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16. Abstract

The UMOT, or Unified Mechanism of Travel, is proposed as an alternative to traditional approaches for modeling interactions between travel and urban structure. The aim of the approach is to assist policy makers in dealing with such issues as the short and long term effects of energy prices and availability; predicting changes in urban structure; forecasting car ownership levels; and the optimal long-term allocation of resources to the various travel modes.

The UMOT approach focusses on regularities in household daily travel time and money expenditures, which are observed to be transferable both spatially and temporally. The use of total travel expenditures and unit costs eliminates the need in conventional models to calibrate coefficients correlating trip components to separate lists of explanatory variables. The UMOT approach generates simultaneous estimates of travel components, such as daily travel distance, modal shares, car ownership levels and household locational patterns, which are compared with observed data - not calibrated to them - for model validation.

Results of the analyses presented in this report, at both aggregate and disaggregate levels, corroborate previous results. A new development of this study is the formulation, testing and verification of "travel probability fields". These fields describe the spatial distribution of single trips in continuous probabilistic terms. Travel probability fields, as measure of accessibility, are found to be consistent with urban economic models of location decisions. This allows travel patterns to be related theoretically to urban structure through a dynamic feedback process.

The encouraging results of the empirical and theoretical developments presented in this report give real hope for achieving the ultimate objective of a fully dynamic, interactive urban travel/land use model.

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FOREWORD

The work reported herein is one element of a broad program of research in systems analysis whose aim is to provide analysts and decision makers in DOT with far better analytic tools than are now at their disposal. The focus of the program is on understanding the interactions between transportation and the social and economic functioning of society as a non-linear dynamic process. The need is to be able to address a series of tough questions regarding the long and short term roles of transportation in such issues as energy supply and conservation, industrial development, productivity and urban revitalization.

The UMOT project has been concerned primarily with the relationship between urban passenger transportation and land use, although in the coming year we plan to expand this work to include intercity passenger travel as well. It approaches the problem from a fresh viewpoint - that of using observed regularities in the expenditures of travel time and money as constraints to link transportation and urban form. With the past year's research, which explore the travel data from extensive household survey made in Washington, D.C. in 1968, the existence of these regularities has been demonstrated conclusively. The concept is finally gaining greater acceptance as other researchers, both here and abroad, find identical regularities.

Much needs to be done to realize the ultimate potential of this approach. In its present form, only travel decisions are internal to the UMOT process, with locational decisions (i.e. land use) being either fixed or external. Obviously, locational decisions must also be represented as internal variables if the broader effects of transportation on urban form are to be modeled. Gratifying progress was made in this direction during the past year by the explorations in travel fields. The dynamics of the process must also be represented explicitly. A start has been made, as this report indicates, but much work lies ahead. Even so, the model in its present, limited form is beginning to show a remarkable power in several experimental applications to predict travel decisions in both the short and intermediate term.

A recent development is the increasing interest being shown outside DOT in the UMOT model as well as other products of our research program. Cooperative research agreements are now in process in West Germany, the UK and the Netherlands. An international study group has been formed to examine broadly the subject of transportation/land use interactions. Within the Federal government an interagency committee has been established to further research in systems analysis. It is hoped that these actions portend a wider support for research in this general area, and for the development of a new and more realistic family of models having a predictive power more nearly matched to the tasks at hand.

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EXECUTIVE SUMMARY

The UMOT, or Unified Mechanism of Travel, is proposed as an alternative to traditional approaches for modeling interrelationships between travel and urban structure. The aim of the approach is to provide a tool for exploring the short and long term implications of a range of policy decisions involving transit and highway financing, energy costs, urban development, or changes in urban infrastructure.

The Research and Special Programs Administration of the U.S. DOT has provided support for UMOT development for the last three years. The research reported herein was conducted during 1979-1980 and its objectives were twofold: (1) to test the underlying assumptions of the UMOT approach using a detailed transportation planning study dataset, and (2) to extend the model framework toward including dynamic interactions between travel and urban structure.

The UMOT approach focusses on regularities in household travel patterns. Travel time and money expenditures per average traveler have been found particularly to be transferable both spatially and temporally; changes in urban structure or travel time and money costs result in shifts in trip frequencies, lengths and choices among modes. Similarly, changes in household characteristics, such as income and household size, result in changes in the characteristics of the trips that household members actually make on any given day. In all cases, shifts in total travel expenditures per average traveler are predictable.

The use of total travel components and expenditures eliminates the need in conventional models to calibrate coefficients correlating trip rates, mode choice or trip distributions to separate lists of explanatory variables. This is judged to be an important advantage because such coefficients are seldom found to be stable over time or transferable among cities. The UMOT approach generates estimates of travel components, such as daily travel distance, modal shares, car ownership levels and household location patterns, which are compared with observed data - not calibrated to them - for model validation.

Previous tests of the UMOT approach were based mostly on aggregate urban travel data. This report presents results of both aggregate and disaggregate analyses carried out using 1968 household socioeconomic and locational characteristics in two corridors in the Washington, D.C. Metropolitan area. The results of the analyses corroborate previous results:

- The daily travel time expenditures per average traveler display consistent regularities. The frequency distributions are similar for all traveler segments, where the segments are defined by income and car ownership levels.

- It was necessary to derive the daily travel money expenditures per average household indirectly because travel money expenditures were not reported in the home-interview survey. The results, estimated as the product of reported daily travel distanc by mode and travel unit costs, corroborate previously reported regularities.

- The various aspects of daily travel are closely linked: daily travel distance with door-to-door speed; average trip length with daily travel distance; trip rate with average trip time; and proportions of trips by trip purpose with trip rate. These observations emphasize the need to consider such interrelationships simultaneously within a dynamic feedback process.

A new development of this study is the formulation, testing and verification of "travel probability fields". These fields describe the spatial distribution of single trips in continuous probabilistic terms:

- Travel probability fields capture the salient characteristics of trip destination distributions. The scalar parameters of travel probability fields are related in a straightforward manner to the travel measures of average trip speed and average trip length, and to residence distance from the urban center. Moreover, travel probability fields provide additional information on the geographic orientation of travel and urban density functions.

- Travel probability fields are consistent with urban economic models of location decisions. This allows travel patterns to be related theoretically to urban structure. Moreover, there is convincing evidence that the model can be implemented as a component in a simultaneous, dynamic feedback process for forecasting travel and associated changes in urban land-use patterns.

These encouraging conclusions, supported by the results of the empirical and the theoretical developments presented in this report, give real hope for achieving the ultimate objective of a fully dynamic, interactive urban travel/land use model.

* * *

CONTENTS

CHAPTER 1: INTRODUCTION

1.1 Foreword

A new approach to the modeling of urban travel has been proposed to integrate models of urban structure and travel. This approach is called the Unified Mechanism of Travel, or UMOT. It was first conceptualized for the World Bank (1.1) and further developed for the U.S. Department of Transportation and the Federal Republic of Germany Ministry of Transport (1.2).

Previous tests of the UMOT approach were based mostly on aggregate urban travel data. However, it is appropriate to test the accuracy of the approach using a detailed data set from a conventional comprehensive urban transportation planning study. Such data can be used as well to test extension of the conceptual framework to include dynamic links with urban structure. The results of this research are presented herein. First, a short description of the UMOT approach is presented, to serve as a background.

1.2 The UMOT Approach

The UMOT approach is based on the predictable regularities observed in the mean expenditures on travel per traveler and per household, in time and money terms, which can be attributed to such factors as the socioeconomic characteristics of the household, the transport system, and the urban structure. When these regularities are observed to be transferable both between cities and over time, then the expenditures can be regarded as "travel budgets" which, under certain conditions, are applied as constraints on travel behavior.

One useful way of applying travel budgets as constraints is within the microeconomic theory of consumer behavior, where consumer utilities are maximized under explicit constraints. In the UMOT approach, the utility of the spatial and economic opportunities to which a person travels, represented by the average daily travel distance, is maximized under the explicit constraints of time and money budgets allocated to travel.

While based on empirical evidence and supporting theories of consumer behavior (1.2, 1.3), the UMOT process follows several recent concepts and recommendations advanced in system theory (1.4). the unique characteristics of the UMOT approach which distinguish it from conventional travel models are as follows:

- Causality. Causality in travel modeling is typically assumed a priori. For example, it is typically assumed in travel models that car availability per household increases trip generation. Namely, car ownership is the cause, while more trips is the effect. However, it might be also argued legitimately that the need for more travel generates car ownership levels.

  In the UMOT process there are no assumptions about unilateral, fixed, causality. The process is based on a systemwise approach, where all travel components interact with each other and with the transport system through a simultaneous dynamic feedback process. Thus, each component can be both cause and effect, depending on the feedback step.

- Validation. Conventional travel models are calibrated to observed travel choices. Thus, both the independent and dependent variables must be known before such models can be calibrated. For instance, a model which is required to estimate trip rates per household is calibrated (fitted) to the observed trip rates, and the model is then validated by its ability to reproduce the same observations to which it was fitted. This may be regarded as a tautological process.

  In the UMOT process no desired output is ever calibrated to the observed values. The outputs are the expected choices, which are then compared with the observed choices - not fitted to them - for the model's validation.

  For example, the process can be started by assuming that each and every household in the urban area owns, say, 5 cars. Such an assumption, of course, is absurd. Nonetheless, the travel system converges rapidly to the observed car ownership levels, by households' socioeconomic characteristics.

- Transferability. Conventional models usually must be recalibrated in each separate city. The coefficients, fitted to cross-sectional data, are then assumed to remain fixed over time for each city. However, a prerequisite for a model's temporal transferability in one city is considered to be its spatial transferability between cities at one point in time, a condition which is not always met by conventional models.

  The UMOT process is based on relationships that apply to the travel constraints, relationships which have been observed to be transferable both spatially and temporally in one country. There are no fixed coefficients associated with the choices in the UMOT process and the model is activated through all its phases for each endogenous and exogenous change. Even the constraints are not constant, but can vary in response to endogenous and exogenous factors.

- Equilibrium vs. Disequilibrium. Conventional travel models are usually based on the assumption that the demand is in equilibrium with the supply. Thus, by definition, each alternative scenario must reach, or at least approach, equilibrium between demand and supply.

  However, it is equally valid to say that it is the amount of possible disequilibrium associated with alternative futures which generate forces that dynamically change urban structure, often in unexpected ways. The UMOT process attempts to measure, as one of its outputs, the amount of potential disequilibrium affecting households of different socioeconomic and locational characteristics.

While the above distinctions between conventional travel models and the UMOT process are discussed in detail in the remainder of this report, one example is revealing at this stage. Conventional models regard travel distance as a disutility, measured by the time and money costs required to overcome distance between origin-destination pairs. In the UMOT process, travel distance is regarded in utility terms, representing the benefits of access to spatial and economic opportunities within the urban area.

1.3 The Data

The principal purposes of the present study were twofold: to analyze in depth the basic travel/urban relationships that emerge from a comprehensive transportation study data set in order to verify the basic assumptions underlying the UMOT process; and to extend the UMOT framework to include explicitly interactions between travel and urban structure. The data set chosen for analysis was the Washington, D.C. 1968 Comprehensive Transportation Study.

Data from the 1968 Washington, D.C. expanded home-interview survey were stratified by the following dimensions: Income Class, Residential Location (corridor and ring), Household Size, and Car Availability. The original 10 income classes were condensed into 5 classes because of small sample size (less than $4,000 income; $4,000 to $8,000; $8,000 to $12,000; $12,000 to $20,000; and over $20,000 income in 1968 dollars).

The metropolitan area was divided into 6 corridors, each containing 6-7 rings, as shown in Figure 1.1. The distance of each ring centroid to the urban center was weighted by population, on a zonal level.

As a quantitative background to the analyses presented in the remainder of this report, the salient spatial distributions of the population characteristics in the Washington, D.C. Metropolitan Area in 1968 are analyzed in Appendix A. Focus in Appendix A is on income and car availability.

The results of the analyses documented in Appendix A have several implications: First, while income strongly affects car ownership levels, the latter appears to be a better measure for describing the household spatial distributions than the former. Second, a principal part of the diffusion pattern of households by income, both at each ring and versus distance from the center, may be ascribed to car availability. Lastly, an approach is suggested for the spatial allocation of households in an urban area as part of the UMOT process. In essence, the macro-estimates of car ownership levels, combined with the spatial distributions of car

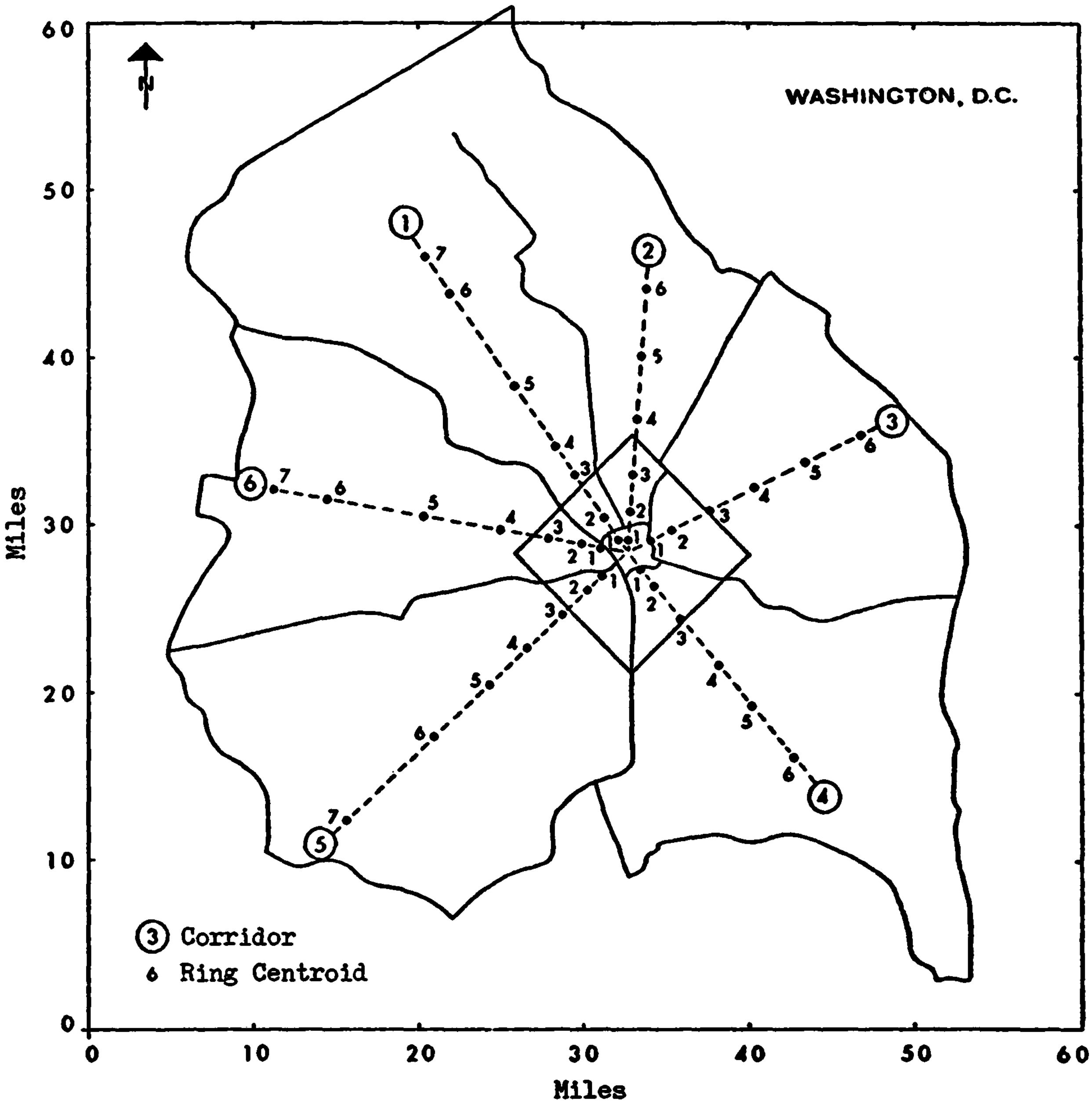

Figure 1.1: Corridors and Rings in the Metropolitan Area of Washington, D.C., 1968

ownership levels, can be used to generate the spatial distribution of households, which can then be iterated through new micro car ownership levels, until convergence between travel demand and system supply within the travel budgets of time and money is reached or approached.

The detailed analyses presented in the following chapters are based on data relating to households residing in <u>two corridors</u> in the Washington, D.C. 1968 Metropolitan area. The first corridor, called <u>North</u>, is part of corridor 1 shown in Figure 1.1, and it includes parts of Georgetown, Chevy Chase, Bethesda and Potomac. The second corridor, called South, is part of corridor 5 in Figure 1.1, and it includes parts of Arlington, Falls Church and Fairfax.

The basic tables provided the following background data on households, travelers and travel components in the two corridors: Travelers per household, daily travel distance per traveler/household, daily travel door-to-door time per traveler/household, trip rate per traveler/household, trip distance, trip time, and door-to-door speed. The stratification dimensions in each table were income, household size, car availability, and residence distance from the urban center. A further stratification was by two trip purposes: work and non-work.

Sample size in the 1968 survey was relatively low, varying between 1 to 4 percent, with an average value of about 3 percent. Hence, 100 households in the tables, detailed in the following chapters and appendices, are actually anywhere between only 1 and 4 households, and large variations in household and travel characteristics are to be expected when the expanded number of households is less than 750-1,000.

The spatial distributions of households in the two corridors are similar, and they follow the general trends of all households in the metropolitan area. For example, the distances of the households' weighted centroids to the urban center are 8.1 and 8.6 miles, respectively.

Table 1.1 summarizes the general socioeconomic and travel characteristics of households and travelers in the two corridors.
The table shows that:

(1) Household size, travelers per household, cars per household, and amount of travel, all increase with household income.

(2) All household and travel characteristics are somewhat higher in the South corridor than in the North corridor. Especially noticeable is the higher speed in the South corridor.

(3) The only travel characteristics which are similar in both corridors are the daily travel time per traveler and trip time.

Table 1.1 : Summary of Household and Travel Characteristics, North and South Corridors, Washington, D.C. 1968

NORTH

	Characteristic	Income Class					Total
		1	2	3	4	5	
Households	Households (HH)	3,990	10,260	15,240	19,060	12,510	61,060
	Household Size	1.58	2.00	2.90	3.41	3.51	2.95
	Cars/HH	0.52	0.71	1.22	1.62	1.83	1.34
	Travelers/HH	1.29	1.53	1.80	2.27	2.69	2.05
	Time/HH, min.	89.95	105.52	124.05	158.43	192.60	143.48
	Distance/HH, m.	15.42	20.08	29.43	41.64	47.87	34.53
	Trips/HH	3.59	4.37	5.34	7.08	8.84	6.32
Travelers	Travelers (TR)	5,150	15,740	27,400	43,190	33,650	125,130
	Time/TR, min.	69.65	68.80	69.01	69.92	71.61	70.02
	Distance/TR, m.	11.90	13.09	16.37	18.38	17.80	16.85
	Trips/TR	2.78	2.85	2.97	3.12	3.29	3.08
Trips	Trips	14,290	44,890	81,420	134,920	110,600	386,120
	Trip Distance	4.30	4.59	5.51	5.88	5.41	5.46
	Trip Time	25.08	24.13	23.22	22.38	21.78	22.69
	Speed, mph	10.29	11.42	14.23	15.77	14.91	14.44
SOUTH							
Households	Households (HH)	5,640	6,990	10,580	12,710	4,900	40,820
	Household Size	1.32	2.71	3.46	3.53	3.62	3.08
	Cars/HH	0.55	1.11	1.43	1.76	2.04	1.43
	Travelers/HH	1.07	1.65	2.18	2.51	2.79	2.11
	Time/HH, min.	62.12	111.69	150.62	186.85	203.12	149.31
	Distance/HH, m.	11.38	31.90	44.32	55.72	62.42	43.36
	Trips/HH	2.87	4.67	6.77	7.94	8.79	6.48
Travelers	Travelers (TR)	6,050	11,520	23,020	31,960	13,680	86,230
	Time/TR, min.	57.93	67.75	69.34	74.30	72.74	70.70
	Distance/TR, m.	10.61	19.35	20.37	22.16	22.35	20.53
	Trips/TR	2.67	2.83	3.11	3.16	3.15	3.07
Trips	Trips	16,180	32,640	71,610	100,950	43,060	264,440
	Trip Distance	3.97	6.83	6.55	7.01	7.10	6.69
	Trip Time	21.67	23.92	22.29	23.52	23.12	23.06
	Speed, mph	10.99	17.14	17.66	17.89	18.44	17.42

The following chapters present the analyses of households and travel characteristics in detail.

1.4 Scope of the Report

This report covers the following subjects:

Chapter 2: Travelers per Household. It is shown that the number of travelers per household during an average weekday is significantly related to the household's socioeconomic and locational characteristics.

Chapter 3: Travel Time. A thorough analysis of the daily travel times per average traveler, by household socioeconomic characteristics, suggest that such times display consistent regularities, confirming previous results.

Chapter 4: Travel Money. An analysis based on indirect measures (e.g., travel distance by mode) suggests that the travel money expenditures per average household are closely related to the household's socioeconomic characteristics, and display consistent regularities, confirming previous results.

Chapter 5: Travel Distance. Since travel distance is a key concept in the UMOT process, special emphasis is given to the analysis of this travel component. The results confirm previous indications, namely that the daily travel distance per average traveler is strongly related to the available door-to-door speed.

Chapter 6: Trips and Their Components. Since conventional urban travel models are based on trips, special analyses were conducted in order to test the hypothesis that all trip components, including trip rate, trip distance and trip time, are interrelated with each other and with the daily travel distance and travel time per average traveler. All previous indications were confirmed.

Chapter 7: Travel Probability Fields and Urban Spatial Structure. Travel probability fields express the probability of trips to terminate within certain geographical areas. The characteristics of travel fields, including their size and shape, are found to be related to the daily travel distance per average traveler and door-to-door speed. Thus, it is proposed that such fields can serve as a direct link between travel generation and distribution, bypassing the need to deal with origin-destination matrices of single trips. An underlying theory of travel probability fields is presented which is based upon economic principles of consumer behavior, and tests of theoretical hypotheses confirm the correspondence between theory and application.

Chapter 8: Dynamic Relationships. Preliminary elements of a dynamic theory to unify the observed travel relationships with population and employment distributions are advanced in this chapter. The theory is based on urban economic principles. While it is recognized that the proposed theory cannot yet account for many activities taking place in an urban area, it appears to capture the dynamic changes of travel and urban structure in response to changes in exogenous factors, such as income, household size, transport system efficiencies and travel costs.

Chapter 9: Conclusions and Recommendations. This chapter summarizes the salient results of the study, and points out their importance and implications to both conventional travel models and the UMOT process. The chapter concludes with recommendations about the best way in which further research of specific subjects can be combined with a practical development of the UMOT process into an operational travel/urban structure model.

Appendices: Detailed analyses are presented in the appendices. Also, preliminary theoretical developments not yet empirically tested are described in the appendices; they are summarized and interpreted in the main body of the report.

CHAPTER 2: TRAVELERS

2.1 Introduction

Travelers are regarded in this report as the building blocks of travel analysis and travel models.

It is not always easy to define who is a traveler. In a general way, each and every person may be regarded as a potential traveler over a period of time, say a month. Therefore, some researchers prefer to relate observed travel to an average person within his/her household. This approach has much merit when travel data are based on a travel diary extended over several days. On the other hand, it can also be argued that a traveler is a person who decides to make a trip on a particular day, by a particular mode, to a particular destination and at a particular time. Hence, it would be preferable to relate travel to those who reported it, especially when the travel data are based on a one-day survey. In such an approach, the problem can then be decomposed into two basic parts: what is the probability that a person having certain socioeconomic characteristics becomes a traveler during a given day, and what are his/her specific travel characteristics during that day ?

Since the data analyzed in this report are based on a one-day survey, the latter approach has been found to be more appropriate, and a traveler is defined as a person who made at least one trip during the survey day by a mechanized mode (private or public modes). Hence, all travelers were identified first, and then stratified by their households' socioeconomic and travel characteristics. The analyses then proceed along two levels. First, the estimated number of travelers per average household per weekday, by the households' socioeconomic characteristics, is derived. Second, the travel characteristics per average traveler are related to his/her household's socioeconomic characteristics. This chapter presents the results of the first level analyses, while the following chapters detail the results of the second level analyses.

2.2 Travelers per Household

Multiple regression analyses were carried out in order to assess the effects of various factors on the estimated number of travelers per household.

The data were stratified by household income, and further stratified within each income group by household size (1, 2, 3-4, and 5+), car ownership levels (0, 1, 2, 3+), and residence distance from the urban center (2-mile increments from zero to 19+ miles).

Household income is used in the UMOT process as a principal input since the travel money budget is related to household income. Also, residence distance is regarded in the UMOT process as an output. Thus, it appears reasonable to consider only household size and car ownrship levels as the independent variables for the estimation and prediction of the number of travelers per household. This limits multicolinearity concerns to only two independent variables, with only marginal loss of information. Furthermore, since car ownership level is a final output of the UMOT process, the possible multicolinearity between household size and car ownership is minimized.

Table 2.1 summarizes the regression results for the North and South corridors. The order of contribution of the various independent variables to the explanation power of the relationships is: (i) household size, (ii) cars per household, (iii) income, and (iv) residence distance from the urban center. It appears that household size and car ownership level capture most of the income and residence distance effects, since the addition of income and residence distance as independent variables adds only marginally to the explanation power of the relationships.

The final selected relationships take the form of:

$$\text{Travelers/HH, North} = 0.418 + 0.436(\text{HH Size}) + 0.412(\text{Cars/HH}), \qquad (2.1)$$

$$\text{Travelers/HH, South} = 0.233 + 0.530(\text{HH Size}) + 0.390(\text{Cars/HH}). \qquad (2.2)$$

Table 2.1 : **Travelers per Household, Linear Regression Statistics, North and South Corridors, Washington, D.C., 1968**

Regression Step		NORTH[1] Household Size	NORTH Cars/HH	NORTH Annual Income	NORTH Residence Distance	SOUTH Household Size	SOUTH Cars/HH	SOUTH Annual Income	SOUTH Residence Distance
1	a	0.572	1.108	0.827	1.164	0.389	1.103	0.755	1.140
	b	0.605	0.734	0.373	0.087	0.684	0.775	0.445	0.130
	R^2	0.269	0.220	0.124	0.032	0.300	0.209	0.178	0.091
	F	699.22	536.60	268.15	63.28	612.85	377.52	309.73	142.50
	t	26.45	23.16	16.38	7.95	24.76	19.43	17.60	11.94
2	a	0.418				0.223			
	b	0.436				0.530			
	c		0.412				0.390		
	R^2	0.317				0.338			
	F	441.18				364.49			
	t	16.43	11.58			16.68	9.03		
3	a	0.160				-0.029			
	b	0.418				0.491			
	c		0.339				0.283		
	d			0.117				0.144	
	R^2	0.326				0.349			
	F	306.33				255.55			
	t	15.73	8.85	5.03		15.11	5.93	5.03	
4	a	0.318				-0.051			
	b	0.453				0.512			
	c		0.386				0.295		
	d			0.103				0.144	
	e				-0.047				-0.017
	R^2	0.333				0.350			
	F	237.31				192.37			
	t	16.45	9.78	4.43	4.55	14.42	6.09	5.04	1.49

Since the actual application of these relationships is to discrete values of household size and car ownership levels, a comparison between the two relationships for combinations of discrete values of household size and car ownership levels is shown in Figure 2.1.

It can be inferred from the above comparison that the two relationships are similar for all practical purposes. Indeed, other investigations have shown that such relationships in Washington, D.C. and Twin Cities were statistically similar and transferable both spatially and temporally (2.1).

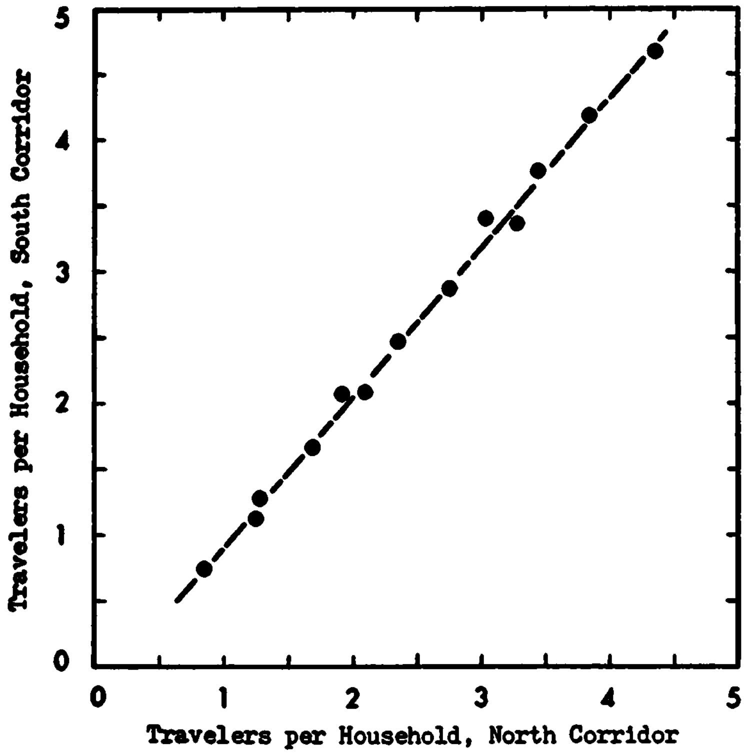

Figure 2.1 : Estimated Travelers per Household, Comparison between the North and South Corridors, Washington, D.C. 1968

CHAPTER 3: TRAVEL TIME EXPENDITURES

3.1 Introduction

In the search for predictable regularities in travel behavior, which are transferable both spatially and temporally, it has been observed that the daily travel time expenditures per average traveler appear to vary within a narrow range (3.1). Further analyses have suggested that even the variations around the mean values are similar in a wide range of cities (3.2, 3.3, 3.4, 3.5, 3.6). Table 3.1 summarizes the daily travel time per average traveler in selected cities by extreme ends of income and car ownership levels, where a traveler is defined as a person who made at least one motorized trip during the survey day.

Table 3.1: Daily Travel Time per Motorized Traveler, hr.

1. BY INCOME		High	Low
Bogota, Colombia		1.05	1.78
Santiago, Chile		1.09	1.52
Singapore		1.14	1.36
2. BY MODE		Car	Transit
Washington, D.C., 1955		1.09	1.27
1968		1.11	1.42
Twin Cities, 1958		1.14	1.05
1970		1.13	1.15
All U.S., 1970		1.06	0.99
3. BY CAR AVAILABILITY			
St. Louis, 1976	0 Car	1.06	
	1 Car	0.99	
	2 Cars	1.05	
	3+ Cars	1.06	
	Average	1.04	
4. BY HH SIZE & CARS	HH Size	1 Car	0 Car
The Nuremberg Region, 1975	1	1.22	1.41
	2	1.25	1.42
	3	1.28	1.36
	4+	1.27	1.35
5. BY DAYS		Survey Day	
Munich, 1976	1st Day	1.15	
	2nd Day	1.16	
	3th Day	1.16	
6. Total			
Toronto 1964		1.09	
Calgary 1971		1.11	
Montreal 1971		1.18	

It was inferred from such observations that travelers, on the average, appear to have a "preferred" daily travel time, of about 1.1 hours at high income levels. Low income travelers, on the other hand, may have to spend more daily travel time, for less travel distance, than high income travelers, depending on the available speeds. Such regularities were regarded as travel time "budgets" allocated by travelers to travel, and applied as explicit constraints in the UMOT process (3.7).

This does not mean that each and every traveler spends a fixed amount of time on his/her travel each day. What the results suggest is that once the mean and the variance of the daily travel time per average traveler belonging to a population segment are known, the probability of an individual traveler to behave as his group can be deduced.

Other researchers have attempted to interpret these observations in different ways. Studies in England and France (e.g., 3.8, 3.9) tested the hypothesis that the daily travel time per average person is fixed over time. Not unexpectedly, such an hypothesis was rejected, mainly because the proportion of travelers vs. population may change over time by such factors as household size, age distribution, and car ownership levels. Supposing, for the sake of argument, that daily travel time per average traveler were absolutely fixed over time, the daily travel time per average person would still be expected to vary with changes in the proportion of travelers. Another consistent misunderstanding of the travel time budget concept is to say that each and every individual traveler spends a fixed amount of travel time each day which equals the average value.

In order to expel such misrepresentations of the travel time budget concept, additional in-depth analyses of travel time expenditures per traveler were carried out. Following are the summarized results of these analyses, while analyses details are provided in Appendix B.

3.2 Daily Travel Time per Average Traveler vs. Average Person

The effect of household size on the daily travel time per average person is shown in Table 3.2 (3.1). Reported travel times per average traveler vary within a narrow range, but travel times per average person very significantly.

Table 3.2 : Daily Travel Time per Person and per Car Traveler, by Household Size, Minneapolis/St. Paul, 1958 and 1970.

Household Size		1	2	3-4	5+	Average
Time per Person, hr	1958	1.04	0.93	0.68	0.46	0.64
	1970	1.09	0.95	0.69	0.57	0.70
Time per Traveler, hr	1958	1.04	1.16	1.18	1.08	1.14
	1970	1.08	1.17	1.14	1.10	1.13

These results support the two-level approach described in Section 2.1: First, the number of travelers per household during a weekday are related to the household's socioeconomic characteristics, in order to determine the probability that a person is a traveler during a specified period. Second, the reported daily travel time per average traveler are related to the travelers who actually traveled.

In the following discussions, data per average traveler refer to the values averaged for travelers belonging to households stratified by their socioeconomic characteristics.

3.3 Analysis of Variance

The first test involved analyses of variance (ANOVA) of the daily travel times per average traveler in the two corridors, stratified by household income, household size, residence location, and car ownership levels. Two and three-way analyses were carried out on travelers' daily travel times (unweighted ovservations). Table 3.3 summarizes the ANOVA results. An example of the results is presented in Appendix B, Figure B.1.

Table 3.3 : Summary of ANOVA, F-values of Daily Travel Time per Traveler, North and South Corridors, Washington, D.C. 1968

Corridor	Number of Observations	Household Size	Car Ownership	Residence Distance	Household Income	3-way Interactions
North	3,872	8.696*	2.397	2.704*		1.108
		9.824*	1.613		1.739	0.877
		8.519*		2.387	1.594	1.185
			1.539	2.838*	1.111	1.586
South	2,538	9.225*	2.365	3.042		1.449
		12.418*	0.751		10.099*	0.917
		12.159*		1.938	8.989*	1.142
			0.253	1.902	5.142*	0.977

(*) - Significant at 0.01 level

It may be inferred from Table 3.3 that:

(1) There are significant differences between the mean daily travel times per average traveler belonging to households of different sizes in both corridors. Such a phenomenon was previously noted in the case of Munich (1.2, p. 98), produced mainly by households with three members. Such households may include three adults who traveled during the survey day or, at the other extreme end, they may include a husband, wife and child, of whom only the husband traveled. Thus, it is advisable to stratify travelers by household size.

(2) Car ownership does not affect significantly the mean daily travel time per average traveler.

(3) Residence distance and household income show conflicting trends in the two corridors.

(4) There are no interaction effects between the various factors. Namely, if a significant difference is noted in one group, it is not affected by, nor does it affect, other groups.

The above results are not fully conclusive for two reasons: First, a few possible outlier cases may have a large effect on both the mean values and the variance in each group. (Although certain data screening was performed previously, at this stage of the analyses it was unclear as to the effectiveness of the screening.) Second, the ANOVA tests deal with the dispersion around mean values, without considering the distributions of travel time by individual travelers. In order to assess such possible effects on the daily travel time per average traveler, the data were stratified by time increments, as described in the next section.

3.4 The Effects of Outlier Cases

Frequency distributions tables of the daily travel time per average traveler in the two corridors were prepared, by 10-minute increments and up to 8-hours travel time, and stratified by income and by car ownership levels (Appendix B, Figure B.2). Investigations of the frequency distributions revealed that a few travelers traveling extremely long periods per day have a substantial influence on the daily travel time per average traveler. Table 3.4 summarizes such effects on both the mean travel times and their variance in the two corridors.

