Session III
Improving Farebox
Recovery Ratios—
Revenue Enhancement Strategies

TRANSIT
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A SIMPLE TECHNIQUE FOR CALCULATING CHANGES IN FARE CATEGORY DISTRIBUTION

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PREFACE

The American Public Transit Association (APTA) and the Urban Mass Transportation Administration (UMTA) are joint sponsors of the Transit Productivity Program. The purpose of this technical assistance program is to support the continuing efforts of the transit industry to improve operating and maintenance practices as well as to strengthen performance monitoring and evaluation, management control and information, and internal and external communications systems. The intent is to provide a broad perspective of productivity improvement, but, at the same time, focus on tested and workable examples of productive management and operating practices within the U.S. transit industry.

The session in which this paper is presented is one of a series of efforts prepared for the Transit Productivity Program. Prepared for presentation at the April 1984 Western Regional Conference of the American Public Transit Association in Portland, Oregon, this paper is intended to provide transit managers with a broad perspective on productivity improvement while focusing on tested and workable examples of productive management and operating practices within the U.S. transit industry.

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THE PROBLEM

The transit industry and its federal supporters have spent millions of dollars studying the effects of fare pricing changes on ridership. These studies have produced useful results in deriving **simple elasticity factors** that can assist in forecasting changes in ridership as a function of price changes for any one of a variety of fare types. However, these simple elasticity factors are limited in that they can be applied only to changes in average fare or **exclusive fare categories.**

But what of non-exclusive, or **substitutable** fare categories? A Tri-Met example is the **set** of adult 2-zone fare riders. There are three **fare categories** within this set. That is to say, there are three ways for riders in the adult, 2-zone set (market) to pay their fare. The following table lists the categories:

	Fare Category	Existing Fare Price	Fare Price/Ride	% Rides	Proposed Fare Price
1.	Adult 2-zone **cash**	$ .75	$.75	10	$ .75
2.	Adult 2-zone **ticket**	6.50 (per 10 rides)	.65	5	7.50
3.	Adult 2-zone **pass**	23.00	.47	12	28.00
				27	

While the above fare categories are not exclusive of each other, neither are they perfect substitutes. The cash fare requires the least initial outlay of money and does not have to be purchased at special outlets. Ticket and pass fares, while cheaper, require a larger cost outlay and a special effort to purchase. In addition, the pass fare requires a commitment to use the transit system a certain number of times to recover the initial investment (break-even).

If the above fare categories **were perfect substitutes,** any price change would (theoretically) send all the riders in the high-fare categories scurrying into the low-priced category. If the fare categories were **completely exclusive,** we could simply choose an appropriate elasticity factor from the several studies that have been done on the subject, then apply this factor in an elasticity formula to estimate the change in ridership as a function of change in fare.

But since these fare categories are neither perfect substitutes nor mutually exclusive, how do we go about calculating rider shifts due to fare pricing changes? We know, for example, that if we raise the fare in one category, some riders in that category will shift to the other fare categories and some riders will discontinue riding (or ride less). How do we estimate these shifts?

The literature is of little help here. I believe this is largely because (to my knowledge) there has never been a good before-and-after **panel survey** of rider response to fare change. Previous studies have sent consultants out on a cold trail of data on fares and ridership changes. Even assuming that such data is of good quality, it measures only changes in **aggregate levels** of ridership (by fare category), not rider shifts among fare categories.

A SOLUTION

The model described herein calculates the changes in ridership within a set of (interrelated) fare categories due to changes in fare (price) in one or more of these fare categories. A **set of interrelated fare categories** is defined as two or more fare categories that vie for the same basic ridership market. The key to this method is use of a price/rider ratio factor ("K") which represents the propensity of riders to purchase one fare substitute over another—even at the same unit (per ride) price.

I wish to emphasize that this model is **empirical.** While it is not without theoretical foundation, neither is it a product of extensive econometric estimating techniques (although hopefully, we may someday be able to do this). The model is also hueristic in that it iterates (toward closure) through a series of steps. To its credit, the model appears (in our testing at Tri-Met) to yield reasonable results. It also allows the user to estimate a distinction between shifts of existing riders (within a fare set) vs. attraction/loss of new/old riders using existing data.

