th} \text{ row of } [A] \times \text{ the } j^{th} \text{ column of } [B])$$

Example 1:

Find the product of matrix A and B.

$$A = \begin{bmatrix} 4 & 1 \\ 1 & 2 \\ -2 & 4 \end{bmatrix}_{3\times 2} \qquad B = \begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix}_{2\times 2}$$

$$[A][B] = \begin{bmatrix} 4 & 1 \\ 1 & 2 \\ -2 & 4 \end{bmatrix} \times \begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} -7 & 16 \\ 0 & 11 \\ 8 & 10 \end{bmatrix}_{3\times 2}$$

Example 2:

As a simple example of a matrix formulation illustrative of the multiplication process consider the pin connected truss of Figure F.1.1.

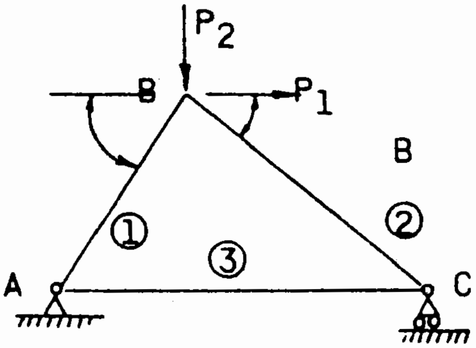


Figure F.1.1

At joint B we have the external forces P_1 and P_2. If we consider the equilibrium of Joint B, Figure F.1.2, we have

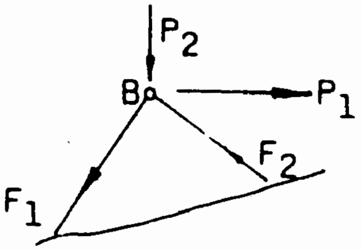


Figure F.1.2

$$\sum F_H = 0$$

or, $$P_1 = F_1 \cos\alpha + F_2 \cos\beta \qquad \text{. . . . . (1)}$$

$$\sum F_V = 0$$

or, $$P_2 = -F_1 \sin\alpha + F_2 \sin\beta \qquad \text{. . . . . (2)}$$

Equations (1) and (2) may be written in matrix form as

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} \cos\alpha & \cos\beta \\ -\sin\alpha & \sin\beta \end{bmatrix} \times \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \qquad \text{. . . . . (3)}$$

If we apply the above role for multiplication we would see that Eq. (3) is the same as Eq.'s (1) and (2), we can write Eq. (3) in an even more abbreviated manner as

$$\{P\} = [A]\,\{F\} \qquad \text{. (4)}$$

$$\{P\} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}, \quad [A] = \begin{bmatrix} \cos\alpha & \cos\beta \\ \sin\alpha & \sin\beta \end{bmatrix}, \text{ and } \{F\} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

Basically then matrix notation, such as Eq. (4), is a short hand way of writing a system of equations.

Example 3:

Suppose we are given the following relationships

$$\begin{aligned} z_1 &= 6y_1 + y_2 & \qquad y_1 &= 3X_1 + 4X_2 \\ z_2 &= 2y_1 - 8y_2 & \qquad y_2 &= -4X_1 + 2X_2 \\ z_3 &= y_1 - 3y_2 & & \end{aligned}$$

and one wishes to express the z values as a function of the x values. This can be done by direct substitution as follows

$$z_1 = 6\ (3X_1 + 4X_2) + 1\ (-4X_1 + 2X_2) = [\ (6)(3) + (1)(-4)]\ X_1 + [(6)(4)+(1)(2)]\ X_2$$
$$= 14X_1 + 26X_2$$
$$z_2 = 2(3X_1 + 4X_2) - 8(-4X_1 + 2X_2) = [(2)(3) + (-8)(-4)]\ X_1 + [(2)(4)+(-8)(2)]\ X_2$$
$$= 38X_1 - 8X_2$$
$$z_3 = 1(3X_1 + 4X_2) - 3(-4X_1 + 2X_2) = (1)(5)+(-3)(-4)\ X_1 + (1)(4)+(-3)(2)\ X_2$$
$$= 15X_1 - 2X_2$$

This operation also could have been performed using matrix multiplication by writing

$$\{Z\}_{3x1} = [A]_{3x2} \{Y\}_{2x1},\quad \{Y\}_{2x1} = [B]_{2x2} \{X\}_{2x1}$$

and then by substitution

$$\{Z\}_{3x1} = [A]_{3x2} [B]_{2x2} \times \{X\}_{2x1} = [C]_{3x2} \{X\}_{2x1}$$

Where

$$[C]_{3x2} = [A]_{3x2} [B]_{2x2} \text{ and } c_{ij} = \sum_{k=1}^{2} a_{ik} b_{kj}$$

Or in general

$$[C]_{LXN} \quad [A]_{LXM} [B]_{MXN} \text{ and } c_{ij} = \sum_{k=1}^{M} a_{ik} b_{kj}$$

$$[C]_{3x2} = \begin{bmatrix} 6 & 1 \\ 2 & -8 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ -4 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 14 & 26 \\ 38 & -8 \\ 15 & -2 \end{bmatrix}$$

$$\therefore \{Z\} = [C]_{3x2} \{x\} = \begin{bmatrix} 14x_1 & 26x_2 \\ 38x_1 & -8x_2 \\ 15x_1 & -2x_2 \end{bmatrix}$$

Notice that the results are identical.

III Matrix Inversion:

The inverse, $(A)^{-1}$, of a square matrix (A) is defined such that

$$[A]\,[A]^{-1} = [A]^{-1}\,[A] = [I] \qquad (5)$$

where (I) is defined as the unit matrix. (I) is a square matrix with 1's on the main diagonal and zeros elsewhere. For example, a 3x3 unit matrix is

$$[I] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We can visualize operations with the inverse matrix by a parallel with division i.e., if

$$z = xy$$

then $x = z/y = z(y)^{-1}$

note that $(y)(y)^{-1} = y/y = 1 \qquad (6)$

observe the similarity of Eq.'s (5) and (6).

Now that we know what the inverse matrix is defined to be, how do we find it? Suppose we are given the set of equations

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= y_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= y_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= y_3 \end{aligned} \quad \underline{\text{or}} \quad \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix}$$

and we wish to express the X as function of Y, that is

$$\begin{aligned} b_{11}y_1 + b_{12}y_2 + b_{13}y_3 &= x_1 \\ b_{21}y_1 + b_{22}y_2 + b_{23}y_3 &= x_2 \\ b_{31}y_1 + b_{32}y_2 + b_{33}y_3 &= x_3 \end{aligned} \quad \underline{\text{or}} \quad \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}$$

what we are asking is that knowing the coefficients of (A), how may one determine the coefficients in the <u>inverse</u> matrix (B)?

Take as the values of Y, $y_1 = 1$, $y_2 = 0$, $y_3 = 0$, then

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = 1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = 0$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = 0$$

$$b_{11}(1) + b_{12}(0) + b_{13}(0) = x_1 \text{ , } x_1 = b_{11}$$
$$b_{21}(1) + b_{22}(0) + b_{23}(0) = x_2 \text{ , } x_2 = b_{21}$$
$$b_{31}(1) + b_{32}(0) + b_{33}(0) = x_3 \text{ , } x_3 = b_{31}$$

Therefore, if we determine the values of x by solving the first set of simultaneous equations, we will have the first column of the (B) matrix. If we then take $y_1 = 0$, $y_2 = 1$, $y_3 = 0$ and solve the first set we obtain the solutions constituting the second column of (B). Therefore if we have N equations in N unknowns we must solve N sets of simultaneous equations to obtain the N columns of the (B) matrix.

The above relationship can be written in matrix form as

$$[A] \{X\} = \{Y\} \text{ and } [B] \{Y\} = \{X\}$$

Notice that if we substitute for (X) in the first of these we obtain

$$[A] [B] \{Y\} = [A][A]^{-1} \{Y\} = \{Y\}$$

and that the only way this relationship can hold is if

$$[A] [A]^{-1} = [I]$$

Therefore, if we can find the inverse of a given matrix by some method, we can check its correctness by multiplying the given matrix by its inverse to see if the unit matrix is obtained. The reader may wish to do this to augment his understanding of matrix multiplication and to verify the preceding inversion. Many volumes have been written on different methods of inversion and simultaneous equations solution. These methods will not be discussed here, and it is left to the initiative of the reader to familiarize himself with them.

IV Symmetry and Transposition:

Two additional definitions of particular importance in structural theory employing matrices are the symmetric matrix and the transpose matrix. A symmetric matrix is a square matrix where the coefficients are symmetrical about the main diagonal (i.e., $a_{ij} = a_{ji}$). For example,

$$\begin{bmatrix} 5 & 6 & 7 & 8 \\ 6 & 4 & 15 & 75 \\ 7 & 15 & 3 & 0 \\ 8 & 75 & 0 & 2 \end{bmatrix} \longleftarrow \text{Main Diagonal}$$

A matrix (B) is defined to be the transpose of the matrix (A) if and only if $b_{ij} = a_{ji}$, or

$$[B] = [A]^T = \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ -4 & 6 \end{bmatrix} \text{ where } [A] = \begin{bmatrix} 3 & 2 & -4 \\ -1 & 4 & 6 \end{bmatrix}$$

or to put it another way, the rows and columns have been interchanged.

Example 4:

Find the transpose of

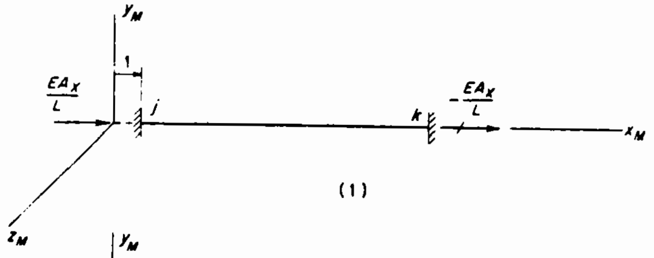
$$A = \begin{bmatrix} 1 & 5 & -6 \\ 3 & 8 & 12 \end{bmatrix}$$

Interchanging Rows & Columns:

$$A^T = \begin{bmatrix} 1 & 3 \\ 5 & 8 \\ -6 & 12 \end{bmatrix}$$

Appendix F.2

Space Frame Member Stiffnesses



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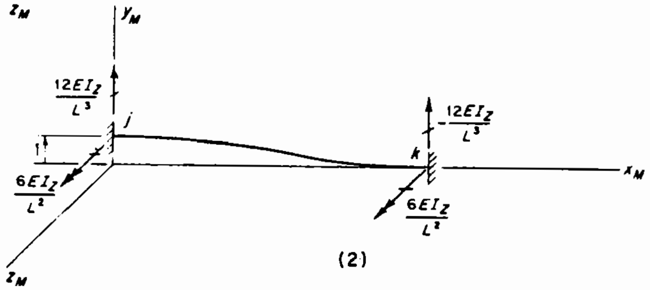

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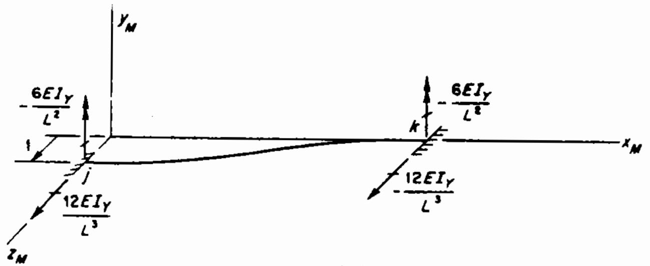

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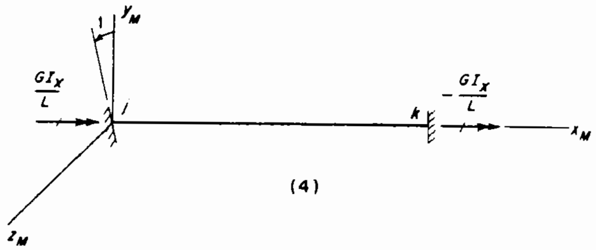

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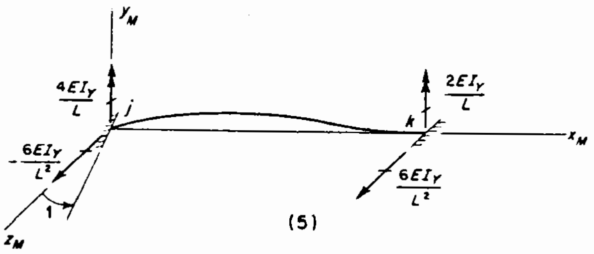