Table 3.4 : Effects of Outliers on the Daily Travel Time per Traveler (TT), North and South Corridors, Washington, D.C. 1968

Corridor	Percent of Travelers	Daily TT min.	Upper TT Bound, hr	Standard Deviation	Standard Error	Coefficient of Variation
North	100	66.83	8	48.25	0.76	0.72
	99.5	65.48	4	44.57	0.70	0.68
	95	60.03	2.6	36.75	0.59	0.61
South	100	67.52	8	50.24	0.91	0.74
	99.5	65.76	4	45.95	0.83	0.70
	95	59.65	2.7	37.82	0.70	0.63

The results in Table 3.4 suggest that:

(1) Five percent of travelers at the tail-end of the distributions affect the mean travel time and coefficient of variation (ratio of standard deviation to mean) by 11 and 18 percent, respectively, in the North corridor, and by 13 and 17 percent, respectively, in the South corridor.

(2) The exclusion of five percent of travelers at the tail-end of the distributions reduces the upper bound of travel times from 8 hours to 2.6 and 2.7 hours in the two corridors, namely a reduction by 67 percent. Furthermore, such an exclusion also brings closer the mean values and coefficients of variation in the two corridors.

(3) Such extreme cases may have a significant effect on the ANOVA results described above.

It would seem reasonable, therefore, to exclude five percent of outlying cases at the tail-end of the distributions. Whenever possible, a better approach is to screen carefully all outlying cases in the original data in order to exclude professional drivers and reporting/coding errors from the data set.

3.5 The Travel Time Frequency Distributions and Coefficients of Variation

Another analysis of similarities between the daily travel time per average traveler involves contingency-table tests applied to the daily travel time frequency distributions.

Figure 3.1 shows such distributions, by income, for the North corridor. Table 3.5 summarizes the results of the tests applied to the original data in the two corridors (upper bound of 8 hours travel time, by 10-minute increments), stratified by household income and by car ownership levels (Appendix B, Table B.1).

Table 3.5 : Summary of Analysis of Daily Travel Time Frequency Distributions per Traveler, Washington, D.C. 1968

BY HOUSEHOLD INCOME - TOTAL

Corridor	X^2	$X^2_{.05}$ critical (df)
North	7.16	49.8 (36)
South	16.78	49.8 (36)
Total	29.30	101.9 (81)

BY CAR OWNERSHIP - TOTAL

Corridor	X^2	$X^2_{.05}$ critical (df)
North	9.05	40.1 (27)

0 CAR vs. 2 CAR

Corridor	X^2	$X^2_{.05}$ critical (df)
North	4.74	16.9 (9)

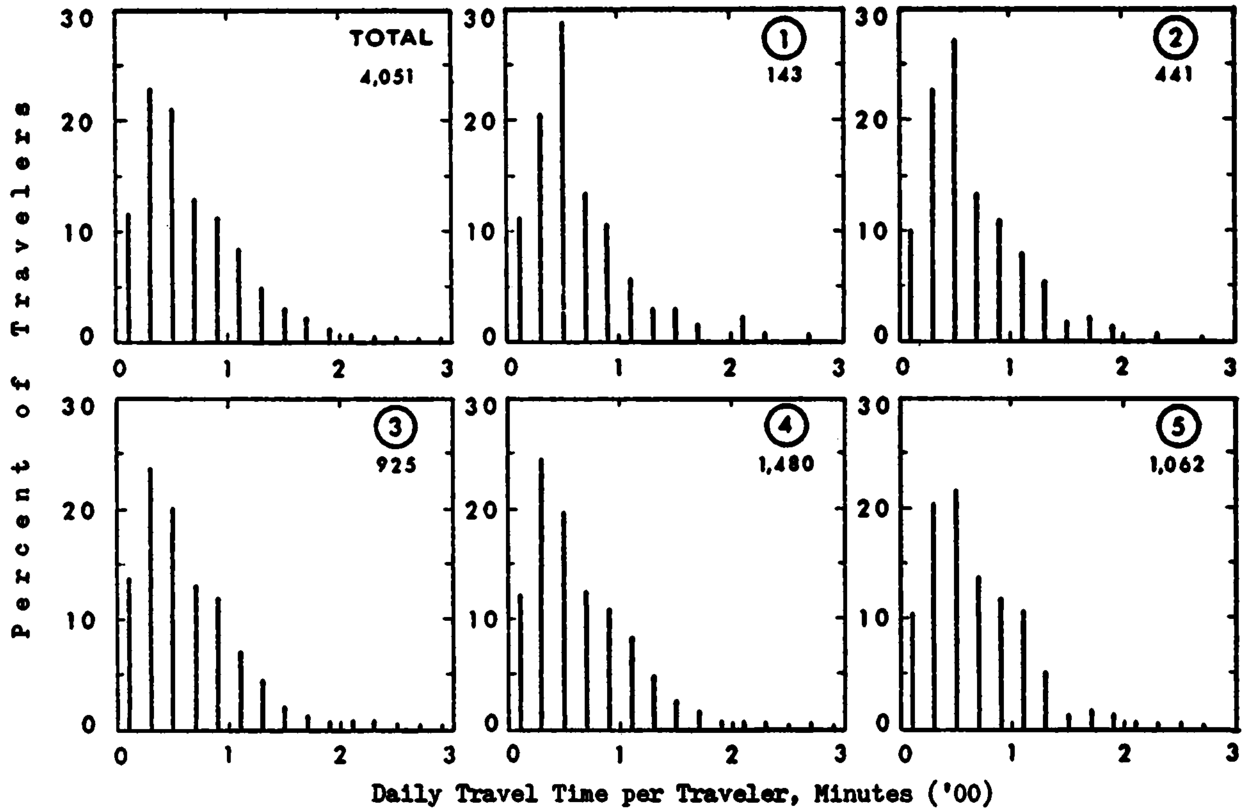

Figure 3.1 : Daily Travel Time per Traveler Frequency Distributions, by 20 Minute Intervals, for Five Household Income Groups and Total (also showing number of cases), North Corridor, Washington, D.C. 1968

There are no significant differences between the daily travel time frequency distributions at the 0.05 confidence level. Although each individual traveler may spend a random amount of time each day, the frequency distributions by the travelers' characteristics, both by income and by car ownership levels, are remarkably similar. This should not be interpreted to mean that each and every traveler spends a fixed amount of time on his/her daily travel; it means that groups of travelers, stratified by at least household income and car ownership levels, display stable behavioral trends. Similar distributions were observed in Canada (3.6).

The coefficient of variation, defined as the standard deviation over the mean, is a dimensionless measure of the amount of dispersion about a mean value. Table 3.6 details the coefficient of variation (CoV) of the daily travel time per average traveler of the five income groups in the two corridors. The table is divided into two parts: The values listed in the first part are derived from the basic travel tables and are based on the expanded samples of daily travel times up to 8 hours per traveler per day. Also shown are the expanded number of households in each income group. The values listed in the second part of Table 3.6 are derived from the ANOVA statistics and are based on the reported, unweighted, travel times up to 4 hours per traveler per day. Also shown are the original number of travelers in each income group.

Table 3.6 : Coefficient of Variation of Daily Travel Time per Traveler, by Household Income, North and South Corridors, Washington, D.C. 1968

ALL TRAVELERS (Unlimited Travel Time, Expanded Sample)

Income Class	NORTH			SOUTH		
	Households	TT, hr	CoV	Households	TT, hr	CoV
1	3,990	1.16	0.69	5,640	0.97	0.88
2	10,260	1.15	0.65	6,990	1.13	0.71
3	15,240	1.15	0.71	10,580	1.16	0.69
4	19,060	1.17	0.70	12,710	1.24	0.68
5	12,510	1.19	0.64	4,900	1.21	0.63
Total	61,060	1.17		40,820	1.18	

ANOVA (Up to 4 hrs. Travel Time, Original Sample)

Income Class	Travelers	TT, hr	CoV	Travelers	TT, hr	CoV
1	138	1.09	0.67	204	0.89	0.70
2	419	1.14	0.63	265	1.08	0.61
3	877	1.10	0.67	595	1.12	0.64
4	1,412	1.13	0.65	1,048	1.20	0.65
5	1,026	1.17	0.60	426	1.22	0.61
Total	3,872	1.13		2,538	1.15	

It is apparent from Table 3.6 that limiting the daily travel time per average traveler by an upper bound of 4 hours reduces both the mean travel time and its CoV. Importantly, the CoV value in the second part of Table 3.6 is similar to the values noted in other cities, as presented in Table 3.7.

Table 3.7 : Coefficient of Variation of the Daily Travel Time per Traveler in Selected Cases and Washington, D.C.

City	Coefficient of Variation
Munich: 1st day	0.57
2nd day	0.56
3th day	0.56
Bogota	0.57
Santiago: 0 car	0.55
1 car	0.60
2+ cars	0.62
Singapore, "Before"(*): Veh. Owning HH	0.60
Non-Veh. HH	0.70
"After" (*): Veh. Owning HH	0.56
Non-Veh. HH	0.51
Washington, D.C. (Excluding outliers):	
North:	0.61
South:	0.63

The effect of sample size on the CoV of the daily travel time per average traveler is shown in Figure 3.2. It is based on the travelers' expanded sample size of income group 3 in the North corridor. The expanded number of travelers is shown on the lower scale, while the original sample size is shown on the upper scale. The observations are stratified by household size, car ownership level and residence distance from the urban center, and they are aggregated at increasing levels towards the right-hand side of the diagram, to reach a total average value of 0.72 for all travelers of income group 3. These results are typical of other income groups as well.

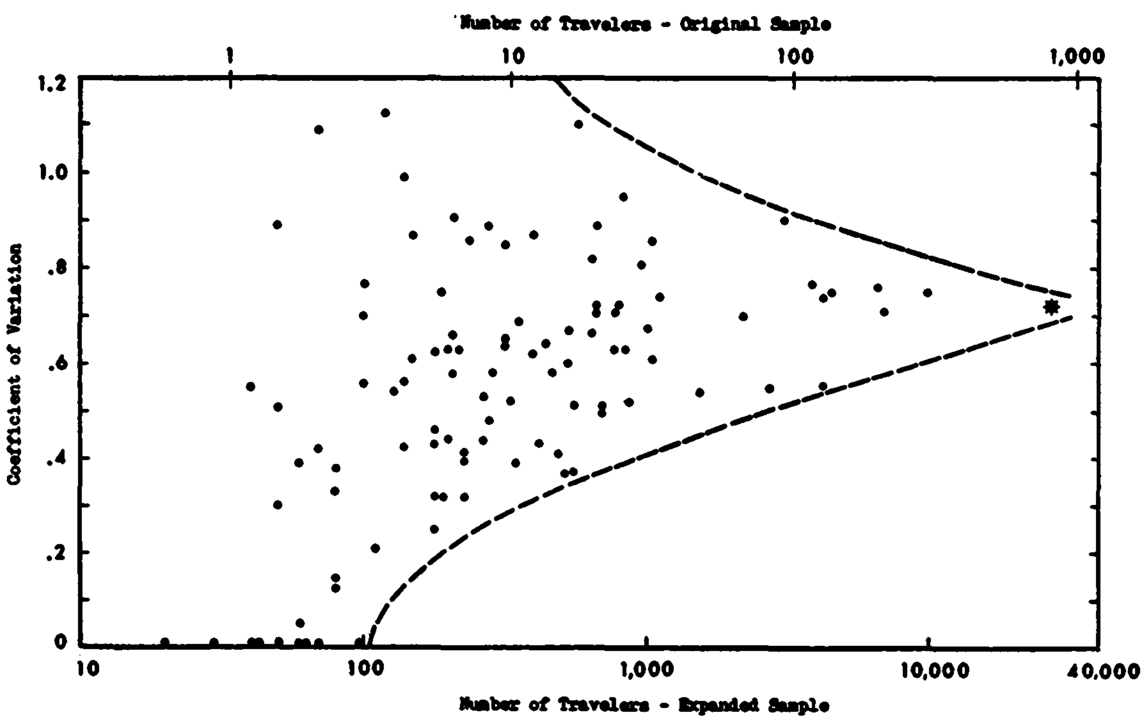

Figure 3.2 : Coefficient of Variation of the Daily Travel Time per Traveler vs. Sample Size, North Corridor, Washington, D.C. 1968

Variations in the CoV tend to decrease as the sample size increases, following well-known trends in sampling theory. For instance, the CoV tends to stabilize at an expanded sample size of about 2,000 travelers, which means a sample size of about 60 travelers. Since the sampling unit is the household, and since there are about two travelers per average household, it can be inferred that the minimum sample size in each population stratum should be about 30 for reliable estimates, which is in agreement with sampling theory and general rules-of-thumb in linear statistical analyses.

Because of the above considerations, the analyses presented in the following sections are based on the cases which include at least 20 households of each population segment. The cutoff was chosen at 20, rather than 30, to avoid discarding possible relevant data.

3.6 Residential Location Effects on Daily Travel Time per Traveler

The question arises as to whether or not residence distance from the urban center affects the daily travel time per average traveler. A plausible argument is that the daily travel time per average traveler should increase with residence distance from the urban center. This argument applies most directly to trips destined for the urban center (suoh as to work), but might it also apply to the daily total travel time per average traveler?

In order to answer the above question, the daily travel times, travel distances, and door-to-door speeds per average traveler were stratified by residence distance from the urban center, as detailed in Appendix 4.4. The daily travel distance per average traveler increases with both household income and residence distance from the urban center (and also with car ownership, which is strongly related to the above two factors). Thus, if the above argument is true, the daily travel time per average traveler would be expected to increase with the daily travel distance, as a function of residence distance. This expectation is tested in gure 3.3, where the daily travel time per average traveler is related to residence distance from the urban center.

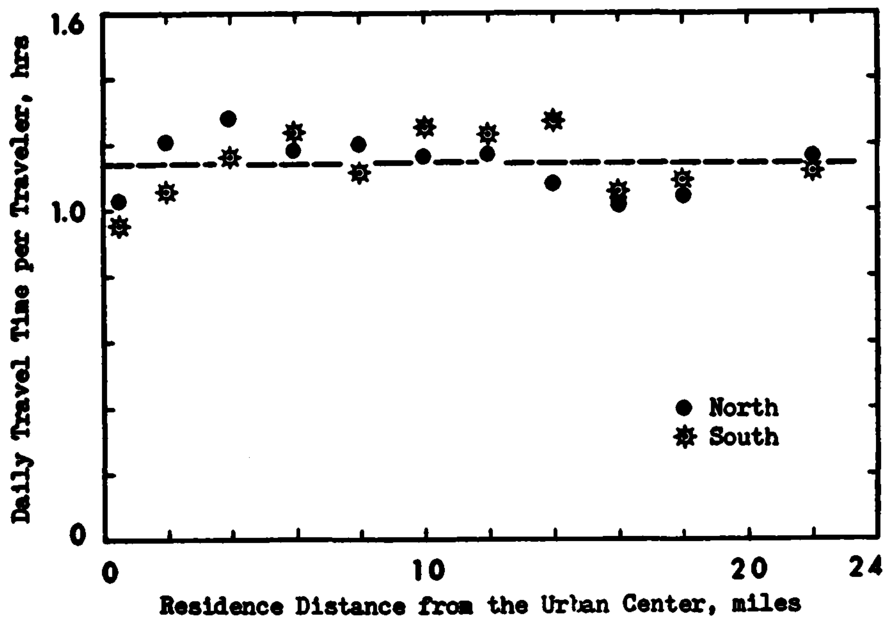

Figure 3.3 : Daily Travel Time per Average Traveler vs. Residence Distance from the Urban Center, North and South Corridors, Washington, D.C. 1968

No relationship between daily travel time per average traveler and residence distance from the urban center is evident in Figure 3.3. A significance test for the slopes of the relationships in the two corridors (Table B.2 of Appendix B) demonstrates zero slope. This result suggests that (i) the daily travel time per average traveler in the two corridors is not affected by residence distance from the urban center, and (ii) travelers in the two corridors are very near to, or at, their preferred daily travel time budget, of about 1.1 hours. Similar results were observed in Canada (3.6).

In conclusion, the above results appear to confirm once again that the daily travel time per average traveler displays predictable regularities. The implications of such results to urban structure are discussed in Chapters 7 and 8.

CHAPTER 4 : TRAVEL MONEY EXPENDITURES

4.1 Introduction

Regularities in travel money expenditures per average household were observed in both urban and interurban travel. Figure 4.1 shows one such example, based on the national household expenditure surveys in the U.S. (4.1). Similar results are also found in other countries, including England, Canada and West Germany, as shown in Table 4.1.

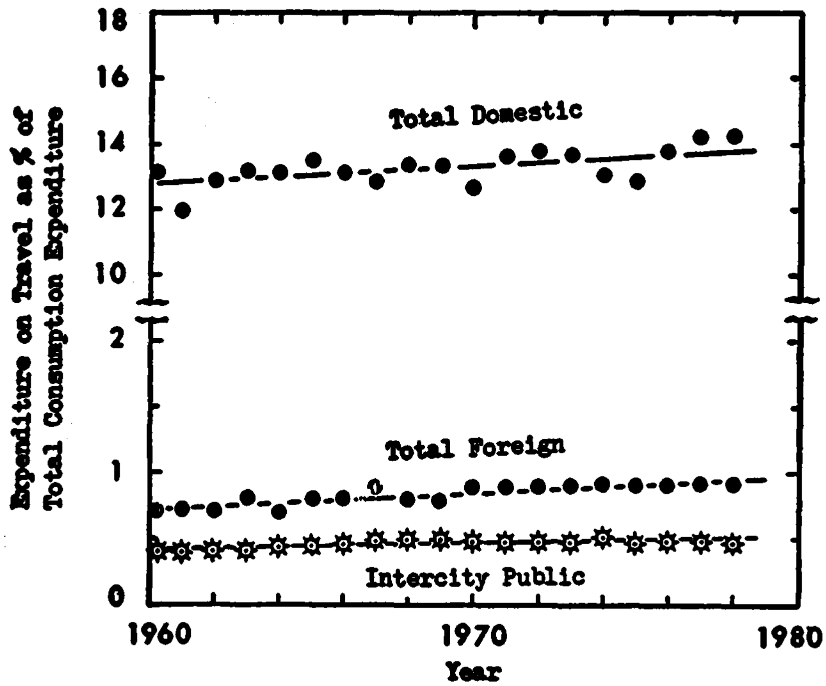

Figure 4.1 : Expenditure on Total Domestic, Total Foreign and Intercity Public Travel, as Percent of Total Consumption Expenditure, U.S. 1960-78

Table 4.1 : Travel Money Expenditures

NATIONWIDE vs. Total Household Expenditures, %

U.S.	1963 - 1975	13.18 ± 0.38
Canada	1963 - 1974	13.14 ± 0.43
U.K.	1972	11.7
West Germany	1971 - 1974	11.28 ± 0.54

URBAN vs. Household Income, %

		With Cars	Carless
Washington, D.C.	1968	11.0	4.2
Twin Cities	1970	10.1	3.4
Nuremberg Region	1975	11.8	3.5

Regularities in travel money expenditures per average household within urban areas were identified in Washington, D.C. and Twin Cities in the U.S. (4.2), and in Nuremberg and Munich in West Germany (4.3). In all these cases, travel money expenditures were found to be related to the socioeconomic characteristics of the households, including household income and car ownership levels. For instance, households in the two U.S. cities whose members traveled by car only spent about 10.5 percent of their daily average income on travel, at all income levels. Households whose members traveled by bus only, on the other hand, spent only 3-5 percent of their daily average income on travel, at all income levels. The same values were observed also in the two cities in West Germany. Thus, a significant gap between the travel money expenditures by carless and by car-owning households was identified, although the money expenditures, percentages, per average household of the two groups appeared to be similar at all income levels. Based on such regularities, the travel money expenditures per average household are regarded as "travel money budgets", complementary to the "travel time budgets", and applied as constraints on travel in the UMOT process (4.3).

Following the analysis of travel time expenditures per average traveler/household in the two corridors, detailed in the previous chapter, an attempt was made to estimate also the travel money expenditures per average household. However, while the door-to-door travel times were reported in the home interview survey, no information on travel money expenditures were available. Hence, the travel money expenditures had to be derived indirectly, as the product of reported travel distance and unit costs, as detailed below.

4.2 Travel Money Expenditures in the Two Corridors

The basic tabulations included data on the daily travel distance per average household, by socioeconomic characteristics. Since the travel distance per average household was total per day, by all motorized modes, no differentiation by mode could be made in this study. In order to identify travelers who use only one mode during their daily travel, either bus or car, it was assumed that travelers of carless households at the lowest two income classes travel by bus only, while travelers of car-owning households at the highest two income classes travel by car only. Although this assumption may not be fully valid, it provides a reasonable basis for a first-approximation estimation of the travel money expenditures.

Table 4.2 summarizes the results of such an estimation, based on the following steps:

(1) The daily travel distances per average household, by income class and car ownership, are assigned to either "bus only" or "car only" categories.

(2) The travel distance by bus is multiplied by the cost per unit distance, 6 cents per passenger-mile in 1968, to result in the total money expenditure per average household. This expenditure is then related to the average daily income per average household (320 days per year).

(3) The travel distance by car requires several additional estimation steps, based on information on car travel in 1968, detailed in Reference 5.2 (pp. 129-131):

(a) The daily passenger-miles of travel are divided by 1.5, the average car occupancy rate, to result in the daily car-miles of travel.

(b) The daily door-to-door speed is multiplied by 1.58, to result in the network speed.

(c) The cost per unit distance of car travel is estimated as a function of speed by the relationship:

$$\text{¢/mile} = 1.683(\text{Speed})^{-0.75} \quad . \tag{4.1}$$

(d) The product of car-miles of travel and unit costs equals the daily money expenditure per average household.

The results of the above steps are summarized in Table 4.2, and they suggest that: (i) The daily travel money expenditure per average carless household is about 3-6 percent of the daily income, and (ii) the daily travel expenditure per car-owning household is about 9-11 percent of the daily income. Both results follow previously observed trends.

Table 4.2: Travel Money Expenditures per Household, North and South Corridors, Washington, D.C. 1968

Mode	Income Group	Corridor	Cars/HH	HH	D/HH	D/Car	$Speed_{dd}$	$Speed_{net}$	$/m	Cost $	Annual Income	Income/day	Cost/Inc. %
Bus	1	North	0	2,300	9.07		7.07		0.06	0.54	3,000	9.38	5.8
		South	0	2,700	8.85		9.50		0.06	0.53	3,000	9.38	5.7
	2	North	0	3,900	11.99		7.50		0.06	0.72	6,000	18.75	3.8
		South	0	890	10.35		8.25		0.06	0.62	6,000	18.75	3.3
Car	4	North	2	9,700	48.99	32.7	16.87	26.7	0.143	4.69	16,000	50.00	9.4
		South	2	6,850	61.39	40.9	19.10	30.2	0.131	5.34	16,000	50.00	10.7
	5	North	3	1,540	72.91	48.6	16.26	25.7	0.147	7.17	22,000	68.75	10.4
		South	3	940	88.90	59.3	22.57	35.7	0.115	6.83	22,000	68.75	9.2

HH - Household; D - Daily Travel Distance; $Speed_{dd}$ - Door-to-Door Speed; $Speed_{net}$ - Network Speed

Nonetheless, such results should be regarded as a rough approximation, awaiting a more detailed analysis on the basis of travel distance stratified by mode, household size, and similar factors.

4.3 Implications

A travel money budget does not mean that each and every household spends a fixed amount of money on travel each day. A travel money budget per average household is a mean value with a distribution around it, caused by such factors as intrinsic differences between households belonging to the same socioeconomic class, and by daily variations in travel behavior per individual travelers belonging to the same household.

Furthermore, the travel money budget is applied in the UMOT process as an upper-bound constraint, say 11 percent of income, to result in a distribution of households by car ownership levels within the same income class. Thus, the proportions of households by car ownership levels, belonging to a particular socioeconomic group, will affect the average travel money constraint, and the process is repeated by iterations until convergence of the travel system is reached or approached. For instance, although the travel money budget of an average household at the lowest income class is assumed to be, say, 11 percent of income at the start of the process, it converges to its lowest bound, say 3 percent of income, at the conclusion of the process. Thus, the budget constraints in the UMOT process are not constant, but interactive components of a dynamic travel system.

The last point is of special importance for the evaluation of potential situations where even the upper-bound money constraint, say 11 percent of income, cannot satisfy the minimum required travel, such as travel to and from work. Such situations are observed in cities of some developing countries, where low income travelers spend a high proportion of their household's income on travel. The same also applies to potential situations in cities of developed countries, where rapidly increasing travel costs may decrease travel distances down to a critical point after which the travel money budget starts to increase above its previously observed upper bound.

The UMOT process shows the potentiality of identifying such situations as one of its outputs. Such cases are particularly important for policy makers and planners, as they may signify the start of relatively rapid changes in urban structure, such as relocation of residences and jobs. This subject is further discussed in Chapter 8.

CHAPTER 5: DAILY TRAVEL DISTANCE PER TRAVELER

5.1 Introduction

The daily travel distance per average traveler is a key notion in the UMOT process. In models used in conventional urban transportation planning processes, travel distance is a final output, following the phases of trip generation, mode choice, trip distribution, and trip assignment. In the UMOT process, daily travel distance per average traveler, by mode, is the first output, after interacting by feedback with such factors as car ownership levels and transport system supply (5.1). Furthermore, travel distance is regarded in conventional models as a disutility, measured by the time and money costs required to overcome distance between origin-destination pairs. In the UMOT process, on the other hand, travel distance is regarded in utility terms, representing the benefits of access to spatial and economic opportunities.

The analyses described in this chapter suggest that the daily travel distance per average traveler is strongly related to just one parameter - the daily door-to-door speed available to the traveler. Speed is affected by many factors, including the efficiency level of the transport system, income, car ownership levels, and residential location. Once speed is estimated as a function of such factors, it provides a strong basis for estimating travel distance per average traveler of a specific population segment. Knowledge of the number of travelers per average household by socioeconomic type, as described in Chapter 2, then allows the estimation of the daily travel distance, by mode, per average household belonging to a certain socioeconomic class.

5.2 Daily Travel Distance per Average Traveler vs. Speed

Figure 5.1 shows daily travel distance vs. daily travel time per average traveler, stratified by residence distance from the urban center (Table C.1 of Appendix C). Daily travel distance per average traveler is unrelated to his/her daily travel time. Thus if an absolutely fixed daily travel time per average traveler would be observed for all travel distances, the distance vs. speed relationship should

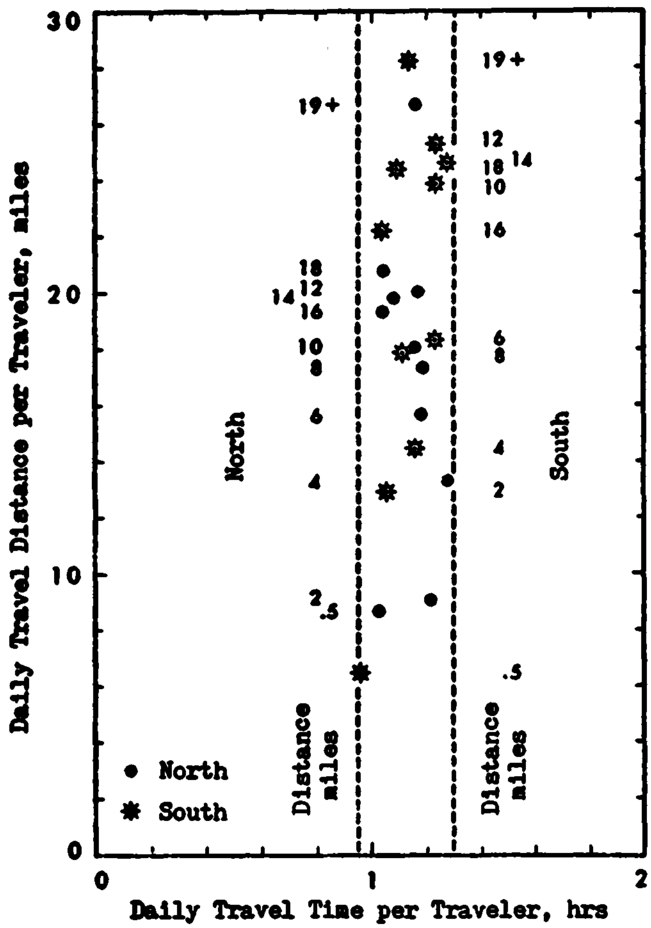

Figure 5.1: Daily Travel Distance vs. Daily Travel Time per Traveler, by Residence Distance from the City Center, North and South Corridors, Washington, D.C. 1968

be a straight line, with a slope equal to the constant travel time and an intercept passing through the origin of the x-y coordinates. Variabilities from such a line would suggest a departure from a constant daily travel time per average traveler.

Based on date presented in Table C.2 of Appendix C, relationships between the daily travel distance per average traveler and door-to-door speed for the two corridors are shown in Figure 5.2(*). The points represent values per average traveler, stratified by household income, car ownership levels and residence distance from the urban center in the North and South corridors.

(*) Since the speeds were derived not independently but as a quotient of travel distance over travel time per average traveler, the relationship of distance vs. speed is a transformation of the relationship of distance vs. time.

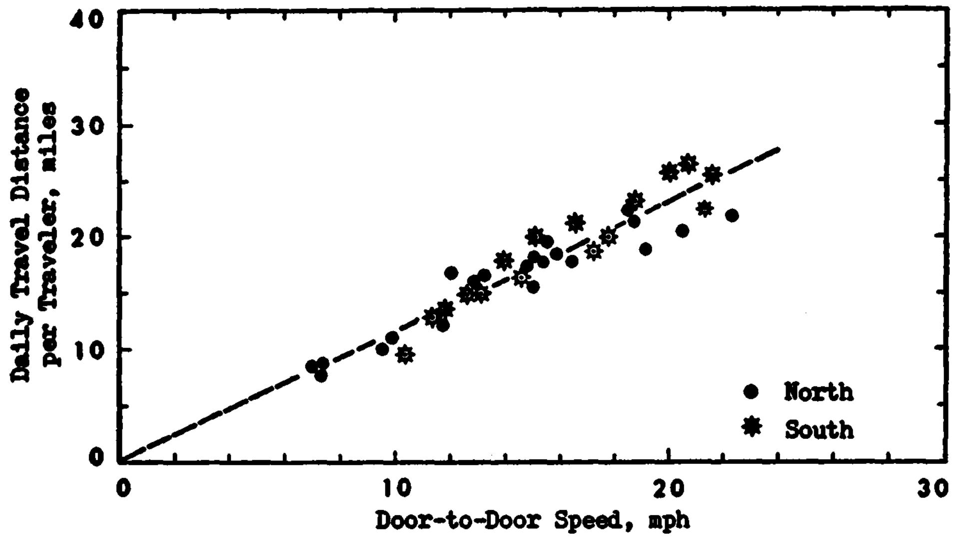

Figure 5.2 : Daily Travel Distance per Traveler, by Residence Distance from the City Center, versus Daily Mean Door-to-Door Speed, North and South Corridors, Washington, D.C. 1968

The regression equations, based on Table C.3 of Appendix C, are:

North Corridor:	Distance/TR = 1.841 + 1.002(Speed),	(R^2= 0.895)	(5.1)
South Corridor:	Distance/TR = -1.639 + 1.277(Speed).	(R^2= 0.905)	(5.2)
North + South :	Distance/TR = 0.409 + 1.125(Speed)	(R^2= 0.892)	(5.3)

Each relationship is highly significant. Results of tests of equality between sets of coefficients in two linear regressions (Chow test) show that there are no significant differences between the relationships for the North and South corridors at the 0.05 confidence level (Table C.3): Travelers in the North and South corridors appear to behave similarly with respect to their daily travel distance vs. door-to-door speed.

The above results refer to values per average traveler belonging to households of different socioeconomic groups. However, it was shown in Chapter 3 that the daily travel time per individual traveler is not a fixed value, but rather a distribution around a mean value. Hence, the daily travel distance per average traveler is expected to display similar distributional characteristics. Indeed, this is the case. Table 5.1 details the coefficient of variation (CoV) of the daily travel distance and daily travel time per average traveler, by household income, based on all observations including outliers.

Table 5.1 : Coefficient of Variation of Travel Distance per Traveler, North and South Corridors, Washington, D.C. 1968

Income Class	NORTH Distance Miles	NORTH CoV Distance	NORTH CoV Time	SOUTH Distance Miles	SOUTH CoV Distance	SOUTH CoV Time
1	11.94	0.79	0.69	10.61	1.03	0.88
2	13.09	0.98	0.65	19.35	0.96	0.71
3	16.37	0.86	0.71	20.37	0.91	0.69
4	18.38	0.86	0.70	22.16	0.79	0.68
5	17.80	0.80	0.64	22.35	0.72	0.63

The CoVs of travel distances are consistently higher than those of travel time. This might be the result of the fact that it is more easy to transfer money than time between days. Since travel distance can be considered as the output of interactions between the travel time and money budgets, the daily travel distance per average traveler is expected to display more variability than the respective travel time.

5.3 The Daily Travel Distance per Household

The daily travel distance per average household, by socioeconomic characteristics, are detailed in Table C.3 of Appendix C. These data are summarized in Figure 5.3 as a function of household income and car ownership levels in the two corridors.

The relationships in Figure 5.3 follow expected trends: Daily travel distance increases with both household income and car ownership levels. However, travel distance can be related to households' socioeconomic characteristics through their travelers. It has been shown in Section 2.2 that the number of travelers per household is related to the household's socioeconomic characteristics. Thus, the daily travel distance per average household can be captured by relationships describing the expected number of travelers per household.

Indeed, such a relationship is evident in Figure 5.4, which is summarized from data described in Table C.3. The observations are stratified by household income and car ownership levels. The number of travelers per household captures such factors as household size, household income and car ownership levels.