The model is readily adaptable to a microcomputer on any of the electronic speadsheet ("calc") software available. For those analysts, such as myself, who can never seem to get access to a microcomputer, the model can be employed easily with a hand calculator—either programmable or non-programmable.

Nomenclature

The following symbols are used consistently throughout this paper:

R – Rides	One-way originating (linked) trips per time period (day, week, year, etc.). Rides per fare category can also be expressed as a percent of total rides (for all fare categories).
F – Fare	Price per ride. F can represent either the average fare level for the set or fare level for each fare category.
K – Price/ Rider Ratio	A measure of the propensity of riders in a common set to distribute themselves between two fare categories—even at a common price.
E – Elasticity	Simple "shrinkage ratio" formula that equates elasticity as the percent change in ridership divided by percent change in fare. (Note that other elasticity formulas could be used here as well. However, these formulas are usually more precise than the data used to estimate them.)

Subscripts:

c – cash fare paying rides

p – pass fare rides

t – ticket fare rides

1 — existing (former) values

2 — future (after) values (or 2nd iteration)

3 — future (after) values (or 3rd iteration)

K Factors

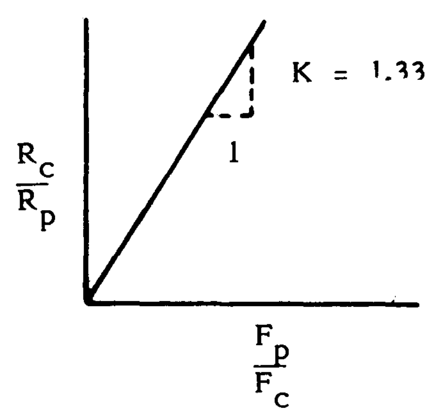
The principle of the K factors can be explained best by using the adult 2-zone fare set as an example. If the per-ride cost of both the pass fare and the cash fare happened to be equal, then the K factor relating pass to cash would simply be the ratio of the ridership (percentages) of cash to pass. Thus:

$$K_{pc} = \frac{R_c}{R_p} \quad \text{(assuming common pricing)}$$

However, since the real prices (per ride) for cash and pass rides are, indeed, different, we normalize the above equation to account for this difference in fares.

$$K_{pc} = \frac{R_c}{R_p}\,\frac{F_c}{F_p} \qquad \textbf{or} \qquad \frac{R_c}{R_p} = K_{pc}\,\frac{F_p}{F_c}$$

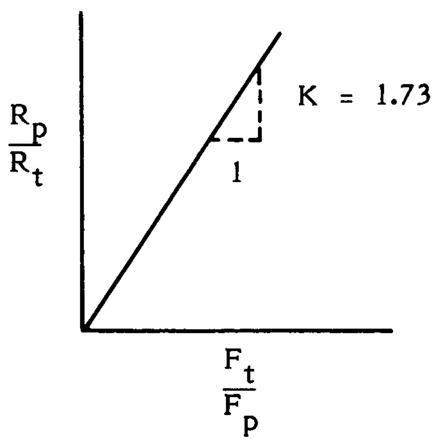
This latter function may be plotted as shown:

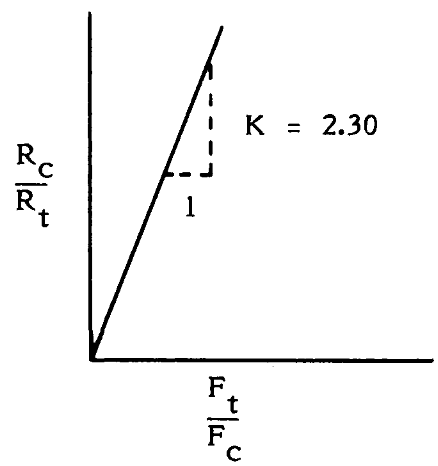


What this relationship says is that if cash and pass prices (per ride) were equal, 1.33 times as many riders would choose cash over pass as their fare payment.