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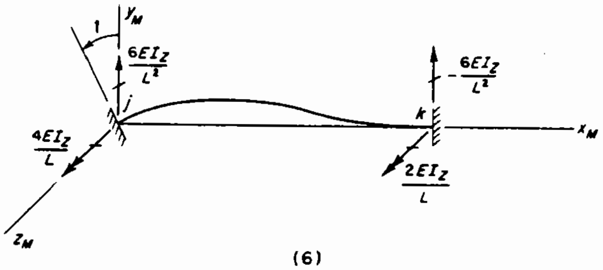
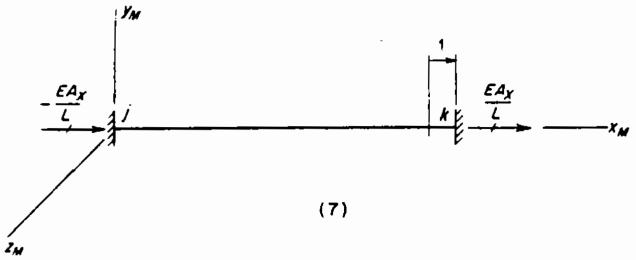


(6)



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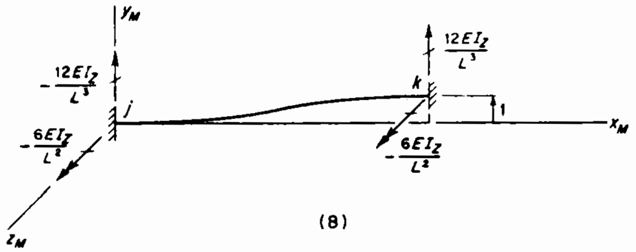

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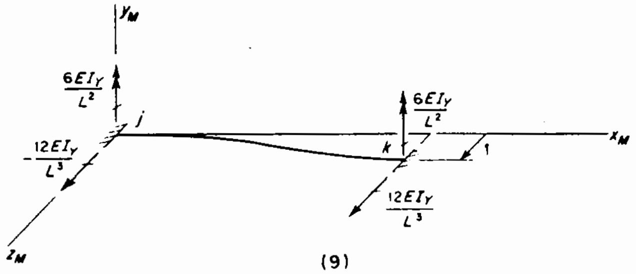

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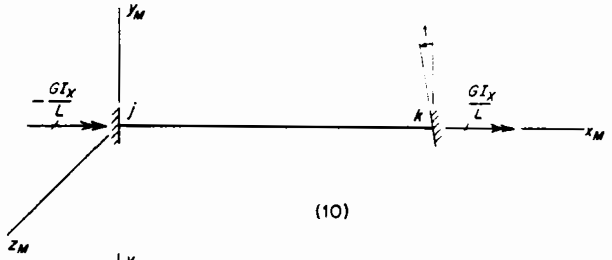

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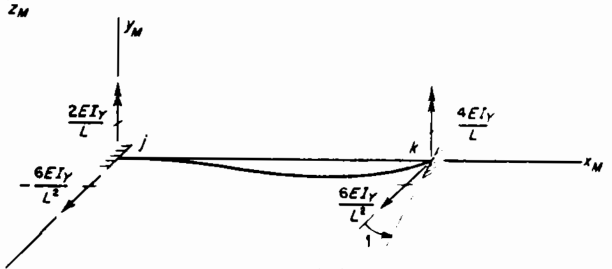

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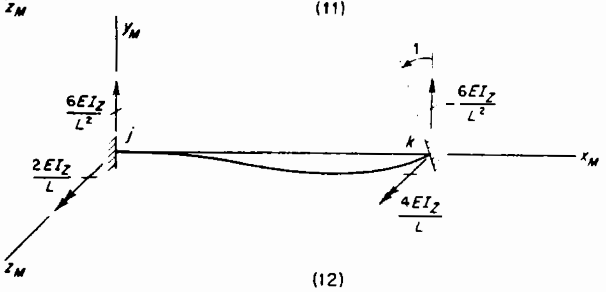


(12)

Space Frame Member Stiffness Matrix

$$
S_M = \begin{array}{c|cccccc:cccccc|}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
\hline
1 & \frac{EA_X}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{EA_X}{L} & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & \frac{12EI_Z}{L^3} & 0 & 0 & 0 & \frac{6EI_Z}{L^2} & 0 & -\frac{12EI_Z}{L^3} & 0 & 0 & 0 & \frac{6EI_Z}{L^2} \\
3 & 0 & 0 & \frac{12EI_Y}{L^3} & 0 & -\frac{6EI_Y}{L^2} & 0 & 0 & 0 & -\frac{12EI_Y}{L^3} & 0 & -\frac{6EI_Y}{L^2} & 0 \\
4 & 0 & 0 & 0 & \frac{GI_X}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{GI_X}{L} & 0 & 0 \\
5 & 0 & 0 & -\frac{6EI_Y}{L^2} & 0 & \frac{4EI_Y}{L} & 0 & 0 & 0 & \frac{6EI_Y}{L^2} & 0 & \frac{2EI_Y}{L} & 0 \\
6 & 0 & \frac{6EI_Z}{L^2} & 0 & 0 & 0 & \frac{4EI_Z}{L} & 0 & -\frac{6EI_Z}{L^2} & 0 & 0 & 0 & \frac{2EI_Z}{L} \\
\hdashline
7 & -\frac{EA_X}{L} & 0 & 0 & 0 & 0 & 0 & \frac{EA_X}{L} & 0 & 0 & 0 & 0 & 0 \\
8 & 0 & -\frac{12EI_Z}{L^3} & 0 & 0 & 0 & -\frac{6EI_Z}{L^2} & 0 & \frac{12EI_Z}{L^3} & 0 & 0 & 0 & -\frac{6EI_Z}{L^2} \\
9 & 0 & 0 & -\frac{12EI_Y}{L^3} & 0 & \frac{6EI_Y}{L^2} & 0 & 0 & 0 & \frac{12EI_Y}{L^3} & 0 & \frac{6EI_Y}{L^2} & 0 \\
10 & 0 & 0 & 0 & -\frac{GI_X}{L} & 0 & 0 & 0 & 0 & 0 & \frac{GI_X}{L} & 0 & 0 \\
11 & 0 & 0 & -\frac{6EI_Y}{L^2} & 0 & \frac{2EI_Y}{L} & 0 & 0 & 0 & \frac{6EI_Y}{L^2} & 0 & \frac{4EI_Y}{L} & 0 \\
12 & 0 & \frac{6EI_Z}{L^2} & 0 & 0 & 0 & \frac{2EI_Z}{L} & 0 & -\frac{6EI_Z}{L^2} & 0 & 0 & 0 & \frac{4EI_Z}{L}
\end{array}
$$

Appendix F.3

FORMULAS FOR COLUMN SHEARS AND MOMENTS

The formulas for the shears and moments for prismatic and non-prismatic columns are included in this attachment. These formulas may also be used to calculate the required stiffness terms for period calculations by substitution in unit displacements as described in Section 3.3.1.

a. Prismatic

i. Fixed-fixed

$$V_i = \frac{12EI}{l^3} \Delta i$$

$$M_i = \frac{6EI}{l^2} \Delta i$$

ii. Fixed-pinned

$$V_i = \frac{3EI}{l^3} \Delta i$$

$$M_i = \frac{3EI}{l^2} \Delta i = lV_i$$

b. Non-Prismatic

i. Fixed-fixed

$$V_i = \frac{EI_c \left[(K_T(1+C_T)+K_B(1+C_B) \right]}{l^3} \Delta_i$$

$$M_{T_i} = \frac{EI_c K_T \left[1+C_T\right]}{l^2} \Delta i$$

$$M_{B_i} = \frac{EI_c K_B \left[1+C_E\right]}{l^2} \Delta i$$

ii. Fixed-Pinned

$$V_i = \frac{EI_c K_B(1-C_B C_T)}{l^3} \Delta i$$

$$M_i = lV_i = \frac{EI_c K_B(1-C_B C_T)}{l^2} \Delta i$$

where:

I_c = minimum moment of inertia along the member for bending in the transverse direction in ft^4.

K_T = stiffness factor - top

K_B = stiffness factor - bottom

C_T = carry-over factor - top

C_B = carry-over factor - bottom

September 1974 NO. SM 45

Appendix G.1

OFFICE OF STRUCTURES
RESEARCH AND DEVELOPMENT
STRUCTURE MECHANICS

APPLICATIONS OF THE 1973 CALIFORNIA EARTHQUAKE DESIGN CRITERIA

By:

Roy A. Imbsen

Thomas A. Nelson

Robert N. Chittenden

Yee-Jung Ho

David W. Coats

Summary of Case Studies

Case 1 South Turlock O.C. Br. No. 39-191

- Spans-2 Equal (162)
- Bent-single column 25'
- Skew-Zero
- Stiffness Index = 7.46

Case 1A

- Bent - Changed to 40' column
- Stiffness Index = 2.87

Case 2 Davis Road U.C. Br. No. 23-154

- Spans-3 Unbalanced (73,154,119)
- Bent-Single column 23'
- Skew-45° Abutments only
- Stiffness Index = 2.89

Case 2A

- Skew-Changed to Zero
- Stiffness Index = 3.13

Case 3 Route 113/80 Separation Br. No. 23-177

- Spans-2 Unequal (232, 203)
- Bent-2 columns 26'
- Skew-60°
- Stiffness Index = 2.98

Case 3A

- No skew
- Stiffness Index = 4.93

Case 4 Von Karman Avenue O.C. Br. No. 55-612

- Spans-3 Approx. Equal (121,120,135)
- Bent-3 columns 24'
- Skew-0°
- Stiffness Index = 1.86

Case 4A

- Changed skew to 45°, Abutments & Bents
- Stiffness Index = 1.79

Case 4B

- Changed Bent 3 to 40' columns, no skew.
- Stiffness Index = 1.52

Case 4C

- Changed skew to 45°, Abutments & Bents
- Stiffness Index = 1.47

Case 5 North Connector O.C. Br. No. 57-879

- Spans-8 (192,223,223,212,215,226,226,195)
- Bents-single columns (67,46,61,58,45,33,45)
- Curve-700'

Case 6 North Turlock Overhead (Modified) Br. No. 38-144R

- Spans-3 (100,136,100)
- Bents-2 columns 36' (Pinned)
- Skew-None
- Stiffness Index = 1.65

Case 6A

- Change skew to 45°, Abutments and Bents
- Stiffness Index = 1.31

Case 6B

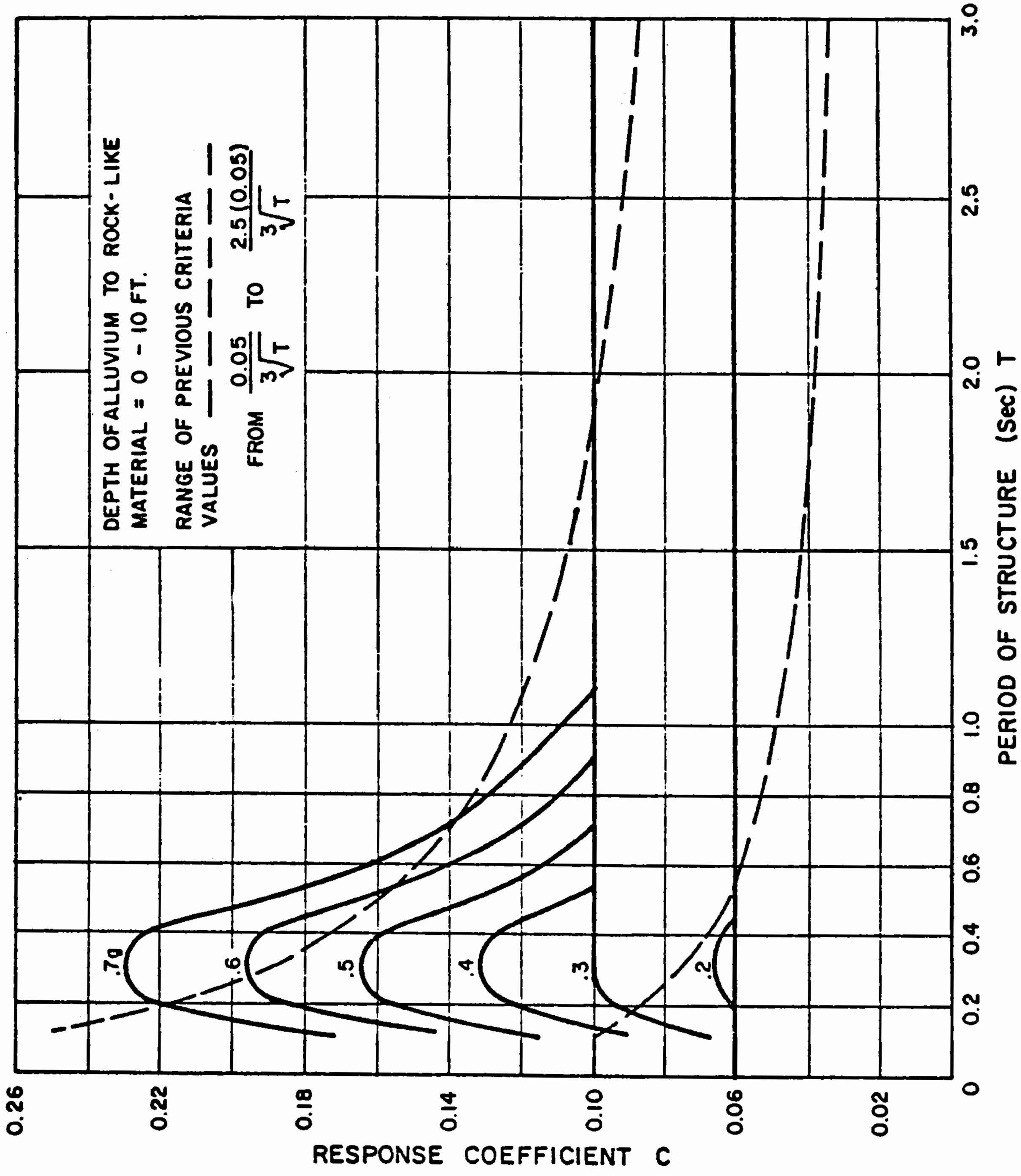
- Columns fixed
- No skew
- Stiffness Index = 3.00