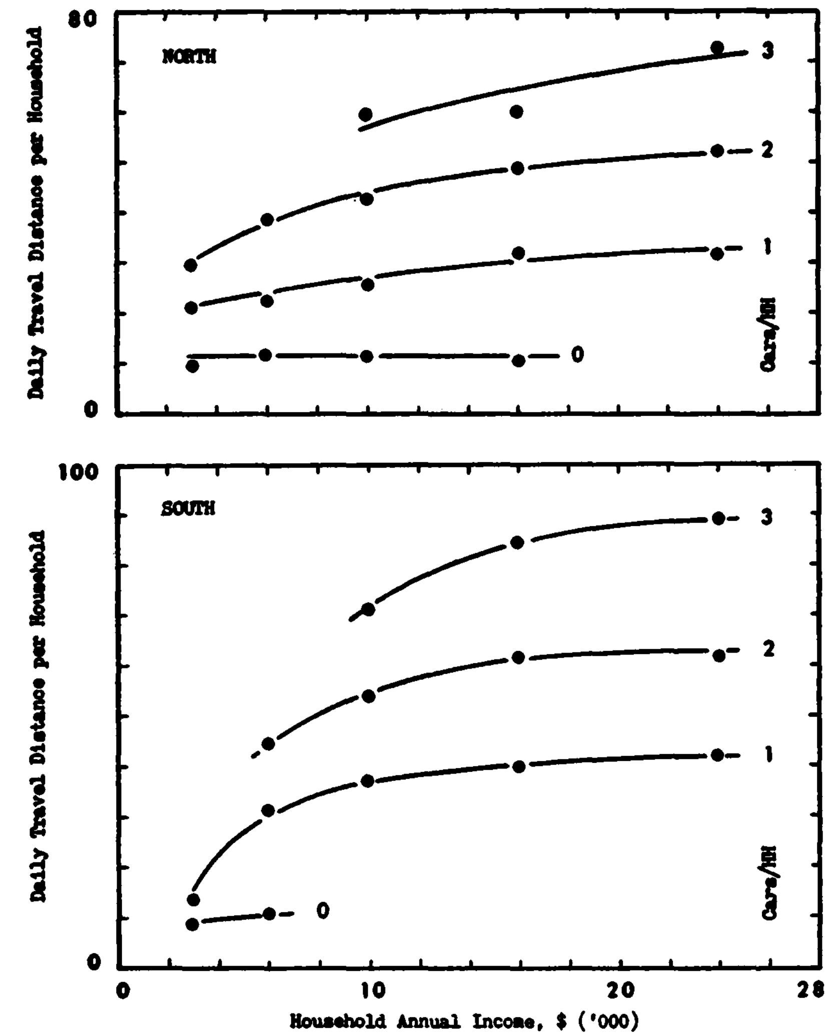

Figure 5.3: Daily Travel Distance per Household, by Household Income and Car Ownership, North and South Corridors, Washington, D.C. 1968

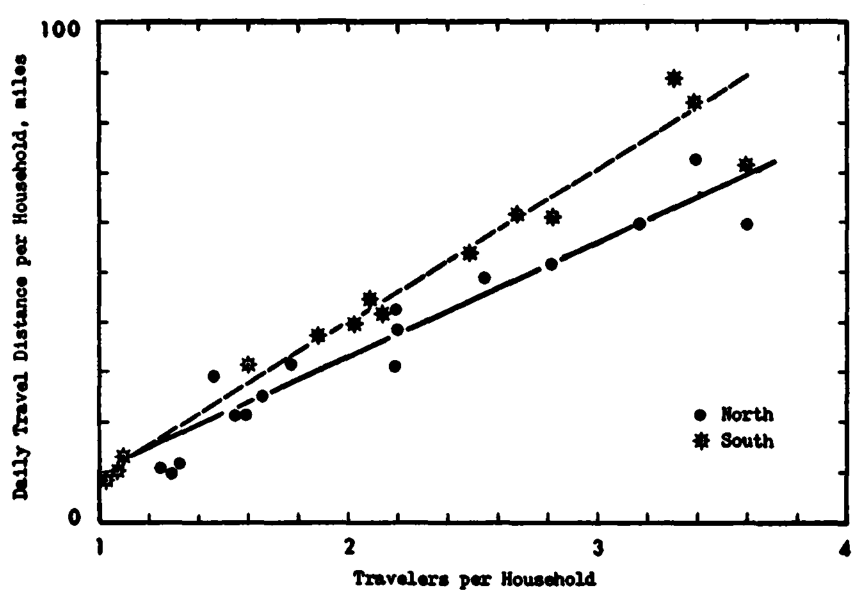

Figure 5.4: Daily Travel Distance per Household vs. Number of Travelers per Household, North and South Corridors, Washington, D.C. 1968

Results of statistical tests, detailed in Appendix C, show that the relationships in each corridor are highly significant at the 0.05 confidence level, but they are dissimilar. However, another explanatory factor has to be considered, namely the door-to-door speed available to the travelers, as discussed in Section 5.2. The daily travel distance per average household can be regarded as an output from an interactive process where household size, household income, car ownership levels, number of travelers per household, and speed, all interact with each other to result in the daily travel distance per traveler and, hence, also per household.

CHAPTER 6: TRIPS AND THEIR COMPONENTS

6.1 Introduction

It has been shown in Section 5.2 that the daily travel distance per average traveler is strongly related to his/her daily door-to-door speed. This travel distance can be apportioned between trip rate and trip distance in various ways. It is therefore important to determine travelers' trade offs between the two travel components under varying travel conditions.

The definition of a trip in conventional urban transportation planning processes is ambiguous to a large extent since trips are linked/chained/ clustered and combined into "tours" in various ways during the calibration phase of the models. Thus, the basic travel data in an urban area can vary according to the chosen definition of a trip. In contrast, travel components that remain unchanged by any definition are the total daily travel components, such as the daily travel distance, and the daily travel time and money expenditures per traveler/household. Therefore, whatever the definition of a trip in a conventional model is, valuable insights are gained if the trip is treated simultaneously with its respective trip distance within the total daily travel distance, and with its respective trip time and trip cost within the total daily travel time and money expenditures. Examples of such a simultaneous treatment are presented in the following sections.

6.2 Relationships Among Trip Rate, Trip Distance and Daily Travel Distance

Table D.1 of Appendix D details the trip rate, trip distance, daily travel distance and speed per average traveler, stratified by household income and residence distance from the urban center. (Income and residence distance display the widest range of trip distance and daily travel distance values.) Relationships between trip distance and daily travel distance per average traveler are depicted in Figure 6.1.

Statistical tests are detailed in Table D.2: Each relationship, for each corridor, was found to be significant at the 0.05 confidence level. Furthermore, the two relationships, in the two corridors, are found to be statistically similar through tests of equality between regression coefficients.

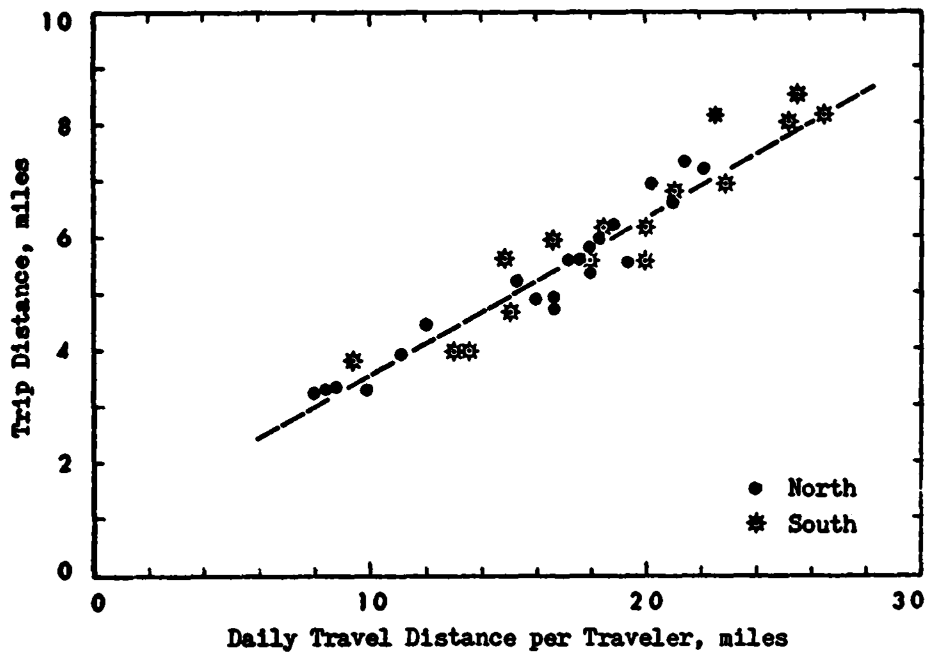

Figure 6.1 : Average Trip Distance vs. Daily Travel Distance per Traveler, by Income and Residence Distance from the City Center, North and South Corridors, Washington, D.C. 1968

These results suggest the following interactions: (i) travelers tend to increase their daily travel distance with increase in door-to-door speeds (Section 5.2), (ii) travelers tend to increase their trip distance with increase in the daily travel distance. The question as to whether or not they also increase their trip rate under such circumstances is addressed below.

6.3 Trip Rate vs. Daily Travel Distance

Figure 6.2, based on data detailed in Appendix D, depicts the relationships between trip rate and daily travel distance per average traveler in the two corridors, stratified by household income and residence distance from the urban center.

Statistical tests are detailed in Table D.2: The link between trip rate per average traveler and his/her daily travel distance is very weak. It may be inferred that when travelers have the opportunity to increase their daily travel distance in a given urban area, as a result of increases in daily door-to-door speeds, they prefer to increase trip distance over trip rate. A test for the correlation between trip rate and trip distance is described in the following section.

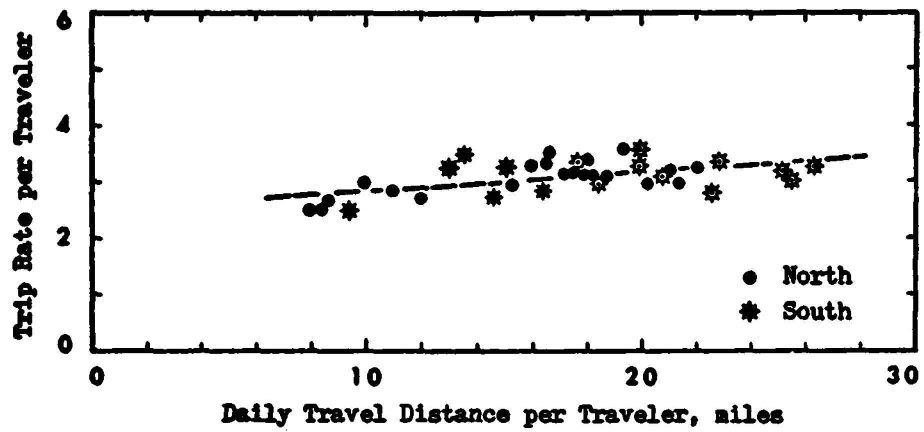

Figure 6.2 : Daily Trip Rate vs. Daily Travel Distance per Traveler, by Income and Residence Distance from the City Center, North and South Corridors, Washington, D.C. 1968

A test for a possible correlation between trip rate and trip distance per average traveler in the two corridors is also detailed in Table D.2 of Appendix D, based on data listed in Table D.3. No significant relationship was found between the trip rate and trip distance at the 0.05 confidence level. There appears to be little or no trade-offs between trip rate and trip distance within the total daily travel distance per average traveler.

6.4 Trip Rate vs. Trip Time

An additional test was carried out in order to assess whether there is a correlation between trip rate and trip time per average traveler. Such a relationship is expected on the basis of the narrow range of daily travel time per average traveler observed in the two corridors, suggesting an inverse relationship between trip rate and trip time.

Results of this test, detailed in Table D.2, show that there is a significant relationship between trip rate and trip time per average traveler at the 0.05 confidence level. Moreover, the relationships in the two corridors are statistically similar.

These results corroborate the results presented in the previous sections: (i) Trip rate is inversely proportional to the trip time within the daily travel time per average traveler. (ii) Trip distance is related directly to the daily travel distance per average traveler. (iii) Daily travel time per average traveler is related directly to the door-to-door speed. Thus, speed is a principal link between travel components.

Such relationships suggest underlying behavioral mechanisms. These relationships can also be useful in conventional models based on trips because they provide controlling totals for the various separate submodels, such as trip generation, mode choice, and trip distribution. For example, a transfer of a traveler from car to bus is expected to result in the following changes in his/her travel: (a) an increase in the daily travel time, for less daily travel distance, (b) an increase in trip time for a given trip distance, thus a reduction in trip rate, (c) a reduction in trip rate is expected to reduce the proportions of discretionary trips and, hence, also affect trip distance.

6.5 Trips by Purpose

Further examples of interrelationships among travel components are detectable through analyses of travel data by trip purpose. The basic data were stratified into nondiscretionary travel (work and business) and discretionary travel (all other purposes). The data for the North corridor are summarized in Table D.3 of Appendix D.

The interactions between the travel components by the above two purposes are summarized below. Table 6.1 presents the travel components, by the two purposes and total, stratified by car ownership levels(*).

(*) Summation of travelers by purpose does not equal the total number of travelers because individual travelers may travel for either one, or both, purposes.

Table 6.1 : **Daily Travel Characteristics per Average Traveler, by Trip Purpose and Car Ownership, North Corridor, Washington, D.C. 1968**

TOTAL

Cars/HH	TR	D	T	v	R	d	t
0	11,170	8.89	72.86	7.32	2.33	3.84	31.38
1	46,570	15.67	69.47	13.53	3.05	5.12	22.74
2	55,470	18.91	69.79	16.26	3.24	5.86	21.59
3+	11,900	19.36	70.59	16.46	3.22	5.99	21.87
Tot/Avg	125,110	16.85	70.02	14.44	3.09	5.42	22.92

WORK

Cars/HH	TR	D	T	v	R	d	t
0	7,500	7.30	64.78	6.76	1.87	3.89	34.70
1	25,130	12.08	53.73	13.49	1.91	6.31	28.08
2	24,730	16.34	58.44	16.78	1.97	8.27	29.60
3+	5,970	15.49	54.62	17.02	1.95	7.95	28.07
Tot/Avg	63,330	13.50	56.96	14.22	1.93	6.94	29.46

NON-WORK

Cars/HH	TR	D	T	v	R	d	t
0	5,460	8.19	60.20	8.16	2.21	3.73	27.68
1	32,590	13.08	57.83	13.57	2.88	4.54	20.04
2	42,010	15.36	57.76	15.96	3.11	4.95	18.57
3+	8,540	16.14	60.19	16.09	3.12	5.16	19.23
Tot/Avg	88,600	14.15	58.17	14.60	2.97	4.74	19.74

TR - **Travelers**
D - **Travel Distance**
T - **Travel Time**
v - **Door-to-Door Speed**
R - **Trip Rate**
d - **Trip Distance**
t - **Trip Time**

Results show that:

(1) Total Daily Travel: Travelers of carless households spend more daily travel time for less travel distance than travelers of car-owning households, and their trip time is longer, for less and shorter trips during the day.

(2) Daily Work Trips: These trends also apply to work trips.

(3) Daily Non-Work Trips: These trends also apply to non-work trips. Furthermore, the proportion of travelers traveling for non-work purposes increases with car ownership levels.

(3) General Comments: The average trip time to work by travelers of car owning households is remarkably stable, following the trend noted for the total daily travel time (but not constant, since each value is the average of a distribution). Furthermore, the availability of more cars increases the door-to-door speed, which is reflected in both the total daily travel distance and the trip distance to work and for non-work purposes.

Table 6.2 summarizes the same travel components as in Table 6.1, but this time by household income. Travelers of households with increasing income are able to "purchase" higher speeds and thus travel longer daily travel distance. The longer daily travel distance allows the travelers to make more and longer trips, where the preferences lean towards the latter.

Table 6.2 : Daily Travel Characteristics per Average Traveler, by Trip Purpose and Household Income, North Corridor, Washington, D.C. 1968

TOTAL

Income	TR	D	T	v	R	d	t
1	5,150	11.94	69.65	10.29	2.78	4.30	25.08
2	15,740	13.09	68.80	11.42	2.85	4.59	24.13
3	27,400	16.37	69.01	14.23	2.97	5.51	23.22
4	43,190	18.38	69.92	15.77	3.12	5.88	22.38
5	33,650	17.80	71.61	14.91	3.29	5.41	21.78

WORK

Income	TR	D	T	v	R	d	t
1	1,840	12.02	68.77	10.49	1.97	6.11	34.95
2	8,680	9.37	55.00	10.22	1.91	4.90	28.73
3	15,430	12.92	56.54	13.71	1.97	6.55	28.66
4	22,090	15.11	57.61	15.74	1.94	7.80	29.72
5	15,260	14.26	56.13	15.24	1.90	7.49	29.49

NON-WORK

Income	TR	D	T	v	R	d	t
1	3,720	10.57	62.32	10.18	2.87	3.68	21.73
2	10,160	12.28	59.61	12.36	2.78	4.41	21.42
3	18,260	13.64	55.77	14.67	2.79	4.89	19.97
4	31,200	14.73	55.98	15.79	2.95	4.99	18.97
5	25,280	15.08	61.44	14.73	3.23	4.67	19.04

In conclusion, there appears to be increasing evidence to suggest that the travel components of trip distance, time and rate, are strongly interrelated both with the travelers' socioeconomic characteristics and with each other. These travel components typically are treated separately within different sub-models, such as trip generation, trip mode choice and trip distribution in conventional planning processes. In the UMOT process they are interrelated within a single model framework.

The importance of interrelating all travel components within one framework can be demonstrated by showing how the spatial distribution of trips is governed by the travel budgets and the available door-to-door speeds on a daily basis. This subject is elaborated in the following chapter.

CHAPTER 7: TRAVEL PROBABILITY FIELDS AND URBAN SPATIAL STRUCTURE

7.1 Introduction

Models of person movement within urban areas have generally used the concept of individual trips, as discussed in Section 6.1. Examples include not only the familiar gravity, intervening opportunities and entropy-maximization models employed for traffic analyses in early transportation planning studies, but also the complex disaggregate models developed more recently to describe jointly trip generation, trip distribution and mode split.

It has been pervasively demonstrated that many important causal factors in travel demand are not present in such trip-based models. Many researchers have argued that it is necessary to abandon the trip-based approach in favor of an approach involving total household activity patterns (e.g., 7.1, 7.2, 7.3, 7.4, 7.5, 7.6, 7.7).

It has been shown that the spatial distribution of activity sites visited by households in a particular residential location can be depicted by a probability density function described in locational coordinates (7.8). Certain properties of the density functions were shown to be related to the residential location relative to the urban center(s), the socioeconomic characteristics of the households, and to travel speeds and costs. It was further shown that bivariate normal distributions positioned at one standard deviation (or another isoprobability contour) are effective representations of the spatial distributions of trip destinations (7.8). Using examples computed for the Washington, D.C., U.S.A. and Nuremberg, F.R.G. metropolitan areas, it was shown that: (1) the major axis of the ellipse tends toward an urban center; (2) the ellipses are more elongated, the farther the origin households are located from the urban center; (3) car travel fields are more elongated than transit travel fields for the same households; and (4) the direction of the major axis of the ellipse is also affected by available transportation systems supply such as bus routes.

Such travel fields are methodologically consistent with attempts made in urban geography to describe the spatial distribution of activity patterns (7.2). A probabilistic approach has been proposed in which trips result from the likelihood of contact between two fields, expressed by a joint probability function, with trip distances and travel opportunities represented as component marginal distributions (7.9). This approach also suggested the possibility of bivariate normal distribution of activities, namely that trip density rings would be elliptical. In related efforts, travel costs and speeds have been described by continuous, deterministic functions in geographical space (7.10, 7.11, 7.12). Such functions are intended to relate trip generation and distribution to features of urban structure.

These empirical results hold forth a promise that travel patterns might be effectively related in an activity behavior sense to urban spatial structure. Toward that end, a residential location-travel generation model is proposed herein which is consistent with the standard mathematical models of urban residence and activity location (7.13, 7.14, 7.15, 7.16, 7.17, 7.18).

7.2 An Underlying Behavioral Theory

It is postulated that the trips generated by a homogeneous group of households defined by income, car ownership and socioeconomic stratum, as well as locational proximity, are influenced by a small number of discrete urban centers. Each center defines a spatial distribution of trip attractors. Regardless of the number of centers, they can be considered hierarchically (7.19). The highest-order center probably accounts for the greatest number of trips for the household group; presumably it is a Central Business District (CBD). Lower order centers might be approximated by one or two centers describing the distribution of, say, convenience shopping and recreation attractors.

It is appropriate to consider only the highest-order center for the initial model specification. This center can be interpreted either as the focus of household employment, which is consistent with earlier urban economic models, or as the focus of economic opportunities or other reasons for agglomeration, which is consistent with more recent generalizations (7.20). The latter approach is more relevant in the present circumstance.

According to empirical evidence, the density of activity sites falls off as a monotone decreasing convex function from such an urban center. Thus, the density of activity sites representing trip attractors can be depicted relative to the spatial x-y plane as shown in Figure 7.1.

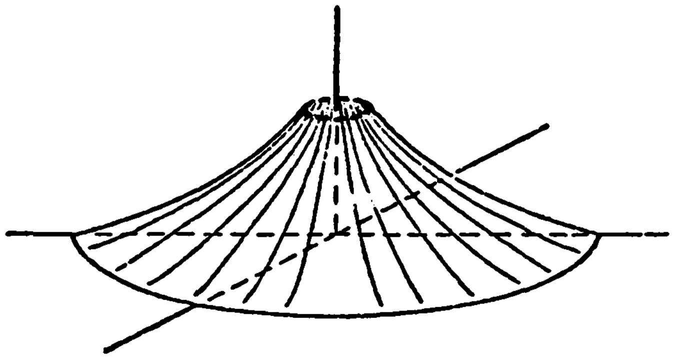

Figure 7.1

In the specific cases of population or employment densities, it has been shown that the density functions of Figure 7.1 can be approximated by a negative exponential function (7.16, 7.21, 7.22, 7.23, 7.24). However, when considering all activity sites together, there is no clear evidence concerning the fits of various functional forms. Consequently, the model proposed herein requires only the general convexity property depicted in Figure 7.1 and relies on piecewise linearization. The mathematical form of the activity site density function is estimated as one of the outputs of the test of the model.

Densities in the immediate vicinity of the urban center are ignored in the present approach, as depicted by the surface truncation in Figure 7.1. Empirical evidence is not conclusive regarding such central densities, and the mathematical complexities needed to account for them (e.g., 7.25, 7.26, 7.27, 7.28) are not justified for initial model development.

The area that can <u>potentially</u> be covered by trips originating from a household located at distance r_0 from the urban center is generated by projections from a cone in x-y-time space. This cone, shown in Figure 7.2, has its apex at the household location and an angle ϕ to the vertical given by $\tan\phi =$ some measure of transportation system speed. Such a geometric depiction of time and space has been developed in detail (7.1, 7.2) and has been adopted and extended in analyses of accessibility (7.6, 7.29, 7.30).

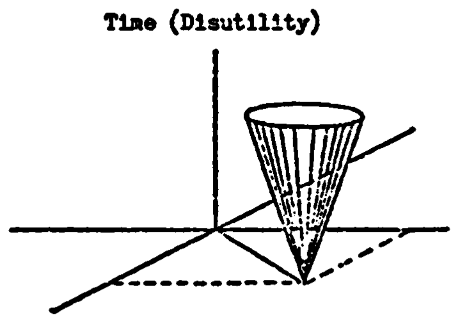

Figure 7.2

Superimposing the travel possibilities cone (Figure 7.2) and the density of attractors surface (Figure 7.1), the intersection of the two surfaces circumscribes an approximately elliptical area on the density of attractors surface, shown in Figure 7.3. The analytics of the geometry are greatly simplified by approximating the exponential density surface by a tilted plane in the neighborhood of the household location, as also shown in Figure 7.3. This approximation assumes that there is a negligible decline in travel attractions in a direction transverse to the urban center in the neighborhood of the household. The intersection of the surfaces is then precisely an ellipse.

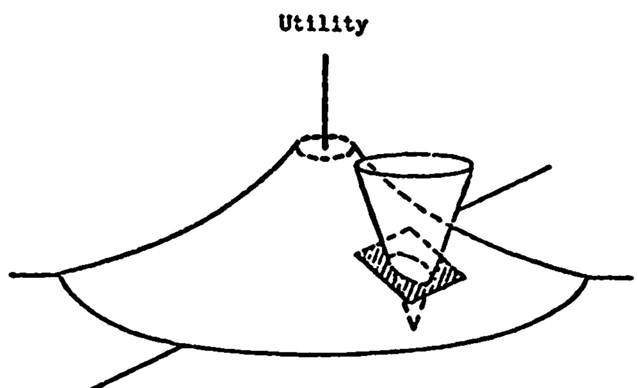

Figure 7.3

It is proposed that the volume within the cone under the ellipse is related to the benefits of travel achievable by a household located at r_0. The geometric interpretation of these travel benefits is facilitated by referring to Figure 7.4.

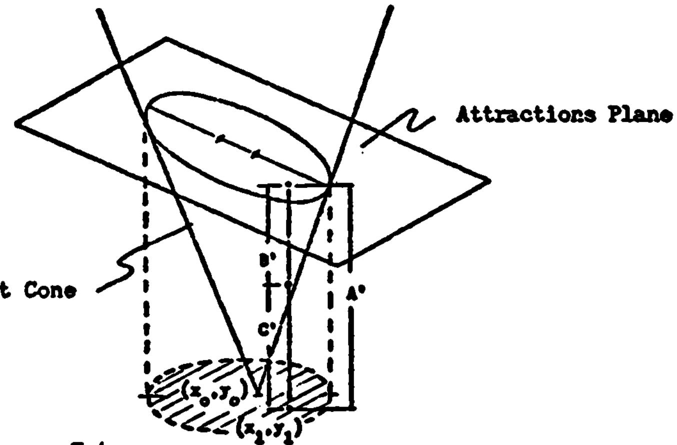

Figure 7.4

Consider a trip from the household location at (x_0,y_0) to any point (x_1,y_1) in the x-y plane. The vertical projection A' from (x_1,y_1) to the tilted attractions plane represents the gross benefits attributed to the trip. (That is, the activities at the trip destination are the gross benefits associated with the trip.) The vertical projection C' from (x_1,y_1) to the surface of the travel time or disutility cone represents the costs attributed to the trip. The projection B'= A'- C' represents the net benefits of the trip. The integral of all such trips with B'> 0 is the volume of the cone beneath the tilted plane, the total net travel benefit associated with residential location (x_0,y_0). Trips to points outside the ellipse will not be made because the costs of such trips are greater than the gross benefits. At the boundary of the ellipse costs just equal benefits, and the longest trips to the boundary are in the direction of the urban center.

Such a representation of net benefits is essentially a three-dimensional analog to consumer surplus. Through this representation, calculations of travel benefits as functions of the parameters of the spatial distribution of travel attractions and travel efficiency functions can proceed using the analytic geometry of conics and right circular cones. In this initial model specification, the parameters of the former include the density of activities at the urban center and the rate at which activity site density decreases with distance from the center; the single parameter of the latter is a measure of travel speed. All of the calculations are relative to household distance from the urban center.

Figure 7.5 depicts a cross-section through the urban center and household location.

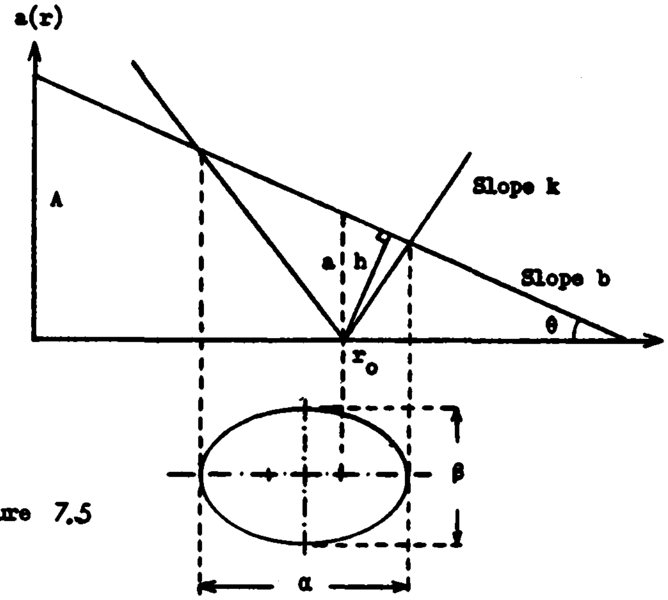

Figure 7.5

Equations for the axes of the ellipse formed by the intersection of the plane and the cone are:

$$\alpha = \frac{ak}{k^2 - b^2} \tag{7.1}$$

and

$$\beta = \frac{a}{\sqrt{k^2 - b^2}} \tag{7.2}$$

where:

- a = level of attractions at the household location,
- b = slope of the attraction distribution,
- k = slope of the travel possibilities cone (inverse of travel speed),
- α = major axis of ellipse which is oriented along the ray from the urban center, and
- β = minor axis of ellipse which is parallel to pivot line through household focus.

Furthermore, the minimum distance from the household location point to the plane of the attraction distribution is given by

$$h = \frac{a}{\sqrt{b^2 + 1}} \quad , \tag{7.3}$$

a quantity derived using the cosines and tangents of the angle θ in Figure 7.5.

Thus, the volume of the cone beneath the plane is

$$B = \frac{\pi}{3}\alpha\beta h$$

$$= \frac{\pi}{3}\frac{a^3}{\sqrt{b^2+1}}\frac{k}{(k^2-b^2)^{3/2}} \quad . \tag{7.4}$$

Equation (7.4) is a measure of net travel benefits achieved at distance r from the center, where a, b and k are functions of distance. A corresponding measure of travel time expenditure consistent with the attainment of such travel benefits is the average length of trips to all points within the elliptical travel field multiplied by the average speed:

$$\gamma = \frac{2}{3}k\sqrt{\pi\alpha\beta}$$

$$\gamma = \frac{2}{3}\frac{ak^{3/2}\pi^{1/2}}{(k^2-b^2)^{3/4}} \quad . \tag{7.5}$$

Here is assumed that trip frequency does not vary significantly so that there is a proportional relationship between average trip distance and total travel time expenditures. Equation (7.5) relates this average distance to the elliptical travel area.

Rearranging (7.5),

$$(k^2-b^2)^{3/2} = \frac{4\pi a^2k^3}{9\gamma^2} \quad , \tag{7.6}$$

and substituting expression (7.6) into expression (7.4) for travel benefits yields

$$B = \frac{3\gamma^2}{4k^2}\frac{a}{\sqrt{1+b^2}} \quad . \tag{7.7}$$

Finally, introducing the linear approximation for attractions as a function of the opportunity, A, at the urban center,

$$a(r) = A - br \quad , \tag{7.8}$$

net travel benefits achievable at location r are

$$B = \frac{3\tau^2}{4k^2}\frac{(A - br)}{\sqrt{1 + b^2}} \quad . \tag{7.9}$$

Thus, for the linear approximation, travel benefits are proportional to the square of travel time expenditures, are also proportional to the square of average travel speed, and are an increasing function of the opportunity level at the urban center. However, benefits are a decreasing function of the rate at which activities decrease with distance from the center.

The elongation of the travel ellipse is given by

$$\begin{aligned}\frac{\alpha}{\beta} &= \frac{k}{\sqrt{k^2 - b^2}} \\ &= (1 - \frac{b^2}{k^2})^{-1/2} \quad .\end{aligned} \tag{7.10}$$

Thus, the elongation increases with distance r provided that the ratio b/k does, that is, provided that there is a more rapid increase in speed with distance than decrease in the slope of the density of travel attractors. This condition is expected to be satisfied in the case of linear attraction densities: speeds increase with distance while the slope of the density of travel attractors is constant. This condition might be satisfied for exponential density functions at locations sufficiently far from the urban center; at locations near the center it is not expected to be satisfied. Thus, for exponential density functions, travel fields implied by the present model might become less elongated, reach a minimum elongation, then become more elongated as distance increases from the urban center. Finally, equation (7.10) implies that the travel ellipses for car travel are more elongated than those for transit travel if the speed of car travel is greater than the speed of transit travel, assuming that speeds are the same in all directions from the household. This might not be the case for transit travel because radial routes might have better service than transverse routes. In such a case the circular cone could be replaced by an elliptical cone, but this complicates the analytics and is a subject for future research.

7.3 The Underlying Theory in Probabilistic Choice Form

The elliptical travel fields in the prior approach can also be generated through application of a random utility model of household trip-making behavior. This random utility model reinforces the prior approach by clarifying certain analytical points not readily derivable in the prior analytics, and directly addresses the definition of travel probability fields as iso-probability contours of binomial normal distributions functions.

Consider a household located at any point in the urban space. It is convenient to define this arbitrary point as the pole in a polar coordinate system (r,θ) where the coordinate angle θ is measured relative to the vector connecting the household location and the urban center, as shown in Figure 6.

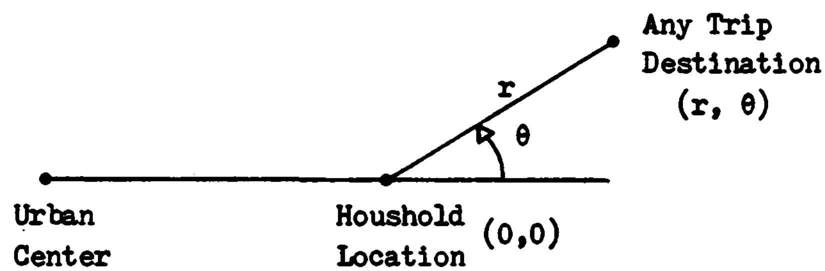

Figure 7.6

Assuming once again that the negative exponential density function for travel opportunities can be approximated in the neighborhood of the household location by a plane tilted in the direction of the urban center, the level of economic opportunities at trip destination (r,θ) is

$$u = a - br\text{Cos}\,\theta \;. \tag{7.11}$$

From a deterministic viewpoint, the household's decision whether or not to make a trip to destination (r,θ) depends upon whether or not travel benefits, attributed to the opportunities at (r,θ), are greater than the cost of the trip. A trip is made whenever

$$u > kr \;, \tag{7.12}$$

That is, whenever

$$a - brCos\ \theta > kr\ , \tag{7.13}$$

or whenever

$$r < \frac{a/k}{(1 + b/k\ Cos\ \theta)}\ . \tag{7.14}$$

Now the critical distance at which cost just equals benefits is given by

$$r = \frac{a/k}{(1 + b/k\ Cos\ \theta)} \tag{7.15}$$

which is the equation of an ellipse in polar coordinates with the pole (household location) at one focus and eccentricity given by

$$e = b/k\ . \tag{7.16}$$

Since the eccentricity of the ellipse is defined as

$$e = \frac{\sqrt{\alpha^2 - \beta^2}}{\alpha} \tag{7.17}$$

where α and β are the major and minor axes, respectively, equation (7.16) can be compared with the eccentricity of the ellipse derived in the prior model: Squaring both sides of equation (7.17)

$$e^2 = 1 - \frac{\beta^2}{\alpha^2}\ , \tag{7.18}$$

substituting b/k for e (equation (7.16)) and rearranging,

$$\frac{\alpha}{\beta} = (1 - \frac{b^2}{k^2})^{-1/2}\ . \tag{7.19}$$

A comparison of equations (7.10) and (7.19) shows that the same results are obtained in the deterministic and random utility approaches.

Random components might be associated with both the density of economic opportunities, as reflected in the locations of trip attractors, and the household's perception of travel benefits and costs. First, let

$$q = q(r,\theta)r\ dr\ d\theta \tag{7.20}$$

denote the probability that a trip attractor is located in the differential area centered at trip destination (r,θ) at distance r from the household. Second, introduce a random component of utility, ϵ, assumed to be distributed independently of q. The decision to make a trip to (r,θ) then depends upon whether benefits are greater than costs, taking into account the random utility component and the probability of finding a trip attractor:

$$\Pr(\text{trip}) = q\Pr(u + \epsilon - kr > 0) , \tag{7.21}$$

or

$$\Pr(\text{trip}) = q\Pr(\epsilon > kr - a + br\cos\theta) . \tag{7.22}$$

Iso-probability contours for trip destinations are then defined by

$$kr - a + br\cos\theta = \text{Constant} = C_o \tag{7.23}$$

which is the equation of an ellipse in the polar coordinate system:

$$r = \frac{(C_o + a)/k}{(1 + b\cos\theta)} . \tag{7.24}$$

To demonstrate iso-probability contours, it is usually assumed that a random component such as ϵ is distributed normally. Equation (7.21) for the probability of making a trip to a particular destination becomes

$$\Pr(\text{trip}) = q[1 - N(\frac{kr - u - \mu}{\sigma})] , \tag{7.25}$$

where N( ) denotes the standardized normal density function with mean μ and standard deviation σ. However, the forms of the iso-probability contours for (7.16) are more apparent when the normal distribution is approximated by a logistic:

$$N(t) \doteq \frac{1}{1 + e^{-\delta t}} , \tag{7.26}$$

where it has been shown that for the value of $\delta = 1.70174$

$$\left| N(t) - \frac{1}{1 + e^{-\delta t}} \right| < 0.00946 \tag{7.27}$$

for all t, and that value of δ minimizes the bound (7.31).

Thus,

$$\Pr(\text{trip}) \doteq \frac{q}{1 + e^{(\delta/\sigma)(kr - u - \mu)}} , \tag{7.28}$$

and it is clear that the iso-probability contours are described by constant values of the exponent:

$$(\delta/\sigma)(kr - u - \mu) = \text{Constant} = C_1 , \tag{7.29}$$

or, rearranging terms and substituting expression (7.11) for u,

$$r = \frac{(\sigma C_1/\delta + a + \mu)/k}{(1 + b/k \operatorname{Cos} \theta)} \tag{7.30}$$

which is once again the equation of an ellipse. The shape of the ellipse is determined by the parameters b/k defining the eccentricity or elongation, and for given values of μ, σ, δ and a, the constant C_1 determines the area. Conversely, the constant C_1 can be set so that the iso-probability contour corresponds to a particular volume under the probability density function.