Now, it is probable that this function is not really linear, at least not over the entire range of possible fare price combinations. Enterprising readers are invited to plot actual curve sections if data is available. However, if we assume that the function is linear over the range of fare changes likely to occur, then the above function has the advantage of being derived using only present (not historical) data.

In a similar manner, the other K factors can be derived for the pass-ticket and cash-ticket combinations.





$$K_{pt} = \frac{R_p}{R_t} \frac{F_p}{F_t} \qquad K_{tc} = \frac{R_c}{R_t} \frac{F_c}{F_t}$$

$$\frac{R_p}{R_t} = K_{pt} \frac{F_t}{F_p} \qquad \frac{R_c}{R_t} = K_{tc} \frac{F_t}{F_c}$$

The values of K calculated above are based on the example data in the fare category distribution table on the first page. Here the K factors show the propensity of riders to choose pass and cash fares over ticket fares.

Application

Assuming that the K factors will remain constant over fare price changes, we can now use them to forecast ridership shifts. The example here is a three-category fare set (with three K factors). However, the technique could also be applied to a two-category fare set (with one K factor) or possibly to a four-category set (with six K factors)—although the algebra on this latter case would be considerably more complex.

1. The first step is to **calculate the K factors** based on the fare category distribution data on page 1.

$$K_{pc} = \frac{R_c}{R_p} \frac{F_c}{F_p} = \frac{10}{12} \times \frac{.75}{.47} = 1.33$$

$$K_{pt} = \frac{R_p}{R_t} \frac{F_p}{F_t} = \frac{12}{5} \times \frac{.47}{.65} = 1.73$$

$$K_{tc} = \frac{R_c}{R_t} \frac{F_c}{F_t} = \frac{10}{5} \times \frac{.75}{.65} = 2.31$$

2. Sum the total (percent) rides for this set of fare categories.

$$R_{TOT(1)} = R_{c1} + R_{p1} + R_{t1}$$

$$= 10 + 5 + 12 = 27\%$$

3. Compute existing average fare for entire set of interrelated ridership categories (F_{AVE}) using a weighted average technique.

$$F_{AVE(1)} = \frac{F_c R_c + F_p R_p + F_t R_t}{R_c + R_p + R_t}$$

$$= \frac{.75(10) + .47(12) + .65(5)}{10 + 12 + 5} = \$.607$$

4. Compute future average fare for the set of fare categories. Note that we initially use existing fare category distributions (since that is all we have) rather than future distributions (which we are trying to predict).

$$F_{AVE(2)} = \frac{F_{c2} R_c + F_{p2} R_p + F_{t2} R_t}{R_c + R_p + R_t}$$

$$= \frac{.75(10) + .57(12) + .75(5)}{10 + 12 + 5} = \$.670$$

5. Compute the new total (percent) rides for this set of fare categories using simple elasticity formula. Use elasticity of E = -.3.

$$R_{TOT(2)} = R_{TOT(1)} \left[E \left[\frac{F_{AVE(2)} - F_{AVE(1)}}{F_{AVE(1)}} \right] + 1 \right]$$

$$= 27.0 \left[-.3 \left[\frac{.670 - .607}{.607} \right] + 1 \right] = 26.49\%$$

6. Using the K factors and new (proposed) fare values for each category, solve for the new rider ratios.

$$\frac{R_c}{R_p} = K_{pc} \frac{F_p}{F_c} = 1.33 \left[\frac{.57}{.75}\right] = 1.01$$

$$\frac{R_p}{R_r} = K_{pt} \frac{F_r}{F_p} = 1.73 \left[\frac{.75}{.57}\right] = 2.28$$

$$\frac{R_c}{R_t} = K_{tc} \frac{F_t}{F_c} = 2.31 \left[\frac{.75}{.75}\right] = 2.31$$