Case 6C

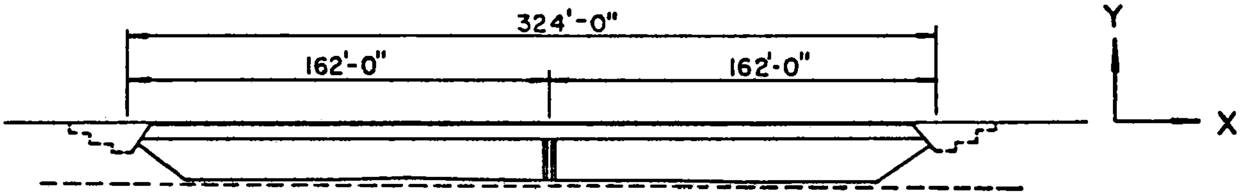
- Columns fixed
- Change skew to 45°, Abutments and Bents
- Stiffness Index = 2.04

RESPONSE COEFFICIENT "C" FOR VARIOUS VALUES OF PEAK ROCK ACCELERATION "A"

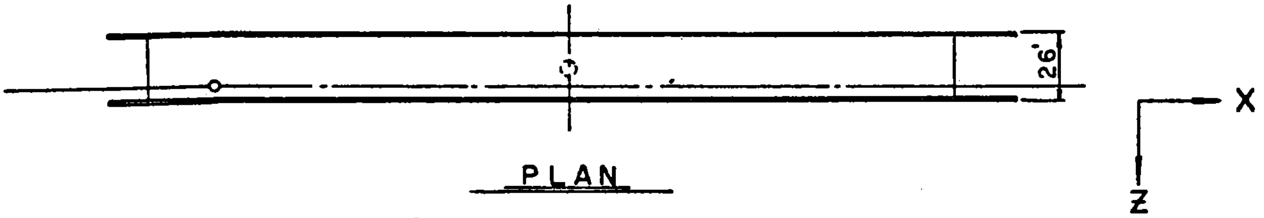
(DEPTH OF ALLUVIUM TO ROCK-LIKE MATERIAL = 0-10 FT)



CASE 1&1A SOUTH TURLOCK OVERCROSSING



ELEVATION



PLAN

STRUCTURAL DATA

SUPERSTRUCTURE:

$A = 42.3\ \text{Ft.}^2$

$I_Y = 1909\ \text{Ft}^4$

$I_Z = 268.4\ \text{Ft.}^4$

$I_X = 546.0\ \text{Ft.}^4$

$E = 3600$ KSI

D.L. = 6.68 K/Ft.

COLUMN (TYPE 5):

$I_{min} = 49\ \text{Ft.}^4$

$K_{TOP} = 11.02 \quad C_{TOP} = 0.404$

$K_{BOT} = 4.92 \quad C_{BOT} = 0.904$

$E = 3500$ KSI

D.L. = 94 K

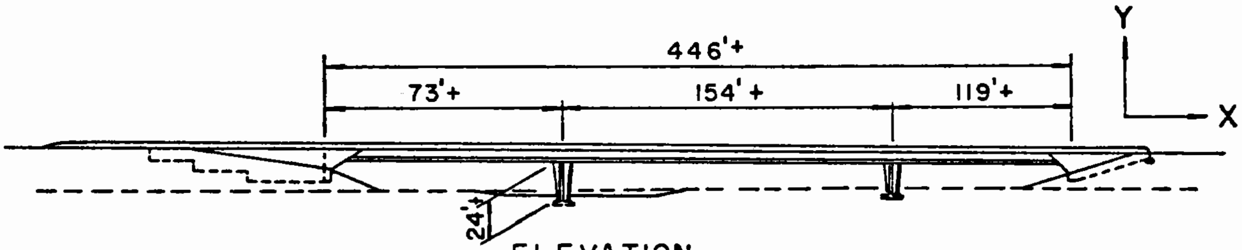
HEIGHT = 25' (CASE 1) & 40' (CASE 1A)

CASE 1/1A

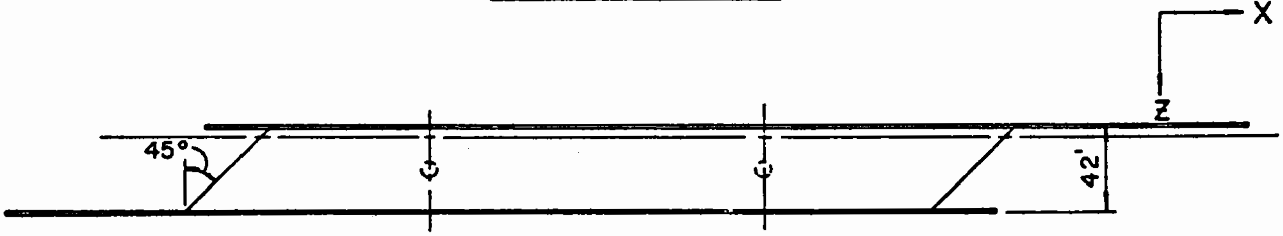
BRIDGE : South Turlock Overcrossing Br. No. 39-131
SPANS : 2 Equal (162,162)
BENT : Single Column Case 1 25'/Case 1A 40'
SKEW : Zero
CURVE : Tangent

BENT	METHOD	PERIOD	SHEAR		MOMENT TOP		MOMENT BASE		COMMENTS
			TRANS.	LONG.	TRANS.	LONG.	TRANS.	LONG.	
2	Formula	.54	212	0	0	0	5300	0	Case 1 25' Column Stiffness Index = 7.46
	Uniform	.41	271	0	2034	0	4762	0	
	Dynamic	.37	270	0	2000	0	4719	0	
	Field Test	.33							
2	Formula	1.14	122	0	0	0	4880	0	Case 1A 40'Column Stiffness Index = 2.87
	Uniform	.67	142	0	2181	0	3527	0	
	Dynamic	.58	153	0	2270	0	3742	0	

CASE 2 DAVIS ROAD ON-RAMP UNDERCROSSING



ELEVATION



PLAN

STRUCTURAL DATA

SUPERSTRUCTURE :

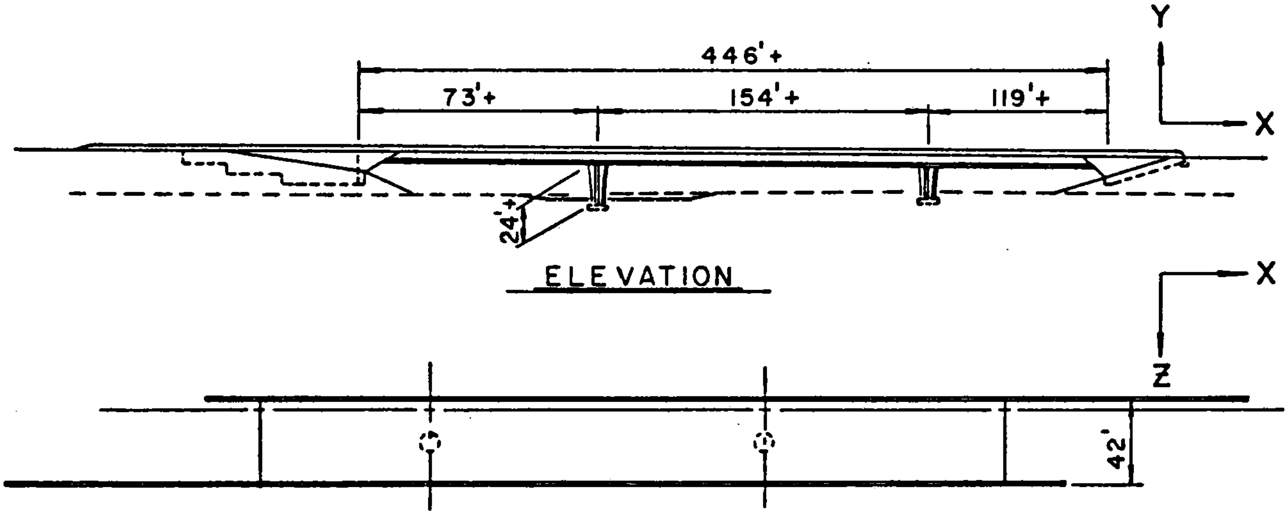
A = 67.57 $Ft.^2$
I_Y = 9416.8 $Ft.^4$
I_Z = 294.5 $Ft.^4$
I_X = 884.3 $Ft.^4$
E = 3850 KSI
D.L. = 10.47 K/Ft.

COLUMN (TYPE 8):

I_{min} = 34.22 $Ft.^4$
K_{TOP} = 8.68 C_{TOP} = 0.442
K_{BOT} = 4.74 C_{BOT} = 0.809
E = 4360 KSI
D.L. = 96.0 K

NOTE: COLUMN BOTTOMS PINNED IN LONGITUDINAL DIRECTION

CASE 2A DAVIS ROAD ON-RAMP UNDERCROSSING



ELEVATION

PLAN

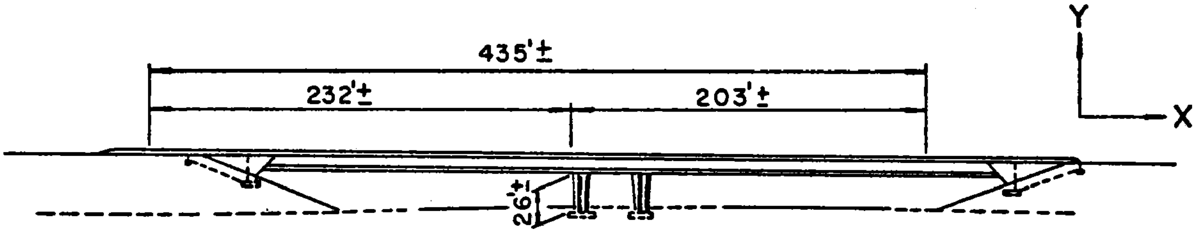
STRUCTURAL DATA - SAME AS CASE 2

CASE 2/2A

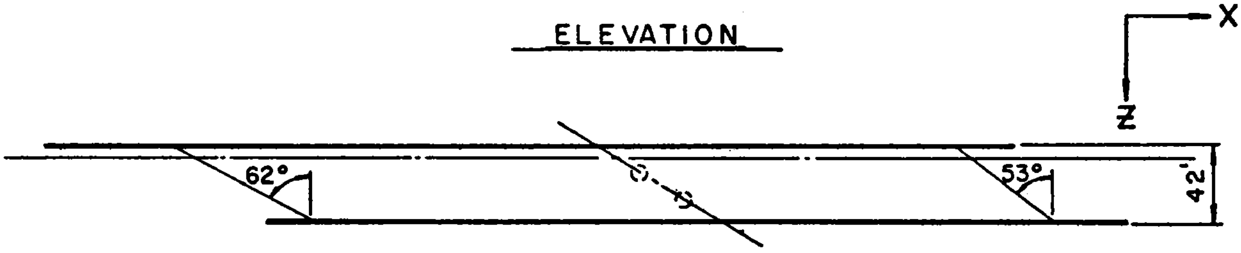
BRIDGE : Davis Road Undercrossing Br. No. 23-154
SPANS : 3 Unbalanced (73, 154, 119)
BENT : Single Column 23'
SKEW : Case 2 45° Abutments Only/Case 2A No Skew
CURVE : Tangent