Equation (7.28) shows that, holding the direction θ constant, trip frequency falls off with distance as 1-L(r), where L(r) is a logistic function. For large r trip frequency is approximately exponential:

$$\Pr(\text{trip}) \doteq q e^{-(\delta/\sigma)(k - b + b\operatorname{Cos} \theta)r} . \tag{7.31}$$

The coefficient in the exponent, the rate of decline, is greatest when $\theta = 0$, i.e., in the direction away from the urban center. It is smallest when $\theta = \pi$, i.e., in the direction toward the urban center.

Consequently, it is appropriate to employ probability distribution theory in order to analyze the travel ellipses revealed in travel survey data on trip distributions. If shorter length trips are more frequent than longer trips for the same trip purpose, two-dimensional bell-shaped probability distributions, such as the bivariate normal distribution, would be consistent with the elliptical model in that concentric ellipses represent iso-probability contours for these distributions. A test of the model for a given urban environment and household group would involve analysis of the parameters of the distributions fit to the observed travel data.

7.4 Parameters of the Computed Travel Probability Fields

Calculations were performed in accordance with techniques used extensively in the field of geography to study population distributions (7.32):

(1) Households' Centroid. Given the X and Y corrdinates of each sampled household's residence location, the weighted households' centroid is calculated:

$$\bar{X} = \frac{\Sigma w_i X_i}{\Sigma w_i} \quad , \qquad \bar{Y} = \frac{\Sigma w_i Y_i}{\Sigma w_i} \quad . \tag{7.32}$$

where: X_i, Y_i = coordinates of residence location,
w_i = expansion factor of sample.

(2) Trip Destinations' Centroid. Given the x and y coordinates of each trip destination, generated by specific households, the weighted trip destination centroid is calculated:

$$\bar{x} = \frac{\Sigma w_i x_i}{\Sigma w_i} \quad , \qquad \bar{y} = \frac{\Sigma w_i y_i}{\Sigma w_i} \quad . \tag{7.33}$$

It should be noted that trips for purpose Home were excluded, in order to derive the centroid of trips destinations away from home.

(3) Standard Deviation of Trip Destinations. The standard deviation of trip destination distributions about their respective centroids is calculated:

$$\sigma_x = \sqrt{\frac{\Sigma w_i x_i^2}{\Sigma w_i} - \bar{x}^2} \quad , \qquad \sigma_y = \sqrt{\frac{\Sigma w_i y_i^2}{\Sigma w_i} - \bar{y}^2} \quad . \tag{7.34}$$

(4) Correlation Coefficient. The correlation coefficient between the two distributions is calculated:

$$r = \frac{\Sigma w_i x_i y_i - \dfrac{(\Sigma w_i x_i)(\Sigma w_i y_i)}{\Sigma w_i}}{\sqrt{\Sigma w_i x_i^2 - \dfrac{(\Sigma w_i x_i)^2}{\Sigma w_i}} \sqrt{\Sigma w_i y_i^2 - \dfrac{(\Sigma w_i y_i)^2}{\Sigma w_i}}} \quad . \qquad (7.35$$

(5) Angle of Rotation. In order to find the new coordinate system along which σ_x and σ_y take the maximum and minimum values, the angle of

rotation between the initial and the new coordinate systems is calculated:

$$\alpha = \tfrac{1}{2}\ \mathrm{arctg}\left[\frac{2r\sigma_x\sigma_y}{\sigma_x^2 - \sigma_y^2}\right] . \qquad (7.36)$$

(6) Transformation of Coordinates. The initial coordinates of all trip destinations are transformed to the new coordinate system.

(7) Maximum/Minimum Standard Deviations. The standard deviations would take maximum/minimum values in the new coordinate system:

$$\sigma'_x = \sqrt{\frac{\Sigma w_i x_i'^2}{\Sigma w_i} - \bar{x}'^2} \ ; \qquad \sigma'_y = \sqrt{\frac{\Sigma w_i y_i'^2}{\Sigma w_i} - \bar{y}'^2} \ . \qquad (7.37)$$

where w_i is the expansion factor. σ'_x and σ'_y describe an ellipse within which about 68 percent of activity sites are expected to be located.

(8) Distance Indices. Two distances are calculated from the above values. The first distance, d_c , is the distance from the households' centroid to the activity site centroid, called "Centroid Distance":

$$d_c = \sqrt{(\bar{X} - \bar{x})^2 + (\bar{Y} - \bar{y})^2} \ . \qquad (7.38)$$

The second distance, d_σ , called "Standard Distance", is defined as:

$$d_\sigma = \sqrt{\sigma_x^2 + \sigma_y^2} = \sqrt{\sigma_x'^2 + \sigma_y'^2} \ . \qquad (7.39)$$

which can be regarded as a bivariate measure of activity site dispersion.

Figures 7.7 and 7.8 illustrate typical examples of the travel probability fields computed in the two corridors. Once again, the dimensions of household stratification were income and residence distance from the urban center, resulting in the computation of travel probability fields for each of five different income classes at each of five different distances. Small sample sizes precluded a finer grained distance breakdown and use of household size or car ownership as additional segmentation dimensions.

Appendix E., Table E.1, details the trip destination frequency distributions along the original X and Y axis, stratified by Work, Non-Work and Total purposes for a typical household segment: income group 3 residing in the North corridor at a distance class 7-11 miles from the urban center. Figure E.1 of Appendix E, shows these frequency distributions graphically.

It was found in the analyses typified in Table E.1 and Figure E.1 that:

(1) The trip frequency distributions along both and x-and y-axes show a symetrical shape, for both trip purposes and for the total. This is an important result.

(2) Each frequency may be generally approximated by a normal distribution; in the case of the household segment analyzed in Table E.1 and Figure E.1, the χ^2 test for similarity is accepted at the 0.05 confidence level along the X-axis, and is rejected along the Y-axis.

It can be inferred that part of the dispersion of destinations is attributable to the dispersion of households within the analyzed band. A more detailed analysis may reduce the dispersion.

The travel probability fields for the middle income segment residing in the two corridors at increasing distances from the urban center are shown in Figure 7.7. For clarity purposes only every second residence distance is shown. Similar results were found for the other income segments and distances. The basic travel characteristics of this household segment and the key parameters of the six computed travel probability fields are listed in Table E.2 of Appendix E.

The travel probability fields for households in different income segments located at the same distance from the urban center are shown in Figure 7.8. Again, for clarity purposes only every second income segment is shown. Similar results were found for the other distances and income segments. The basic travel characteristics of these three household segments and the key parameters of the three computed travel probability fields are listed in Table E.3 of Appendix E.

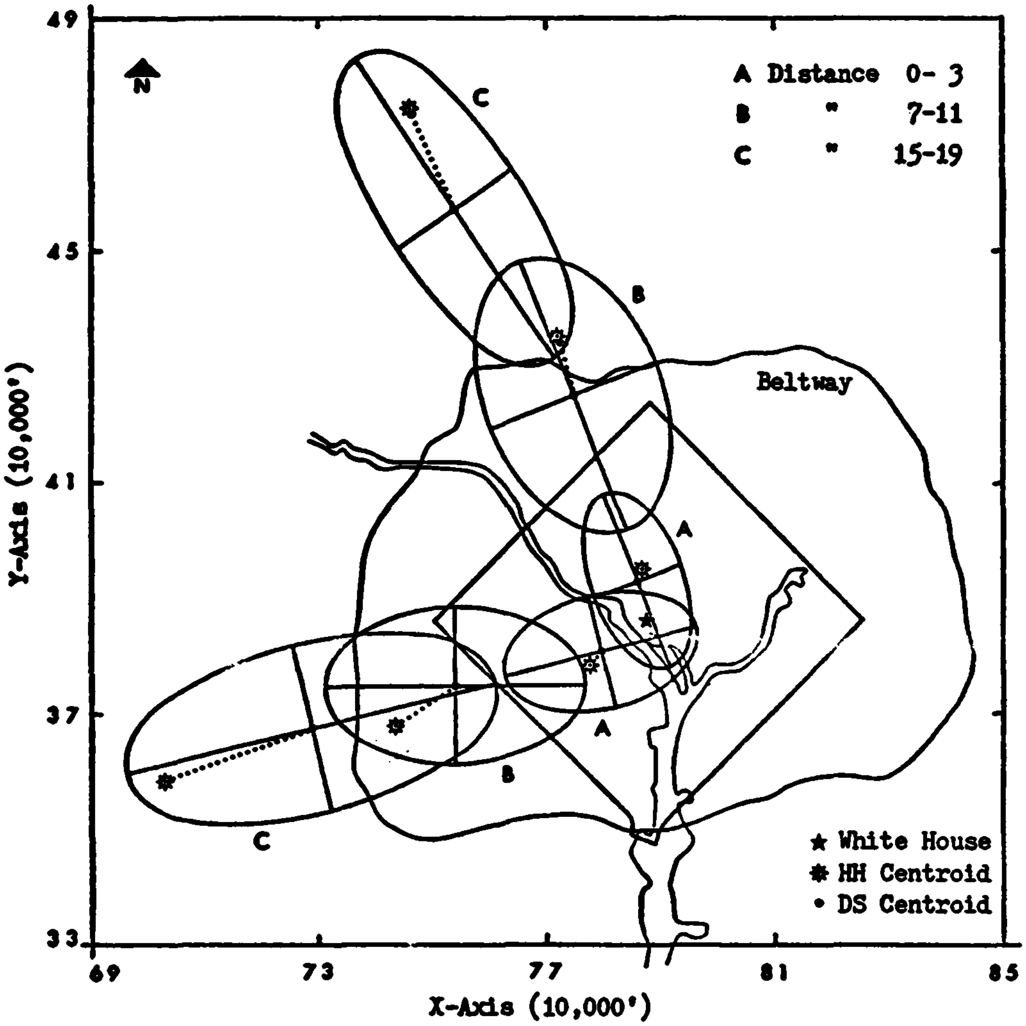

Figure 7.7 : Travel Probability Fields, Households of Income Class $ 8-12,000 Residing in Corridors North and South at Distance Classes 0-3, 7-11 and 15-19 miles from the City Center, Washington, D.C. 1968 (HH - Households, DS - Destinations)

It is apparent from the results typified in Figures 7.7 and 7.8 that the ellipses are directed approximately toward the urban center. This orientation is consistent with the proposed theory.

In order to quantitatively test relationships among the parameters of the travel probability fields and the travel characteristics of the households for which the fields are computed, regression analyses were conducted using data for all household income segments at all distances from the urban center in both corridors. The data are listed in Table E.4 of Appendix E.

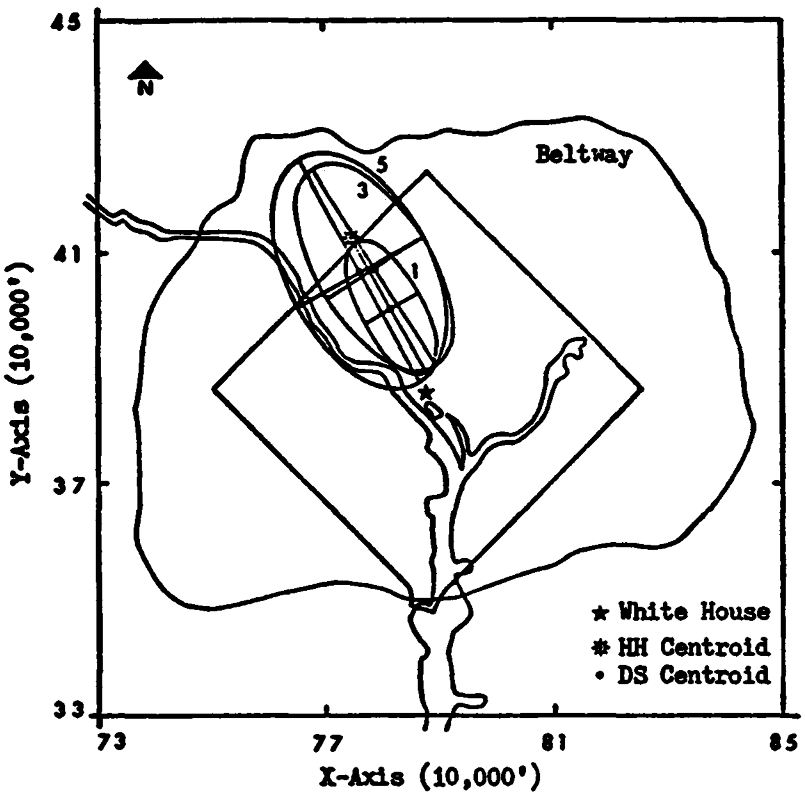

Figure 7.8 : Travel Probability Fields, Households of Income Classes \$ 0-4,000, \$ 8-12,000 and \$ 20,000 and Over Residing in Corridor North at Distance Class 3-7 miles from the City Center, Washington, D.C. 1968 (HH - Households, DS - Destinations)

It is apparent from Figures 7.7 and 7.8 that both the centroid distance (equation (7.47)) and the standard distance (equation (7.44)) tend to increase with both distance from the urban center (Figure 7.7) and income (Figure 7.8). It has been noted in Section 5.2 that average daily door-to-door travel speed is strongly related to both income and residence distance from the urban center, and tests were conducted to determine the strength of the relationships between speed and the two travel probability field distance indices.

The relationships between centroid distance and speeds and between standard distances and speeds for the two corridors are shown in Figure 7.9. Each data point in Figure 7.9 represents a particular combination of household income and distance from the urban center. The detailed regression results are listed in Table E.5 of Appendix E.

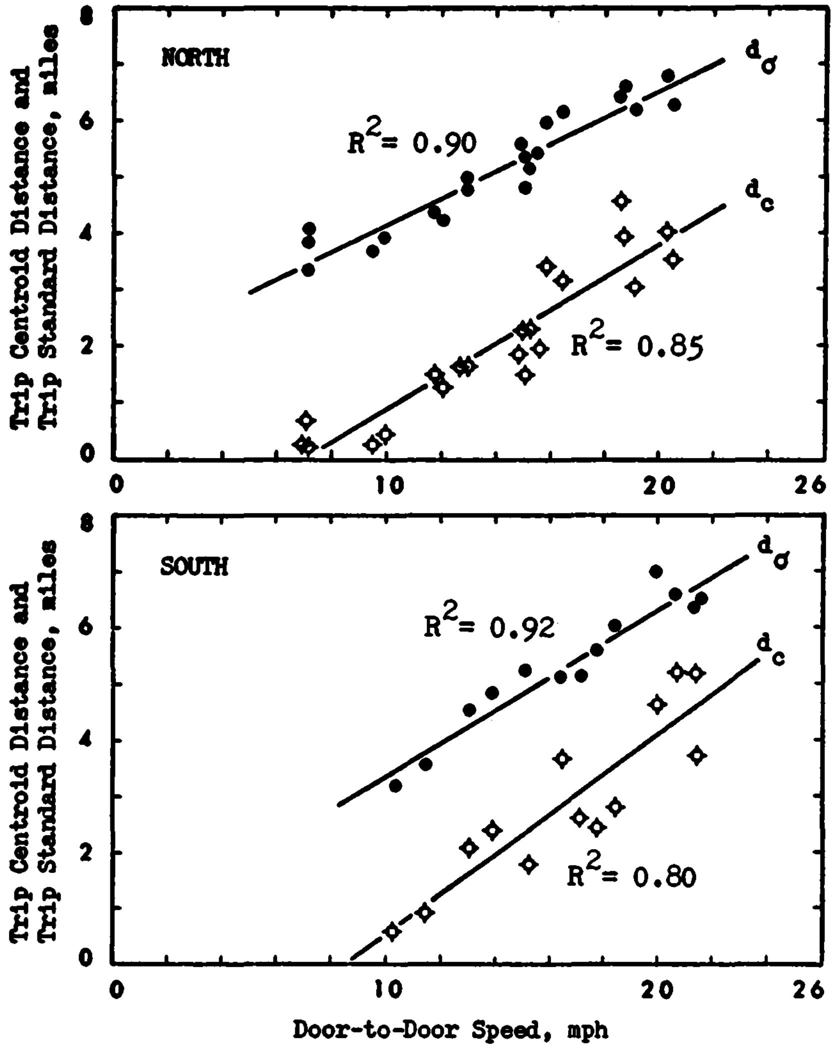

Figure 7.9: Trip Centroid Distance (d_c) and Trip Standard Distance (d_σ) versus Daily Mean Door-to-Door Speed, by Household Income and Residence Distance from the City Center, North and South Corridors, Washington, D.C. 1968

These results indicate:

(1) Each relationship is highly significant, with R^2 values of 0.80 and above, as indicated in Figure 7.9. No significant curvilinearities are apparent.

(2) The relationships of centroid distance versus speed are statistically similar in the two corridors, as determined by tests of linear regression coefficient equality (<u>7.33</u>).

(3) The North and South corridor relationships of standard distance versus speed are at the threshold of being similar at the 0.05 confidence level (F-value = 2.30; critical F-value = 2.29).

These results accentuate the usefulness of door-to-door travel speed as an indicator of the dispersion of households' activity sites. Speed appears to be a key variable in understanding interactions between travel demand and urban structure.

If travel probability fields are to be considered an effective representation of activity dispersion, the probability field distance indices should also be related to the average trip length for each household segment. The relationships between cetroid distances and average trip length and between standard distance and average trip length are shown in Figure 7.10 and are detailed in Appendix E, Table E.4.

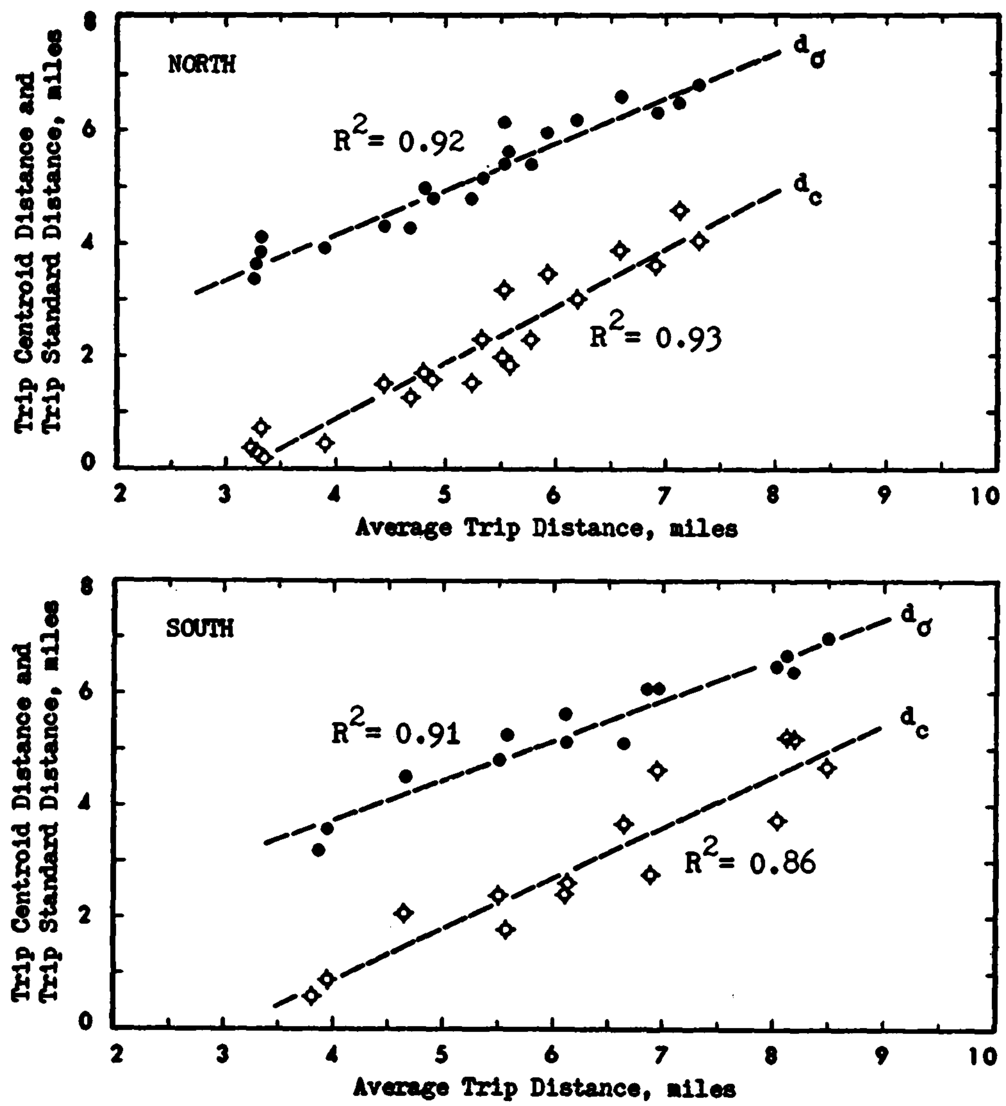

Figure 7.10: Trip Centroid Distance (d_c) and Trip Standard Distance (d_σ) versus Average Trip Distance, by Household Income and Residence Distance from the City Center, North and South Corridors, Washington, D.C. 1968

The regression analyses indicate:

(1) Each relationship is highly significant, with R^2 values equal to or greater than 0.86. Again, simple linear relationships are the most effective representations.

(2) The relationships of centroid distance versus average trip length are statistically similar in the two corridors.

(3) However, the relationships of standard distance versus average trip length are statistically dissimilar in the two corridors.

These results are consistent with those previously cited: the scalar parameters of travel probability fields are directly and simply related to other scalar travel veriables. The identification of differences in relationships between corridors holds forth the promise that certain parameters of travel probability fields capture variations in urban structure. This topic is explored in the following section.

7.5 Tests of Urban Density Function Hypotheses

The model development of Section 7.2 included a derived expression for the elongation of the travel ellipse:

$$\frac{\alpha}{\beta} = \left(1 - \frac{b^2}{k^2}\right)^{-1/2} \qquad (7.10)$$

where

b = slope of the activity-site density function,
k = inverse of travel speed,
α = major axis of ellipse, and
β = minor axis of ellipse.

Rearranging equation (7.10) and substituting the maximum and minimum standard deviations determined empirically (equation (7.37)) for α and β, respectively,

$$b = CK\left[1 - \left(\frac{\sigma'_x}{\sigma'_y}\right)^{-2}\right]^{1/2} \qquad (7.40)$$

where C is a proportionality constant necessitated by the measurement of speed in arbitrary units.

Thus, estimates of the slope of the activity-site density function can be generated at various distances from the urban center in each corridor from travel probability field parameters and estimates of travel speed. Computations of travel probability fields for different income strata provide multiple estimates of the slope parameter at each distance.

Regression analyses of density function slope versus distance determined that the slopes are concave functions of distance from the urban center in both corridors. This is consistent with robust evidence of the concept in urban economics and geography that densities decline at an ever-decreasing rate from the urban center.

Both exponential and power functions were fitted to the data in each corridor. Details are listed in Table E.6 of Appendix E. Results for the North corridor are shown in Figure 7.11. From these results it is impossible to choose between the exponential and power representations. Goodness-of-fit measures are equivalent; and both graphs exhibit some degree of curvilinearity: convexity in the semi-log plot and concavity in the log-log plot.

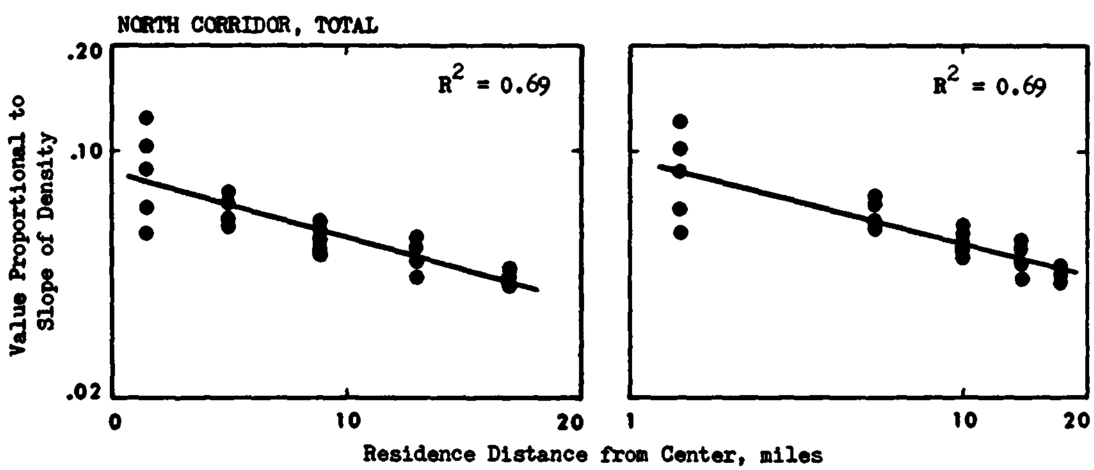

Figure 7.11

Results for the South corridor are shown in Figure 7.12. In this case the exponential representation is best. That is, the density curve for activity sites is inferred to be an exponential function, although the goodness-of-fit measure does not support a compelling argument. Nevertheless, this result is consistent with robust empirical evidence concerning the distributions of population and jobs in urban areas (7.16, 7.21, 7.22, 7.23, 7.24).

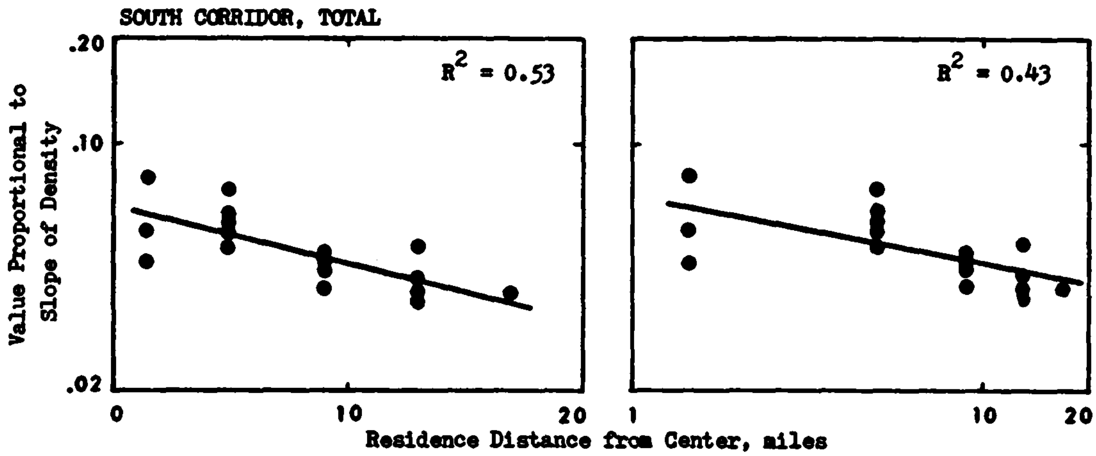

Figure 7.12

In order to refine the analyses and test hypotheses concerning differences in distributions of jobs and other types of activities within the urban area, separate travel probability field calculations were performed for work and non-work trips. Data limitations prevented any further breakdown into trip purposes. The travel probability fields for work and non-work purposes are shown in Figure 7.13 for the middle income segment in the North corridor. These results are typical of the other income segments in both the North and South corridors.

(Figure 7.13 is shown next page)

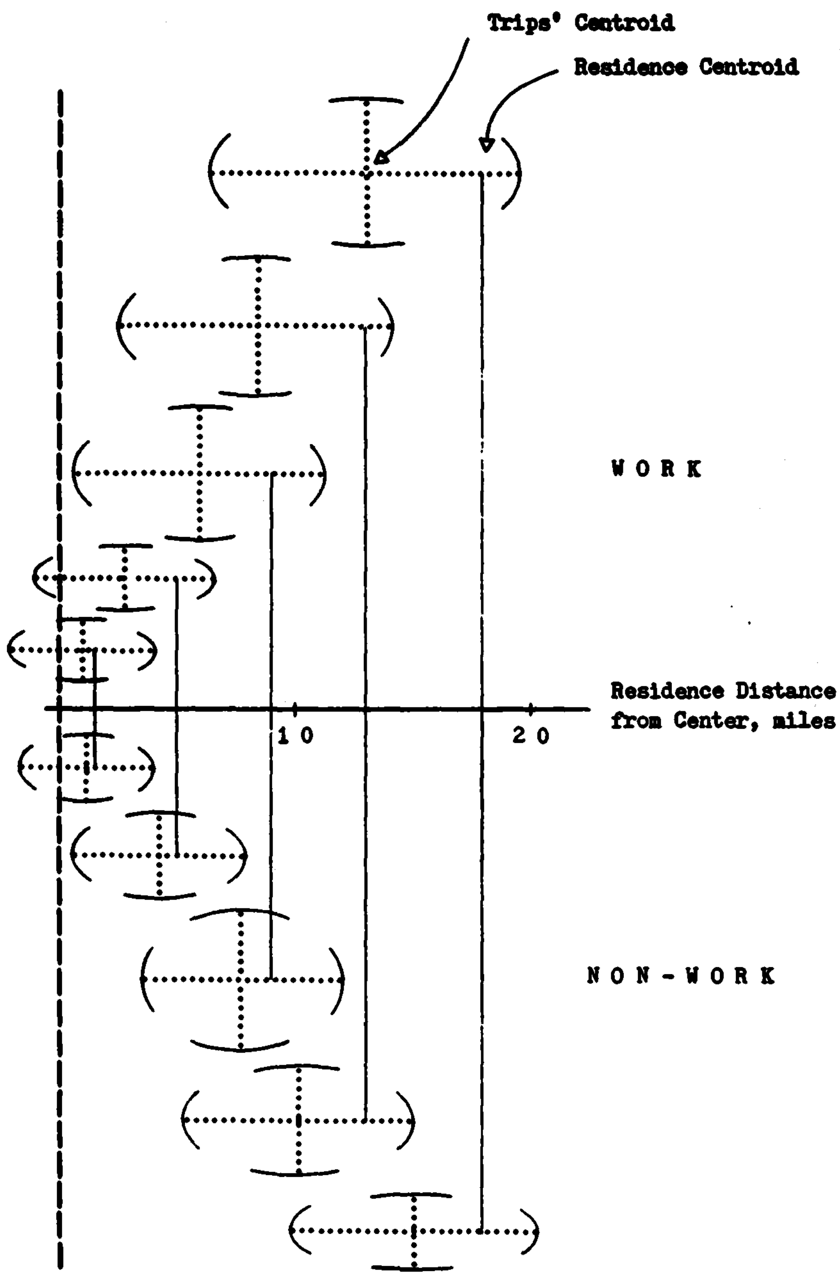

Figure 7.13: Travel Probability Fields for Work and Non-Work Purposes by Residence Distance from the Center, Middle Income Segment in the North Corridor, Washington, D.C., 1968

Using equation (7.40) to estimate density function slopes for work and non-work activity sites, regression analyses of slopes versus distance from the urban center revealed that power functions best represent the densities of both work and non-work activities in the North corridor (Figure 7.14). Details are provided in Table E.6 of Appendix E. Thus, separate consideration of work and non-work activities eliminates the ambiguity between common functional forms displayed in Figure 7.11 and improves the goodness-of-fit, as demonstrated through comparison of R^2 values in Figures 7.11 and 7.14.

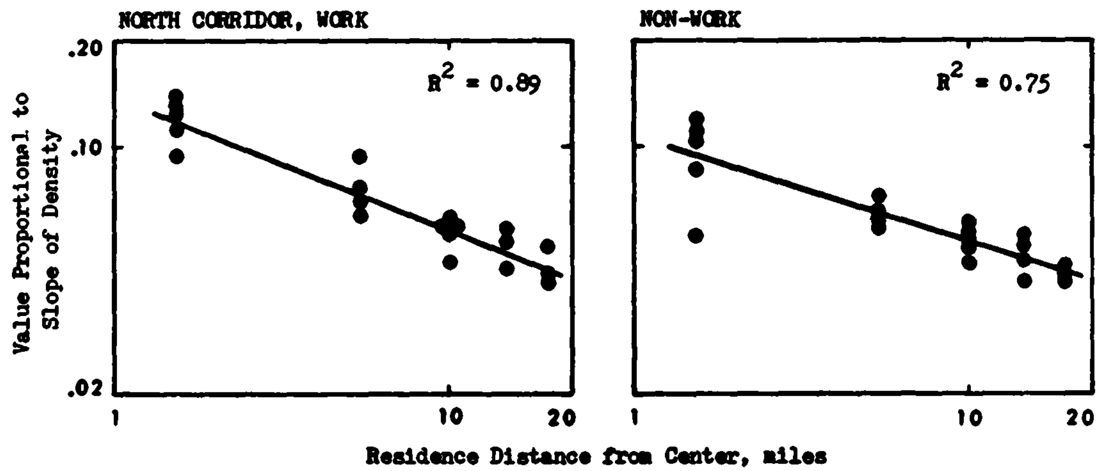

Figure 7.14

Results from analyses of South corridor data revealed that exponential functions best represent the densities of both work and non-work activities (Figure 7.15). This is consistent with results shown in Figure 7.12 for total travel, but goodness-of-fit values are improved dramatically by considering work and non-work activities separately.

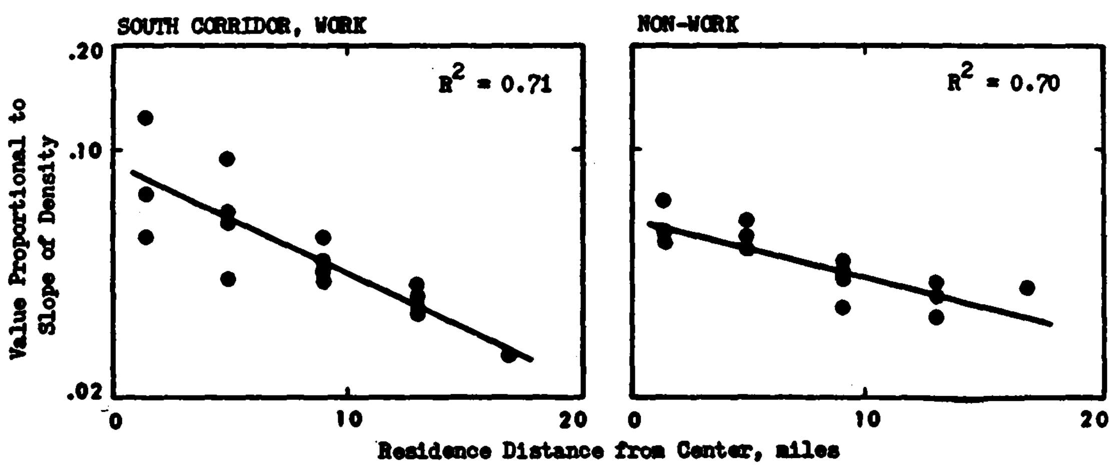

Figure 7.15

Finally, density function slopes were directly compared for work versus non-work activities in the two corridors. The work and non-work regression curves for both corridors are superimposed in Figure 7.16. In both cases the density of work activities declines more rapidly in the vicinity of the urban center than does the density of non-work activities. More specifically, the work activities density function is steeper than the non-work activities density function within approximately fifteen miles of the urban center in the case of the North corridor and within approximately ten miles of the center in the case of the South corridor. Assuming that work activities are strongly related to the density of jobs, while non-work activities are distributed more in keeping with the distribution of population, these results are as expected; empirical evidence has shown that density functions for jobs are steeper than those for populations in most metropolitan areas (7.34). Moreover, the more pronounced flattening of the work activities density function in the south corridor relative to the North corridor is consistent with the recognized land-use effects of major urban expressways: job densities decay less rapidly from the urban center along corridors in which land use accessibilities are enhanced by highly efficient transportation systems; such is the case in the present South corridor.