7. Using these ratios and the new total (percent) rides, solve for the new rider distributions by fare category.

$$R_{c2} = R_{TOT(2)} \left(1 + \frac{1/R_c}{R_t} + \frac{1/R_c}{R_p} \right)$$

$$= 26.49 \left(1 + 1/_{2.31} + 1/_{1.01} \right) = 10.93\%$$

$$R_{p2} = R_{TOT(2)} \left(1 + \frac{R_c}{R_p} + \frac{1/R_p}{R_t} \right)$$

$$= 26.49 \left(1 + 1.01 + 1/_{2.28} \right) = 10.82\%$$

$$R_{t2} = R_{TOT(2)} \left(1 + \frac{R_c}{R_t} + \frac{R_p}{R_t} \right)$$

$$= 26.49 \quad (1 + 2.31 + 2.28) = 4.74\%$$

8. As a check, make certain the percentage of rides by individual fare category sum to the total for the set.

$$R_{c2} + R_{p2} + R_{t2} = 10.93 + 10.82 + 4.74 = 26.49\%$$

$$R_{TOT(2)} = 26.49\%$$

9. We can now use these new fare category distribution factors to recompute new average fare, new total percent ridership, and revised fare category distribution factors. We begin this second iteration by repeating step 4 to recompute average fare.

$$F_{AVE(3)} = \frac{F_{c2} R_{c2} + F_{p2} R_{p2} + F_{t2} R_{t2}}{R_{c2} + R_{p2} + R_{t2}}$$

$$= \frac{.75(10.93) + .57(10.92) + .75(4.74)}{26.49}$$

$$= \$.676$$

10. Using the new average fare, recompute the new total (percent) rides for this set of fare categories.

$$R_{TOT(3)} = R_{TOT(1)} \left[E \left[\frac{F_{AVE(3)} - F_{AVE(1)}}{F_{AVE(1)}} \right] + 1 \right]$$

$$= 27.0 \left[-.3 \left[\frac{.676 - .607}{.676} \right] + 1 \right] = 26.17\%$$

11. Using the rider ratios from step 7 and the new total (percent) rides, solve for new rider distributions by fare category.

$$R_{c3} = R_{TOT(3)} \left(1 + \frac{1/R_c}{R_t} + \frac{1/R_c}{R_p} \right)$$

$$= 26.17 \left(1 + 1/2.31 + 1/1.01 \right) = 10.80\%$$

$$R_{p3} = R_{TOT(3)} \left(1 + \frac{R_c}{R_p} + \frac{1/R_p}{R_t} \right)$$

$$= 26.17 \left(1 + 1.01 + 1/2.28 \right) = 10.69\%$$

$$R_{t3} = R_{TOT(3)} \left(1 + \frac{R_c}{R_t} + \frac{R_p}{R_t}\right)$$

$$= 26.17 \quad (1 + 2.31 + 2.28) = 4.68\%$$

Although we could perform another iteration, the results would change little from the values already computed. Comparing the values obtained in Step 11 with those of the first table, we note the following:

1. We increased the average fare in this set from \$.607 to \$.676 (11.4 percent) and lost 3.1 percent of our riders.

2. We raised fares in both pass and ticket categories and lost riders in both categories. In the ticket category, we raised fares 15 percent and lost 6.4 percent riders. In the pass category, we raised fares 22 percent and lost 11 percent riders. The cash category increased 8 percent with riders shifting from the other two categories.

RIDERSHIP/REVENUE SHIFTS

WITH FARE LEVEL CHANGES

FARE POLICY ISSUES
(EVALUATION CRITERIA)