BENT	METHOD	PERIOD	SHEAR		MOMENT TOP		MOMENT BASE		COMMENTS
			TRANS.	LONG.	TRANS.	LONG.	TRANS.	LONG.	
2	Formula	.58	213	0	0	0	5112	0	Case 2 45° Skew at Abutments only Stiffness Index 2.89
	Uniform	.47	161	0	1316	0	2553	0	
	Dynamic	.36	158	51	1238	1226	2540	0	
3	Formula	.63	244	0	0	0	5856	0	
	Uniform	.47	245	0	1982	0	3742	0	
	Dynamic	.36	245	40	1907	913	3774	0	
2	Formula	.58	213	0	0	0	5112	0	Case 2A No Skew Stiffness Index 3.13
	Uniform	.40	194	0	1841	0	2831	0	
	Dynamic	.34	188	0	1751	0	2730	0	
3	Formula	.63	244	0	0	0	5856	0	
	Uniform	.40	282	0	2518	0	4060	0	
	Dynamic	.34	278	0	2456	0	3977	0	

CASE 3 ROUTE 113/80 SEPARATION (WEST)



ELEVATION



PLAN

STRUCTURAL DATA

SUPERSTRUCTURE :

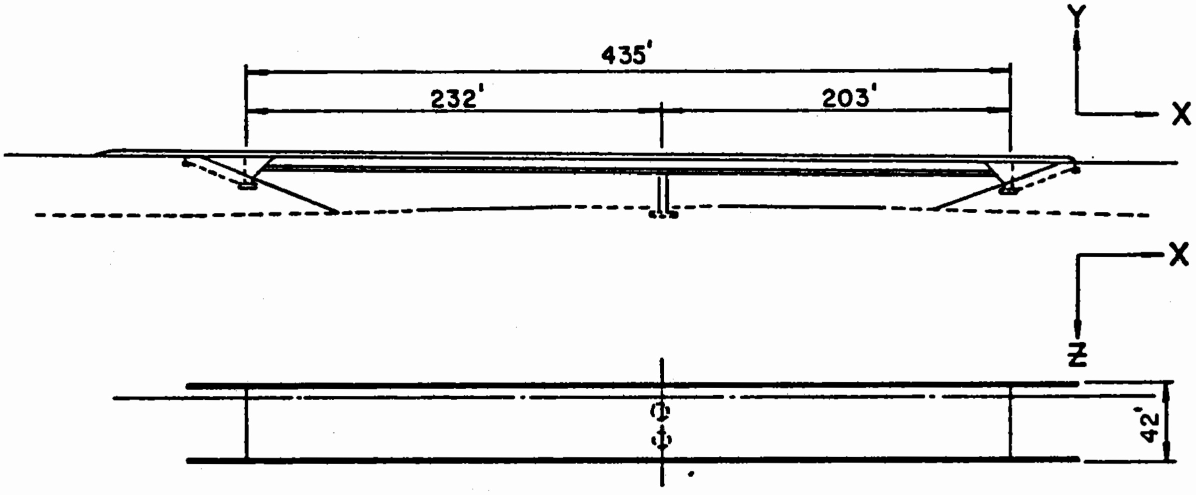
A = 80.41 Ft.^{2}
I_Y = 11092.3 Ft.^{4}
I_Z = 718.6 Ft.^{4}
I_X = 1791 Ft.^{4}
E = 3850 KSI
D.L. = 12.40 ^{K}/Ft.

COLUMN (TYPE 8) :

I_{min} = 34.22 Ft.^{4}
K_{TOP} = 8.22 C_{TOP} = 0.449
K_{BOT} = 4.70 C_{BOT} = 0.787
E = 4360 KSI
D.L. = 102 K

NOTE: COLUMN BOTTOMS PINNED IN
ALL DIRECTIONS

CASE 3A ROUTE 113/80 SEPARATION (WEST)



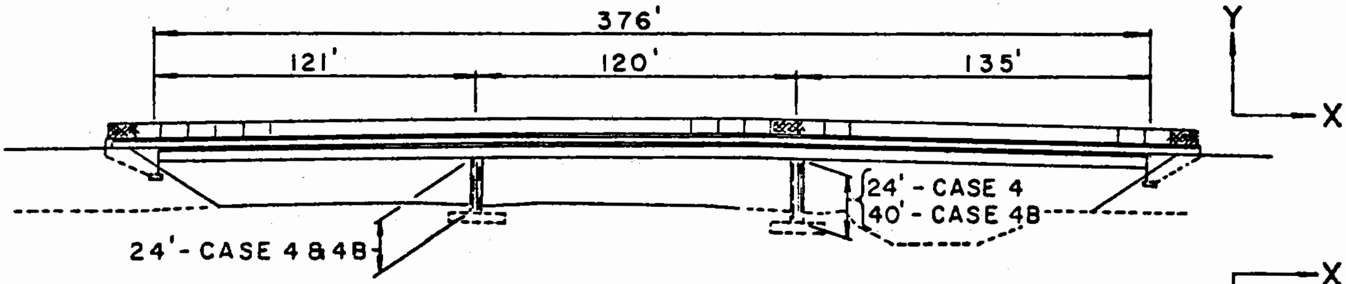
STRUCTURAL DATA - SAME AS CASE 3A

CASE 3/3A

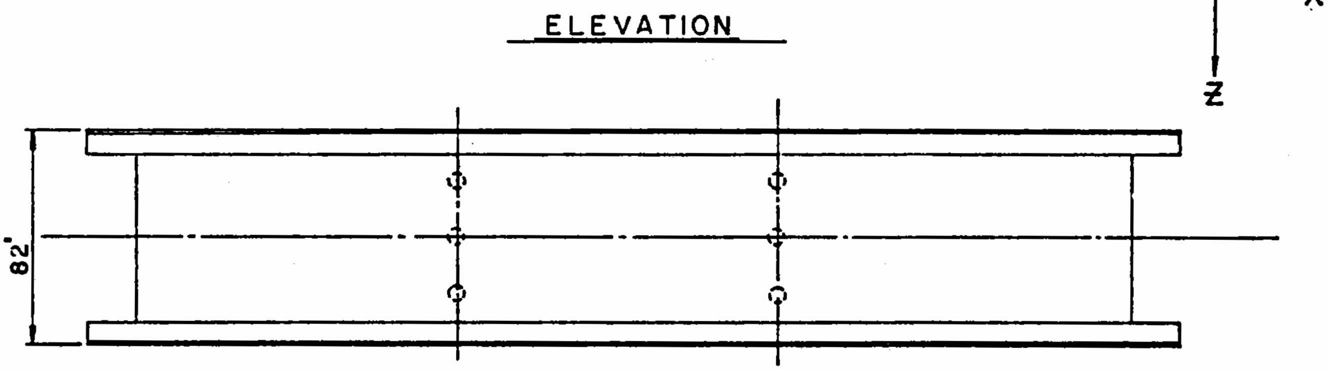
BRIDGE : Route 113/80 Separation Br. No.
SPANS : 2 Unequal (232,203)
BENT : 2 Columns 26'
SKEW : 60°
CURVE : Tangent

BENT	METHOD	PERIOD	SHEAR		MOMENT TOP		MOMENT BASE		COMMENTS
			TRANS.	LONG.	TRANS.	LONG.	TRANS.	LONG.	
2 Lt.	Formula	.52	216	0	5616	0	0	0	Case 3 60° Skew Stiffness Index 2.98
	Uniform	.63	158	4	4025	108	0	0	
	Dynamic	.55	134	145	3356	3670	0	0	
2 Rt.	Formula	.52	216	0	5616	0	0	0	
	Uniform	.63	127	4	3384	113	0	0	
	Dynamic	.55	105	122	2751	3219	0	0	
2 Lt.	Formula	.52	216	0	5616	0	0	0	Case 3A No Skew Stiffness Index 4.93
	Uniform	.49	243	3	6188	83	0	0	
	Dynamic	.46	262	5	6564	113	0	0	
2 Rt.	Formula	.52	216	0	5616	0	0	0	
	Uniform	.49	209	3	5583	86	0	0	
	Dynamic	.46	226	4	5907	105	0	0	

CASE 4&4B VON KARMAN AVENUE OVERCROSSING



ELEVATION



PLAN

STRUCTURAL DATA

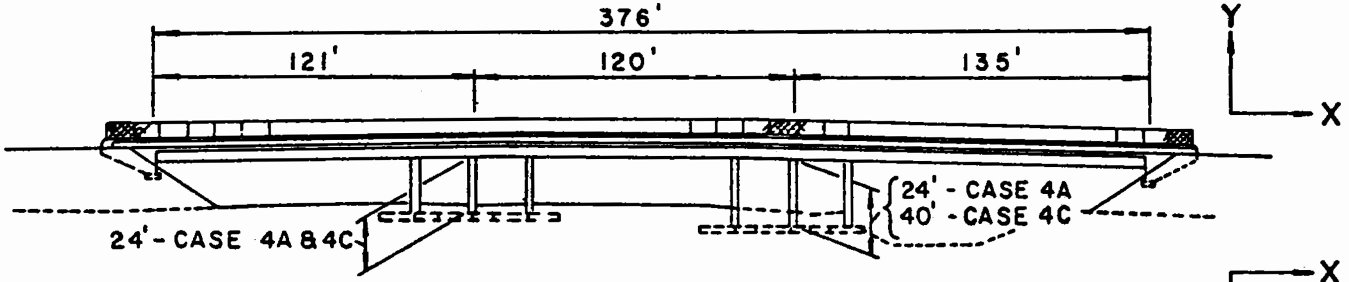
SUPERSTRUCTURE :

$A = 123.34\ Ft.^2$
$I_Y = 65550\ Ft.^4$
$I_Z = 526.98\ Ft.^4$
$I_X = 116.98\ Ft.^4$
$E = 3000\ KSI$
$D.L. = 18.50\ {}^K/Ft.$

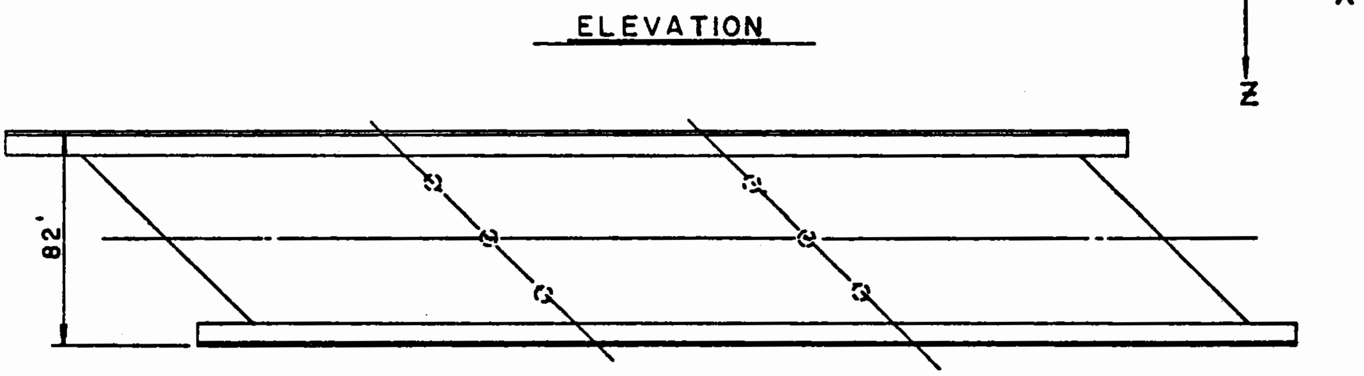
COLUMN (4' DIA.) :

$A = 12.57\ Ft.^2$
$I = 12.57\ Ft.^4$
$E = 3000\ KSI$
$D.L. = 45.0^*\ K$

CASE 4A & 4C VON KARMAN AVENUE OVERCROSSING



ELEVATION



PLAN

STRUCTURAL DATA - SAME AS 4 & 4B

CASE 4/4A

BRIDGE : Von Karmon Avenue Overcrossing Br. No. 55-612
SPANS : 3 Approx. Equal (121, 120, 135)
BENT : 3 Columns per bent 24'
SKEW : Case 4 0° Skew / Case 4 A 45° Skew
CURVE : Tangent