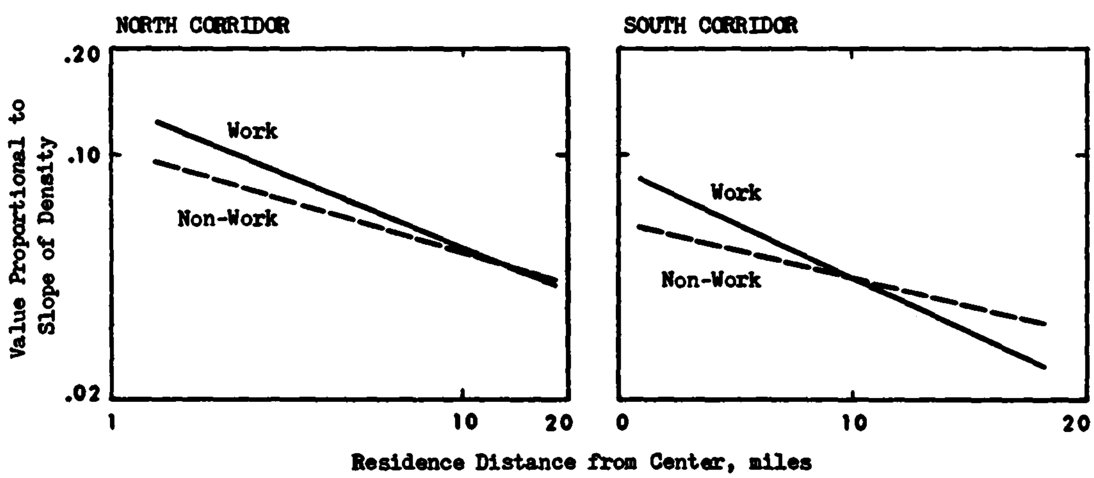

Figure 7.16

7.6 Rent-Bid Functions

The initial proposed household location choice model assumes that households have utility functions with three components dependent on residential location; housing, travel benefits (i.e., accessibility), and (negative) housing cost. Additional utility components will not affect residential location if they are separable with respect to the spatial components.

Utility for a household located at distance r from an urban center is thus given by

$$U = H(q) + B - pq \qquad (7.41)$$

where

$H = H(q)$ = utility of housing space $q(r)$,

$B = B(r)$ = travel benefits obtainable at location r, and

$p = p(r)$ = price of housing at r.

Here it is assumed that the utility of general consumption is separable and linear.

Assuming that the utility of housing space is logarithmic as in the standard model of the New Urban Economics (7.35).

$$U = \gamma \log(q) + B - pq \; . \qquad (7.42)$$

Income enters the present form of the proposed model as a stratification variable, car ownership can be treated likewise, as might other types of household characteristics. (The subscript i, referring to household stratum i, has been dropped from the utility components and overall utility level for reasons of simplicity.)

The household then chooses its housing such that

$$\underset{q(r)}{\mathrm{Max}}\, U = \underset{q(r)}{\mathrm{Max}} \left[\gamma \log(q) + B - pq \right] . \qquad (7.43)$$

The necessary and sufficient condition for such a utility maximum is

$$q = \frac{\gamma}{p} \; , \qquad (7.44)$$

and achieved utility U* is found by substituting the utility maximizing solution (35) into the utility function (33):

$$U^* = \gamma \log \frac{\gamma}{p} + B - \gamma \quad . \tag{7.45}$$

Using expression (9) for approximate travel benefits, achieved utility becomes

$$U^* = \gamma \log \frac{\gamma}{p} - \gamma + \frac{3\gamma^2}{4k^2} \frac{A - br}{\sqrt{1 + b^2}} \quad . \qquad (7.46$$

Solving for housing price p(r),

$$p(r) = \gamma \exp[-1 - \frac{U^*}{\gamma} + \frac{3\gamma^2}{4k^2} \frac{A - br}{\sqrt{1 + b^2}}] \quad . \tag{7.47}$$

Equation (38) describes the rent-bid function for the particular socio-economic stratum of households in question. Since achieved utility U* must be constant in the definition of the rent-bid function, equation (38) can be written

$$p(r) = C \exp[- \frac{3\gamma^2}{4k^2} \frac{br}{\sqrt{1 + b^2}}]$$

$$= C\, e^{-\lambda r} \quad , \tag{7.48}$$

where C and λ represent spatially-independent parameters at first approximation. Thus, the rent-bid function implied by the proposed residential location model is a negative exponential in terms of distance r from the urban center.

Furthermore, residential density is also a negative exponential function of distance:

$$\frac{1}{q(r)} = \frac{p(r)}{\gamma} \tag{7.49}$$

$$= \frac{C}{\gamma} e^{-\lambda r} \quad ,$$

which implies that residential density and the density of economic opportunities are log-linearly related.

Results (7.48) and (7.49) are consistent with the standard model of the new urban economics.

Thus, inputs to the model proposed in this chapter are the locations of trip attractors, such as jobs and shops, described by continuous spatial functions, and speeds of travel at various locations. Outputs are bid-rent functions describing the prices households are willing to pay for housing at various locations and the resulting density of housing at these locations. Implementation of this model then allows feedback linkages within the UMOT process to relate causal factors in housing and activity location decisions to actual locations through the modeling of traffic congestion, the separation between residences and jobs, and housing supply.

As a preliminary step in the development of an operational location model within the UMOT process, Appendix F presents an interpretation of the model of the present chapter. This interpretation involves development of the comparative statistics properties of the model and derivation of residential location patterns for various simplifying assumptions. Results show that the model is consistent with concepts from the fields of social ecology and urban geography, although further research is required.

* * *

CHAPTER 8: DYNAMIC RELATIONSHIPS

8.1 Introduction

It has been shown in the previous chapters that consistent relationships emerge from a comprehensive urban transportation data set which confirm relationships previously identified in less detailed data from other urban areas. These relationships suggest underlying behavioral mechanisms which motivate urban travelers within the urban system.

Most conventional travel and urban structure models deal with three different time scales: short, medium, and long. The short time scale refers to travel decisions on a daily basis, such as whether or not to make a certain trip, to what destination, by what mode, and so on. The medium time scale refers to travel decisions which may have a medium-term effect on travel, such as whether or not to purchase a car. The long time scale refers in most cases to major household decisions with long-term effects, such as residence location. It is also recognized that many medium and long-term decisions are interlinked, such as residence relocation coupled with the purchase of a car. The same can also be said about short and medium-term decisions.

When reviewing this issue from the viewpoint of urban structure, the various interactions taking place in the city can be divided into fast interactions, on a daily basis, such as the recurring daily activities, and the slow interactions, which may take years to become evident, such as the dispersion of population into the suburbs. The fast interactions can be viewed within a closed daily system, especially when based on an average weekday. The slow interactions, however, are more difficult to deal with, mainly because they involve dynamic changes within a potentially open system.

Such factors as varying reaction (or relaxation) times of different sub-systems often affect the total urban system in unexpected and counterintuitive ways. For example, observations that households tend to disperse from the urban center outwards with increases in income and speed intuitively lead to the conclusion that households would tend to gravitate back towards the urban center when travel costs increase and speeds decrease. It may, therefore, come as a surprise to realize that this is not the case; while some households would move back to the urban center, it is typically the jobs which follow residences outwards at a more rapid pace, with a net result of more dispersion of activities than before.

The UMOT travel process at the present stage of development, as outlined in Figure 8.1 (8.1), combines short and medium-term interactions by a dynamic feedback process. For example, a change in travel costs will affect not only the daily travel per traveler/household, but

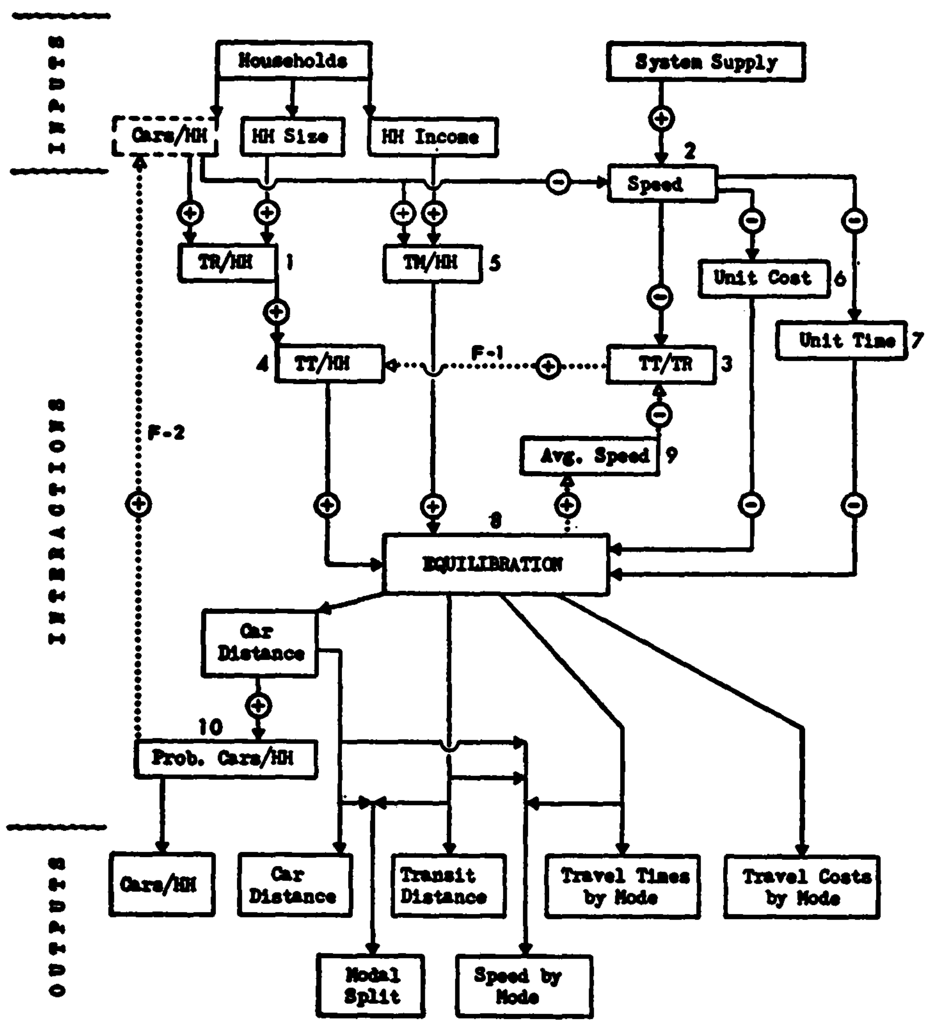

Figure 8.1 : Flow Chart of the Interactions between Travel Demand, System Supply and Car Ownership, the UMOT Model

⟶ Input/Output flow

5 Interaction. Effect of Input on Output is expressed by ⊕ or ⊖

······▸ Feedback

also car ownership levels, all which interact simultaneously. Hence, a policy maker can evaluate the possible effects of increases in travel costs, although the process does not yet provide information concerning how much time will elapse between various levels of effects. The reaction times of sub-systems to endogenous and exogenous changes are a crucial parameter in a fully dynamic model. This is especially true when dealing with changes in urban structure. Unfortunately, few data are available on such reaction times; it is a neglected part in the study of urban structure.

At the present stage of development, therefore, only the so-called fast interactions can be modeled in the UMOT process. Presently, no reaction times, in real time terms, can be specified. The theoretical framework proposed in the remainder of this chapter should, therefore, be regarded as an intermediate stage in the development of a fully dynamic model. The lack of reaction times might be addressed through the use of simulations, as recommended in Chapter 10.

8.2 Theoretical Framework

Urban systems pose one of the great challenges of our time. Since nature and function of urban systems affect the lives and welfare of most of us, any effort at a better understanding leading to improved prediction and control seems worth undertaking, even if the following results of such an analysis fall inevitably short of the levels of aspiration that might have been entertained. Urban analysis can certainly benefit from an infusion of new ideas and techniques, ranging from anthropology to catastrophe theory (8.2).

When all is said and done, however, it is probable that the principal hope would still seem to lie in the patient application of economic analysis to the complexities of urban systems. For there are indeed inescapable economic relationships at the heart of the complexities of urban systems and no serious scientific effort can afford to overlook these. The theoretical development in Appendix G is frankly economic and uses nothing more esoteric than the new urban economics (8.3). The model of Appendix G (summarized in the following discussion) is proposed as a starting point in which relevant urban economic principles are brought forth for scrutiny with regard to the UMOT process.

The introduction of dynamics in the new urban economics logically began with the introduction of time in exploring the impact on equilibrium densities and rents due to changes in population, income, transport costs, and so forth. Indeed, the development of the theoretical underpinnings of the UMOT process has followed similar lines: comparative static properties of the urban location model of Chapter 7 are discussed in Appendix F.

However, urban economists are no longer restricting themselves to comparative statics. Linear programming models have been developed which treat time and space discretely (8.4, 8.5), and from these approaches models have been developed which consider urban space in a more realistic continuous fashion (8.6, 8.7). The latter, continuous approach is applied herein.

In Appendix G. a model is formulated to describe how the size of the residential area of a city expands or contracts in response to the growth of population, of income and to changes in the value of surrounding agricultural land and in transportation costs and number of travelers per household.

The model is based upon a standard new urban economics model of residential choice (8.8, 8.9, 8.10, 8.11) and uses a household utility function which generates exponential housing densities consistent with empirical evidence (8.12, 8.13, 8.14). At one important stage in the model development, a linear relationship emerges between growth rates of exogenous and endogenous economic variables in the short term (Appendix G, equation (20)):

$$\frac{dN}{dt}/N + (1+\phi)\cdot\frac{dy}{dt}/y = \frac{dS_1}{dt}/S_1 + \phi\cdot\frac{dK}{dt}/K + \phi\cdot\frac{dm}{dt}/m + (1+\frac{\phi}{2})\cdot\frac{dA}{dt}/A \qquad (8.1)$$

where:

N = total number of households in urban area,

y = household income,

S_1 = agricultural land rent (land rent at urban periphery),

K = commuting cost per unit distance,

m = number of commuters per household,

A = urban residential land area, and

$$\phi = \frac{\lambda R F'(\lambda R)}{F(\lambda R)} \tag{8.2}$$

a short-term constant in terms of:

λ = a coefficient approximately fixed in the short term,

R = the radius of the urban area = $\frac{A}{\pi \gamma}$, where γ denotes the proportion of land used for housing, and

$$F(\lambda R) = \frac{1}{\lambda^2 R^2}(e^{\lambda R} - 1 - \lambda R). \tag{8.3}$$

Equation (8.1) thus shows how the rate of growth of the urban area is related to rates of growth in such factors as population, household income, number of workers per household and travel costs:

(1) Derived model results show that urban land area is expected to expand at a rate less than population growth. With area extension, population density is expected to remain constant at the urban fringe while higher density is generated in the interior of the urban area, resulting in greater overall density. "Urban sprawl" of recent decades must therefore be attributed to causes other than purely demographic ones.

(2) Urban land area is expected to expand with expansion of household income, but at a faster rate. This is potentially one root of the postwar growth in urban areas.

(3) Growth in the number of commuters per household due to increased participation in the labor force of married women, due to increased numbers of childless couples living together, or due to increased numbers of single-person households acts as a contracting factor on urban area expansion.

(4) The effects of changes in travel costs depend upon the characteristics of the urban area in question. Generally, however, increases in travel costs lead to higher inner-city densities and slow down area expansion attributable to demographic and income factors. The effect is more pronounced in larger urban areas. Results show that

postwar decreases in real travel costs explain much of the land area expansions of large urban areas.

At this stage, however, the possible "ratchet effect" in a fully dynamic process should be noted, where a reversal in travel cost trends does not mean a corresponding reversal in urban structure trends. When considering the reaction time of the various urban sectors, such as the housing stock and job locations, to exogenous factors it might well happen that the net effect of an increase in travel costs will result in the dispersion of jobs outward, towards residences, thus decreasing residence-work trip lengths.

Calculation of the joint effect of changes in many or all of the model explanatory variables simultaneously requires further development of feedback relationships. Steps for accomplishing the theoretical and empirical aspects of such further research are outlined in Appendix G.

The relevance of urban economic principles in explaining dynamic phenomena is further demonstrated in Appendix H. An exercise is presented therein in which economic principles are used qualitatively to explain the effects of a recent occurrence, the temporary slow down in housing-market transactions due to substantial increases in financing costs and reduced supply. The effects are viewed in terms of recent increases in travel costs and are shown to apply mainly to locations and relocations of office-type businesses, with particular locational patterns implied in various labor market situations.

8.3 The UMOT/Urban Interactions

As already mentioned in Chapter 1, the UMOT travel process is based on the minimum number of assumptions, including those of an objective function and the constraints. Hence, no *a priori* assumptions about the expected outputs can be made. That is, since the UMOT process is not calibrated to the observed choices, there is no way by which one can force the process to produce the observed choices. Exercises carried out with the UMOT travel process produced some results which appeared to be counterintuitive at first sight. For example, the scenario which provides a free transit system resulted in an increase in travel distance by both transit and private modes. Only after careful evaluation was it realized that the increase in transit travel is brought about mainly by walking trips transferring to transit, while free transit allows the diversion of monies from transit to car travel. Indeed, there are now several cases to suggest that this is happening in real life (8.15, 8.16). In short, several seemingly counterintuitive results which later were found to take place under real conditions, increased the confidence in the potential development of the UMOT process into an operational real-to-life model.

The UMOT travel process has been the subject of previous research, and its schematic structure is shown in Figure 8.1. The conceptual structure of the UMOT/Urban Interactions process is shown in Figure 8.2 where the UMOT travel part is shown in the upper-left-hand portion.

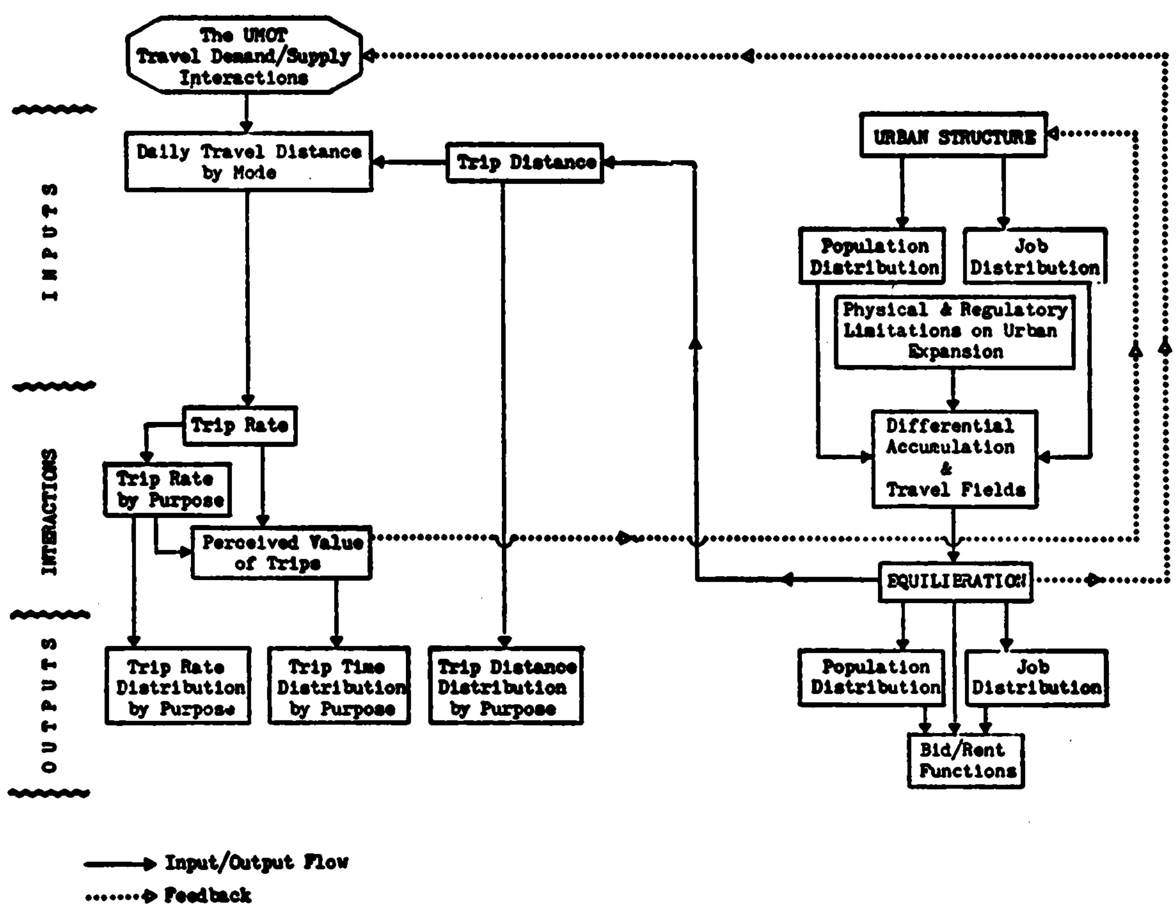

Figure 8.2 : Schematic Structure of the UMOT/Urban Interactions Process and its Link with the UMOT Travel Model

Two characteristics of the schematic structure of the UMOT/Urban Interactions process depicted in Figure 8.2 are of special interest:

(1) The principal link to the UMOT travel process is through the trip distance. Trip distance has been found to be more sensitive to both the transport system and urban structure than trip rate, thus serving as a responsive link between travel, transport system and urban structure.

(2) Two principal feedback loops are proposed at this stage of development. The first, smaller loop, is between trip rate and trip distance within the total travel distance, as discussed in Chapter 6. The second loop is between urban structure and the UMOT travel process, where changes in the spatial distributions of households and activity attractors are likely to affect the amount and patterns of generated travel, and vice versa.

An important part which is still missing in the above structure to make it fully dynamic is the reaction times of the various urban subsystems. However, data on such reaction times are meager. Therefore, one possible way to proceed with the development of a fully dynamic model is through simulations, where values of reaction times will be tested in order to identify their critical ranges. The outputs of such simulations can then be compared with available empirical data. Such outputs can also suggest which factors and corresponding reaction times are the most critical ones for the dynamic modeling of urban structure. This subject is further outlined in Chapter 9.

CHAPTER 9: CONCLUSIONS AND RECOMMENDATIONS

9.1 Conclusions

The empirical results presented in this report are compatible with results observed previously in analyses of more aggregate data. Moreover, the empirical results combine with results from the theoretical developments presented herein to support several important conclusions:

(1) The daily travel time expenditures per average traveler display predictable regularities. The daily travel time frequency distributions are similar for all traveler segments in the two case-study corridors, where the segments are defined by income and car ownership levels. The means and coefficients of variation effectively define the daily travel time frequency distributions for different traveler segments.

(2) The daily travel characteristics of an average traveler are closely interlinked: daily travel distance with door-to-door speed; average trip length with daily travel distance; trip rate with average trip time; and proportions of trips by trip purposes with trip rate. These results emphasize the need to consider explicitly such interrelationships within a simultaneous, dynamic feedback process.

(3) Travel probability fields capture the salient characteristics of trip destination distributions. The scalar parameters of travel probability fields are related in a straightforward manner to the travel demand measures of average trip speed and average trip length, and to residence distance from the urban center. Moreover, travel probability fields provide additional information on the geographic orientation of travel and the density of urban activity sites. Results are consistent with independent evidence concerning the mathematical forms of urban density functions.

(4) Travel probability fields are consistent with urban economic models of location decisions. Tests are positive concerning an urban economic interpretation of travel probability fields as measures of accessibility. This allows travel patterns to be theoretically related to urban structure.

(5) Preliminary assessments of the dynamic properties of the urban economic model relating travel and urban structure lead to the conclusion that the model can be implemented as a component in a simultaneous, dynamic feedback process for forecasting travel and associated changes in urban land-use patterns.

These encouraging conclusions, supported by the results of the empirical and the theoretical developments presented in this report, appear to justify the extension of the UMOT from an experimental process to a developmental model. Several recommendations for the required stages for such an extension are outlined below.

9.2 Recommendations

Three principal phases are required in order to extend the UMOT process into a developmental model:

I. The UMOT Travel Sector: Complete the travel sector as quasi-dynamic model, interrelating travel components such as car ownership, daily travel time, distance and speed, within a feedback process of mutual interdependence.

II. The UMOT Urban Sector: Complete the urban structure sector as a quasi-dynamic model, link it with the travel model, and explore the interactions between the two. Components to be interrelated include the densities of residences, jobs, and other activity sites, and travel distances and speeds.

III. The UMOT Travel/Urban Model: Complete the fully dynamic transportation/urban structure model, merging the travel and urban sectors.

Each phase would include extensions of the theoretical base, empirical verification, model construction (including computer programming) and model demonstration and verification.

Specific subjects for further attention are the use of catastrophe theory to model sudden changes in outputs (e.g., urban structure) brought by continuous changes in the inputs (e.g., travel costs); the use of bifurcation theory to model the self-structuring properties of urban areas; the levels of input and output stratifications required for different planning purposes; and the model's responsiveness and sensitivity to a wide range of changes in the inputs. A further line of exploration involves the reaction times associated with various urban structure components, and it is possible that dynamic simulations may assist in understanding some of the dynamics of urban areas.

* * *

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APPENDICES

APPENDIX A : POPULATION DISTRIBUTIONS

Figure A.1 summarizes the proportions of households by income class, cumulative and separate, within each ring of Figure 1.1. Detailed data are listed in Table A.1.

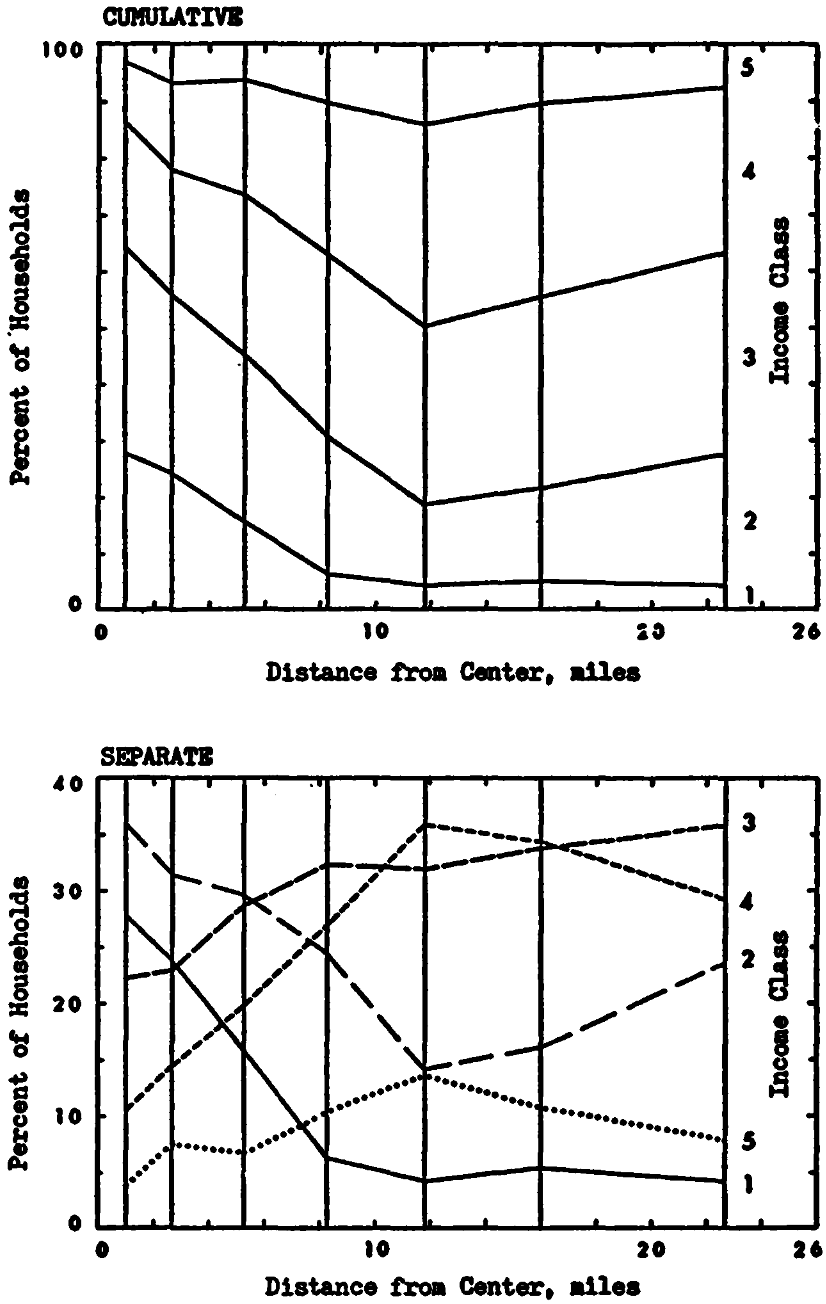

Figure A.1: Cumulative and Separate Percentage of Households, by Income Class, within Each Ring, vs. Distance from City Center, Washington, D.C. 1968

Figure A.1 displays well-known trends. Low income households are concentrated near the urban center, while high income households are dispersed outwards. However, the diagram also shows a relatively high level of household diffusion for each income class; high income households are also found near the center, while low income households are also found far from the center.

Table A.1: Household Distribution, by Income Level, Washington, D.C. 1968

Ring	Distance from Center, miles	Household Income Class											
		1		2		3		4		5		Total	
		HH	%	HH	%	HH	%	HH	%	HH	%	HH	%
1	0.9	14,335	27.7 13.0	18,731	36.3 8.5	11,429	22.1 4.6	5,328	10.3 2.6	1,851	3.6 2.4	51,674	100.0 6.0
2	2.7	35,149	23.8 32.0	46,459	31.4 21.0	33,791	22.8 13.5	21,531	14.5 10.7	11,093	7.5 14.6	148,023	100.0 17.2
3	5.3	38,799	15.6 35.3	73,719	29.7 33.3	71,115	28.6 28.4	48,624	19.6 24.0	16,246	6.5 21.4	248,503	100.0 28.9
4	8.3	12,338	6.1 11.2	49,483	24.6 22.4	64,943	32.3 26.0	54,108	26.9 26.8	20,416	10.1 26.9	201,288	100.0 23.5
5	11.8	5,595	4.1 5.1	19,140	14.1 8.7	43,380	31.9 17.4	48,564	35.8 24.0	19,175	14.1 25.3	135,854	100.0 15.8
6	16.0	2,701	5.4 2.5	8,048	16.0 3.6	17,009	33.9 6.8	17,150	34.2 8.5	5,293	10.5 7.0	50,201	100.0 5.9
7	22.8	951	4.1 0.9	5,518	23.5 2.5	8,351	35.6 3.3	6,837	29.1 3.4	1,808	7.7 2.4	23,465	100.0 2.7
Total		109,868	100.0	221,098	100.0	250,018	100.0	202,142	100.0	75,882	100.0	859,008	100.0

APPENDIX A

Table A.2 details the household spatial distribution by car availability versus distance from the urban center. The relationship between the number of cars per average household and household income class, and stratifications of car ownership levels by income class, are shown graphically in Figures A.2 and A.3, respectively. The interrelationship between income and car availability is shown in Figure A.4.

Table A.2: Household Distribution, by Car Availability, Washington, D.C. 1968

Ring	Distance from Center, miles	Cars per Household								Total	
		0		1		2		3+			
		HH	%	HH	%	HH	%	HH	%	HH	%
1	0.9	33,522	64.9	16,039	31.0	1,688	3.3	425	0.8	51,674	100.0
			18.7		4.1		0.7		0.8		6.0
2	2.7	63,290	42.8	67,802	45.8	14,441	9.7	2,490	1.7	148,023	100.0
			35.3		17.3		6.1		5.1		17.2
3	5.3	59,948	24.1	130,345	52.5	49,543	19.9	8,667	3.5	248,503	100.0
			33.4		33.3		20.8		17.7		28.9
4	8.3	14,779	7.3	96,285	47.8	74,220	36.9	16,004	8.0	201,288	100.0
			8.2		24.6		31.1		32.7		23.5
5	11.8	4,946	3.6	51,682	38.0	65,819	48.5	13,407	9.9	135,854	100.0
			2.8		13.2		27.6		27.4		15.8
6	16.0	2,024	4.0	19,370	38.6	22,786	45.4	6,021	12.0	50,201	100.0
			1.1		4.9		9.5		12.3		5.9
7	22.8	973	4.2	10,430	44.4	10,107	43.1	1,955	8.3	23,465	100.0
			0.5		2.6		4.2		4.0		2.7
Total		179,482	100.0	391,953	100.0	238,604	100.0	48,969	100.0	859,008	

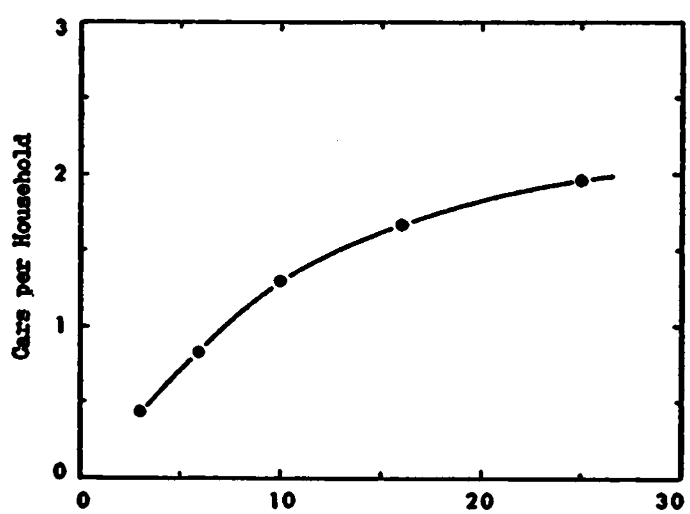

Figure A.2: Car Availability per Household vs. Household Annual Income, Washington, D.C. 1968

APPENDIX A

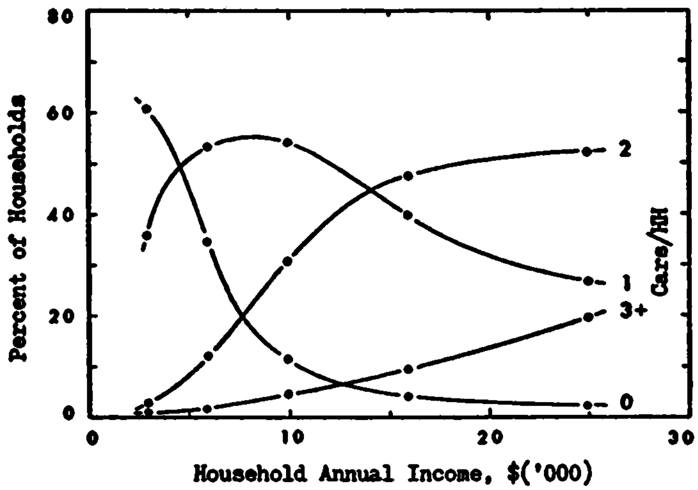

Figure A.3: Percent of Households by Car Availability vs. Household Annual Income, Washington, D.C. 1968

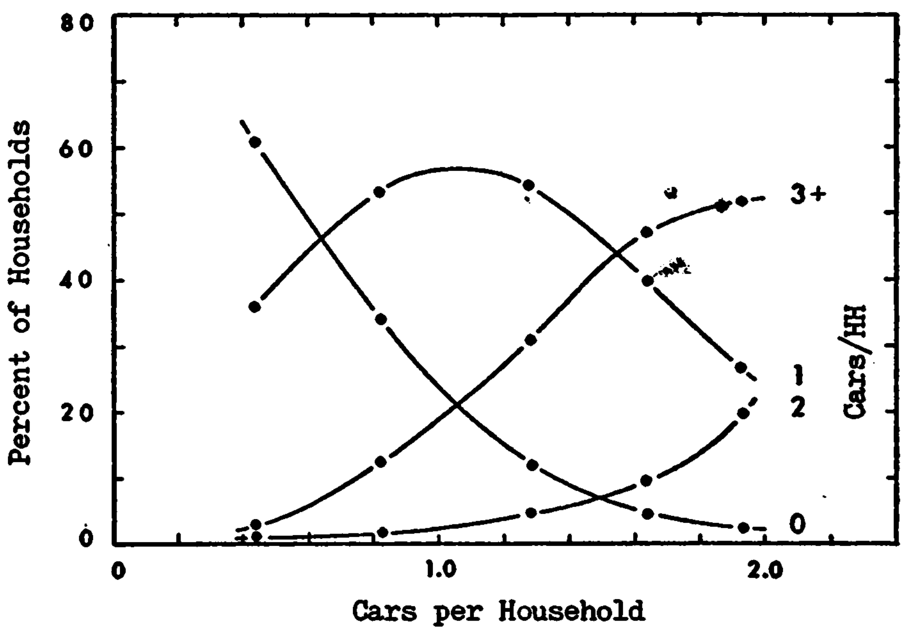

Figure A.4: Percent of Households by Car Availability vs. Average Car Availability, Washington, D.C., 1968

The relationships depicted in Figure A.4 are consistent with those found earlier (2.1) to be transferable both between cities and over time.