1. Revenues
2. Riders
3. Equity
4. Operations

MEANS TO EFFECT/AFFECT FARE POLICY

1. Marketing
2. Fare Collection
3. Fare Pricing

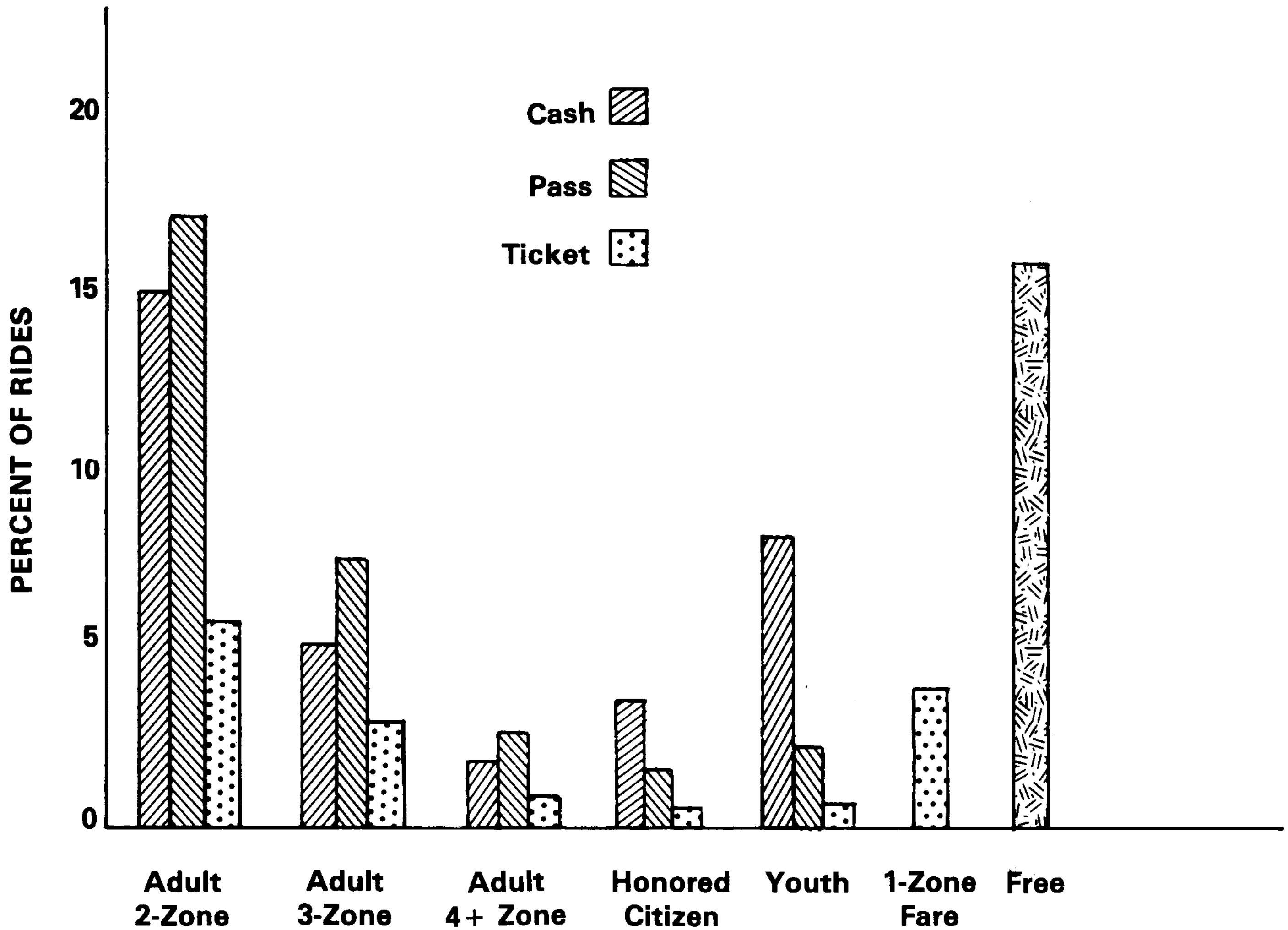
FARE PRICING MECHANISMS

1. By Distance
2. By Time
3. By Prepayment Type
4. By Socioeconomic Group

TRI-MET FARE STRUCTURE

	CASH	10-RIDE TICKET	VALID AS TRANSFER	MONTHLY PASS
Adult All-Zones	$1.25	$11.50	2½ hours	$40.00
Adult 3-Zone	1.00	9.00	2 hours	32.00
Adult 2-Zone (1 or 2 zones)	.75	6.50	1½ hours	23.00
Short Hopper 1-Zone	NA	5.00	1 hour	NA
24-Hours All-Zones	NA	2.50	24 hrs. (unlimited rides)	
Youth All-Zones	.50	4.50	2½ hours	15.00
Retarded Citizen (all hours, all zones)	.25	NA	2½ hours	6.00
Honored Citizen (weekdays 7-9 a.m., 4-6 p.m.)		Same as "Adult" fare		
Honored Citizen (all other hours, all zones)	.25	NA	2½ hours	6.00