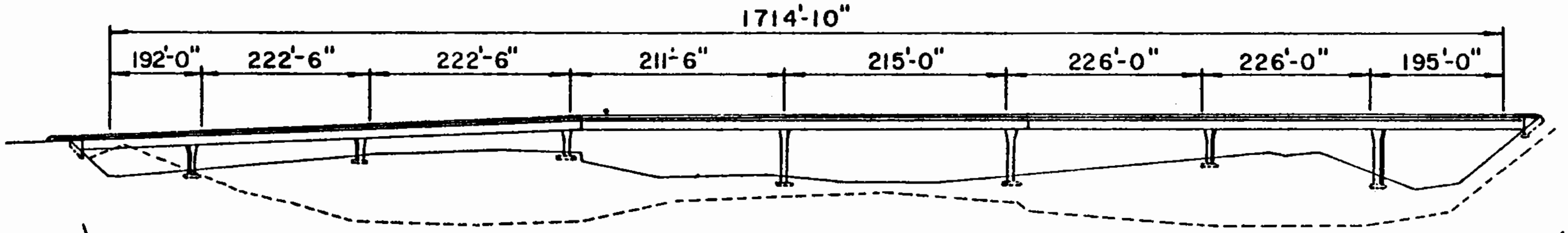
BENT	METHOD	PERIOD	SHEAR		MOMENT TOP		MOMENT BASE		COMMENTS
			TRANS.	LONG.	TRANS.	LONG.	TRANS.	LONG.	
2	Formula	.45	129	0	1548	0	1548	0	Case 4 0° Skew Stiffness Index 1.86
	Uniform	.34	70	0	831	0	834	0	
	Dynamic	.32	74	0	884	0	887	0	
3	Formula	.47	133	0	1596	0	1596	0	
	Uniform	.34	74	0	880	0	883	0	
	Dynamic	.32	78	0	942	0	945	0	
2	Formula	.45	129	0	1548	0	1548	0	Case 4A 45° Skew Stiffness Index 1.79
	Uniform	.35	73	12	858	105	932	174	
	Dynamic	.33	83	15	988	188	1012	178	
3	Formula	.47	133	0	1596	0	1596	0	
	Uniform	.35	77	10	886	84	956	150	
	Dynamic	.33	81	9	934	100	1002	123	

CASE 4B/4C

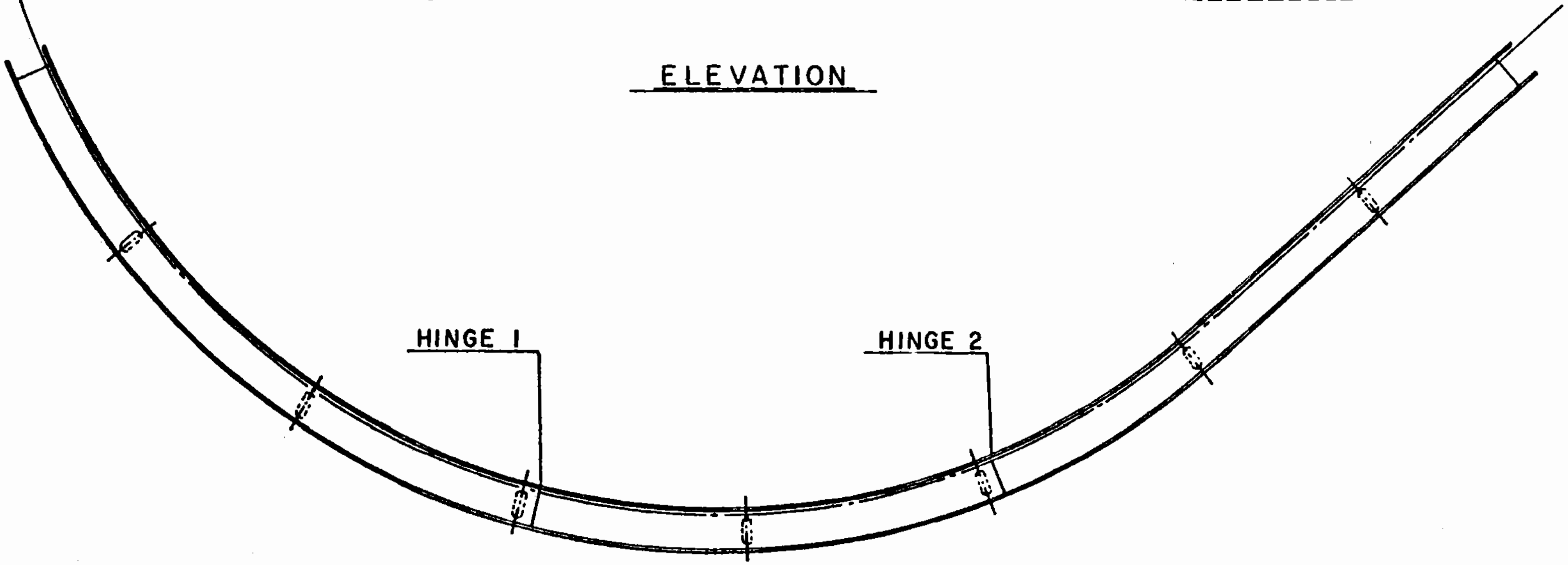
BRIDGE : Von Karmon Avenue Overcrossing Br. No. 55-612
SPANS : 3 Approx. Equal (121, 120, 135)
BENT : 3 Columns per Bent. Case 4B & 4C - 40' Columns at Bent 3
SKEW : Case 4B No skew / Case 4C 45° Skew
CURVE : Tangent

BENT	METHOD	PERIOD	SHEAR		MOMENT TOP		MOMENT BASE		COMMENTS
			TRANS.	LONG.	TRANS.	LONG.	TRANS.	LONG.	
2	Uniform	.38	84	0	996	0	998	0	Case 4B
	Dynamic	.36	90	0	1070	0	1073	0	
3	Uniform	.38	20	0	393	0	394	0	Stiffness Index 1.52
	Dynamic	.36	22	0	426	0	428	0	
2	Uniform	.39	88	14	1010	121	1106	210	Case 4C
	Dynamic	.36	94	24	1134	329	1125	258	Stiffness Index 1.47
3	Uniform	.39	21	3	423	58	437	73	
	Dynamic	.36	20	2	389	47	410	53	

CASE 5 NORTH CONNECTOR OVERCROSSING



ELEVATION



PLAN

STRUCTURAL DATA

SUPERSTRUCTURE :

$A = 75.6 \text{ Ft.}^2$

$I_Y = 8318.0 \text{ Ft}^4$

$I_Z = 764.9 \text{ Ft.}^4$

$I_X = 2454.0 \text{ Ft.}^4$

$E = 4100$ KSI

D.L. = 13.74 K/Ft.

COLUMNS : NON-PRISMATIC

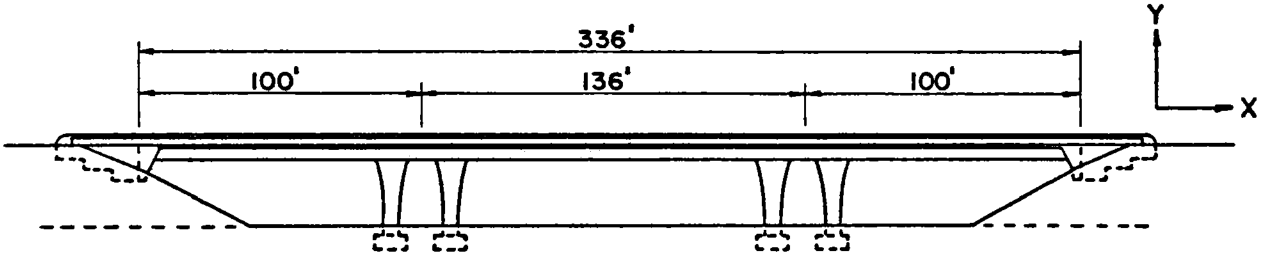
E = 4100 KSI

CASE 5

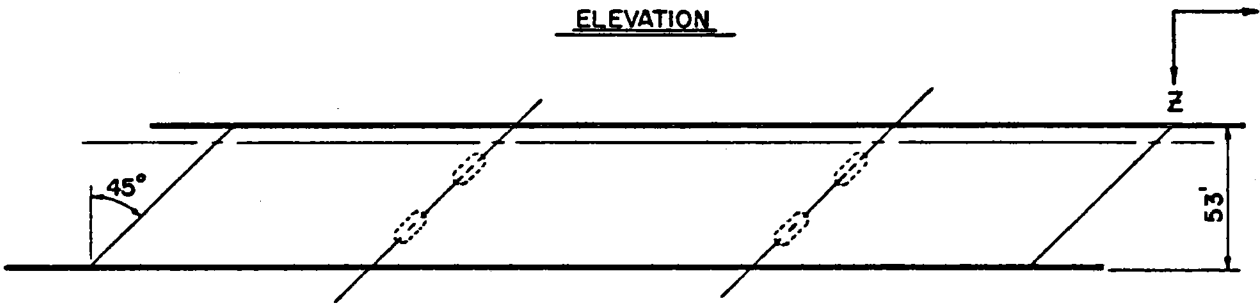
BRIDGE : North Connector Overcrossing Br. No. 57-879
SPANS : 8 (192,223,223, 212, 215, 226, 226, 195)
BENT : Single Column (67, 46, 61, 58, 45, 33, 45)
SKEW: Bents Radial
CURVE: 700'

BENT	METHOD	PERIOD	SHEAR		MOMENT TOP		MOMENT BASE		COMMENTS
			TRANS.	LONG.	TRANS.	LONG.	TRANS.	LONG.	
2	Uniform	1.01	226	217	1772	4699	8,300	5005	
	Dynamic	.98	346	166	3251	4160	12,166	3782	
3	Uniform	1.01	307	285	889	4423	11,249	5158	
	Dynamic	.98	344	385	2160	6629	11,432	6479	
4	Uniform	1.01	349	123	611	2727	15,200	2883	
	Dynamic	.98	267	233	1209	5064	12,119	5538	
5	Uniform	1.01	316	28	1666	791	16,509	819	
	Dynamic	.98	383	212	3036	6223	18,937	5775	
6	Uniform	1.01	273	131	1493	4006	15,267	4030	
	Dynamic	.98	379	255	3559	7535	19,496	7832	
7	Uniform	1.01	364	319	371	6446	15,709	7719	
	Dynamic	.98	378	303	2304	5497	15,606	7915	
8	Uniform	1.01	159	142	1975	4750	8,542	4631	
	Dynamic	.98	124	180	2320	6041	6,224	5613	

CASE 6A & 6C NORTH TURLOCK OVERCROSSING (MODIFIED)



ELEVATION



PLAN

STRUCTURAL DATA

SUPERSTRUCTURE : SAME AS CASE 6 & 6B

COLUMNS : SAME AS CASE 6 & 6B
CASE 6A-COLUMNS PINNED
CASE 6C-COLUMNS FIXED

CASE 6/6A

BRIDGE : North Turlock Overhead (Modified) Br. No. 38-144R
SPANS : 3 (100, 136, 100)
BENT : 2 Columns per Bent 36' (Columns Pinned)
SKEW : Case 6 0° Skew / Case 6A 45° Skew
CURVE : Tangent

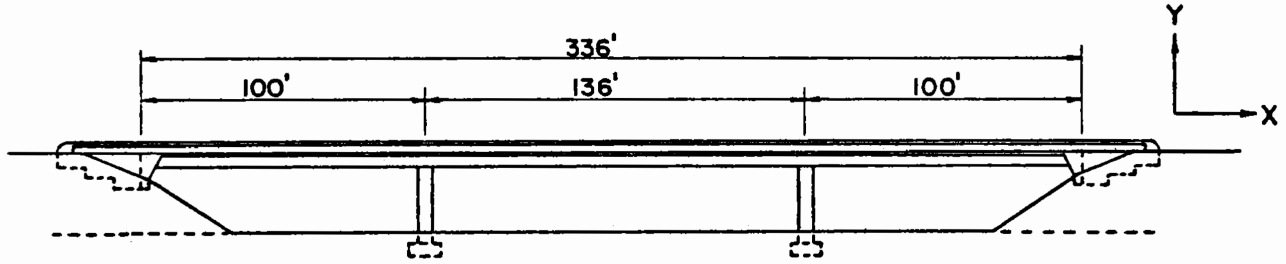
BENT	METHOD	PERIOD	SHEAR		MOMENT TOP		MOMENT BASE		COMMENTS
			TRANS.	LONG.	TRANS.	LONG.	TRANS.	LONG.	
2	Uniform	0.44	55	2	1973	71	0	0	Case 6 No Skew
	Dynamic	0.40	62	2	2076	77	0	0	Stiffness Index 1.65
3	Uniform	0.44	55	2	1973	71	0	0	
	Dynamic	0.40	62	2	2076	77	0	0	
2	Uniform	0.50	20	23	737	845	0	0	Case 6A 45° Skew
	Dynamic	0.43	55	18	1854	567	0	0	Stiffness Index 1.31
3	Uniform	0.50	20	23	737	845	0	0	
	Dynamic	0.43	55	18	1854	567	0	0	