APPENDIX A

Figure A.5 shows the cumulative and separate proportions of households by car availability versus distance from the urban center. Comparing this figure with Figure A.1, the spatial distribution of households by car availability, shows stronger polarization than by income. Car availability appears to be a better discriminator of household location than income.

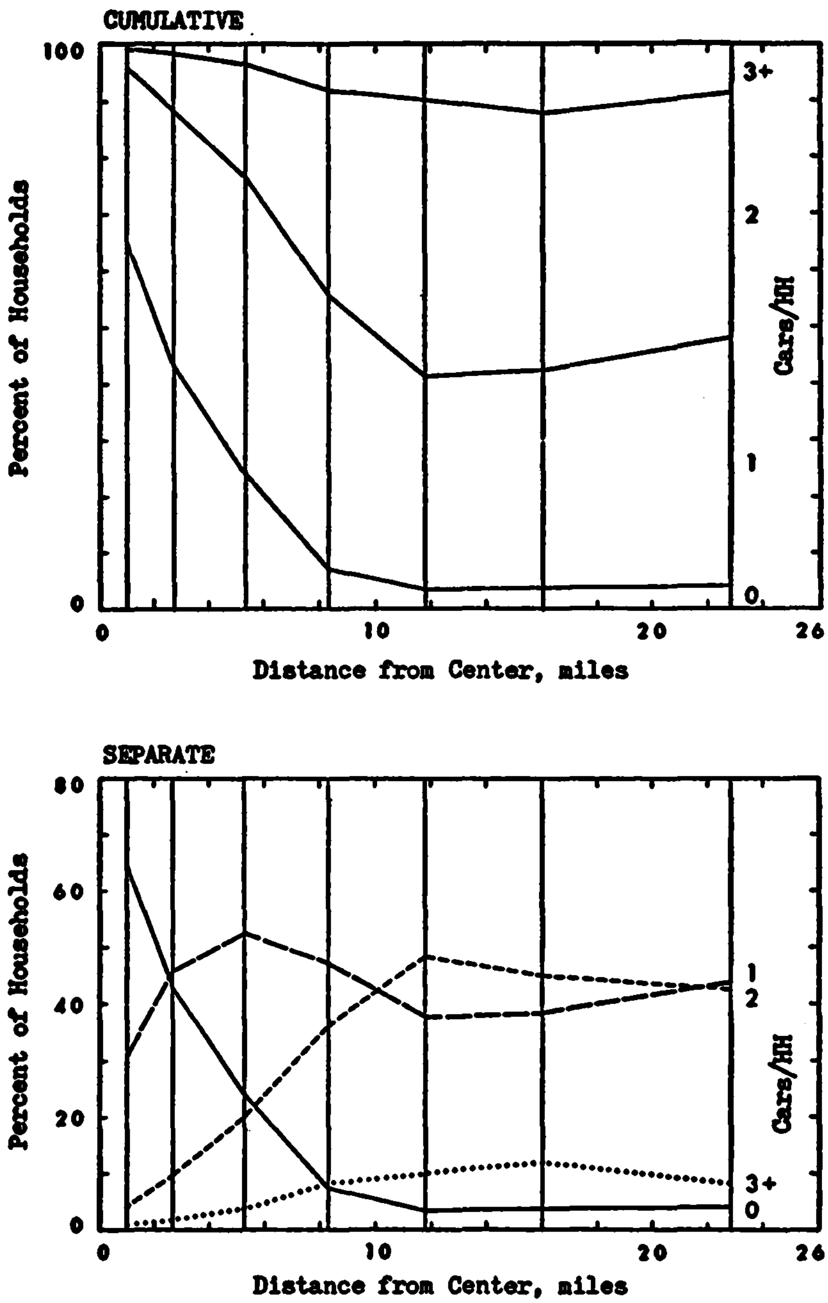

Figure A.5: Cumulative and Separate Percentage of Households, by Car Availability, within Each Ring, vs. Distance from City Center, Washington, D.C. 1968

APPENDIX A

The relationship between car ownership and residential location is also explored in Figure A.6, where the longitudinal distribution of households, versus distance from the urban center, are shown by income class and by car availability.

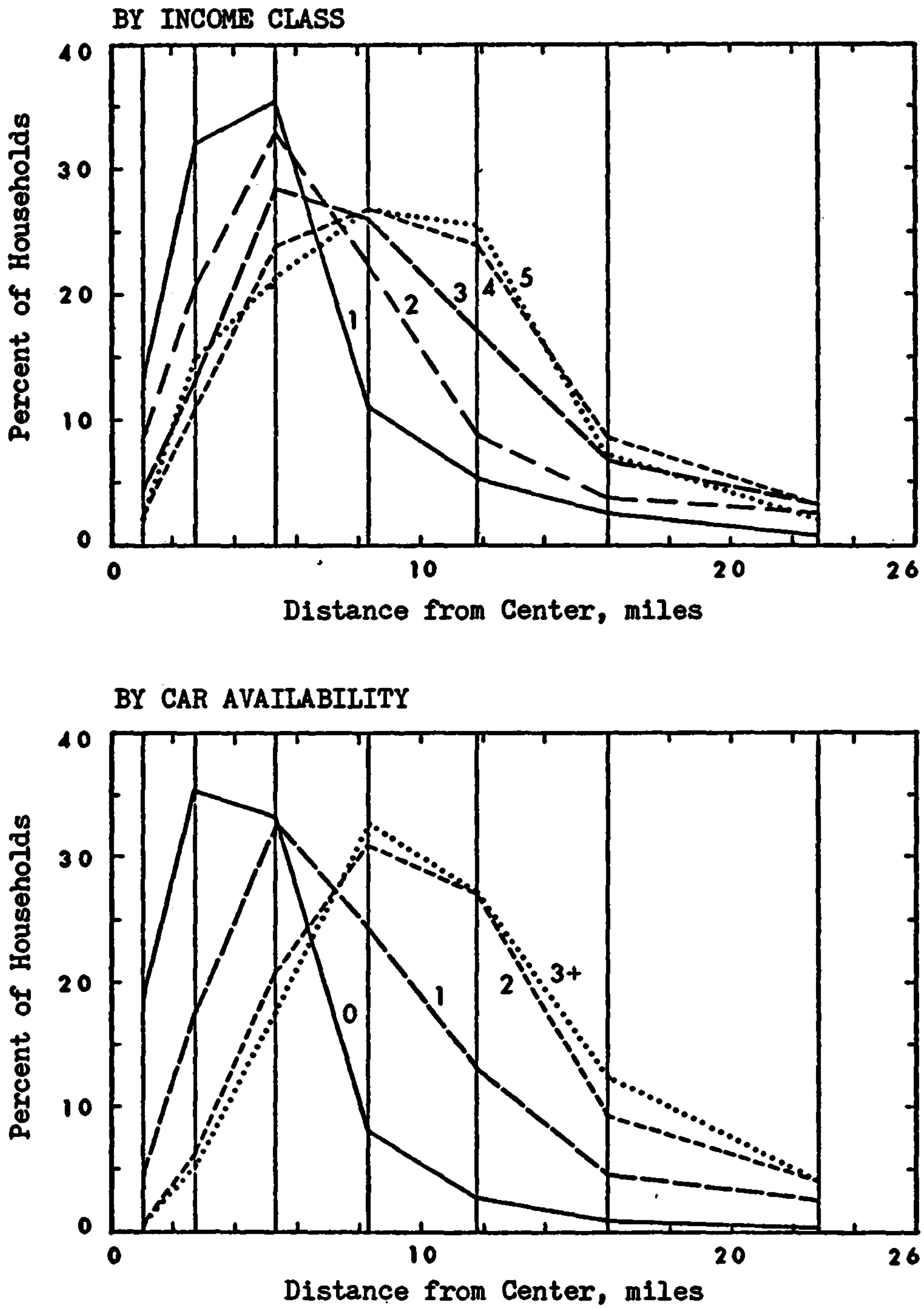

Figure A.6: Household Distribution, by Income Class and Car Availability, vs. Distance from City Center, Washington, D.C. 1968

APPENDIX A

A more rigorous analysis of the spatial distributions of households than the one shown in Figure A.6 is obtainable using the statistical technique of "standard distance" commonly applied in describing the expansion or contraction of population spatial distributions over time (7.32). The standard distance in the present case was computed as follows:

(1) Households' Centroid. Given the X and Y coordinates of each sampled household's residence location, the weighted households' centroid is calculated:

$$\bar{X} = \frac{\Sigma w_i X_i}{\Sigma w_i} \quad , \quad \bar{Y} = \frac{\Sigma w_i Y_i}{\Sigma w_i} \tag{A.1}$$

where: X_i, Y_i = coordinates of residence location,
w_i = expansion factor.

(2) Standard Deviation. The standard deviation of household distributions about their respective centroids is calculated:

$$\sigma_x = \sqrt{\frac{\Sigma w_i X_i^2}{\Sigma w_i} - \bar{X}^2} \quad , \quad \sigma_y = \sqrt{\frac{\Sigma w_i Y_i^2}{\Sigma w_i} - \bar{Y}^2} \tag{A.2}$$

(3) Standard Distance. The standard distance is then defined as:

$$\text{Standard Distance} = \sqrt{\sigma_x^2 + \sigma_y^2} \tag{A.3}$$

Standard distances were calculated for the household spatial distributions by income and by car availability, and the results are detailed in Table A.3 and are summarized in Figure A.7. Figure A.7 shows that the household distributions by income are aligned in a Northwesterly direction, and the standard distance increases gradually with income. The centroids of the household distributions by car availability, on the other hand, are geographically more concentrated, while the differentiation between the distributions are pronounced.

The relationships between standard distance and income class and car availability within each income class are also shown graphically in Figures A.8 and A.9. Here it is demonstrated again that spatial variations in the standard distance by car availability surpasses variations by income class.

APPENDIX A

Table A.3: Standard Distance of Households, by Income and Car Availability, Washington, D.C. 1968

Characteristic	Income Class 1			Income Class 2			Income Class 3			Income Class 4			Income Class 5		
	σ_x	σ_y	S.D.	σ_x	σ_y	S.D.	σ_x	σ_y	S.D.	σ_x	σ_y	S.D.	σ_x	σ_y	S.D.
By Income	4.39	4.08	6.00	5.80	5.17	7.77	6.65	6.36	9.20	7.23	6.57	9.77	6.18	6.25	8.80
Cars/HH 0	3.67	3.57	5.12	3.80	3.58	5.22	3.47	3.38	4.84	2.93	3.11	4.28	2.88	2.79	4.01
1	5.16	4.72	6.99	6.27	5.60	8.40	6.38	6.03	8.78	6.58	5.73	8.73	5.01	5.00	7.07
2	6.92	5.67	8.95	7.61	6.53	10.03	7.72	7.51	10.77	7.78	7.48	10.80	6.40	6.72	9.28
3+	5.04	4.40	6.68	7.96	6.70	10.40	8.06	7.51	11.01	8.11	7.53	11.07	7.18	7.10	10.10

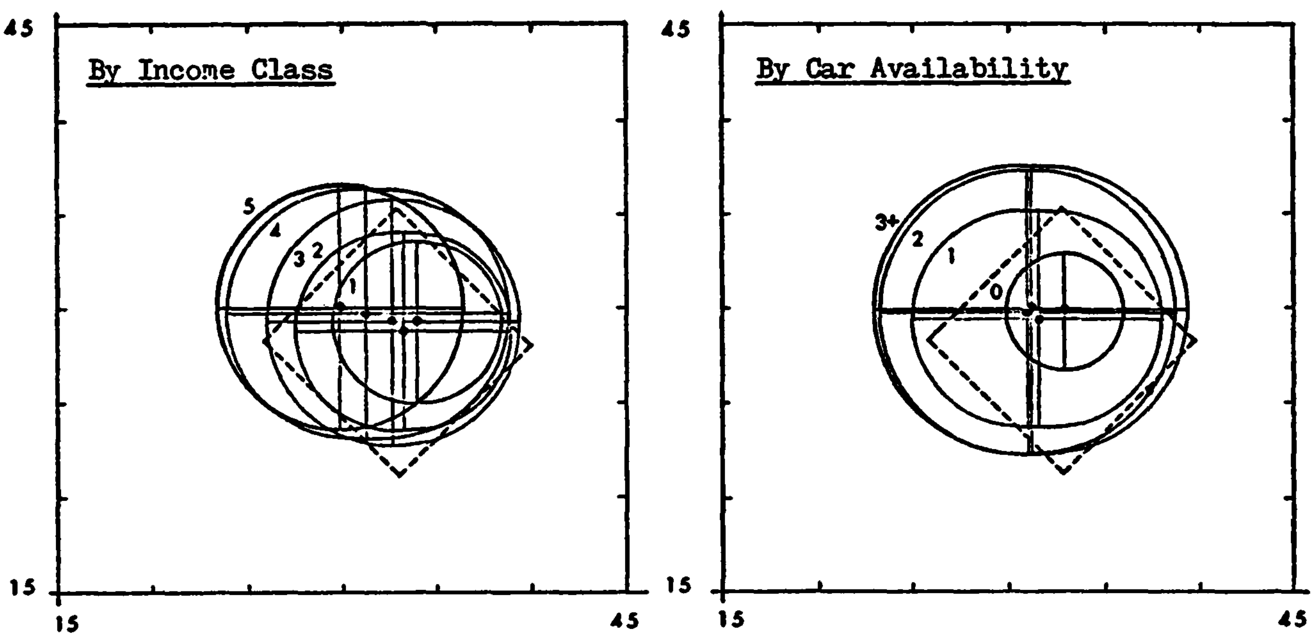

Figure A.7: Standard Distance of Household Distributions along the X and Y Coordinates, by Income Class and Car Availability, Washington, D.C. 1968 (See Figure 1 for complete map)

APPENDIX A

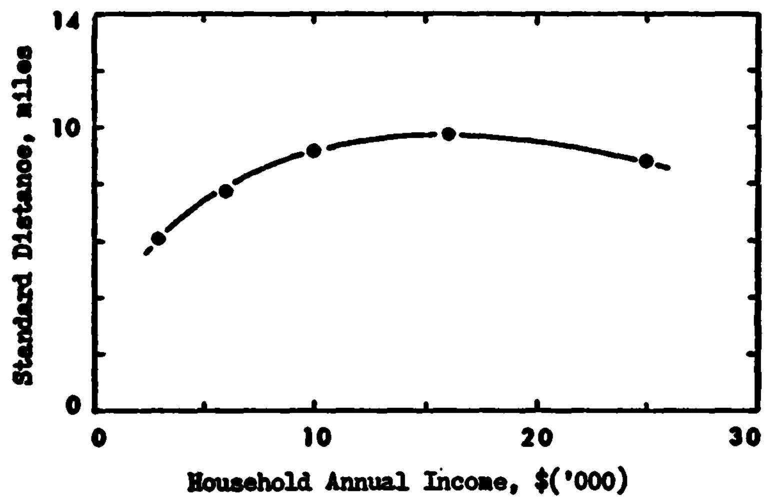

Figure A.8: Standard Distance, by Household Income, Washington, D.C. 1968

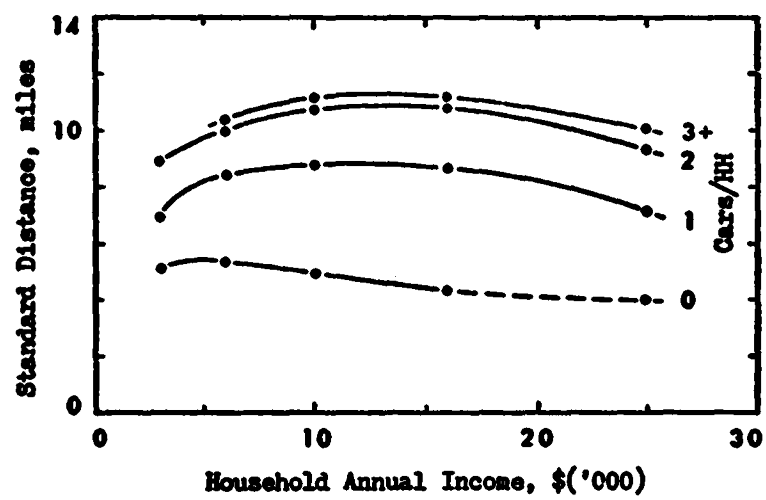

Figure A.9: Standard Distance, by Household Income and Car Availability, Washington, D.C. 1968

APPENDIX A

Table A.4 summarizes a two-way analysis of variance of the standard distance versus income class and car availability. There is a significant difference between the spatial distributions by car availability at the 5 percent level of significance, while there is no significant difference between the spatial distributions by income class.

Table A.4: Analysis of Variance for Standard Distance

Interaction	Sum of Squares	Degrees of Freedom	Mean Square	F ratio	$F_{.05}$ Critical
Between Car Availability	80.08	3	26.69	27.52	3.49
Between Income Class	12.60	4	3.15	3.25	3.26
Error	11.60	12	0.97		
Total	104.28	19			

APPENDIX B : TRAVEL TIME ANALYSIS

ANALYSIS OF VARIANCE: COR1213 INCOME=ALL 9-Jun-80
File TV1213 (Creation date = 29-May-80)

* * * * * * * * * * * * * A N A L Y S I S O F V A R I A N C E * * * * * * * * * * * * *

T MINUTES PER TRAVELLER - TOTAL
by HHSIZE HOUSEHOLD SIZE
NCAR NUMBER OF CARS
INCOME HOUSEHOLD INCOME

* *

Source of variation	Sum of Squares	df	Mean Square	F	Signif of F
Main effects	72938.479	10	7293.848	3.863	0.000
HHSIZE	55640.871	3	18546.957	9.824	0.000
NCAR	9135.742	3	3045.247	1.613	0.184
INCOME	13132.932	4	3283.233	1.739	0.138
2-way interactions	78215.227	30	2607.174	1.381	0.081
HHSIZE NCAR	7268.721	6	1211.453	0.642	0.697
HHSIZE INCOME	22828.204	12	1902.350	1.008	0.439
NCAR INCOME	41100.040	12	3425.003	1.814	0.041
3-way interactions	26492.834	16	1655.802	0.877	0.596
HHSIZE NCAR INCOME	26492.834	16	1655.802	0.877	0.596
Explained	177646.560	56	3172.260	1.680	0.001
Residual	7202432.900	3815	1887.925		
Total	7380079.400	3871	1906.505		

4135 cases were processed.
263 cases (6.4 %) were missing.

Figure B.1 : An Example of the ANOVA Outputs

APPENDIX B

DAILY TRAVEL TIME FREQUENCIES: CON1213 PURPOSE=ALL 4-Jun-80 Page 5
File TV1213 (Creation date = 29-May-80)

• • • • • • • • • • • • • • • • • • C R O S S - - - B R E A K D O W N O F • • • • • • • • • • • • • • • • • •
TIME by NCAR NUMBER OF CARS
Controlling for:
INCOME HOUSEHOLD INCOME Value: 3
Variable averaged: TRAV
• Page 1 of 1

TIME	NCAR: Mean / Count / Sum / Std dev	0 CARS (0)	1 CAR (1)	2 CARS (2)	3+ CARS (3)	Row Total
1	Mean	41.99	29.36	27.33	25.03	28.97
	Count	2	24	12	4	42
	Sum	93.97	704.60	327.95	100.14	1216.74
	Std dev	13.50	9.70	4.77	7.73	8.91
2	Mean	51.59	30.62	23.65	23.87	27.69
	Count	1	44	33	5	83
	Sum	51.59	1307.16	780.83	119.33	2290.51
	Std dev	0.33	7.52	4.86	3.22	7.55
3	Mean	42.32	29.90	25.26	22.53	28.37
	Count	6	83	49	10	148
	Sum	253.95	2481.79	1237.69	225.34	4198.69
	Std dev	13.81	5.81	5.12	1.54	9.13
4	Mean	36.07	27.12	26.00	27.82	27.31
	Count	4	38	23	5	70
	Sum	144.30	1030.66	597.92	139.11	1911.99
	Std dev	11.17	7.46	5.54	10.05	7.45
5	Mean	46.35	33.39	27.47	23.40	31.98
	Count	7	32	22	5	66
	Sum	324.50	1068.88	604.27	116.98	2110.67
	Std dev	13.87	15.50	4.02	1.32	13.71
6	Mean	40.94	31.73	30.26	25.74	31.44
	Count	8	65	35	9	118
	Sum	326.95	2062.41	1089.36	231.70	3710.33
	Std dev	12.58	14.48	8.50	8.19	12.63
7	Mean	36.29	31.58	25.25	20.53	29.31
	Count	3	29	13	4	49
	Sum	108.69	915.74	320.24	83.73	1436.31
	Std dev	0.33	11.34	6.86	1.35	10.16
8	Mean	53.29	30.87	25.06	24.05	28.37
	Count	7	29	26	12	70
	Sum	159.85	895.23	651.50	288.56	1996.14
	Std dev	11.70	11.95	5.07	7.12	10.89
9	Mean	69.67	31.57	26.02	27.90	31.74
	Count	6	25	27	4	62
	Sum	348.03	789.22	702.63	111.58	1967.64
	Std dev	20.85	15.90	5.95	12.01	15.99
10	Mean	43.23	29.97	25.31	24.56	28.34
	Count	5	15	17	8	45
	Sum	216.13	434.50	430.27	196.51	1277.41
	Std dev	7.14	12.02	11.63	4.74	11.61
11	Mean	43.59	34.65	30.51	24.58	32.95
	Count	4	14	10	5	33
	Sum	174.36	485.18	305.06	122.92	1087.48
	Std dev	8.94	9.09	12.91	4.27	10.51
12	Mean	0.00	34.08	33.60	26.03	33.34
	Count	0	14	14	2	30
	Sum	0.00	477.08	470.54	52.06	1000.08
	Std dev	0.03	16.01	13.66	8.75	14.32
13	Mean	0.00	30.63	22.94	0.00	27.64
	Count	0	11	7	0	18
	Sum	0.00	336.98	160.59	0.00	497.57
	Std dev	0.00	13.18	1.16	0.00	8.74
14	Mean	43.82	26.46	26.34	24.26	27.94
	Count	2	10	8	1	21
	Sum	47.64	264.56	210.34	24.26	586.75
	Std dev	16.18	5.36	5.63	0.00	8.07
15	Mean	48.19	34.51	21.86	0.00	30.86
	Count	2	7	7	0	16
	Sum	96.38	248.36	153.09	0.00	493.74
	Std dev	21.21	10.99	1.39	0.00	16.54
16	Mean	0.00	35.30	23.05	24.26	31.14
	Count	0	9	5	1	13
	Sum	0.00	317.71	62.44	24.25	404.41
	Std dev	0.00	15.15	0.56	0.00	14.01
17	Mean	0.00	32.22	24.20	0.00	29.54
	Count	0	4	2	0	6
	Sum	0.00	124.87	48.41	0.00	177.28
	Std dev	0.00	22.91	6.17	0.00	18.47
18	Mean	0.00	36.01	29.14	0.00	32.04
	Count	0	3	3	0	6
	Sum	0.00	104.04	94.82	0.00	192.86
	Std dev	0.00	12.50	4.71	0.00	10.54
19	Mean	0.00	43.76	21.55	19.84	28.39
	Count	0	1	1	1	3
	Sum	0.00	43.76	21.55	19.84	85.15
	Std dev	0.00	0.00	0.00	0.00	13.34
20	Mean	0.00	27.56	44.93	0.00	36.25
	Count	0	2	2	0	4
	Sum	0.00	55.12	89.87	0.00	144.99
	Std dev	0.00	6.59	19.64	0.00	15.41
21	Mean	0.00	30.43	0.00	0.00	30.47
	Count	0	2	0	0	2
	Sum	0.00	60.86	0.00	0.00	60.85
	Std dev	0.00	10.65	0.00	0.00	10.55
22	Mean	0.00	25.07	26.88	0.00	25.79
	Count	0	3	2	0	5
	Sum	0.00	75.20	53.77	0.00	128.97
	Std dev	0.00	5.21	7.54	0.00	5.37
23	Mean	0.00	34.10	22.08	0.00	26.09
	Count	0	1	2	0	3
	Sum	0.00	34.10	44.15	0.00	78.25
	Std dev	0.00	0.00	0.70	0.00	6.95
24	Mean	0.00	26.41	58.82	0.00	37.22
	Count	0	2	1	0	3
	Sum	0.00	52.83	58.82	0.00	111.65
	Std dev	0.00	3.05	0.00	0.00	18.83
25	Mean	22.60	25.00	0.00	0.00	23.80
	Count	1	1	0	0	2
	Sum	22.60	25.00	0.00	0.00	47.60
	Std dev	0.00	0.00	0.00	0.00	1.70
26	Mean	0.00	22.60	0.00	0.00	22.60
	Count	0	1	0	0	1
	Sum	0.00	22.60	0.00	0.00	22.60
	Std dev	0.00	0.00	0.00	0.00	0.00
28	Mean	0.00	0.00	22.90	0.00	22.90
	Count	0	0	1	0	1
	Sum	0.00	0.00	22.50	0.00	22.90
	Std dev	0.00	0.00	0.00	0.00	0.00
29	Mean	0.00	0.00	0.00	23.32	23.32
	Count	0	0	0	1	1
	Sum	0.00	0.00	0.00	23.32	23.32
	Std dev	0.00	0.00	0.00	0.00	0.00
32	Mean	0.00	36.45	0.00	0.00	36.45
	Count	0	2	0	0	2
	Sum	0.00	72.91	0.00	0.00	72.91
	Std dev	0.00	37.44	0.00	0.00	37.44
39	Mean	0.00	0.00	21.55	0.00	21.55
	Count	0	0	1	0	1
	Sum	0.00	0.00	21.55	0.00	21.55
	Std dev	0.00	0.00	0.00	0.00	0.00
41	Mean	0.00	0.00	24.41	0.00	24.41
	Count	0	0	1	0	1
	Sum	0.00	0.00	24.41	0.00	24.41
	Std dev	0.00	0.00	0.00	0.00	0.00
Column Total	Mean	48.55	30.85	26.57	24.41	29.62
	Count	59	471	323	77	925
	Sum	2895.77	14571.35	8582.65	1879.92	27399.71
	Std dev	13.72	11.46	7.45	6.07	11.29

Figure B.2 : An Example of Travel Time Frequency Distributions

APPENDIX B

Table B.1 : Travel Time Frequency Distributions

BY HOUSEHOLD INCOME, NORTH AND SOUTH CORRIDORS (%)

Corridor	Income Class	Time Increments, 10 minutes										Total
		1-2	3	4	5	6	7	8	9	10-11	12-47	
North	1	11.2	11.9	8.4	11.9	16.8	6.3	7.0	6.3	7.7	12.5	100
	2	9.8	14.1	8.6	10.9	12.9	6.1	7.0	7.5	8.6	14.5	100
	3	13.5	16.0	7.6	7.1	12.8	5.3	7.6	6.7	8.5	14.9	100
	4	12.0	13.2	11.1	8.0	11.6	5.9	6.5	6.6	8.4	16.7	100
	5	10.1	11.5	8.9	8.7	12.8	6.6	7.0	7.1	10.4	16.9	100
South	1	18.7	18.7	12.3	8.7	11.4	4.6	5.5	5.9	5.0	9.2	100
	2	17.7	12.6	5.8	9.3	11.1	4.3	5.8	8.1	10.9	14.4	100
	3	14.9	13.9	6.8	7.0	11.6	5.9	7.3	6.8	9.5	16.3	100
	4	13.6	10.9	8.0	7.9	12.7	5.3	6.1	6.7	9.3	19.5	100
	5	12.8	12.2	7.3	6.9	12.8	5.3	7.9	9.1	7.4	18.3	100

BY CAR OWNERSHIP (%)

Corridor	Cars/HH	1-2	3	4	5	6	7-8	9	10-11	12-14	15+	
North	0	6.7	11.4	6.7	10.6	19.6	11.4	8.2	9.4	7.9	8.1	100
	1	12.1	14.2	9.4	8.5	11.6	13.3	6.6	8.1	8.4	7.8	100
	2	12.3	13.3	9.3	8.9	11.6	12.8	6.7	9.2	8.7	7.2	100
	3+	9.7	12.5	10.7	5.2	15.5	13.4	7.0	10.2	7.3	8.5	100

APPENDIX B

Table B.2 : Daily Travel Time per Traveler, by Residence Distance from the Urban Center, Washington, D.C. 1968 (Statistics)

	North	South	Total
Σx	112.50	112.50	225.00
Σy	12.52	12.51	25.03
Σxy	125.895	129.575	255.470
Σx^2	1,624.250	1,624.250	3,248.500
Σy^2	14.324	14.335	28.659
n	11	11	22
$\bar{x}$	10.227	10.227	10.227
$\bar{y}$	1.138	1.137	1.138
a	1.185	1.102	1.143
b	-0.005	0.003	-0.0005
R^2	0.132	0.052	0.0016

$$F_{North} = 1.612, \quad F_{South} = 0.326 \quad) \quad F^{Critical}_{.05(1,9)} = 5.12$$

$$F_{Total} = 0.022 \quad) \quad F^{Critical}_{.05(1,20)} = 4.35$$

APPENDIX C : TRAVEL DISTANCE ANALYSES

Table C.1 : Daily Travel Distance and Time per Traveler, by Residence Distance From the Urban Center, North and South Corridors, Washington, D.C. 1968

NORTH

Income		0-1	1-3	3-5	5-7	7-9	9-11	11-13	13-15	15-17	17-19	19+	Total
1	Households	360	1,760	200	350	670	370	100	120	20	30	20	3,990
	Travelers	560	1,850	290	660	690	490	150	270	20	60	100	5,150
	Distance	10.93	7.74	7.46	14.00	13.77	17.18	24.40	18.74	22.70	1.80	22.84	11.94
	Time	60.58	76.19	58.39	80.44	61.77	59.39	103.78	55.15	75.00	32.50	76.25	69.65
2	Households	1,810	3,160	710	910	830	710	590	510	20	770	230	10,260
	Travelers	2,390	4,800	1,090	1,330	1,180	1,270	1,080	920	20	1,160	490	15,740
	Distance	7.83	9.06	12.79	11.35	13.81	24.51	18.08	16.98	26.30	18.56	20.50	13.09
	Time	64.27	76.65	72.00	53.34	63.20	85.61	65.28	63.19	70.00	58.45	61.52	68.80
3	Households	1,060	1,860	840	1,720	1,440	3,050	2,570	1,040	30	1,230	410	15,240
	Travelers	1,270	2,590	1,460	2,770	2,840	5,790	5,190	2,050	60	2,430	950	27,400
	Distance	9.08	7.44	12.22	15.85	20.54	15.91	18.78	16.72	19.59	20.21	24.75	16.37
	Time	71.08	65.36	72.96	73.16	81.98	64.90	71.79	61.60	53.00	59.30	70.82	69.01
4	Households	630	1,410	1,090	2,270	3,680	3,530	3,380	1,420	120	1,090	450	19,060
	Travelers	880	2,490	2,390	4,510	9,470	8,260	7,520	3,340	310	2,650	1,360	43,190
	Distance	9.68	11.44	14.61	16.57	18.02	17.81	20.58	21.90	15.13	22.15	27.72	18.38
	Time	52.49	71.23	79.72	70.86	74.06	69.46	68.83	64.36	52.40	64.70	66.23	69.92
5	Households	470	1,320	840	2,170	3,060	2,150	1,470	430	190	200	210	12,510
	Travelers	1,140	3,200	1,970	5,620	8,780	5,880	4,050	1,300	600	470	640	33,650
	Distance	7.97	10.52	14.87	16.99	16.92	19.61	21.52	24.07	21.00	29.16	35.59	17.80
	Time	45.91	67.75	84.69	73.61	70.23	72.25	70.22	77.24	64.25	89.84	82.65	71.61
Tot./ Avg.	Households	4,330	9,510	3,680	7,420	9,680	9,810	8,110	3,520	380	3,320	1,320	61,060
	Distance	8.68	9.05	13.38	15.76	17.39	18.07	20.05	19.81	19.40	20.84	26.72	16.45
	Time, min	61.92	72.32	76.66	70.50	72.25	69.44	70.19	64.63	60.49	62.47	69.60	69.83

SOUTH

Income		0-1	1-3	3-5	5-7	7-9	9-11	11-13	13-15	15-17	17-19	19+	Total
1	Households	1,280	2,670	1,130	210	90	-	-	50	-	20	200	5,640
	Travelers	1,280	2,780	1,200	250	130	-	-	70	-	70	270	6,050
	Distance	6.34	10.87	14.35	6.64	12.79	-	-	5.01	-	14.75	14.64	10.61
	Time	49.48	56.79	70.49	56.72	82.79	-	-	28.33	-	30.00	58.75	57.93
2	Households	340	1,430	1,180	1,160	340	560	160	420	-	20	1,380	6,990
	Travelers	340	1,930	1,900	1,590	880	860	310	780	-	70	2,860	11,520
	Distance	6.70	16.04	13.51	19.93	19.91	19.99	15.76	22.72	-	12.41	25.93	19.35
	Time	81.30	67.33	65.14	70.39	70.49	63.70	50.95	88.35	-	26.67	64.30	67.75
3	Households	80	640	1,880	1,820	1,490	1,500	510	660	140	140	1,720	10,580
	Travelers	80	1,220	3,920	3,710	3,540	3,050	1,180	1,490	290	410	4,140	23,020
	Distance	7.22	12.52	14.14	15.97	15.67	21.79	26.22	26.50	23.88	28.71	30.79	20.37
	Time	75.39	69.53	69.94	67.93	61.16	68.37	78.97	74.36	64.40	69.65	73.38	69.34
4	Households	50	800	1,820	2,240	1,520	1,950	1,760	1,260	280	260	760	12,710
	Travelers	100	1,380	3,720	5,760	4,260	4,630	4,650	3,490	770	760	2,450	31,960
	Distance	2.74	14.27	14.76	19.65	19.06	26.46	25.97	24.82	22.08	24.48	30.06	22.16
	Time	55.00	69.56	69.95	80.39	65.24	82.60	75.14	77.68	63.09	63.59	70.63	74.30
5	Households	-	280	210	1,040	940	580	790	350	90	190	440	4,900
	Travelers	-	500	540	2,920	2,770	1,450	2,350	970	180	520	1,480	13,680
	Distance	-	14.07	19.91	19.97	19.36	24.31	25.12	25.23	19.61	23.37	28.10	22.35
	Time	-	70.80	84.64	77.84	74.16	80.79	71.07	67.46	54.42	73.32	56.57	72.74
Tot./ Avg.	Households	1,750	5,820	6,220	6,470	4,380	4,590	3,220	2,740	510	630	4,500	40,820
	Distance	6.35	12.94	14.43	18.29	17.91	23.87	25.29	24.59	22.14	24.39	28.20	19.64
	Time	57.00	63.21	69.63	73.91	66.53	75.42	73.55	76.31	61.92	65.63	67.84	69.44

APPENDIX C

Table C.2 : Daily Travel Distance and Door-to-Door Speed per Average Traveler, North and South Corridors, Washington, D.C. 1968 (For sample size over 20 households)