TRI-MET
FARE CATEGORY DISTRIBUTION



SUBSTITUTABLE FARE CATEGORIES

Fare Categories Representing a Common Market of Riders

Example: **Adult 2-Zone Riders**

1. **Cash**
2. **Ticket**
3. **Pass**

Set of substitutable fare categories (items 1–3 above)

EXCLUSIVE FARE CATEGORIES

Fare Categories Representing a Unique Market of Riders

Example: **Adult 2-Zone Riders vs. Adult 3-Zone Riders**

Honored Citizens vs. Youth

ELASTICITY FORMULAS

$$E = \frac{\dfrac{R_2 - R_1}{R_1}}{\dfrac{F_2 - F_2}{F_1}} = \frac{\Delta R / R_1}{\Delta F / F_1}$$

"Shrinkage Ratio"

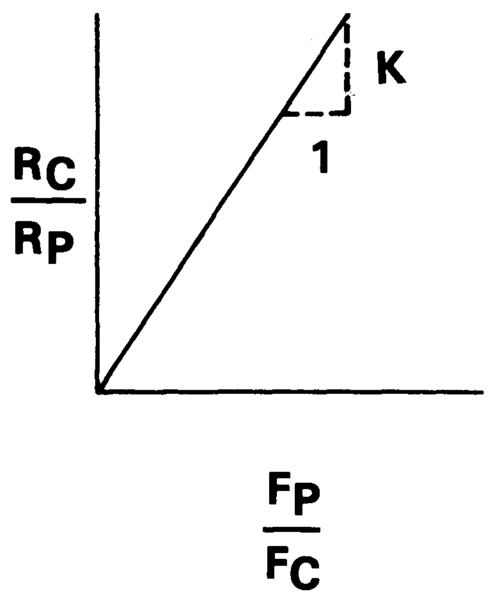
$$E = \frac{\log R_2 - \log R_1}{\log F_2 - \log F_1}$$

"Arc Elasticity"

$$E = \frac{(R_2 - R_1)\ (F_2 + F_1)}{(R_2 + R_1)\ (F_2 - F_1)}$$

"Midpoint Elasticity"

PRICE/RIDER RATIO



$$R_C = K_{PC} \frac{F_P}{F_C}$$

OR

$$K_{PC} = \frac{R_C}{R_P} \frac{F_C}{F_P}$$

TOTAL RIDES PER SET OF FARE CATEGORIES

$$R_{TDT} = R_C + R_P + R_T$$

CALCULATION OF AVERAGE FARE

$$F_{AVE} = \frac{F_C R_C + F_P R_P + F_T R_T}{R_C + R_P + R_T}$$

STEPS

1. Calculate K factors.
2. Sum total % rides for all fare categories in set.
3. Calculate existing average fare.
4. Calculate new average fare using new fares and existing % rides.
5. Calculate new total % rides using old and new average fares, old total % rides, and elasticity factor.
6. Calculate new rider ratios using new fares and K factors.
7. Solve for new % rides by fare category.

EXAMPLE
(FIRST ITERATION)

FARE CATEGORY	OLD FARE	% RIDES	NEW FARE	NEW % RIDES
Cash	$.75	10%	$.75	~10.9%
Ticket	.65	5%	.75	~10.8%
Pass	.47	12%	.57	~ 4.8%
Total/Ave	.61	27%	~.67	~26.5%

~ Approximate Values

EXAMPLE
(SECOND ITERATION)

FARE CATEGORY	OLD FARE	% RIDES	NEW FARE	NEW % RIDES
Cash	$.75	10%	.75	~ 10.8%
Ticket	.65	5%	.75	~ 10.7%
Pass	.47	12%	.57	~ 4.7%
Total/Ave	.61	27%	~ .68	~ 26.2%