CASE 6B/6C

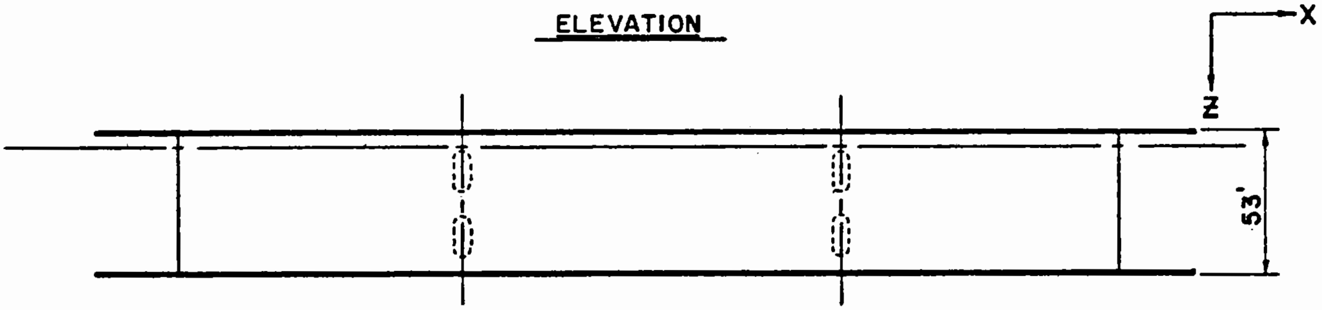
BRIDGE : North Turlock Overhead (Modified) Br. No. 38-144R
SPANS : 3 (100, 136, 100)
BENT : 2 Columns per Bent 36' (Columns Fixed)
SKEW: Case 6B 0° Skew / Case 6C 45° Skew
CURVE: Tangent

BENT	METHOD	PERIOD	SHEAR		MOMENT TOP		MOMENT BASE		COMMENTS
			TRANS.	LONG.	TRANS.	LONG.	TRANS	LONG.	
2	Uniform	0.33	99	4	2140	86	1419	62	Case 6B No Skew
	Dynamic	0.29	105	5	2162	88	1486	70	Stiffness Index 3.00
3	Uniform	0.33	99	4	2140	86	1419	62	
	Dynamic	0.29	105	5	2162	88	1486	70	
2	Uniform	0.40	51	60	1122	1142	735	1002	Case 6C 45° Skew
	Dynamic	0.32	106	38	2182	690	1522	642	Stiffness Index 2.04
3	Uniform	0.40	51	60	1122	1142	735	1002	
	Dynamic	0.32	106	38	2182	690	1522	642	

CASE 6 & 6B NORTH TURLOCK OVERCROSSING (MODIFIED)



ELEVATION



PLAN

STRUCTURAL DATA

SUPERSTRUCTURE :

$A = 71.34$ Ft.^{2}

$I_Y = 15508.2$ Ft.^{4}

$I_Z = 306.63$ Ft.^{4}

$I_X = 906.9$ Ft.^{4}

$E = 3460$ KSI

D.L. = 10.7 $^K/$Ft.

COLUMNS: NON-PRISMATIC

$E = 3460$ KSI

D.L = 127 K

HEIGHT= 36'

CASE 6 - COLUMNS PINNED

CASE 6B-COLUMNS FIXED

For rigid frames a maximum of one-half of the moment caused by earth pressure (lateral) may be used to reduce the positive moment in the beams, in the top slab, or in the top and bottom slab, as the case may be.

When highway traffic can come within a horizontal distance from the top of the structure equal to one-half its height, the pressure shall have added to it a live load surcharge pressure equal to not less than 2 feet (.610 m) of earth.

Where an adequately designed reinforced concrete approach slab supported at one end by the bridge is provided, no live load surcharge need be considered.

All designs shall provide for the thorough drainage of the back-filling material by means of weep holes and crushed rock, pipe drains or gravel drains, or by perforated drains.

1.2.20—EARTHQUAKE STRESSES

In regions where earthquakes may be anticipated, structures shall be designed to resist earthquake motions by considering the relationship of the site to active faults, the seismic response of the soils at the site, and the dynamic response characteristics of the total structure in accordance with the following criteria.

(A) Equivalent Static Force Method

For structures with supporting members of approximately equal stiffness, an equivalent horizontal force (EQ) may be applied to the structure. The distribution of the force shall consider the stiffness of the superstructure and supporting members, abutment restraint, and the deflected position of the structure.

(1) EQ = C.F.W

EQ = The equivalent static horizontal force applied at the center of gravity of the structure.

F = Framing Factor

F = 1.0 for structures where single columns or piers resist the horizontal forces.

F = 0.8 for structures where continuous frames resist horizontal forces applied along the frame.

W = The total dead weight of the structure in pounds (kg).

(2) C = A·R·S/Z

C = Combined Response Coefficient

The calculated coefficient "C" shall not be less than 0.10 for structures with "A" greater than or equal to 0.3 g and 0.06 for structures with "A" less than 0.3g.

Values of coefficients for various depths of alluvium to rock-like material given in Figures 1.2.20A, B, C, and D may be used.

A = Maximum expected acceleration at bedrock at the site.

Seismic risk map of the United States (shown in Figure 1.2.20E, F, G) with following assignment of maximum expected rock acceleration may be used. More exact peak roack acceleration values should be used in areas where "Maximum Expected Rock Acceleration" maps are available.

Zone I	$A = 0.09\ g$
Zone II	$A = 0.22\ g$
Zone III	$A = 0.50\ g$

g = 32.2 ft/sec.^{2} (9.81 m/sec^2)
R = Normalized rock response.
S = Soil amplification spectral ratio.
Z = Reduction for ductility and risk assessment.

$$(3)\quad T = 0.32\sqrt{\frac{W}{P}} \quad \text{or} \quad \left(\sqrt{\frac{W\ (\text{in kg})}{P\ (\text{in } N)}}\right)$$

T = The period of vibration of the structure (sec).
P = Total uniform force, pounds (N) required to cause a one-inch (.025 m) maximum horizontal deflection of the whole structure

The period of vibration may also be computed using dynamic analysis techniques.

RESPONSE COEFFICIENT "C" FOR VARIOUS VALUES OF PEAK ROCK ACCELERATION "A"

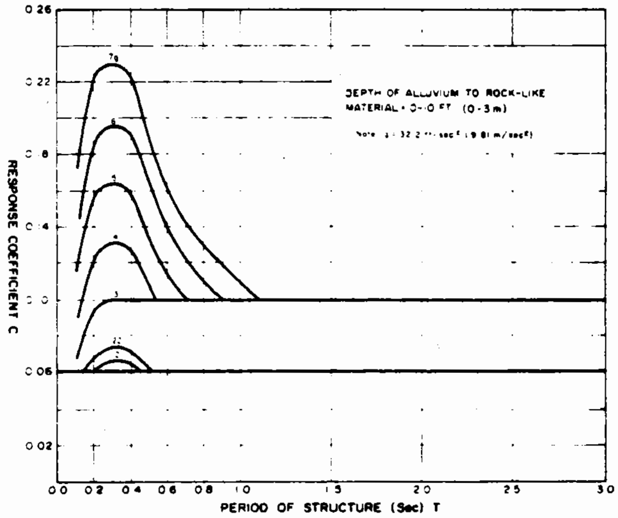


FIGURE 1.2.20A

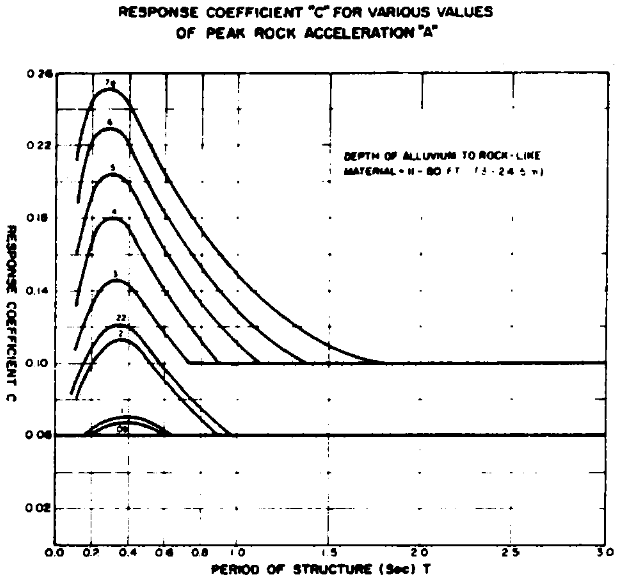

RESPONSE COEFFICIENT "C" FOR VARIOUS VALUES OF PEAK ROCK ACCELERATION "A"

FIGURE 1.2.20B

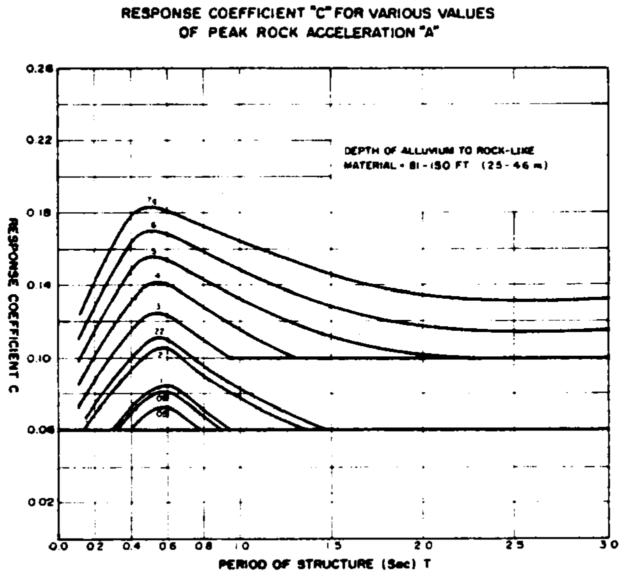
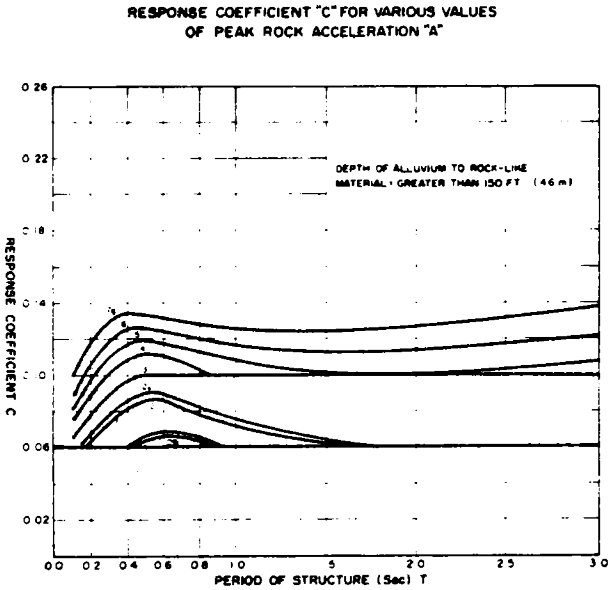

RESPONSE COEFFICIENT "C" FOR VARIOUS VALUES OF PEAK ROCK ACCELERATION "A"

FIGURE 1.2.20C



RESPONSE COEFFICIENT "C" FOR VARIOUS VALUES OF PEAK ROCK ACCELERATION "A"