Income Class	Distance From Center	NORTH Distance	NORTH Speed	SOUTH Distance	SOUTH Speed
1	0- 3	8.46	7.01	9.43	10.40
	3- 7	-	-	12.96	11.46
	7-11	15.31	14.99	-	-
2	0- 3	8.63	7.15	14.66	12.67
	3- 7	11.99	11.66	16.48	14.60
	7-11	19.36	15.52	19.99	17.83
	11-15	17.56	16.39	-	-
	15-19	18.72	19.14	-	-
3	0- 3	7.99	7.12	-	-
	3- 7	16.62	11.98	14.96	13.08
	7-11	17.46	14.84	18.52	17.21
	11-15	18.24	15.84	26.43	20.72
	15-19	20.18	20.49	-	-
4	0- 3	10.99	9.94	13.47	11.80
	3- 7	15.92	12.90	17.75	13.94
	7-11	17.91	14.95	22.93	18.51
	11-15	21.03	18.67	25.52	20.05
	15-19	21.39	20.26	22.63	21.43
5	0- 3	9.87	9.53	-	-
	3- 7	16.46	12.90	19.93	15.18
	7-11	18.02	15.20	21.04	16.53
	11-15	22.10	18.47	25.18	21.56

Table C.3 : Regression Results: Daily Travel Distance vs. Door-to-Door Speed per Average Traveler, North and South Corridors, Washington, D.C. 1968 (For sample size over 20 households)

	NORTH	SOUTH	TOTAL
Σx	294.95	256.97	551.92
Σy	334.21	301.880	636.09
Σxy	5,051.239	5,112.063	10,163.302
Σx^2	4,499.111	4,333.616	8,832.727
Σy^2	5,718.919	6,067.924	11,786.843
n	21	16	37
$\bar{x}$	14.045	16.061	14.917
$\bar{y}$	15.915	18.868	17.192
a	1.841	-1.639	0.409
b	1.002	1.277	1.125
R^2	0.895	0.905	0.892
F	162.5	132.9	288.5
F_N^*	1.425		
F_S^*		1.065	

(*) - vs. Pooled

APPENDIX C

Table C.4 : Daily Travel Distance per Household, by Income and Car Ownership, North and South Corridors, Washington, D.C. 1968

NORTH

Income	Cars per Household														
	0			1			2			3 +			Total		
	HH	D	TR	HH	D	TR	HH	D	TR	HH	D	TR	HH	D	TR
1	2,300	9.07	1.05	1,390	21.18	1.56	220	29.62	1.46	70	62.85	3.27	3,990	15.42	1.29
2	3,900	11.99	1.32	5,430	22.08	1.57	910	38.85	2.21	20	203.54	5.00	10,260	20.08	1.54
3	1,920	10.88	1.25	8,800	25.52	1.65	3,920	42.67	2.19	590	59.91	3.17	15,240	29.43	1.80
4	670	10.29	1.29	7,450	31.89	1.77	9,700	48.99	2.55	1,240	59.70	3.60	19,060	41.64	2.27
5	180	21.72	1.97	3,750	31.44	2.18	7,040	51.82	2.82	1,540	72.91	3.40	12,510	47.87	2.69
Tot/Avg.	8,970	11.07		26,820	27.20		21,790	48.15		3,460	66.51		61,060	34.54	

SOUTH

1	2,700	8.85	1.03	2,800	13.29	1.10	140	21.77	1.33	-	-	-	5,640	11.38	1.07
2	890	10.35	1.08	4,700	31.56	1.60	1,210	44.56	2.09	190	60.43	2.63	6,990	31.90	1.65
3	380	21.90	1.80	5,970	37.27	1.88	3,680	53.99	2.49	550	71.76	3.60	10,580	44.32	2.18
4	120	16.76	1.72	4,450	39.80	2.03	6,850	61.39	2.68	1,280	84.38	3.39	12,710	55.72	2.51
5	40	57.36	4.00	930	41.80	2.14	2,990	60.58	2.82	940	88.90	3.31	4,900	62.42	2.79
Tot/Avg.	4,130	11.07		18,850	33.11		14,870	57.65		2,960	81.93		40,820	43.36	

APPENDIX D : ANALYSES OF TRIP COMPONENTS

Table D.1 : Daily Travel Characteristics per Average Traveler, North and South Corridors, Washington, D.C. 1968 (For sample size above 20 households)

Income	Distance From Center	NORTH D	NORTH R	NORTH d	NORTH t	NORTH v	SOUTH D	SOUTH R	SOUTH d	SOUTH t	SOUTH v
1	0- 3	8.46	2.54	3.33	28.56	7.01	9.43	2.48	3.81	28.56	10.40
	3- 7	-	-	-	-	-	12.96	3.27	3.97	20.77	11.46
	7-11	15.31	2.91	5.26	21.02	14.99	-	-	-	-	-
2	0- 3	8.63	2.57	3.36	28.21	7.15	14.66	2.59	5.65	26.74	12.67
	3- 7	11.99	2.69	4.46	22.92	11.66	16.48	2.77	5.96	24.51	14.60
	7-11	19.36	3.50	5.53	21.36	15.52	19.99	3.27	6.12	20.58	17.83
	11-15	17.56	3.17	5.54	20.27	16.39	-	-	-	-	-
	15-19	18.72	3.02	6.20	19.43	19.14	-	-	-	-	-
3	0- 3	7.99	2.45	3.26	27.45	7.12	-	-	-	-	-
	3- 7	16.62	3.53	4.71	23.57	11.98	14.96	3.21	4.66	21.39	13.08
	7-11	17.46	3.13	5.58	22.56	14.84	18.52	3.02	6.14	21.39	17.21
	11-15	18.24	3.08	5.92	22.46	15.84	26.43	3.25	8.14	23.58	20.72
	15-19	20.18	2.91	6.93	20.31	20.49	-	-	-	-	-
4	0- 3	10.99	2.81	3.91	23.61	9.94	13.47	3.42	3.94	20.06	11.80
	3- 7	15.92	3.28	4.85	22.60	12.90	17.75	3.22	5.51	23.70	13.94
	7-11	17.91	3.10	5.78	23.21	14.95	22.93	3.32	6.90	22.38	18.51
	11-15	21.03	3.19	6.59	21.19	18.67	25.52	3.01	8.49	25.39	20.05
	15-19	21.39	2.92	7.33	21.67	20.26	22.63	2.77	8.18	22.90	21.43
5	0- 3	9.87	2.99	3.30	20.79	9.53	-	-	-	-	-
	3- 7	16.46	3.37	4.88	22.70	12.90	19.93	3.57	5.58	22.06	15.18
	7-11	18.02	3.38	5.33	21.13	15.20	21.04	3.17	6.65	24.13	16.53
	11-15	22.10	3.10	7.13	23.16	18.47	25.18	3.14	8.02	22.32	21.56

D - Daily Travel Distance, miles
R - Trip Rate
d - Trip Distance, miles
t - Trip Time, minutes
v - Door-to-Door Speed, mph

APPENDIX D

Table D.2 : Statistical Tests of Regression Results: Daily Travel Characteristics per Average Traveler, North and South Corridors, Washington, D.C. 1968 (For sample size above 20 households)

Relationship		NORTH	SOUTH	TOTAL
d vs. D	n	21	16	37
	R^2	0.921	0.891	0.913
	F	220.2	114.0	380.6
	F_N^*	1.77		
	F_S^*		0.41	
R vs. D	n	21	16	37
	R^2	0.441	0.091	0.254
	F	14.20	1.41	11.55
	F_N^*	1.53		
	F_S^*		0.72	
R vs. d	n	21	16	37
	R^2	0.186	0.000	0.056
	F	4.20	0.006	2.05
	F_N^*	1.20		
	F_S^*		0.92	
R vs. t	n	21	16	37
	R^2	0.368	0.234	
	F	10.82	2.95	14.44
	F_N^*	1.19		
	F_S^*		0.60	

(*) - Pooled

D = Daily travel distance per traveler

R = Daily trip rate per traveler

d = Average trip distance

t = Average trip time

APPENDIX D

Table D.3 : Daily Travel Characteristics per Average Traveler, by Trip Purpose, North Corridor, Washington, D.C. 1968

Income		TOTAL				WORK				NON-WORK			
		0	1	2	3+	0	1	2	3+	0	1	2	3+
1	TR	2,420	2,170	320	240	1,060	510	190	90	1,600	1,800	160	150
	D	8.63	13.59	20.23	19.20	8.32	15.11	22.63	15.90	7.51	12.11	13.91	21.27
	T	73.29	66.95	71.88	54.43	75.16	65.00	56.38	41.35	60.81	62.31	77.12	62.63
	v	7.07	12.18	16.89	21.17	6.65	13.94	24.08	23.08	7.41	11.66	10.83	20.38
	R	2.47	3.11	2.78	2.91	1.88	1.93	2.29	2.50	2.48	3.19	2.86	3.16
	d	3.50	4.38	7.28	6.61	4.43	7.81	9.90	6.35	3.03	3.79	4.87	6.74
	t	29.70	21.55	25.87	18.73	39.97	33.60	24.67	16.51	24.55	19.51	26.97	19.83
2	TR	5,130	8,510	2,000	100	3,570	3,910	1,180	20	2,540	6,270	1,250	100
	D	9.12	14.11	17.58	40.71	6.24	11.41	11.75	30.14	9.69	12.01	17.06	34.68
	T	72.82	65.76	68.56	126.00	62.52	51.76	42.88	65.00	59.51	56.88	69.29	113.00
	v	7.51	12.87	15.39	19.38	5.99	13.22	16.45	27.82	9.77	12.67	14.77	18.41
	R	2.37	2.97	3.44	5.60	1.83	1.98	1.94	2.00	2.22	2.79	3.67	5.20
	d	3.85	4.74	5.12	7.27	3.41	5.75	6.05	15.07	4.36	4.30	4.65	6.67
	t	30.75	22.12	19.95	22.50	34.19	26.09	22.08	32.50	26.77	20.36	18.89	21.73
3	TR	2,410	14,530	8,580	1,880	1,850	8,370	4,190	1,030	830	9,730	6,390	1,300
	D	8.70	15.46	19.51	18.92	8.46	12.18	16.68	11.73	6.34	12.61	15.27	18.05
	T	71.47	66.84	72.22	67.97	64.27	53.57	62.16	43.97	63.76	53.73	56.25	63.44
	v	7.30	13.88	16.21	16.70	7.90	13.64	16.10	16.01	5.97	14.08	16.29	17.07
	R	2.10	3.01	3.12	3.10	1.95	1.95	2.02	1.96	1.74	2.81	2.86	2.94
	d	4.14	5.13	6.25	6.09	4.35	6.23	8.24	6.00	3.64	4.48	5.33	6.14
	t	34.03	22.20	23.14	21.90	33.01	27.41	30.70	22.48	36.56	19.09	19.64	21.59
4	TR	860	13,180	24,690	4,450	720	7,930	11,250	2,190	430	9,230	18,410	3,130
	D	7.96	18.03	19.24	16.60	5.95	13.04	17.21	14.87	6.02	14.55	15.29	13.19
	T	77.80	75.11	68.43	61.25	57.82	54.39	60.21	55.83	59.39	60.53	54.98	48.00
	v	6.14	14.41	16.87	16.26	6.18	14.38	17.15	15.98	6.08	14.42	16.69	16.48
	R	2.48	3.14	3.15	3.06	1.77	1.86	2.01	1.92	2.02	2.89	3.00	3.01
	d	3.21	5.74	6.11	5.43	3.37	7.01	8.56	7.77	2.98	5.04	5.11	4.39
	t	31.35	23.91	21.73	20.04	32.68	29.26	29.96	29.15	29.37	20.96	18.36	15.97
5	TR	350	8,180	19,880	5,230	300	4,410	7,920	2,640	60	5,560	15,800	3,860
	D	11.05	14.41	18.35	21.46	12.38	10.40	15.46	17.34	4.38	12.97	15.35	17.22
	T	67.82	69.59	70.51	79.17	74.82	53.31	56.33	58.15	30.00	60.17	60.51	67.51
	v	9.77	12.43	15.61	16.26	9.85	11.71	16.46	17.89	8.76	12.93	15.22	15.30
	R	2.00	3.06	3.38	3.36	2.00	1.88	1.89	1.96	2.00	3.01	3.31	3.22
	d	5.52	4.71	5.42	6.38	6.14	5.53	8.16	8.86	2.19	4.31	4.64	5.35
	t	33.91	22.75	20.84	23.55	37.41	28.35	29.73	29.72	15.00	19.98	18.28	20.98

TR - Travelers
D - Daily Travel Distance
T - Daily Travel Time
v - Door-to-Door Speed
R - Trip Rate
d - Trip Distance
t - Trip Time

APPENDIX E : TRAVEL PROBABILITY FIELD ANALYSES

Table E.1 : The Spatial Distribution of Trip Destinations, Households of Income Group 3 Residing in Corridor North at Distance Class 7-11 miles from the City Center, Washington, D.C. 1968

X-Coordinate	No. of Destinations			Y-Coordinate	No. of Destinations		
(10,000)	Work	Non-Work	Total	(10,000)	Work	Non-Work	Total
72.0 - 72.9	-	1	1	39.0 - 39.9	9	9	18
73.0 - 73.9	4	5	9	40.0 - 40.9	10	16	26
74.0 - 74.9	5	6	11	41.0 - 41.9	19	40	59
75.0 - 75.9	10	43	53	42.0 - 42.9	36	58	94
76.0 - 76.9	23	87	110	43.0 - 43.9	22	114	136
77.0 - 77.9	43	109	152	44.0 - 44.9	4	57	61
78.0 - 78.9	38	67	105	45.0 - 45.9	18	31	49
79.0 - 79.9	25	31	56	46.0 - 46.9	4	4	8
80.0 - 80.9	7	3	10	47.0 - 47.9	2	9	11
81.0 - 81.9	3	2	5				
82.0 - 82.9	1	3	4				
Total	159	357	516	Total	124	338	462
Mean	77.827	77.346	77.494	Mean	42.855	43.337	43.208
σ	1.643	1.434	1.518	σ	1.880	1.593	1.689
$Skewness_1$	0.199	- 0.107	- 0.004	$Skewness_1$	0.189	- 0.102	- 0.173
$Skewness_2$	- 0.316	- 0.322	- 0.012	$Skewness_2$	- 1.029	- 0.307	- 0.519
$Skewness_a$	- 0.250	0.192	0.073	$Skewness_a$	0.281	0.067	0.053
$Kurtosis_a$	3.425	4.223	3.785	$Kurtosis_a$	2.623	3.377	3.065
χ^2	3.10	4.84	5.68	χ^2	5.52	15.81	15.28
df	3	4	4	df	2	2	4

Footnote

For clarity purposes, extreme outlier cases are not shown in Table E.1 and Figure E.1, although all cases were included in the analyses shown in the remainder of the Appendix and in the tables and figures of Chapter 7.

APPENDIX E

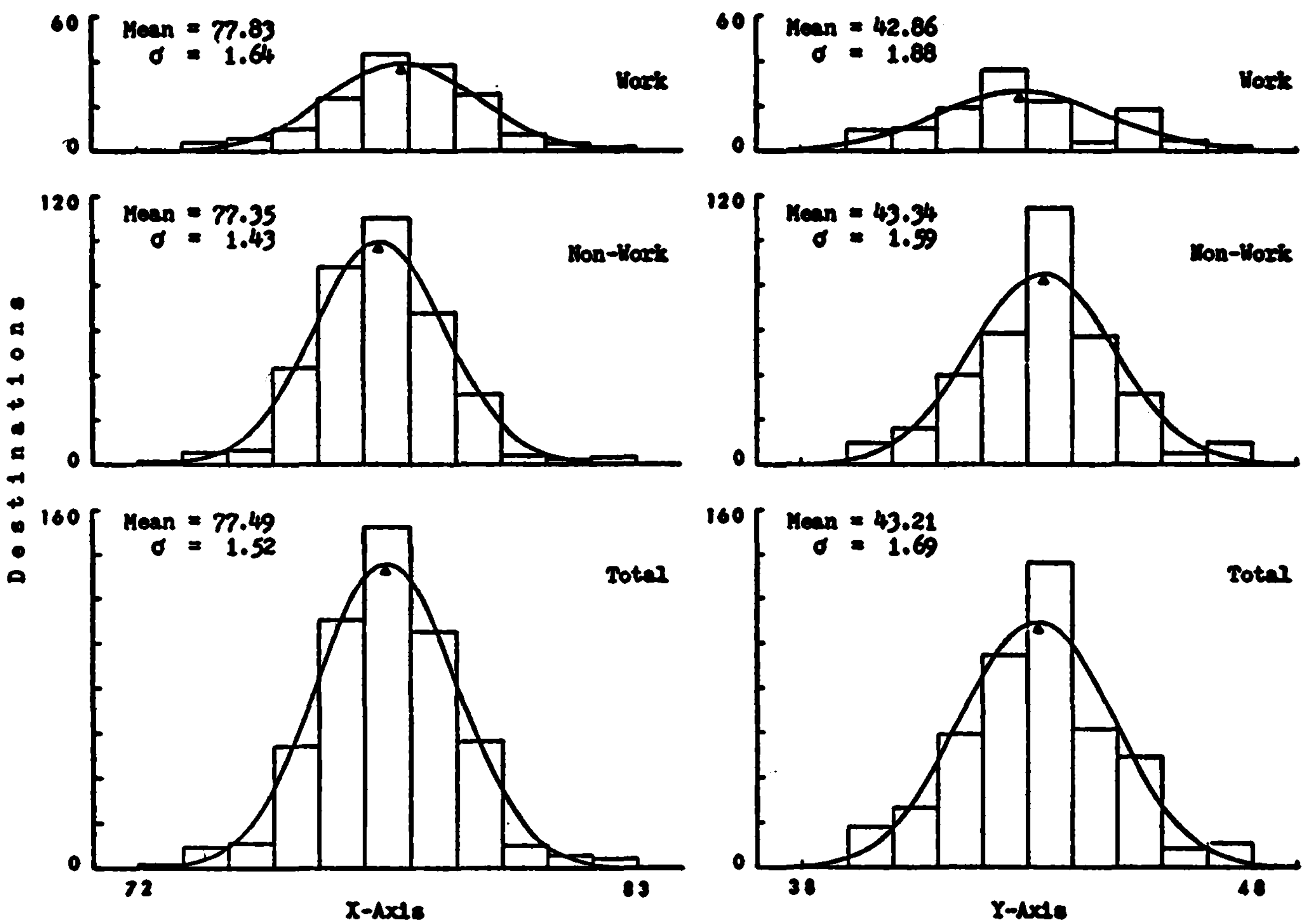

Figure E.1 : The Distribution of Trip Destinations, by Trip Purpose, along the X and Y axes, Households of Income Class $ 8-12,000 Residing in Corridor North, Distance Class 7-11 miles from the City Center, Washington, D.C. 1968 (For clarity, extreme outlying observations are excluded; all other figures and tables include all observations)

APPENDIX E

Table E.2 : Travel Characteristics and Travel Probability Fields, Households of Income Class 3 Residing in Corridors North-West and South-West at Distance Classes 0-3, 7-11 and 15-19 miles, Washington, D.C. 1968

Characteristics	Distance from City Center, miles					
	Corridor North-West			Corridor South-West		
	0 - 3	7 - 11	15 - 19	0 - 3	7 - 11	15 - 19
Households	62	158	47	18	84	13
Household Size	1.47	3.16	3.75	2.05	3.51	3.79
Cars/Household	0.66	1.44	1.43	1.09	1.51	1.73
Travelers/Household	1.35	1.84	1.94	1.82	1.99	2.39
Travel Time/Traveler, hr	1.10	1.19	0.85	1.13	1.12	1.06
Travel Distance/Traveler, m	7.85	18.16	19.84	11.96	20.36	27.57
Door-to-Door Speed, mph	7.15	15.32	23.30	10.55	18.15	25.91
Centroid Distance, d_c, m	0.21	1.84	3.54	0.51	2.59	5.15
Standard Distance, d_σ, m	3.32	5.58	6.28	3.81	5.10	6.84
Destinations(*)	114	522	172	56	321	49

(*) Excluding "Home"

APPENDIX E

Table E.3 : Travel Characteristics and Travel Probability Fields, Households of Three Income Classes Residing in Corridor North-West at Distance Class 3-7 miles from the City Center, Washington, D.C. 1968

Characteristics	$ 0-3,000	$ 8-12,000	$ 20,000 & Over
Households	12	60	77
Household Size	1.73	2.35	3.16
Cars/Household	0.76	0.98	1.67
Travelers/Household	1.88	1.81	2.58
Travel Time/Traveler, hr	1.20	1.11	1.21
Travel Distance/Traveler, m	10.64	12.96	16.08
Door-to-Door Speed, mph	8.84	11.71	13.29
Centroid Distance, d_c, m	2.33	1.25	1.58
Standard Distance, d_σ, m	2.59	4.22	4.79
Destinations (*)	43	177	387

(*) Excluding "Home"

APPENDIX E

Table E.4 : Parameters of Travel Probability Fields and Travel Characteristics per Average Traveler, by Household Income and Residence Distance from the Urban Center, North and South Corridors, Washington, D.C. 1968 (for sample size over 20 households)

Income	Distance From Center	NORTH					SOUTH				
		D	d_c	d_σ	d	v	D	d_c	d_σ	d	v
1	0-3	8.46	0.68	3.80	3.34	7.01	9.43	0.52	3.17	3.81	10.40
	3-7	-	-	-	-	-	12.96	0.88	3.54	3.97	11.46
	7-11	15.31	1.46	4.75	5.25	14.99	-	-	-	-	-
2	0-3	8.63	0.15	4.07	3.36	7.15	-	-	-	-	-
	3-7	11.99	1.46	4.29	4.45	11.66	-	-	-	-	-
	7-11	19.36	1.93	5.38	5.53	15.52	19.99	2.43	5.60	6.12	17.83
	11-15	17.56	3.18	6.12	5.54	16.39	-	-	-	-	-
	15-19	18.72	3.00	6.18	6.20	19.14	-	-	-	-	-
3	0-3	7.99	0.21	3.32	3.26	7.12	-	-	-	-	-
	3-7	16.62	1.25	4.22	4.71	11.98	14.96	2.04	4.49	4.66	13.08
	7-11	17.46	1.84	5.58	5.58	14.84	18.52	2.59	5.10	6.14	17.21
	11-15	18.24	3.44	5.92	5.93	15.84	26.43	5.20	6.61	8.14	20.72
	15-19	20.18	3.54	6.28	6.94	20.49	-	-	-	-	-
4	0-3	10.99	0.40	3.88	3.91	9.94	-	-	-	-	-
	3-7	15.92	1.65	4.98	4.86	12.90	17.75	2.37	4.80	5.51	13.94
	7-11	17.91	2.27	5.36	5.78	14.95	22.93	2.77	6.02	6.90	18.51
	11-15	21.03	3.91	6.57	6.59	18.67	25.52	4.63	6.99	8.49	20.05
	15-19	21.39	4.00	6.77	7.32	20.26	22.63	5.15	6.37	8.18	21.43
5	0-3	9.87	0.21	3.62	3.30	9.53	-	-	-	-	-
	3-7	16.46	1.58	4.79	4.88	12.90	19.93	1.73	5.22	5.58	15.18
	7-11	18.02	2.25	5.14	5.34	15.20	21.04	3.69	5.04	6.65	16.53
	11-15	22.10	4.57	6.45	7.13	18.47	25.18	3.70	6.45	8.02	21.56

D - Daily Travel Distance per Traveler

d_c - Trip Centroid Distance

d_σ - Trip Standard Distance

d - Trip Distance

v - Door-to-Door Speed

APPENDIX E

Table E.5 : Regression Results: Travel Probability Field Parameters vs. Travel Characteristics per Average Traveler, by Household Income and Residence Distance from the Urban Center, North and South Corridors, Washington, D.C. 1968 (for sample size over 20 households)

Relationship		NORTH	SOUTH	TOTAL
d_c vs. d	n	21	14	35
	R^2	0.909	0.862	0.900
	F	192	77	289
	F^*_N	1.79		
	F^*_S		0.51	
d_σ vs. d	n	21	14	35
	R^2	0.923	0.931	0.861
	F	235	161	198
	F^*_N	3.25		
	F^*_S		2.23	
d_σ vs. d_c	n	21	13	34
	R^2	0.922	0.783	0.833
	F	234	39	160
	F^*_N	4.20		
	F^*_S		0.43	
d_c vs. v	n	21	13	34
	R^2	0.850	0.802	0.834
	F	108	44	164
	F^*_N	1.56		
	F^*_S		0.58	
d_σ vs. v	n	21	13	34
	R^2	0.902	0.920	0.855
	F	174	116	187
	F^*_N	2.31		
	F^*_S		1.60	

APPENDIX E

Table E.6 : Attractor-Density vs. Residence Distance Relationships, by Trip Purpose, North and South Corridors, Washington, D.C. 1968

Corridor	Trip Purpose	Sample Size	Power: $b = \alpha r^{\beta}$			Exponential: $b = \alpha e^{\beta r}$		
			α	β	R^2	α	β	R^2
North	Total	21	0.097	-0.26	0.69	0.087	-0.042	0.69
	Work	21	0.14	-0.41	0.89	0.11	-0.064	0.79
	Non-Work	21	0.11	-0.30	0.75	0.091	-0.047	0.69
South	Total	17	0.071	-0.19	0.43	0.066	-0.036	0.53
	Work	16	0.11	-0.39	0.63	0.091	-0.071	0.71
	Non-Work	15	0.071	-0.21	0.68	0.065	-0.038	0.70

APPENDIX F

INTERPRETATIONS OF THE URBAN LOCATION MODEL

1. INTRODUCTION

In the model proposed in Chapter 7 to relate travel probability fields to urban spatial structure, the rent-bid function for a particular socio-economic stratum of households is found to be (equation (7.48)):

$$p(r) = C \exp(-\lambda r)$$
$$= C \exp[-(\frac{3}{4} \frac{\tau^2}{k^2} \frac{b}{\gamma\sqrt{b^2+1}})r] \qquad (1)$$

where

- τ - travel time expenditures,
- k - inverse of speed,
- b - rate of decline of economic opportunities from the urban center, approximated at distance r from the center,
- γ - attraction coefficient for housing space, a taste parameter,
- r - distance to urban center, and
- C - a function of τ, k, b and γ.

Furthermore, for locations (i.e., ranges of distance r) in which the particular socioeconomic stratum of households outbids other strata, residential density was determined to be proportional to rent-bid (equation (7.49)):

$$\frac{1}{q(r)} = \frac{p(r)}{\gamma}$$
$$= \frac{C}{\gamma} \exp(-\lambda r)$$
$$= \frac{C}{\gamma} \exp[-(\frac{3}{4} \frac{\tau^2}{k^2} \frac{b}{\gamma\sqrt{b^2+1}})r] \qquad (2)$$

where

q(r) - housing space at location distance r from the urban center.

The purpose of the present memorandum is to interpret the model of Chapter 7 as a precursor to its linkage dynamically to other models within the UMOT process. This interpretation involves the comparative statics properties of the model and the relative residential locations implied for various socioeconomic strata of households.

2. TRAVEL EXPENDITURES

A preliminary question to the model interpretation involves whether or not the model is consistent with the possibility that a socioeconomic stratum of households might exhibit relatively constant travel budgets throughout its range of residential location. It was proposed that travel expenditure τ has the form (equation (7.5)):

$$\tau = \frac{2}{3}\,\frac{ak^{3/2}\,\pi^{1/2}}{(k^2-b^2)^{3/4}} \tag{3}$$

where

a - level of economic opportunities at the household location distance r from the urban center

Consider a stratum of households located in a distance range $r_1 \leq r \leq r_2$ from the urban center. In moving outward from the center along this range, the level of economic opportunities decreases, $da/dr = -b < 0$, while average speed increases $dk/dr < 0$. From expression (3) it is readily shown that $d\tau/da > 0$, since it is assumed that $k > b$. Thus, changes in the level of economic opportunities lead to decreases in travel time expenditures as distance increases to the urban center. The effect due to changes in average travel speeds is determined by calculating

$$\frac{d\tau}{dk} = \left(\frac{2}{3}\,a\pi^{1/2}\right)\left[\frac{(3/2)k^{1/2}(k^2-b^2)^{3/4} - (3/2)k^{5/2}(k^2-b^2)^{-1/4}}{(k^2-b^2)^{3/2}}\right]$$

$$= (a\pi^{1/2})\left[(k^2-b^2)^{-3/4}k^{1/2} - (k^2-b^2)^{-7/4}k^{5/2}\right] \tag{4}$$

Multiplying both sides of equation (4) by the positive quantity - $[(a\pi^{1/2})(k^2-b^2)^{3/4}k^{-1/2}]$ reveals that

$$\text{Sign}\left[\frac{d\tau}{dk}\right] = \text{Sign}\left[(k^2-b^2) - k^2\right], \tag{5}$$

or

$$\frac{d\tau}{dk} < 0$$

Thus, the effect due to changes in travel speed is an increase in travel expenditures as distance to the urban center increases.

Consequently, the effects of changes in the level of economic opportunities and changes in speed qualitatively compensate within a range $r_1 \leq r \leq r_2$ in which it can be assumed the b is constant. Potentially, the model is consistent with relatively constant travel time expenditures for a stratum of households defined by income, number of household members, number of workers, or other socioeconomic variables. The degree to which exact numerical compensation occurs depends upon the model parameter values as well as speed-distance relationships, $1/k = f(r)$.

The model also implies that observations of relatively constant travel time expenditures over time in light of overall increases in travel speed can be attributed to either or both of two phenomena: First, residential locations remain fixed and the levels of economic opportunities at the locations decrease. Second, residential locations shift outward so that the effect of reduced levels of economic opportunities just compensates temporal plus spatial increases in speed. The first phenomenon might be descriptive of the situations of inner-city residents whose locational opportunities are restricted by the supply of low-cost housing; while the second phenomenon might be more applicable in the cases of residentially-mobile strata. The motivation for households to move outward from the urban center with increases in speed is captured as a desire for increased quantity and quality of housing.

3. COMPARATIVE STATICS

Comparative statics analyses of equations (1) and (2) are complicated because of the interrelationships among the variables through the travel budget τ. However, some analytical progress is gained by substituting equation (3) for τ into the expression for λ implied by equation (1):

$$\lambda = \frac{3}{4}\frac{\gamma^2}{k^2\gamma}\frac{b}{\sqrt{b^2+1}}$$

$$= \frac{\pi a^2}{3\gamma}\frac{b}{\sqrt{b^2+1}}\frac{k}{(k^2-b^2)^{3/2}} \qquad (6)$$

Focussing on changes in the rate of decline of residential density for a particular socioeconomic stratum as a function of changes in overall travel speed,

$$\frac{d\lambda}{dk} = \frac{\pi a^2}{3\gamma}\frac{b}{\sqrt{b^2+1}}\left[\frac{(k^2-b^2)^{3/2} - 3k^2(k^2-b^2)^{1/2}}{(k^2-b^2)^3}\right]$$

$$= \frac{\pi a^2}{3\gamma}\frac{b}{\sqrt{b^2+1}}\left[(k^2-b^2)^{-3/2} - 3k^2(k^2-b^2)^{-5/2}\right] \qquad (7)$$

Thus, multiplying both sides of equation (7) by the positive quantity

$$\frac{3\gamma}{\pi a^2}\frac{\sqrt{b^2+1}}{b}(k^2-b^2)^{5/2}$$

reveals that

$$\text{Sign}\left[\frac{d\lambda}{dk}\right] = \text{Sign}\left[(k^2-b^2) - 3k^2\right] \qquad (8)$$

or

$$\frac{d\lambda}{dk} < 0$$

That is, the rate of decline of bid-rent and potential residential density is a decreasing function of unit travel time, or an increasing function of travel speed. For an unchanged density of economic opportunities, the model predicts that improvements in travel efficiency lead to increase in the rate of decline of residential density for any particular stratum of households. Conversely, increased traffic congestion or decay in the level of transit service leads to increased residential dispersion.

This result is different at first approximation from that obtained using the conventional model of the New Urban Economics, where it is assumed that all employment is concentrated in the CBD. In the basic conventional model with one mode of transportation, improvements in traffic efficiency typically lead to increased housing dispersion, since commuters to the CBD are able to locate further from the CBD, consuming more housing space for the same cost while enjoying equal or greater leisure time than before the travel time improvements. Depending upon the rent-bid function and the marginal utilities of time and housing space, time spent commuting to work might decrease, remain about the same, or even increase. However, the introduction of alternative transportation modes into the conventional model might lead to different results (e.g., Dendrinos, 1976), as might the consideration of travel money as well as time costs (Stucker, 1975). In the case of money costs, a decrease in the cost of commuting to the CBD leads to an increase in a household's disposable income, and the household might allocate a portion of this surplus income to the purchase of higher density housing at increased cost per unit of housing space.

However, increases in overall speed in absence of temporal changes in other explanatory variables is unlikely. For example, according to the argument advanced in Section 2 of this memorandum, observations of constant travel time budgets in light of increases in overall speed is consistent with the simultaneous decentralization of economic opportunities. Moreover, such increases in speed are likely to be accompanied by shifts in the populations of the socioeconomic strata of households. Specifically, increases in speed have been shown to be consistent with relative increases in the strata with more cars available per traveler (Zahavi, 1979), which is consistent with higher levels of real income. As is proposed in the following Section 4, the income effect is associated with increased attraction to quantity and quality of housing, an effect compensatory to the effect due to increases in speed alone.

4. SOCIOECONOMIC ASPECTS

Determining the qualitative nature of the relationships between the explanatory variables of the exponent λ and characteristics defining socioeconomic strata of households allows certain conclusions to be drawn concerning how various strata of households are located relative to the urban center. This is possible because economic theory predicts that the stratum of households with the steepest rent-bid function, that is, with the highest numerical value of the exponent λ in equation (1), will outbid other strata for the locations closest to the urban center, the stratum with the second steepest rent-bid function will locate in the second-closest ring to the center, and so on.

The analysis pursued through equation (8) in Section 3 concludes that the exponent λ is an increasing function of travel speed. Assuming that the demand for travel speed, particularly car travel speed, increases with income (i.e., travel speed is a superior economic good), higher-income households will exhibit a steeper rate of decline of density. This indicates that such households will locate closer to the urban center than lower-income households. (Another way to view this is to hypothesize that higher-income households are able to sustain higher travel speeds through purchase of greater numbers of cars per traveler.)

However, it is also apparent that the exponent λ is a decreasing function of γ, the attraction of housing. Since housing quantity and quality is known to have a high income elasticity of demand, γ is an increasing function of income. This indicates that higher-income households will tend to locate further from the urban center than lower-income households.

The contrasting effects of travel speed and housing attraction make it impossible to conclude how average household income is related to residential distance from the urban center. Nevertheless, this analysis provides guidlines for interpreting the observed relationship shown in Figure 1. The range of distance given by $0 \leq r < r_o$ can be interpreted as locations for which the speed effect is foremost (i.e., a net accessibility influence); the range of distance $r > r_o$ can be interpreted as

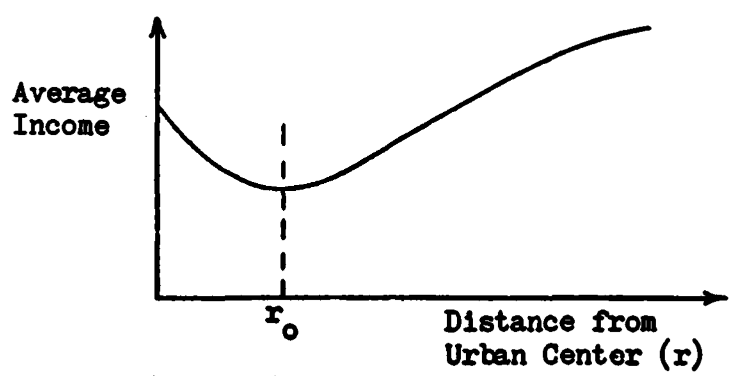

Figure 1

locations for which the housing effect is foremost. The concavity of the curve in Figure 1 is potentially related to the convexity of the typical speed-distance relationship (Zahavi, 1979, Figure 5.5-1). Further research is required.

5. DENSITY GRADIENT FOR A SINGLE-STRATUM CITY

Due to the interrelationships among speed, distance, and the exponent of the residential density function for any socioeconomic stratum, the density gradient is a complicated function even in the simplified case of homogeneous households. This is in contrast to the conventional New Urban Economics model, in which density gradients for multi-strata cities are complicated envelopes of various exponential curves, but the gradient for a single-stratum city is coincident with the simple negative exponential density function for that stratum. The form of the gradient in the ULOM is perhaps best visualized if it is assumed that speed increases with distance from the center in three discrete steps. This leads to a discontinuous gradient of the general shape depicted in Figure 2.

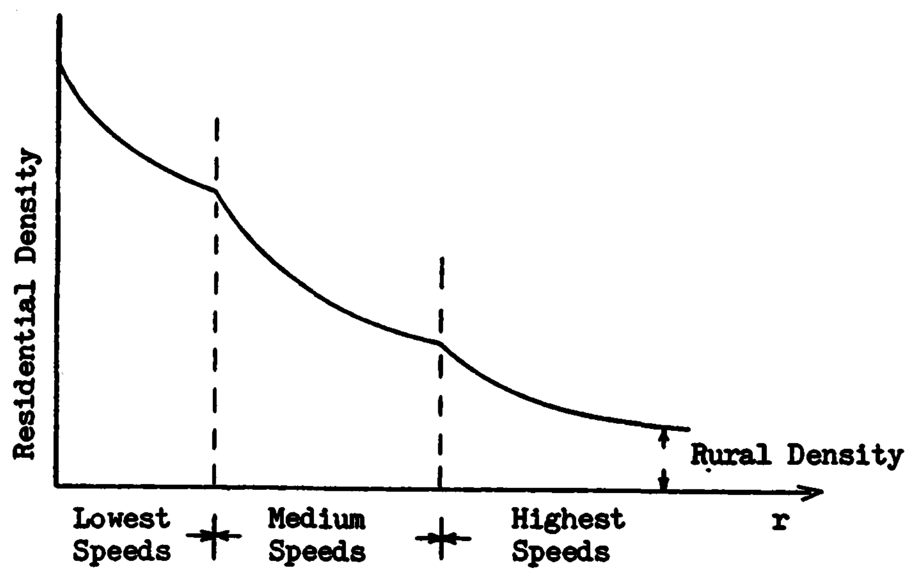

Figure 2

From Figure 2 it is apparent that introducing function of speed versus distance logically can lead to the type of gradient depicted in Figure 3. A potential subject for further research is the empirical testing of such a gradient form.

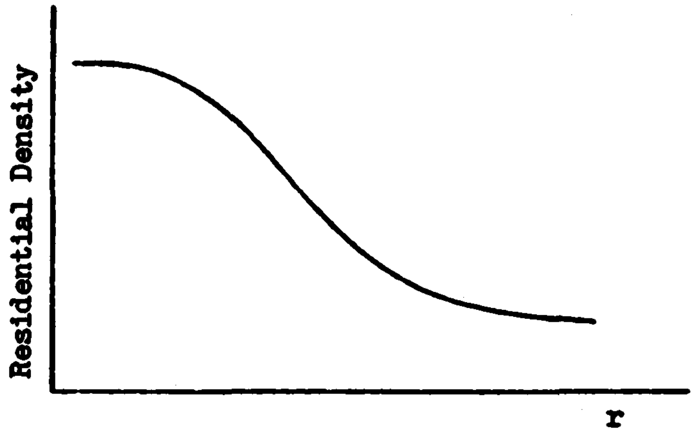

Figure 3

6. A SIMPLIFIED SECTORIAL APPLICATION

Assume that the area served by an urban center is divided spatially into sectors, each sector being the residential domaine of a particular socio-economic stratum of households. Also assume that travel speeds are constant throughout the sector.

This sectorial view of residential land use was originally proposed by Hoyt (1933, 1939), who found that high and low income neighborhoods occupied distinct subareas distributed sectorially. More recently, Anderson and Egeland (1961) found in a comparative study of the applicabilities of the concentric zone (Burgess, 1925), sectorial and multiple-nuclei (Harris and Ullman, 1945) theories of urban social space, that socioeconomic status varies principally by sector, age and family characteristics vary by concentric zone, and minority group isolation is a cluster phenomena. This superpositioning of sectors, zones and segregated areas is still often used by urban geographers in explaining spatial distributions of population and housing characteristics (cf., Berry and Horton, 1970). In the present application, the segmentation of socioeconomic strata represents the sectorial pattern, the density and rent-bid functions represent the (continuous) zonal pattern, and the potential for separate urban centers, particularly ones specializing in ethnic and minority opportunities and satelite areas, represents the segregated or multiple-nuclei pattern.

Let the sector for a particular socioeconomic stratum be of width θ radians. Then the total number of households within the sector located at distance R or less from the urban center is given by

$$H(R) = \int_0^R \frac{1}{q(r)}\,\theta r\,dr$$

$$= \frac{C}{\gamma}\int_0^R \exp\left[-\left(\frac{3}{4}\frac{\gamma^2}{k^2}\frac{b}{\gamma\sqrt{b^2+1}}\right)r\right]\theta r\,dr \tag{9}$$

using expression (2). Integrating by parts,

$$H(R) = \frac{C\,\theta}{\gamma\lambda^2}\left[1 - (1 + R\lambda)e^{-\lambda R}\right] \tag{10}$$

and, for the total number of households residing in the sector,

$$H_t = \int_0^\infty \frac{1}{q(r)}\,\theta r\,dr$$

$$= \frac{C\,\theta}{\gamma\lambda^2} \tag{11}$$

Solving equation (11) for the constant C,

$$C = \frac{\gamma\,\lambda^2\,H_t}{\theta} \tag{12}$$

Substituting for the constant C in expression (10) yields

$$H(R) = H_t\left[\,1 - (1 + R\lambda)e^{-\lambda R}\right] \tag{13}$$

for the population within the sector living within distance R of the urban center. For the simplest case of a single socioeconomic class (i.e., no division of the area into sectors), $\theta = 2\pi$ and H_t = total households in the urban area.

If it is found that the assumptions underlying this simplified approach are consistent at first approximation for a particular class of urban environment, various analytical sensitivity analyses can be pursued.

* * *

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APPENDIX G

THE SHORT-RUN DYNAMICS OF URBAN RESIDENTIAL GROWTH UNDER A REGIME OF EXPONENTIAL DENSITIES

1. This analysis is based on a standard model of residential choice, as developed by Alonso (1964), Beckmann (1957; 1969), Muth (1969), and others, and has many points in common with Anas (1978). Thus housing production is assumed to be described by a Cobb-Douglas production function. A more realistic housing production function is derived in Beckmann-Buttler (1980), but this refinment is not necessary at this stage. This model also uses a standard utility function; it differs, however, in assuming a utility function that generates an exponential density rather than an ordinary logarithmic utility function. We do not consider the case where city size is governed by a prescribed utility level, an approach which is necessarily incomplete without making further assumptions about how income is generated in the city.

Instead we consider various scenarios where the development of area is considered in relation to the given development of the following exogenous variables: number of households N, household income y, number of commuters per household m, commuting costs per distance k, and agricultural land rent s_1.

2. As is customary we start with a "representative household" and thus consider a one-class city. The analysis of the competition of several household types for housing and the dynamics of the internal structure, i.e., of the dividing lines between areas occupied by households of different characteristics will be the subject of later explorations of the UMOT process.

The approach focuses on the same general urban dynamics as in Allen, et al.(1978, 1979), but uses a different technique. The difference is that between a behavioral model based on economic micro-theory and a model based on macro-relationships.

3. The standard model of residential choice in a city which is essentially unimodal (but permits local employment as well, as long as there

is some commuting to the center from every distance in the city) considers the household utility function

(1) $$u = u(y - pq - kmr, q)$$

where the first argument is general consumption and the second is housing space consumed. The following notation is used:

u	utility
y	household income
p	housing rent
q	amount of housing space
k	commuting cost per person per unit distance
m	number of commuters per household
r	distance from the CBD.

The specification of the form of the utility function is to be consistent with observed elasticities of housing demand and with the empirically validated exponential density law:

(2) $$u = \frac{1}{y}\left[y - pq - kmr\right] + a \log q.$$

Thus the marginal utility of consumption is assumed to be inversely proportional to income and constant and the utility of housing is considered to be logarithmic. Other specifications are possible but would yield an exponential density only through approximations.

This approach differs from that of Appendix 8.1 only in the treatment of the utility of consumption; the treatment of the utility of housing is identical in both approaches. The resolution of the theoretical differences is directly related to the linkage of models of fast and slow interactions between travel and urban structure, as discussed in Section 9.1 of the present report. It is a subject for further research.

4. Utility maximization with respect to housing yields the housing demand function

(3) $$q = \frac{ay}{p}$$

which states that the budget share of housing is constant and equal to a, the attraction coefficient in the utility for housing. Insertion of (3) in (2) yields an "indirect utility function"

(4) $$v = 1 - a - \frac{kmr}{y} + a \log ay - a \log p.$$

This function describes achieved utility v as a function of income and of housing rent.

Clearly households are better off, ceteris paribus, the higher the income y and the lower the housing rent p. They are also better off the lower commuting cost. For a given income achieved utility is a decreasing function of the number of commuters. We note in passing that when households can afford at least one unit of housing then achieved utility is also an increasing function of the attraction coefficient a.

5. In a city with households having the same income and the same number of commuters per household achieved utility must be equal at all locations in equilibrium. This generates the equilibrium condition

(5) $$u_o = 1 - a + a \log a - \frac{kmr}{y} + \log y - a \log p$$

and this condition determines the rent bid function p for housing at various distances from the CBD.

(6) $$p = p(r) = aye^{\frac{1-a}{a} - \frac{u_o}{a}} e^{-\frac{km}{ay} r}$$

(6a) $$p(r) = p(0)\, e^{-\frac{km}{ay} r}$$

The price bid for housing falls exponentially with distance from the center. The housing rent gradient is proportional to commuting cost k per person and also to the number of commuters per household. It is inversely proportional to income. Put another way: The "half distance" at which rent is half the level of that in the center is proportional to income and inversely proportional to transportation cost and number of commuters. We can also say that what matters is not commuting cost as such, but the marginal utility of commuting costs as given by $\frac{km}{y}$.

The rent level at the center cannot be determined without bringing in the availability of land and the number of households to be accommodated, i.e., macro-variables. For p(0) contains achieved utility u_o, which is an unknown at this stage.

6. To determine the land area that is available for housing requires an analysis of housing supply as a function of both housing and land rents. This requires a brief analysis of landlord behavior. As usual in economics we assume that they are motivated by profit maximization. While the decision to invest in housing must be based on a comparison of the present value of future rent revenues with the initial cost of construction plus the present value of maintenance costs taking due regard of the expected lifetime of the housing investment, it will simplify the analysis greatly if the present values are converted into steady streams of rent and of costs and an assumption of myopia is made: present levels of (real) rent and (real) costs are expected to continue forever. Actually a less stringent assumption is sufficient: the current ratio of housing rent per unit of space to housing costs per unit of space (capital being measured this way) is expected to continue forever. This latter assumption is made here.

Consider a Cobb-Douglas production function for housing with land and capital as the only inputs. Additional factors in the production function do not change the analysis in any significant way.

(7) $$h = bc^{\beta}$$

where

- h amount of housing supplied per unit area of land
- c amount of capital invested per unit area of land
- b productivity coefficient $b > 0$
- β output elasticity $0 < \beta < 1$.

Let capital be available at a constant unit price w independent of location. The profit per unit area is then

(8) $$ph - wc = pbc^{\beta} - wc.$$

Landlords invest an amount of capital which maximizes this profit. The results are as follows:

(9) $$h = b^{\frac{1}{1-\beta}} (\frac{\beta}{w})^{\frac{\beta}{1-\beta}} p^{\frac{\beta}{1-\beta}}.$$

Housing supply is governed by a constant elasticity function in terms of the ratio of housing rent to capital costs w. The supply elasticity is $\frac{\beta}{1-\beta}$.

Profit achieved per unit of land area becomes land rent s. Straightforward arithmetic yields

(10) $$s = (1-\beta) (\frac{\beta}{w})^{\frac{\beta}{1-\beta}} (bp)^{\frac{1}{1-\beta}}$$

Land rent turns out to be a power function of housing rent with an exponent $\frac{1}{1-\beta} > 1$. The land rent gradient is thus steeper than the housing rent gradient.

7. The size of the residential area is now determined by two conditions:

1) At the city boundary land rent in housing must equal the rent bid for agricultural land q_1, assumed here as given.
2) The total demand for housing must equal the amount supplied within the urban radius R.

Using (6a) in (10) condition 1) assumes the form

(11) $$s_1 = (1-\beta) (\frac{\beta}{w})^{\frac{\beta}{1-\beta}} b^{\frac{1}{1-\beta}} \cdot p(0)^{\frac{1}{1-\beta}} \cdot e^{-\frac{km}{a(1-\beta)y} R}$$

This permits the determination of p(0) from s_1 once the city radius R is known.

The density ρ of residential population in terms of households per unit of land area is obtained by dividing housing demand into housing supply

(12) $$\rho = \rho(r) = \frac{h(r)}{q(r)} = \frac{\text{housing space/land}}{\text{housing space/household}} = \frac{\text{households}}{\text{land}}$$

Assume also that a constant fraction γ of land is used for residential purposes. Other local land uses include streets, parks, local businesses and local facilities.

The total number of households housed in a city of radius R is then

(13) $$N = \int_0^R 2\pi\gamma\, r\, \rho(r)\, dr.$$

Conditions (11) and (13) determine the two unknowns R and p(0). Substituting (3) and (9) in (12) yields

$$\rho = \frac{hp}{ay} = \frac{b^{\frac{1}{1-\beta}} \left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\beta}} p(0)^{\frac{1}{1-\beta}}}{ay}\; e^{-\frac{km}{ay(1-\beta)} r} \quad \text{or}$$

(14) $$\rho = \frac{q_1}{(1-\beta)ay}\; e^{\frac{km}{ay(1-\beta)} \cdot [R-r]}$$

Thus all parameters of the housing production function drop out except the output elasticity β.

Density is seen to be proportional to the level of agricultural rent in relation to household income at the city limits, $r = R$. It rises from there towards the center exponentially at a rate proportional to cost and number of commuters per household, and inversely proportional to household income. Alternatively we may say that density rises proportionately to the marginal disutility of commuting per unit distance.

Substitution of (14) in (13) yields the final equilibrium condition

(15) $$N = \frac{2\pi\gamma s_1}{(1-\beta)ay} \int_0^R e^{\frac{km}{ay(1-\beta)} [R-r]}\, dr.$$

Integration yields

(16) $$N = \frac{2\pi\gamma s_1}{(1-\beta)ay} \cdot \frac{1}{\lambda^2}\, [e^{\lambda R} - 1 - \lambda R]$$

where

(17) $$\lambda = \frac{km}{ay(1-\beta)}.$$

Equation (16) may be rewritten

(16a) $aNy = \frac{2\pi\gamma R^2}{1-\beta} \; q_1 \; F(\lambda R)$

where

(17) $F(x) = \frac{1}{x^2}\,(e^x - 1 - x) = \sum_{n=2}^{\infty} \frac{x^{n-2}}{n!}$

is an increasing function. Here the left-hand side represents aggregate expenditure on housing. Dividing once more by s_1 considered as an index for land prices

(16b) $\frac{aNy}{s_1} = \frac{2\pi\gamma R^2}{1-\beta} \; F(\frac{kmR}{ay(1-\beta)})$.

The left-hand side is now a measure of total housing demand. The right-hand side represents housing supply. It is proportional to the residential area $\pi\gamma R^2$ but the proportionality factor depends on the elasticity $\frac{1}{1-\beta}$ of the housing supply and on the marginal utility of commuting $\frac{km}{y}$.

8. In the following we consider how this equilibrium shifts over time. We assume that the actual movement of the observable variables equals their equilibrium movement. This does not require that actual values are at their equilibrium levels. Consider an adaptive law of motion

(18) $\dot{R} = \mu\,[R^* - R]$

where R* is the equilibrium level of R determined by (16). Adaptive equations of this type are commonly assumed in economic dynamics. Differentiating (18) with respect to time

$$\ddot{R} = \mu\,[\dot{R}^* - \dot{R}].$$

In the absence of significant acceleration or deceleration, the motion of the actual values equals that of the equilibrium values

(19) $\dot{R} = \dot{R}^*$.

This is the hypothesis used here.

While acceleration and deceleration is, of course, to be expected, it represents a second-order effect whose magnitude is small compared to the first-order growth rates, particularly when averaged over some time.

The role of second derivatives in economic dynamics is generally to explain fluctuations of the business cycle variety. Such fluctuations may in fact occur in urban growth but in this study our focus is on trends, i.e., averages over time rather than short-term fluctuations.

Taking logs in (16) and differentiating

$$\frac{\dot{N}}{N} + \frac{\dot{y}}{y} = \frac{2\dot{R}}{R} + \frac{\dot{q}_1}{q_1} + \frac{F'}{F} \cdot [\dot{\lambda}R + \lambda\dot{R}] = 2\,\frac{\dot{R}}{R} + \frac{\dot{q}_1}{q_1} + \frac{\lambda RF'}{F} \cdot (\frac{\dot{\lambda}}{\lambda} + \frac{\dot{R}}{R})$$

$$\frac{\dot{N}}{N} + \frac{\dot{y}}{y} = \frac{\dot{q}_1}{q_1} + \phi \cdot \frac{\dot{\lambda}}{\lambda} + (2 + \phi)\,\frac{\dot{R}}{R} = \frac{\dot{q}_1}{q_1} + \phi \cdot [\frac{\dot{k}}{k} + \frac{\dot{m}}{m} - \frac{\dot{y}}{y}] + (1 + \frac{\phi}{2})\,\frac{\dot{A}}{A}$$

where

(17a) $$\phi = \frac{\lambda RF'(\lambda R)}{F(\lambda R)} > 0 \quad \text{is a short-term constant}$$

and

$$A = \gamma\pi R^2$$

is residential area. Finally

(20) $$\frac{\dot{N}}{N} + (1+\phi)\,\frac{\dot{y}}{y} = \frac{\dot{q}_1}{q_1} + \phi \cdot \frac{\dot{k}}{k} + \phi \cdot \frac{\dot{m}}{m} + (1 + \frac{\phi}{2})\,\frac{\dot{A}}{A}$$

Equation (20) represents a linear relationship between growth rates of endogenous and exogenous economic variables in the short run. Notice that ϕ is not a constant but according to (17a) is itself a function of λR, i.e., of $\frac{kmR}{y}$ which is considered here as changing slowly. In the long run the relationship is nonlinear. The coefficient ϕ is in fact a monotone increasing function of its argument which rises from zero to infinity. In equation (23), for instance, the sensitivity of average growth in area to growth in transportation cost becomes larger as the argument rises over time and the ratio $\frac{2\phi}{2+\phi}$ approaches its limiting value of 2.

9. First, let urban population grow exogenously at a given rate $\frac{\dot{N}}{N}$ and suppose household income, agricultural land values and cost of commuting to remain unchanged. Then

$$(21) \qquad \frac{\dot{A}}{A} = \frac{1}{1 + \frac{\phi}{2}} \frac{\dot{N}}{N} < \frac{\dot{N}}{N} \quad \text{since} \quad \phi > 0.$$

Urban land area expands but at a rate less than population growth. The reason is that with area extension, density remains constant at the fringe but higher densities are generated in the interior of the city. This results in greater overall density and hence a slower growth rate of urban area.

Urban sprawl of recent decades must therefore be attributed to causes other than purely demographic ones.

10. Second, let household (real) income increase while population and all other factors remain constant. Then

$$(22) \qquad \frac{\dot{A}}{A} = \frac{1 + \phi}{1 + \frac{\phi}{2}} \cdot \frac{\dot{y}}{y}$$

Urban area expands with household real income, but at a faster rate. Here is one root of the postwar growth in urban areas.

11. Population and income remain constant while commuting cost changes

$$(23) \qquad \frac{\dot{A}}{A} = - \frac{2\phi}{2+\phi} \frac{\dot{k}}{k}.$$

The sensitivity of urban areal change to transportation cost change depends on the size of ϕ. Now

$$\phi(0) = 0 \quad \text{and} \quad \phi' > 0 \qquad \lim_{x \to \infty} \phi(x) = 0$$

from which it follows that

$$\left(\frac{2\phi}{2+\phi}\right)' > 0 \qquad \text{and}$$

$$0 \leqq \frac{2\phi}{2+\phi} \leqq 2.$$

For large values of λR, e.g., large city radii, a fall in transportation cost can induce twice the relative change in urban area.

Equation (23) goes some way towards explaining past urban areal expansion as a result of motorization, which represented a substantial fall in transportation cost. It also predicts that increases in transportation cost due to rising gasoline prices represents a contracting force. In the presence of other expansionary factors such as population and income growth, this could result merely in a slowdown of expansion. Very substantial increases in gasoline costs would probably be necessary in order that commuters reduce the distances travelled and the urban fringe reverts to persons no longer commuting to the inner city. These forces in turn may motivate the further decentralization of business and shifting of plant locations to the suburbs. But this is a topic for another paper.

12. Growth in the number of commuters per household due to increased participation in the labor force of married women and due to the increase in childless parties living together and of single person households also acts as a contracting factor on urban area expansion. For a larger number of commuters acts mathematically in the same way as an increase in commuting cost. Both are factors increasing the attractiveness of residing in the inner city. The equation relating $\frac{\dot{A}}{A}$ to $\frac{\dot{m}}{m}$ is identical with (23).

13. As cities expand, agricultural land becomes scarcer and its value rises. Assuming all other factors to remain constant (20) yields

$$(24) \qquad \frac{\dot{A}}{A} = -\frac{2}{2+\phi}\,\frac{\dot{s}_1}{s_1}.$$

The implied elasticity of demand for urban land is

$$-\frac{\dot{A}}{A} \Big/ \frac{\dot{s}_1}{s_1} = \frac{2}{2+\phi} < 1.$$

Therefore, the use of land for residential purposes must be classified among the "necessities" whose demand elasticity is less than 1. Of course, the long-term growth of urban area depends on supply conditions for agricultural land as well as on the demand elasticities shown here.

14. To explore the full dynamics of the system in a longer run requires recognition of feedbacks relationships between

the number of commuters m and household income y

the productivity of labor y and the size of the city N

the growth rate of population and the relative level or growth rate of utility achieved in the urban area.

In addition, the roles of technical change would have to be considered in the long run.

Here the focus of the analysis is on the short run: This is appropriate when the process of urban growth is to be studied with regard to implications for and possibilities of economic policy. A closed model for the long run necessarily assumes that the process is left to work itself out without outside interference.

Consider, however, the effects of interaction when area and some other variables are treated as endogenous. We consider first the relationship between household income and number of commuters m

$$y = y \cdot m^{\eta} \qquad \eta > 0.$$

Taking the logarithmic derivative

$$(25) \qquad \frac{\dot{y}}{y} = \eta \cdot \frac{\dot{m}}{m}.$$

In equation (20) introducing exogenous change of m while allowing the variable y to change endogenously

$$(26) \qquad (1+\phi)\,\frac{\dot{y}}{y} = \phi\,\frac{\dot{m}}{m} + \left(1 + \frac{\phi}{2}\right)\frac{\dot{A}}{A}.$$

Upon substitution of (25) the response of area to changes in commuters becomes

$$(27) \qquad \frac{\dot{A}}{A} = \frac{(1+\phi)\eta - \phi}{1 + \frac{\phi}{2}} \frac{\dot{m}}{m}$$

We consider the following cases.

If $\eta = 1$

$$(27a) \qquad \frac{\dot{A}}{A} = \frac{1}{1 + \frac{\phi}{2}} \frac{\dot{m}}{m} > 0.$$

This assumes that the second wage earner contributes proportionately to household income, probably an overstatement.

If $\eta = \frac{\phi}{1+\phi} < 1$

then

$$(27b) \qquad \frac{\dot{A}}{A} = 0.$$

Here the income effect cancels out the substitution effect. If $\eta < \frac{\phi}{1+\phi}$

$$(27c) \qquad \frac{\dot{A}}{A} < 0.$$

The substitution effect is stronger and induces a contraction of urban areas, ceteris paribus.

Suppose next that population growth in the city is related to the growth of household income

$$(28) \qquad \frac{\dot{N}}{N} = \nu \frac{\dot{y}}{y}.$$

Substitution of (28) in (20) while holding all other variables constant

$$(1+\phi+\nu). \frac{\dot{y}}{y} = (1 + \frac{\phi}{2}) \frac{\dot{A}}{A} \qquad \text{or}$$

$$(29) \qquad \frac{\dot{A}}{A} = \frac{2(1+\phi+\nu)}{2+\phi} \frac{\dot{y}}{y}.$$

The right-hand fraction is clearly greater than one

$$\frac{2 + 2\phi + 2\nu}{2+\phi} > 1$$

implying a growth rate of urban areas larger than the growth rate of household income and larger than in (22).

Actually a relationship (28) may arise also when population growth is exogenous and income growth endogenous, i.e., when the causal relationship is reversed. In that case $\frac{1}{\nu}$ represents the response of income growth to the rising scale of production. If urban systems operate under decreasing returns to scale, $\frac{1}{\nu}$ will be negative and this would serve to decrease the response of area growth to population growth below the level predicted by (21), as seen by comparing (21) with (29).

Which interpretation of (28) is appropriate and which sign of ν is to be expected depends on the nature of the city under consideration.

15. This paper has shown how urban residential area changes under the impact of one exogenous change while holding all or all but one of the other variables constant. Empirical testing is hampered by the fact that

1) All these exogenous variables such as population, household income, number of commuters, commuting costs and agricultural land values have changed simultaneously in recent times.

2) These changes have been highly correlated making a simple regression analysis not too promising.

In principle, equation (20) may be cast into the format of an equation predicting area growth $\frac{\dot{A}}{A}$ as a function of all other changes

$$\text{(20a)} \qquad \frac{\dot{A}}{A} = \frac{2}{2+\phi}\left(\frac{\dot{N}}{N} - \frac{\dot{q}_1}{q_1}\right) + \frac{1+\phi}{2+\phi}\,\frac{\dot{y}}{y} - \frac{2}{2+\phi}\left(\frac{\dot{k}}{k} + \frac{\dot{m}}{m}\right).$$

Observe, however, that all coefficients on the right-hand side depend on a single parameter ϕ.

Letting $\frac{2}{2+\phi} = \theta$ one has

$$\phi = 2\,\frac{1-\theta}{\theta}$$

$$\frac{1+\phi}{2+\phi} = \frac{2-\theta}{1+\theta} \qquad \text{and}$$

$$\frac{2\phi}{2+\phi} = 2(1-\theta). \qquad \text{Thus}$$

$$\text{(20b)} \qquad \frac{\dot{A}}{A} = \theta \cdot \left[\frac{\dot{N}}{N} - \frac{\dot{q}_1}{q_1}\right] + \frac{2-\theta}{1+\theta}\,\frac{\dot{y}}{y} - 2(1-\theta)\left[\frac{\dot{k}}{k} + \frac{\dot{m}}{m}\right].$$

In this equation θ and hence ϕ could be estimated in principle by nonlinear regression. But could a reasonable value of $\hat{\theta}$ and a sizable R^2 be considered a valid test of this model?

16. The micro dynamics of cities depends on the macro dynamics explored here, but on structural factors as well which determine the allocation of available land and building structures to competing uses by different types of households and firms. These will be the subject of another paper in which the Mills model is considered in a Von Thünen framework and the existence of at least seven possible distinct zones of land use is recognized. However, any detailed structural changes occurring in urban dynamics must be consistent with the overall changes in urban area derived here.

In this paper the variable of interest has been urban area or urban radius considered as a function of other economic variables. In studying the urban transportation problem, it is appropriate to focus on the number of trips and on trip length. These will depend on all the variables considered here and possibly on additional ones. As a starting point let us suppose that the urban radius is a "sufficient statistic" for the other variables, and consider how, e.g., the average trip length depends on the radius of the city. To a first approximation in a monocentric city average commuting distance is proportional to the urban diameter r with a proportionality factor depending on the density gradient. The growth rate of commuting distance is then exactly one half that of urban area or equal to that of the urban radius. Even in the nonmonocentric city area extension inevitably implies the lengthening of average distances between homes and work places, although not a proportional one.

Since the density gradient changes even in the monocentric case the relationship of average trip length to the extension of the city is a more complicated one. A better prediction than the crude linear relationship for average trip length can be made with the present model.

Assuming a fixed number of trips per household independent of location, the average trip length is

$$L = \frac{\int_o^R r\, \rho(r)\; 2\pi r\; dr}{\int_o^R \rho(r)\; 2\pi r\; dr}$$

Substituting formula (14) for residential density and using (17) one has

$$L = \frac{\int_o^R r^2\, e^{-\lambda r}\, dr}{\int_o^R r\, e^{-\lambda r}\, dr}$$

A short calculation yields

(30) $$L = \frac{2}{\lambda}\left[1 - \frac{\frac{1}{2}\lambda^2 R^2}{e^{\lambda R} - 1 - \lambda R}\right]$$ or

(31) $$L = \frac{2}{\lambda}\; G(\lambda R)$$ (say)

where

(32) $$G(x) = 1 - \frac{x^2/2}{e^x - 1 - x}$$

The function G is related to F from (17)

$$G = 1 - \frac{1}{2F}$$

Observe that

(33) $$G(0) = 0 \qquad \lim_{x\to\infty} G(x) = 1$$

and G(x) is monotone increasing and concave.

In empirical work it may therefore be approximated by other monotone increasing concave functions going through the origin, e.g., by power functions. Zahavi has found that trip length varies approximately as the square root of the urban diameter R, and this is consistent with (31) and (32).

While these calculations were based on trips to the center, the relationship is not changed much qualitatively when a certain proportion of trips is local. But such a more detailed model must be based on the microeconomics of urban neighborhoods, i.e., a more detailed analysis of urban structure in the small--the subject of further research effort.

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SOME IMPLICATIONS OF RESIDENTIAL IMMOBILITY FOR URBAN STRUCTURE

1. As the result of recent events the market for urban housing has to cope with three new phenomena:

(1) Transportation cost in particular, commuting cost is rising due to higher energy costs.

(2) Housing construction is down due to high interest costs and recessional reduction in real income.

(3) Transactions in the housing market are severely taxed by the need for refinancing of mortgages at higher interest costs and in some states by the reassessment of property values as the result of such transactions.

The net result of facts (2) and (3) is that for practical purposes the location of households within the urban area is/temporarily fixed. Even when some mobility of individual households occurs the allocation of land to residences will not change dramatically in the future. The exodus to the suburbs has ceased but neither should we expect a significant backward migration to city centers. There are no compelling reasons for giving up available housing in the outer parts of the city and the supply in the inner city cannot be expected to rise sufficiently to permit any significant relocation of households in the aggregate.

The only types of economic activity that can relocate under the impact of fact (1)--rising transportation costs--are therefore business activities. Here again certain types of business cannot change location without incurring prohibitive increases in transportation cost. These include all manufacturing activities whose output is sold through and shipped to the center. What remains are those manufacturing activities that produce for export from the given city and all those economic activities that are called "footloose" in the location theoretic literature. Prominent among these are offices, i.e., administrative activities without strong links to activities located either in the center, or at the urban fringe, or anywhere else. Offices can, therefore, in principle be located anywhere in a city. The question to be examined here is where will they gravitate under the impact of residential immobility and rising commuting costs.

2. Consider that zone of an urban area that lies outside the CBD and outside the manufacturing belt whose product is sold in the CBD. This outer zone is the zone of interest here and will be termed the neutral zone. Denote the minimum and maximum radius of the neutral zone by r_0, r_2 respectively. We consider the location of potential employees for activities in this zone as given. Let the distribution be denoted by

$$b = b(r).$$

Consider also the rent of usable space as given (short run) or as derived from the given rent of land (long run), and denote this space rent by

$$q = q(r).$$

A single employer (monopsonist) could choose its location at any distance r in this zone

$$r_0 \leq r \leq r_2.$$

If labor availability is no problem, then the optimal location is that of minimal rent, i.e., of maximum distance $r = r_2$.

3. Access to labor may depend on a location, however. If the CBD is the dominant factor in the labor market, the opportunity cost of labor is set by the wage rate in the CBD minus commuting cost. At that wage rate local labor plus all labor residing beyond the distance r of the employer will be available. To induce commuting in the reverse direction from locations closer to the CBD, a premium equal to the saving in commuting cost that has been foregone must be added. This means that the firm will be using essentially only labor that commutes from more distant locations.

A monopsonistic employer will now locate just far enough from the maximum distance r_2 inside the neutral zone so as to secure a sufficient labor supply from locations between r and r_2

$$\ell = \int_r^{r_2} 2\pi x \, b(x)dx.$$

4. The assumption that the CBD is the dominant factor in determining opportunity costs of labor must now be questioned. An efficient allocation of labor to employers at various locations implies that the labor market is divided at some critical distance r_1

$$r_0 < r_1 < r_2.$$

At distances $r < r_1$ the preferred employment location is the CBD and this determines the prevailing wage rate, i.e., the maximum net earnings that labor residing at this distance can obtain.

At distances $r > r_1$ the preferred employment location is in the neutral zone. It is reasonable to assume that employment density always exceeds housing density. Therefore, supply and demand of labor cannot be locally balanced throughout the neutral zone. Rather at all locations there must be net commuting, but the prevailing direction of this commuting will in general not be the same everywhere. At least four configurations are conceivable:

(1) Net commuting is towards the center throughout the neutral zone.
(2) Net commuting is away from the center throughout the neutral zone.
(3) Net commuting is toward the boundaries of the neutral zone.
(4) Net commuting is towards some interior location in the neutral zone.

In the direction of this net flow labor cost is rising. Therefore, it must be highest at points where flows converge from two directions or where flows reach a boundary.

Consequently, there is an incentive to move office locations away from peaks where flows converge to troughs where flows diverge. In the absence of scale economies this would then result in the equalization of wages at all locations, and this can be assured only by balancing local labor supply and demand.

5. If scale economies are significant, and in fact large enough so as to justify but one location for all office activities, then clearly the cost minimum is achieved at some point (or distance) interior to the neutral zone.

If two locations are consistent with full economies of scale, these will not be at the boundary of the neutral zone but at two interior locations. The smaller the effects of scale or the larger the market the more locations become feasible, but they will all be in the interior of the neutral zone. Moreover, they will result in a segmentation of the labor markets such that they each achieve approximately equal labor costs. When location rents are considered, then in fact labor costs may be slightly higher in the more distant locations that enjoy lower location rents.

6. These conclusions hold regardless of the type of transportation system used. If labor commutes by automobile, then the most favorable locations for office employment are near freeway interchanges or freeway exits.

If labor commutes by public transportation, then the best locations are at major transit points where several transit lines intersect. This offers an advantage to those commuters who otherwise would have to transfer from one line to another. It is a well-known theorem of general location theory that intersections of the transportation system offer favorable locations for economic activities.

7. A summary of conclusions and predictions from this analysis and a statement of possible tests follows:

(i) Increasing energy costs raise the cost of commuting. Since large scale relocation of households is economically unfeasible, the major economic relief must come from relocation of business activities.

(ii) The most mobile among economic activities are those termed "footloose" in location theory: primarily office activities.

(iii) When links with other offices or with the CBD are weak, offices can be located at any point that is accessible to labor. Other things equal locations of low rent, i.e., at maximum distance from the CBD will be preferred.

(iv) Access to labor imposes constraints. Competition with the CBD implies that the most favorable locations are beyond a certain distance r_1 which is determined as the maximum distance from which labor is drawn primarily to employment locations in the CBD.

(v) A single best location of offices is at a point in the interior of the neutral zone beyond r_1. This point should have minimum total distance from all potential employees.

(vi) As the number of offices increases and economies of scale recede in importance, office locations can be spread out in such a way as to minimize average costs of commuting to the nearest office center.

The empirical questions are then:

(a) Are major locational changes observed to occur in business location rather than residential land uses?

(b) Is it true that the principal movers among businesses are offices?

(c) Do new offices tend to cluster in a single center?

(d) From which point on does settlement in several clusters of occur?

(e) Are there significant differences in the labor market areas of the several office centers?

(f) What additional locational factors can be observed that guide or affect the location of new office centers?

Regarding (f) the introduction of a new public transportation system such as in Washington, D.C. must be expected to give a sharp impetus to the creation of new office centers. A final question arises in this connection:

(g) Can the resulting savings in total commuting costs be estimated?

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