FIGURE 1.2.20D

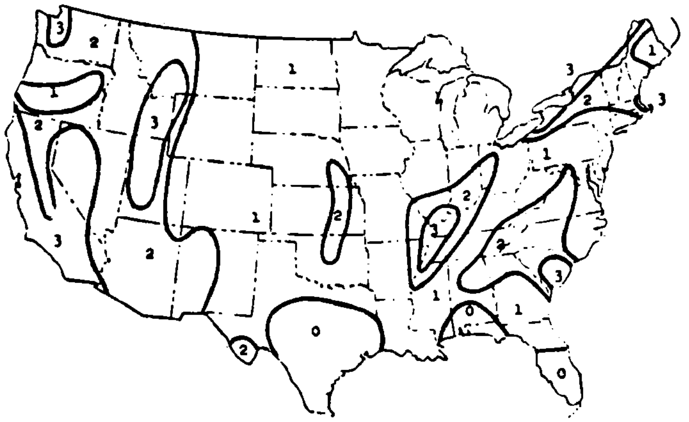


FIGURE 1.2.20E—Seismic risk map of the United States

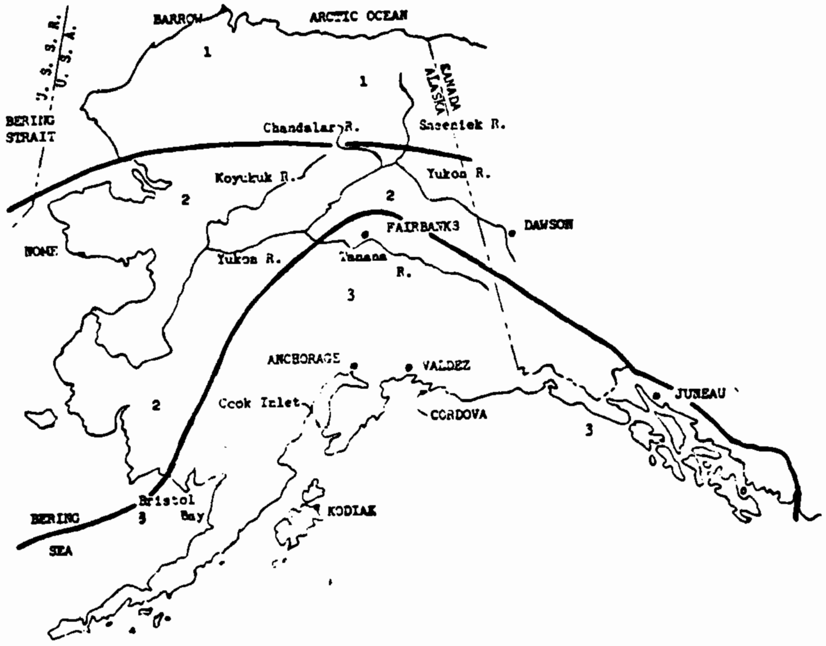


FIGURE 1.2.20F—Seismic zone map of Alaska

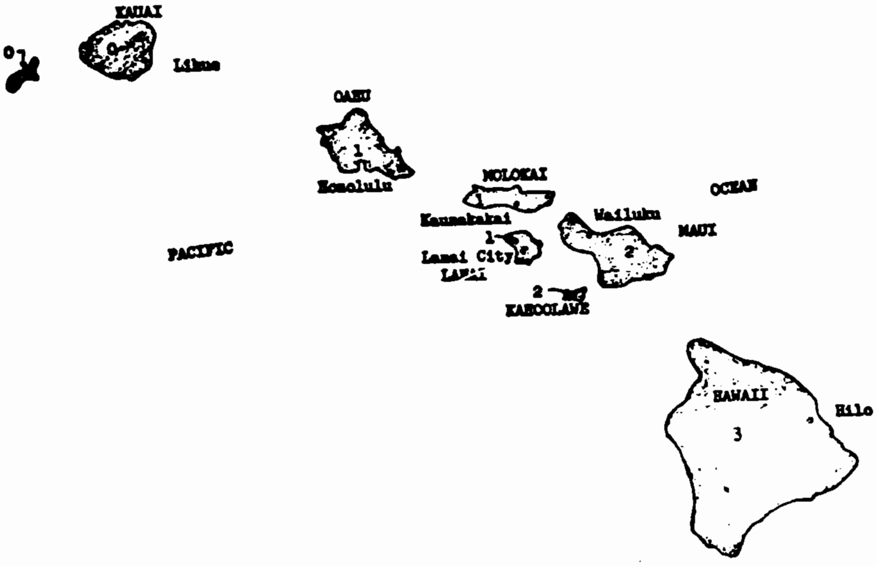


FIGURE 1.2.20G—Seismic zone map of Hawaii

(B) Response Spectrum Method

For complex structures, a response spectrum dynamic approach should be used for seismic analysis.

The combined response curves "C" given in figures 1.2.20A, B, C, D, or equivalent curves, modified by the framing factor "F", may be used as the design response spectrum.

(C) Special Cases

Structures adjacent to active faults, sites with unusual geologic conditions, unusual structures, and structures having a fundamental period greater than 3.0 sec. will be considered special cases. These structures will be required to be designed using current seismicity, soil response and dynamic analysis techniques.

(D) Design of Restraining Features

Restraining features to limit the displacement of the superstructure—i.e. hinge ties, shear blocks, etc.—shall be designed for the following force:

EQ = 0.25 x contributing DL minus column shears due to EQ.

"Contributing DL" is determined by examining the entire frame. For example, a simple span fixed at one end and sliding at the other will have the entire superstructure as the "Contributing DL" for longitudinal forces at the fixed abutment, while one half of the superstructure DL will act at each abutment for transverse forces.

For a frame, such as a 2-span structure, the full length of the bridge should be used as the contribution length in the longitudinal direction. The resulting force can be reduced by deducting the shear in the column due to earthquake.

For hinge restrainers use 0.25 x DL of the smaller of the 2 frames and deduct the column shears due to EQ.

1.2.21—CENTRIFUGAL FORCES

Structures on curves shall be designed for a horizontal radial force equal to the following percentage of the live load, without impact, in all traffic lanes:

$$C = 0.00117 S^2 D = \frac{6.68 S^2}{R} \text{ or } \left(\frac{0.79 S^2}{R}\right)$$

where

C = the centrifugal force in percent of the live load, without impact.
S = the design speed, in miles per hour. (km/hr)
D = the degree of curve.
R = the radius of the curve, in feet. (m)

The effects of superelevation shall be taken into account.

The centrifugal force shall be applied 6 feet (1.829 m) above the roadway surface, measured along the center line of the roadway. The design speed shall be determined with regard to the amount of superelevation provided in the roadway. The traffic lanes shall be loaded in accordance with the provisions of Article 1.2.8.

Appendix H.2

2-16 SEISMIC FORCES

Structures shall be designed to resist earthquake motions by considering the relationship of the site to active faults, the seismic response of soils at the site, and the dynamic response characteristics of the total bridge.

.1 Notation

EQ = Earthquake design force.
A = Maximum expected acceleration at bedrock or "rock-like" material at the site, in g's, as determined by the Engineering Geology Branch.
R = Normalized rock response.
S = Soil amplification spectral ratio.
ARS = 5% damped elastic acceleration response spectrum.
Z = Adjustment factor for ductility and risk assessment.
C = Combined response coefficient = ARS/Z.
W = Dead load of bridge.
T = Fundamental period of vibration, in seconds, of the bridge as a whole.
g = Acceleration due to gravity.

.2 Equivalent Static Force Method

For structures with well balanced spans and supporting bents or piers of approximately equal stiffness, EQ may be assumed as an equivalent uniform static force, equal to CW and applied at the vertical center of gravity of the total structure in any horizontal direction.

For determining C, ARS ordinarily may be taken from the curves in Figures 2-16.2 A, B, C and D. These curves reflect the combined effects of A, R and S for various ranges of alluvium depth over bedrock or "rock-like" material and various values of T. Z shall be taken from Figure 2-16.3.

The ARS curves are valid only for average granular alluvium. The Engineering Geology Branch shall determine alluvium depths. For non-granular alluvium or other unusual soil conditions, they may also furnish a special ARS curve to be used in place of the curves shown here. In no case shall C be less than:

0.10 for structures with $A > 0.3$ g; nor
0.06 for structures with $A < 0.3$ g.

For entry to the ARS curves, T may be computed by the expression, $T = 0.32\sqrt{W/P}$, where P, in units consistent with W, is the total uniform load required to cause a one-inch maximum horizontal deflection of the total superstructure in the direction of EQ.

Distribution of the resulting computed EQ to individual members shall reflect the stiffness of the superstructure and supporting bents or piers, including restraint at the abutments. EQ may be converted to an equivalent uniform load for this purpose, and applied to the structure in the direction of EQ.

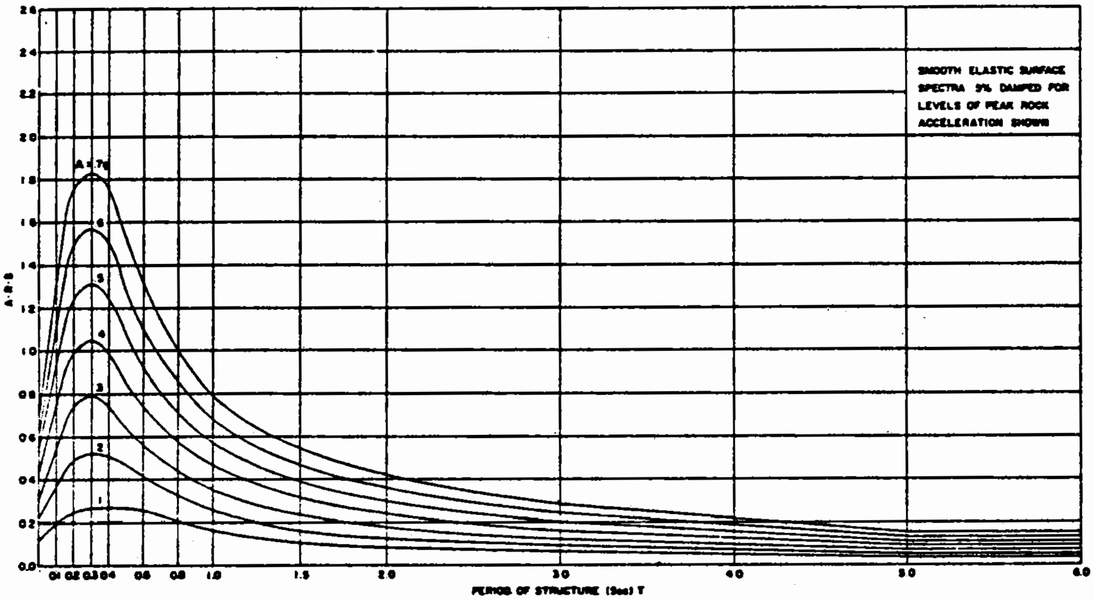


Figure 2-16.2 A

0 - 10' Alluvium

A·R·S SPECTRA

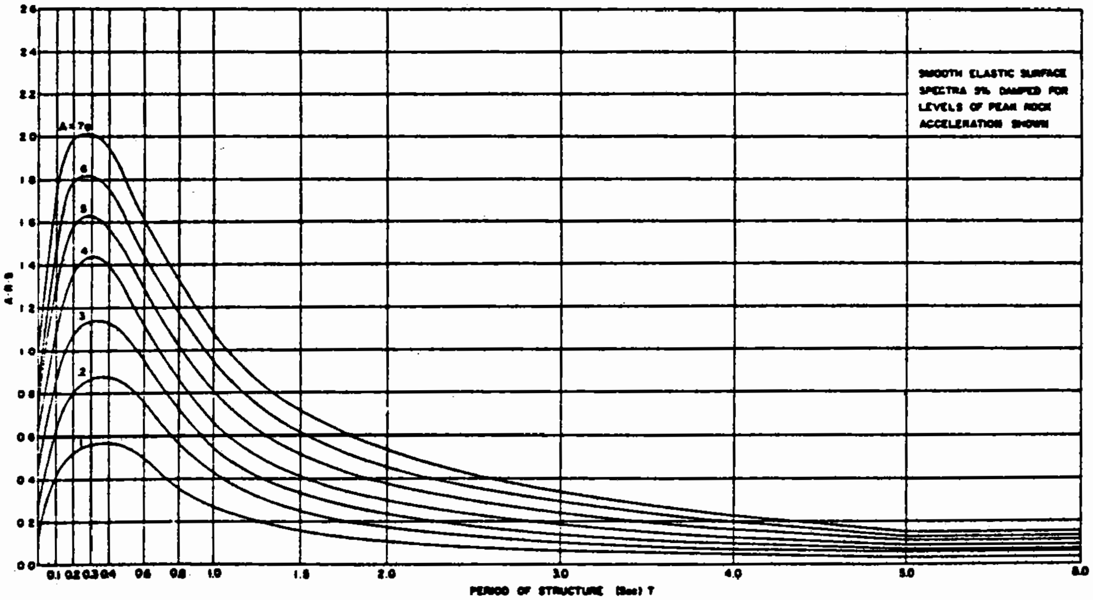


Figure 2-16.2 B

10'- 80' Alluvium

A·R·S SPECTRA

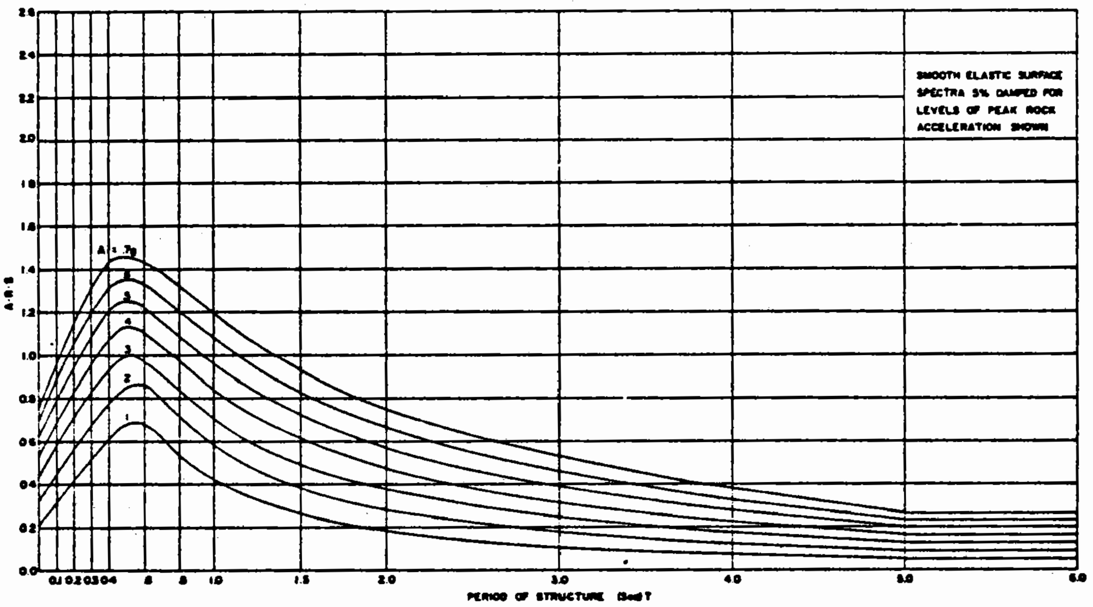


Figure 2-16.2 C

80'-150' Alluvium

A·R·S SPECTRA

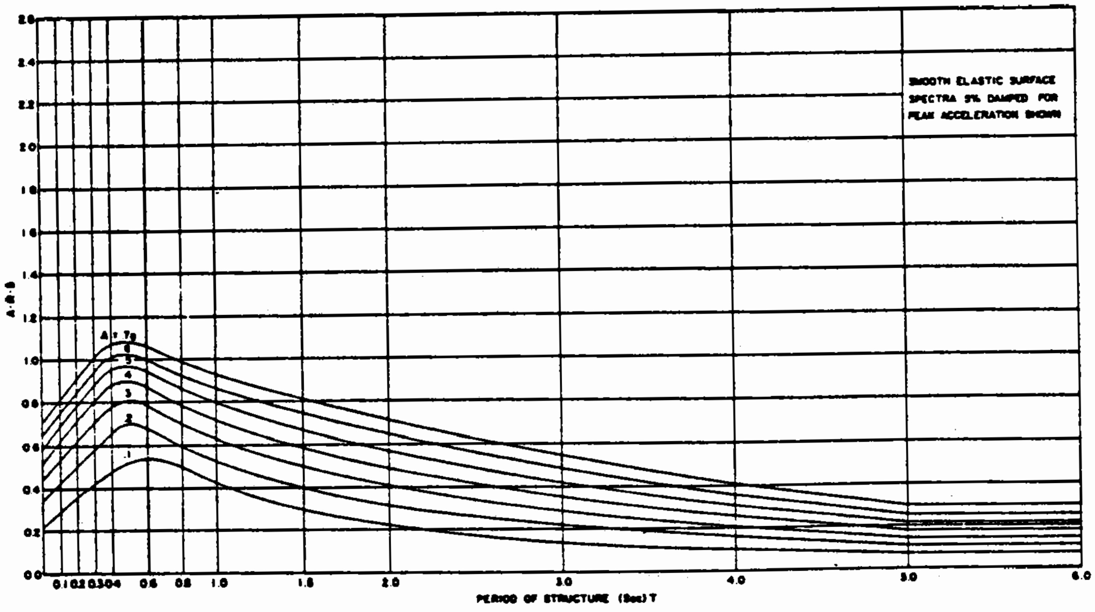


Figure 2-16.2 D

> 150' Alluvium

A·R·S SPECTRA

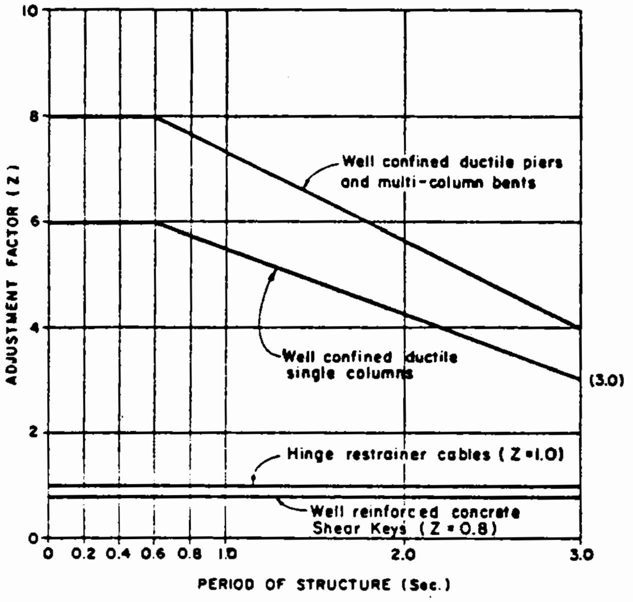
.3 Response Spectrum Method

EQ for structures with significantly irregular configuration or support stiffness may be determined directly as individual member forces by dynamic analysis. This method utilizes a modal analysis based on the application of a response spectrum of ground acceleration to a lumped mass space frame model of the structure.

A 5% damped elastic response curve (ARS), or an equivalent curve furnished by the Engineering Geology Branch, shall be used as the response spectrum loading. Design forces shall then be determined by dividing the resulting elastic forces by an appropriate factor, Z, from Figure 2-16.3.

In no case shall the design force on a column or pier be less than the following fraction of the DL axial force on the member:

0.10 for structures with $A > 0.3g$;
0.06 for structures with $A < 0.3g$.



ADJUSTMENT FOR DUCTILITY AND RISK ASSESSMENT

Figure 2-16.3

.4 Special Cases

Sites adjacent to active faults, sites with unusual geologic conditions, unusual structures, and structures with a fundamental period greater than 3.0 seconds will be considered special cases. In such cases, structures shall be evaluated for EQ by current seismicity, soil response and dynamic analysis techniques.

.5 Restraining Features

Hinge restrainers, shear keys and other restraining features to limit superstructure displacement preferably shall be designed for EQ determined by the response spectrum method applied to the structure as a whole, but in no case for less than:

$EQ = 0.25$ (Contributing DL).

Contributing DL is that portion of total superstructure DL whose response to earthquake motions can directly effect the feature under consideration. For structures with more than one hinge or more than two simple spans, contributing DL may also include a contribution from loads on restrainers beyond the immediately adjacent frames or spans.

When the response spectrum method is used, Z shall be taken from Figure 2-16.3, using the curve for the feature under consideration.

2-17 LOAD COMBINATIONS

The following groups represent various combinations of loads and forces to which a structure may be subjected. Each component of the structure, and the foundation on which it rests, shall be proportioned to withstand safely all group combinations of these loads and forces that are applicable to the particular site and structure type. See Articles 2-2 through 2-16 for the loads and forces. The maximum section required shall be used.

.1 Combinations and Factors

Load combinations are given by:

$$\text{Group } (N) = \gamma \Big[\beta_D \cdot D + \beta_L (L+I) + \beta_C \cdot CF + \beta_E \cdot E + \beta_B \cdot B + \beta_S \cdot SF + \beta_W \cdot W + \beta_{WL} \cdot WL + \beta_{LF} \cdot LF + \beta_P \cdot P + \beta_R (R+S+T) + \beta_{EQ} \cdot EQ + \beta_{ICE} \cdot ICE\Big]$$

Where N = group number,

γ = load factor, see Tables 2-17.1 and 2-17.2,

β = load factor, see Tables 2-17.1 and 2-17.2, and

D = Dead load

L = Live load

I = Live load impact

E = Earth pressure

B = Buoyancy

W = Wind load on structure

WL = Wind load on live load

LF = Longitudinal force from live load

CF = Centrifugal force from live load

P = Prestressing force effects

R = Rib shortening

S = Shrinkage

T = Thermal force

EQ = Seismic force

SF = Stream flow pressure

ICE = Ice pressure

Table 2-17.1

FACTORS FOR LOAD FACTOR DESIGN

GROUP	GAMMA FACTOR	BETA FACTORS													
		D	(L+I) H	(L+I) P	CF	E	B	SF	W**	WL	LF	P	R+S+T	EQ	ICE
I_H	1.30	β_D	1.67	0	1	β_E	1	1	0	0	0	1	0	0	0
I_{PC}	1.30	β_D	0	1	1	β_E	1	1	0	0	0	1	0	0	0
I_{PW}	1.30	β_D	1	1.15	1	β_E	1	1	0	0	0	1	0	0	0
II	1.30	β_D	0	0	0	β_E	1	1	1	0	0	1	0	0	0
III	1.30	β_C	1	0	1	β_E	1	1	1	1	1	1	0	0	0
IV	1.30	β_D	1	0	1	β_E	1	1	0	0	0	1	1	0	0
V	1.25	β_D	0	0	0	β_E	1	1	1	0	0	1	1	0	0
VI	1.25	β_D	1	0	1	β_E	1	1	1	1	1	1	1	0	0
VII	1.00	1	0	0	0	β_E	1	1	0	0	0	1	0	1	0
VIII	1.30	β_D	1	0	1	β_E	1	1	0	0	0	1	0	0	1
IX	1.20	β_D	0	0	0	β_E	1	1	1	0	0	1	0	0	1
X*	1.50	β_D	1.67	0	0	β_E	0	0	0	0	0	1	0	0	0

H denotes H loads. PC denotes P loads on closely spaced girders. PW denotes P loads on widely spaced girders.

β_D = 0.75 when checking columns for minimum axial load and maximum moment or maximum eccentricities.

β_D = 1.00 when checking columns for maximum axial load and minimum moment.

β_D = 1.00 for flexural and tension members and for culverts.

β_E = 0.50 for checking positive moments in rigid frames.

β_E = 1.00 for vertical earth pressure and for rigid culverts.

β_E = 1.30 for lateral earth pressure.

β_E = 1.67 for flexible culverts.

*Group X applies only to culverts. Other Groups do not apply to culverts.

**See Article 2-8 for high wind and moderate wind in individual groups.

(A) Use of Factors

Multiplication of the loads by the gamma and beta factors given in Table 2-17.1 forms the factored level of load magnitude used for Load Factor Design or ultimate strength checks. The same loads, with unit or zero load factors in Table 2-17.2, make up the service level of load magnitude used for Service Load Design or service load checks.

(B) Factor Adjustment

The gamma and beta factors specified for the factored level represent general conditions and should be increased for long spans or unusual structures if, in the engineer's judgment, anticipated loads, service conditions or materials of construction are different than anticipated by the specifications.

(C) Group I Variations

P loads shall be applied only in Group I at the factored level. At this level, Group I in Table 2-17.1 has been subdivided according to the application of the P loads.

Group I_H applies only when live loading does not contain P loads.

Group I_{PC} applies only for P load application to superstructures with closely spaced girders. Closely spaced girders are those with an average spacing less than that for which Footnote 2 to Article 3-1.2(1) of this volume applies.

In using Load Combination I_{PC}, the distribution of the P loads to superstructure elements follows the same rules as apply for the distribution of H loads in accordance with Article 3-1 of this volume.

Group I_{PW} applies only for P load application to superstructures with widely spaced girders and to substructures. Widely spaced girders are those with an average spacing such that Footnote 2 to Article 3-1.2(1) applies. Bent or pier caps shall be considered as substructure elements even though they may be entirely contained within the vertical dimensions of the superstructure.

In using Load Combination Group I_{PW}, only one P load, or one P load and one H load, may be applied to the structure for any one loading case in accordance with Article 2-4.1(A)(6)(b).

.2 Application

Loads shall be applied at the service level for the determination of the bearing area of spread footings based on allowable soil pressure, for the determination of the number of piles to be used based on allowable pile bearing capacity, and for the checking of foundation stability against sliding and overturning. Remaining applications of loads at either the factored level or the service level for the design or checking of structures or structural elements shall be as specified for the various materials or methods of construction in other sections of this volume.

When loads are applied at the service level, the resultant unit stresses shall not exceed the percentage of the basic unit stress for the load combination considered, as given in Table 2-17.2.

P loads shall not be applied at the service level.

Table 2-17.2

FACTORS FOR SERVICE LOAD DESIGN*

GROUP	GAMMA FACTOR	BETA FACTORS													
		D	L+I	CF	E	B	SF	W**	WL	LF	P	R+S+T	EQ	ICE	%
I	1.0	1	1	1	1	1	1	0	0	0	1	0	0	0	100
II	1.0	1	0	0	1	1	1	1	0	0	1	0	0	0	125
III	1.0	1	1	1	1	1	1	1	1	1	1	0	0	0	125
IV	1.0	1	1	1	1	1	1	0	0	0	1	1	0	0	125
V	1.0	1	0	0	1	1	1	1	0	0	1	1	0	0	140
VI	1.0	1	1	1	1	1	1	1	1	1	1	1	0	0	140
VII	1.0	1	0	0	1	1	1	0	0	0	1	0	1	0	133
VIII	1.0	1	1	1	1	1	1	0	0	0	1	0	0	1	140
IX	1.0	1	0	0	1	1	1	1	0	0	1	0	0	1	150

*Not applicable for culvert design. Use Load Factor Design.

**See Article 2-8.1 for high wind and moderate wind in individual groups.

% Indicates percentage of basic unit stress.

No increase in allowable unit stresses shall be permitted for members or connections carrying wind loads only.

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